PRESTRESSED CONCRETE STRUCTURES

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Module – 5: Analysis and Design for Shear and Torsion

Lecture-26: Analysis of Torsion

Welcome back to prestressed concrete structures. This is the fourth lecture of Module 5 on analysis and design for shear and torsion. In this lecture, first, we shall study the stresses in an uncracked beam, due to torsion. Then, we shall study the crack pattern under pure torsion. We shall study the components of resistance for pure torsion. We shall learn about the modes of failure. Finally, we shall learn about the effect of prestressing force.

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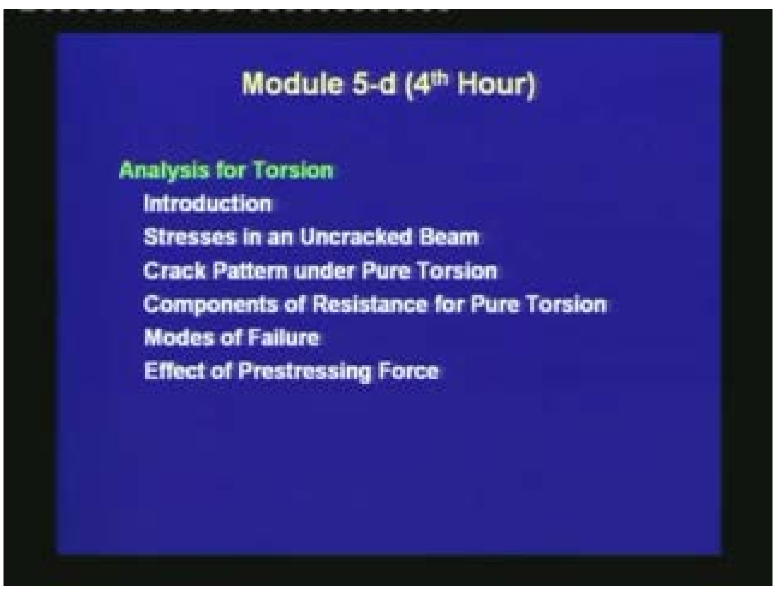

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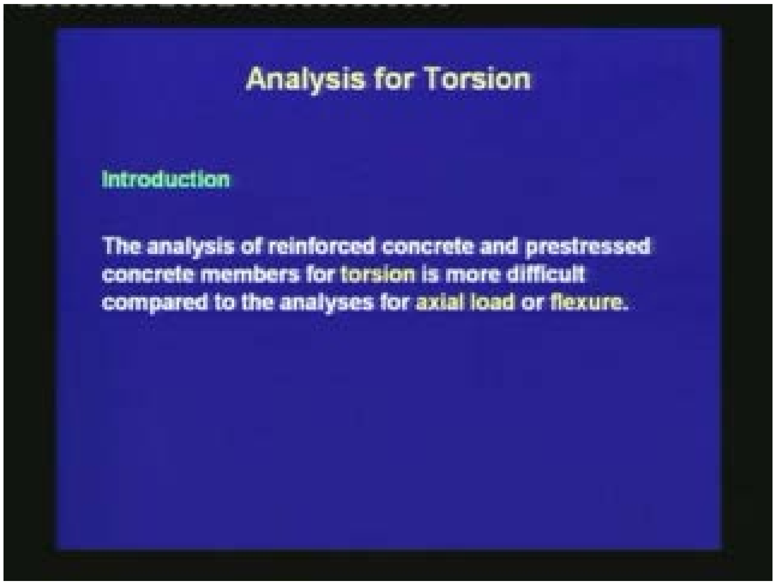

The analysis of reinforced concrete and prestressed concrete members for torsion is more difficult compared to the analyses for axial load of flexure.

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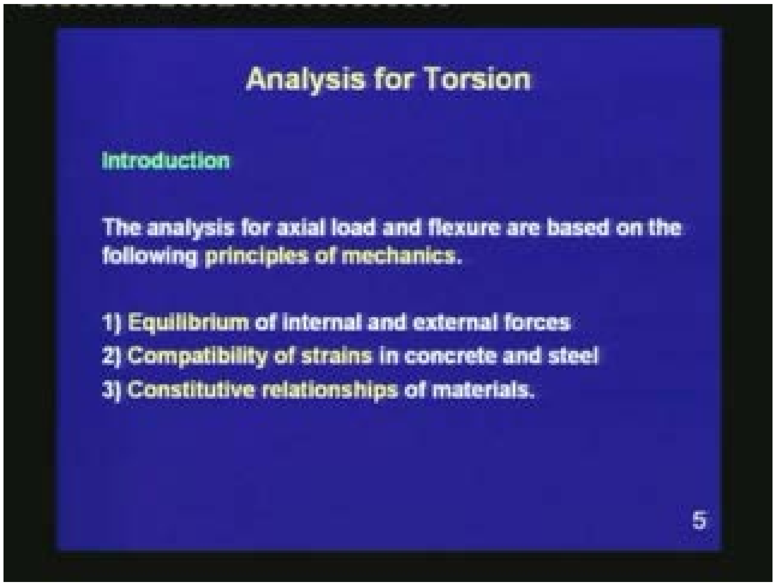

The analysis for axial load and flexure are based on the following principles of mechanics: equilibrium of internal and external forces, compatibility of strains in concrete and steel, and third the constitutive relationships of materials.

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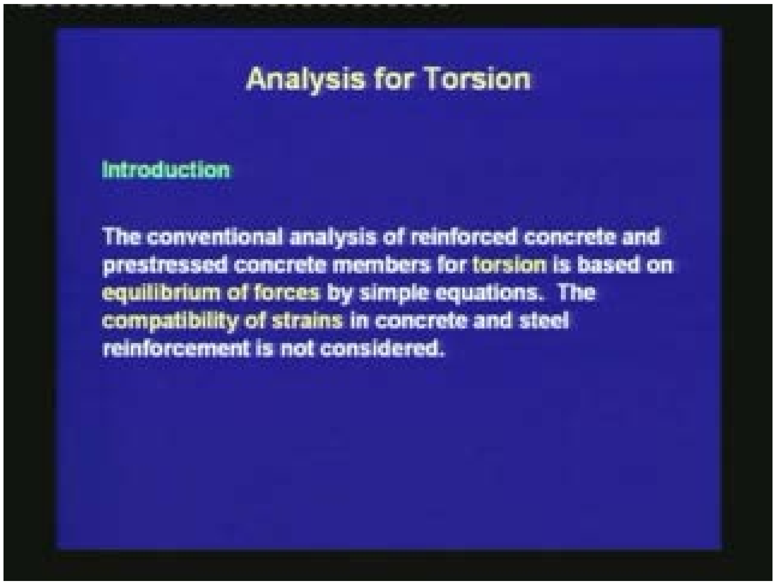

The conventional analysis of reinforced concrete and prestressed concrete members, for torsion is based on equilibrium of forces by simple equations. The compatibility of strains in concrete and steel reinforcement is not considered.

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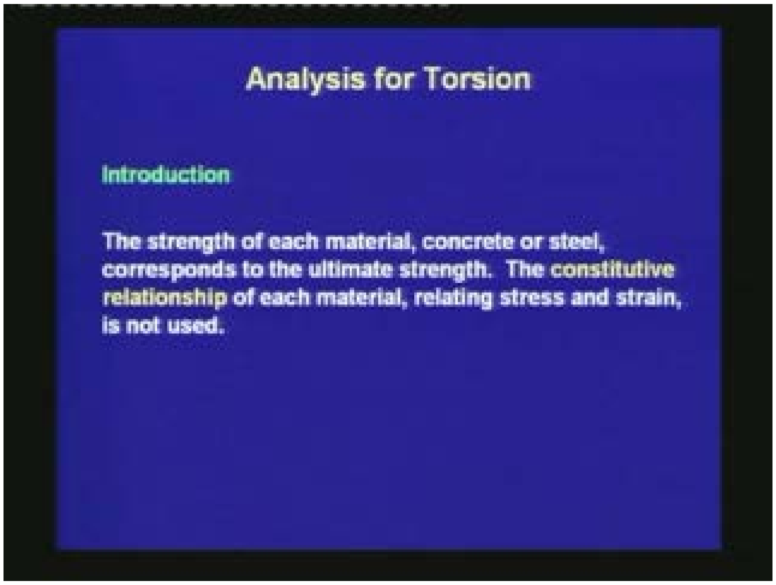

The strength of each material, concrete or steel, corresponds to the ultimate strength. The constitutive relationship of each material, relating stress and strain, is not used. This way, we are doing a simplified analysis for torsion. There are more developed methods for analysis for torsion, which we are not discussing in this lecture.

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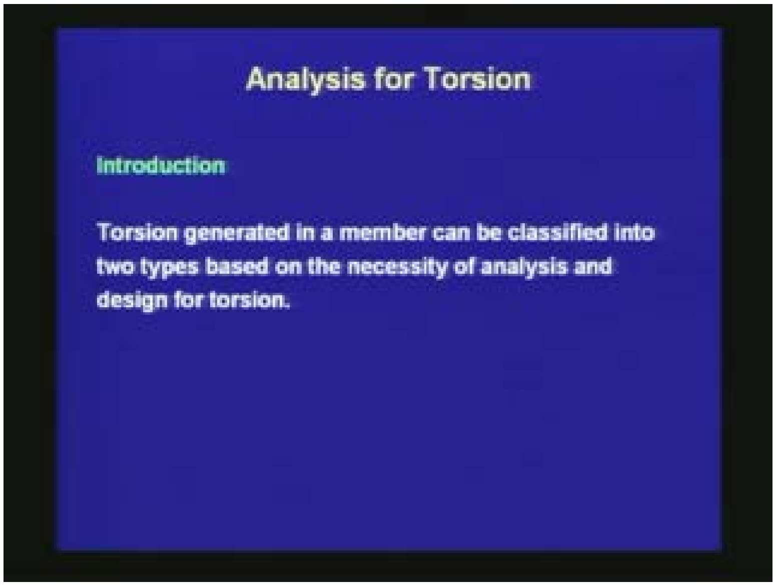

Torsion in a member can be classified into two types based on the necessity of analysis and design for torsion.

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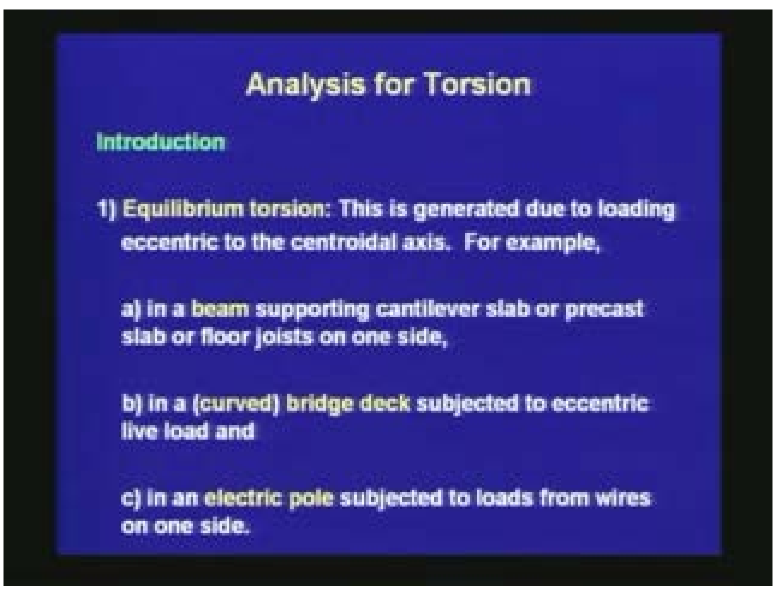

First is the equilibrium torsion. This is generated due to loading eccentric to the centroidal axis. For example, in a beam supporting cantilever slab or precast slab or floor joists on one side; in a bridge deck subjected to eccentric live load (the torsion is higher for a curved bridge deck); and third, in an electric pole subjected to loads from wires on one side.

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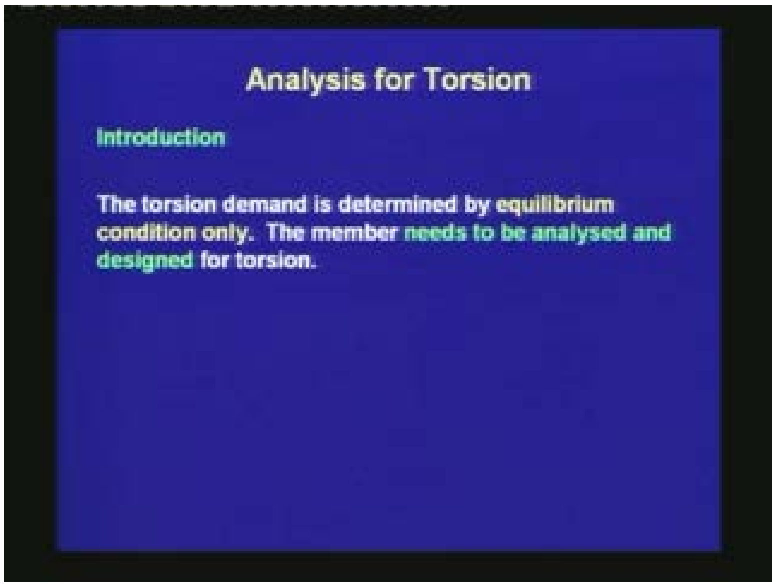

In this type of torsion, the demand is determined by equilibrium condition only. The member needs to be analyzed and designed for torsion.

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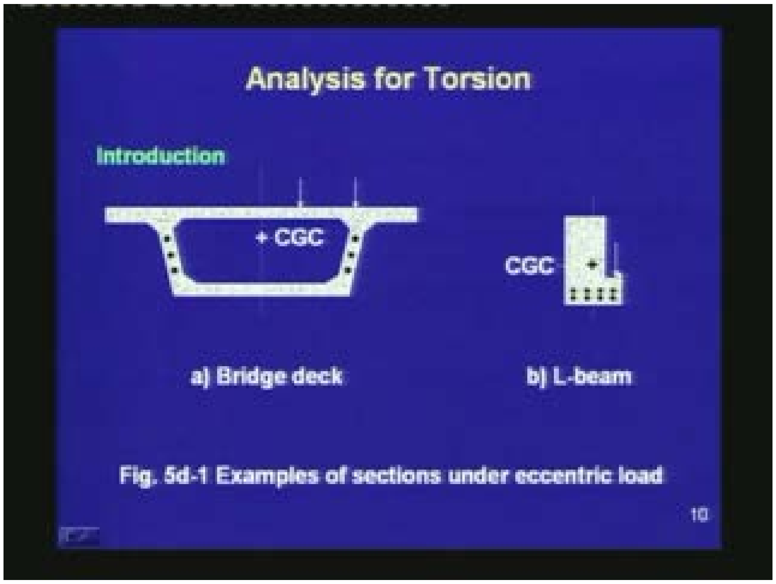

Here, we are seeing two examples where torsion is common. A bridge deck is subjected to torsion, when the wheel loads from the vehicle are eccentric compared to the centroidal axis passing through the CGC. For an L-beam, if it supports floor joists or precast slabs on one side, then the load is also eccentric to the centroidal axis. These sections are subjected to equilibrium torsion.

In equilibrium torsion, the analysis can be done based on equilibrium of the forces only. Usually, the structure is statically determinate. The members have to be designed for torsion, because there is no chance of redistribution.

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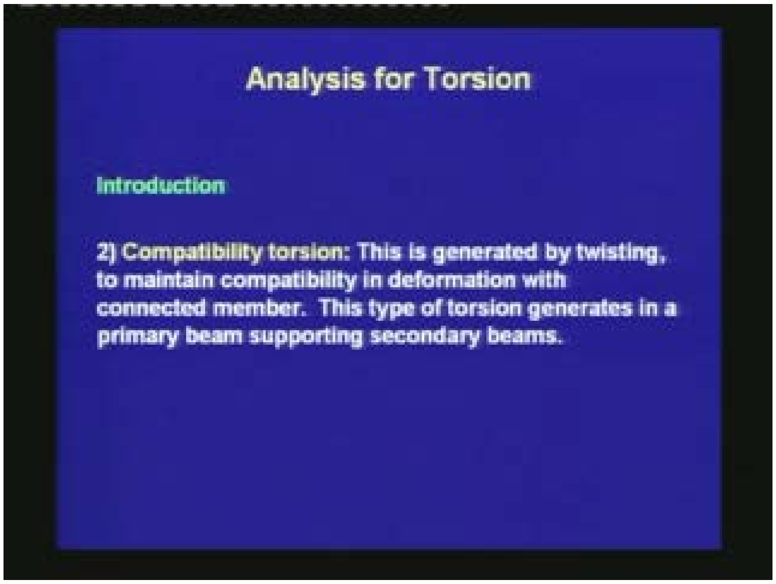

The second type is the compatibility torsion. This is generated by twisting, to maintain compatibility in deformation with the connected member. This type of torsion generates in a primary beam supporting secondary beams, or in the beams of a grid beam system.

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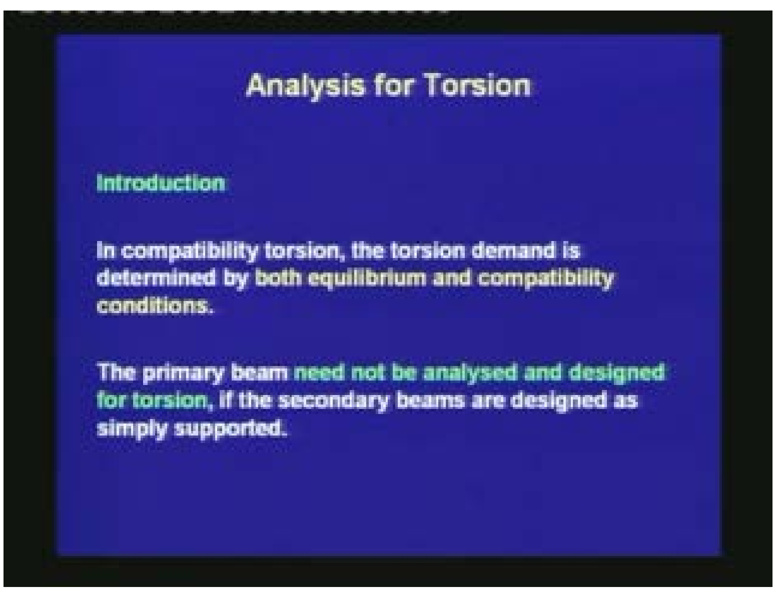

In compatibility torsion, the torsion demand is determined by both equilibrium and compatibility conditions. The primary beam need not be analyzed and designed for torsion, if the secondary beams are designed as simply supported. That means, we are not considering any moment connectivity between the primary beam and the secondary beam. The code allows us to neglect torsion, if it is a case of compatibility torsion.

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Here, the emphasis is laid on equilibrium torsion. To understand the behaviour of a beam under torsion, the presentation will be in the following sequence: first, stresses in an uncracked (which is considered to be homogeneous) rectangular beam without prestressing, under pure torsion (that means, in absence of flexure), with constant torque along the span.

Usually, the civil engineering structures subjected to torsion are non-circular in section. Hence, we shall study rectangular sections. First, to understand the behaviour under torsion, we shall be addressing only pure torsion where we are not considering any bending effect in the beam. We shall also consider a constant torque along the span of the beam.

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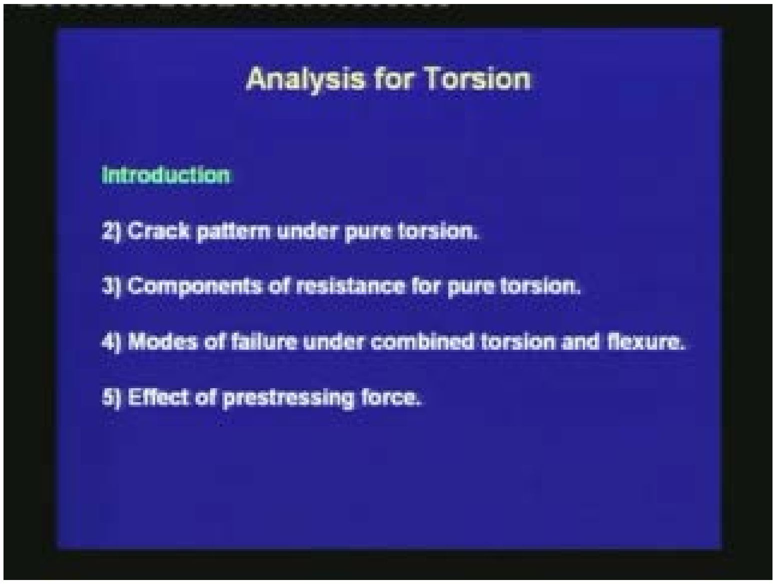

Next, we shall study the crack pattern under pure torsion. We shall study the components of resistance for pure torsion. The modes of failure under combined torsion and flexure will be addressed next. Finally, we shall look into the effect of prestressing force.

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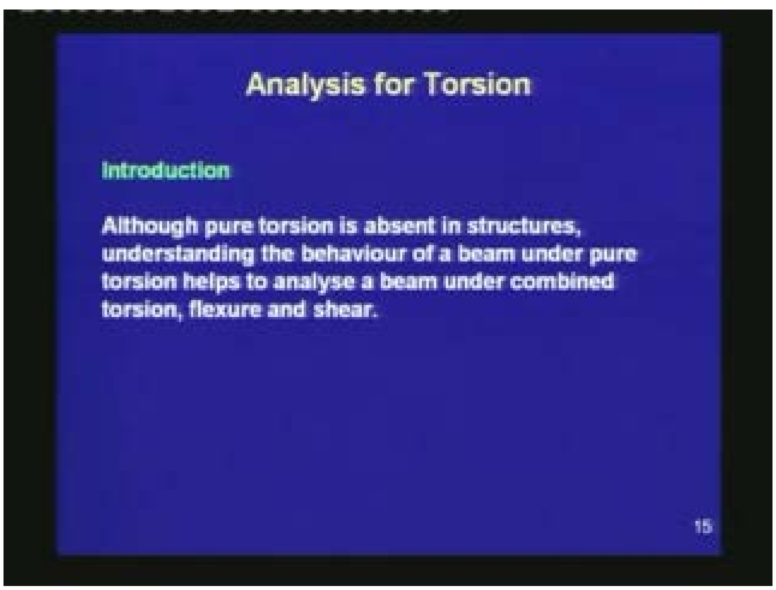

Although, pure torsion is absent in structures, understanding the behaviour of a beam under pure torsion helps to analyze a beam under combined torsion, flexure and shear. As I had mentioned before, the behaviour under torsion is more difficult to understand compared to the behaviour under flexure. We are studying the behaviour only for pure torsion; that means the beam is not subjected to simultaneous bending, or shear due to bending. This is not common in civil engineering structures, but still the understanding of behaviour under pure torsion helps us to analyze a beam under combined torsion, flexure and shear due to the flexure.

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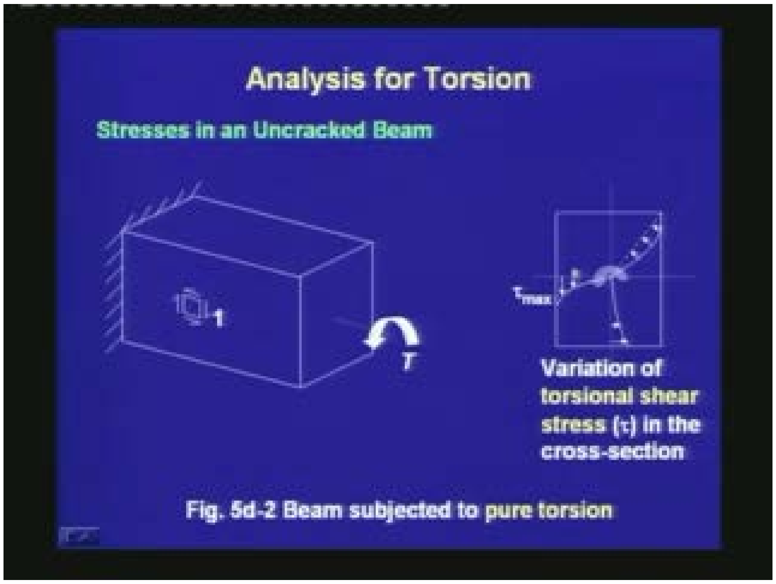

Fig. 5d-2 Beam subjected to pure torsion

First, we are studying the stresses in an uncracked beam, which is subjected to pure torsion. Here, the beam with the rectangular cross-section is subjected to a torque T. The torque is constant throughout its length. If we look into the cross-section of a beam at a location, we will observe that, shear stresses are generated in the cross-section of the beam. These shear stresses will be termed as torsional shear stress. It is different from the flexural shear stress that we had seen in the earlier modules.

The torsional shear stress will be denoted as τ, and its variation is shown in the diagram on the right. It is zero at the center, which is the center of the rotation and as we move out the shear stress increases. It is maximum at the mid-depth of the longer side; it is also high at the mid-depth of the shorter side. The relative magnitudes of these two values on the two sides depend on the ratio of the two sides. Along the diagonal, we will observe that, there is no shear stress at the corner. Thus, the maximum shear stress that occurs in the cross-section is at the mid-depth of the longer side. We shall study the principal stresses and the Mohr’s circle for a member, which is at the mid-depth of a longer side. Here, it is denoted as 1.

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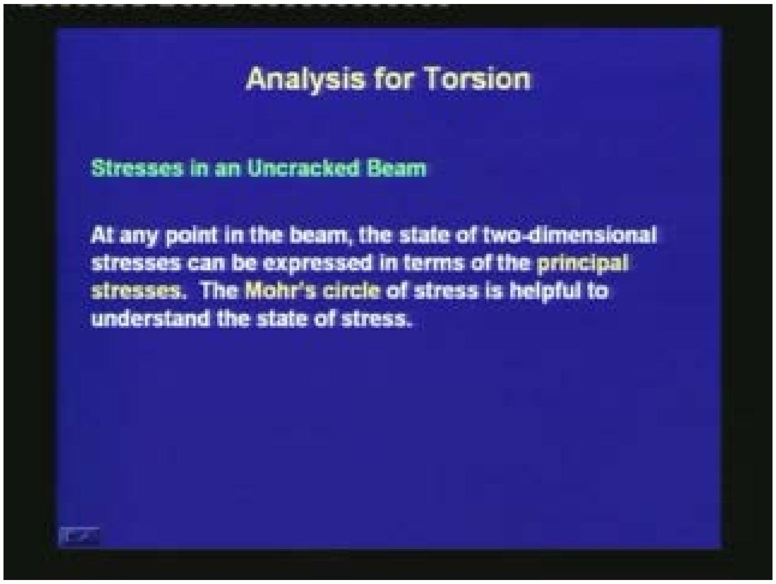

At any point in the beam, the state of two-dimensional stresses can be expressed in terms of the principal stresses. The Mohr’s circle of stress is helpful to understand the state of stress.

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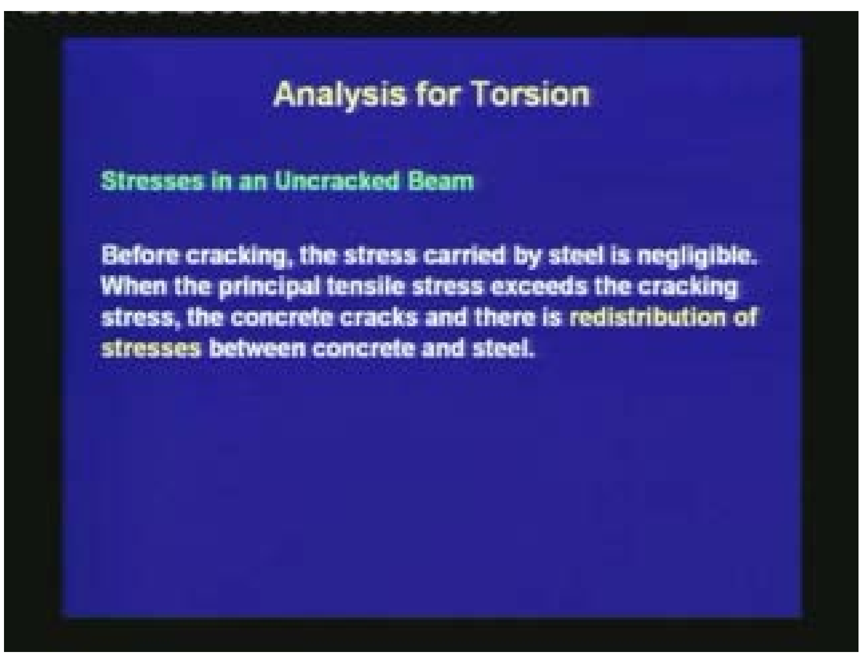

Before cracking, the stress carried by steel is negligible. When the principal tensile stress exceeds the cracking stress, the concrete cracks and there is redistribution of stresses between concrete and steel. It is easy to understand the behaviour first, for an uncracked beam, which is considered to be homogeneous. The presence of steel in an RC member can be neglected, before the cracking of concrete because the stress carried by the steel parts is very small. Thus, before cracking, we are studying the RC beam as a homogeneous beam. If we pick up an element, which is at mid-depth of the longer side, then we are able to understand how the principal stresses are oriented for that element, how the cracks will form and when the maximum principal tensile stress crosses the tensile strength of concrete.

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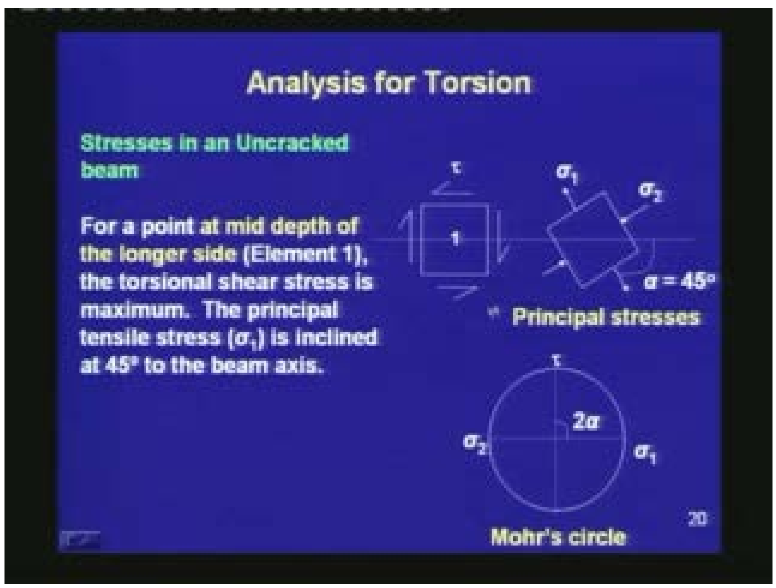

For a point at mid-depth of the longer side, which is denoted as Element 1, the torsional shear stress is maximum. This element is under pure shear stress. We are not considering any stress due to warping. The principal tensile stress is denoted as σ_1 and the principal compressive stress is denoted as σ_2. The stress σ_1 is inclined at 45° to the beam axis. Thus, for the state of pure torsion, the principal tensile stress σ_1 is inclined at an angle α = 45°.

For pure torsion, σ_1 and σ_2 are numerically equal. When we draw the Mohr's circle, it is symmetric about the vertical axis. σ_1 and σ_2 are equidistant from the origin.

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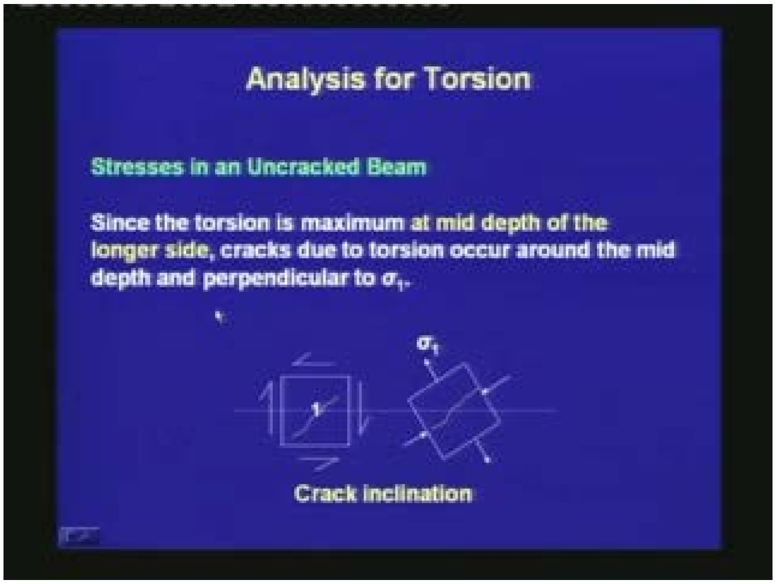

Since the torsion is maximum at mid-depth of the longer side, cracks due to torsion occur around the mid-depth and perpendicular to σ_1. With increasing torque, when σ_1 exceeds the tensile strength of concrete we observe a crack in the concrete. The crack is also inclined at 45° to the axis of the beam.

Next, we are studying the crack pattern under pure torsion.

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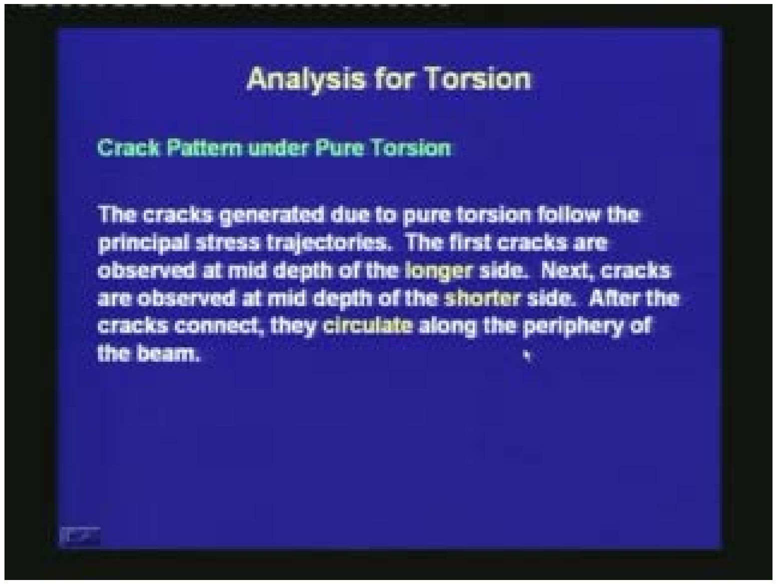

The cracks generated due to pure torsion follow the principal stress trajectories. The first cracks are observed at mid-depth of the longer side. Next, cracks are observed at mid-depth of the shorter side. After the cracks connect, they circulate along the periphery of the beam.

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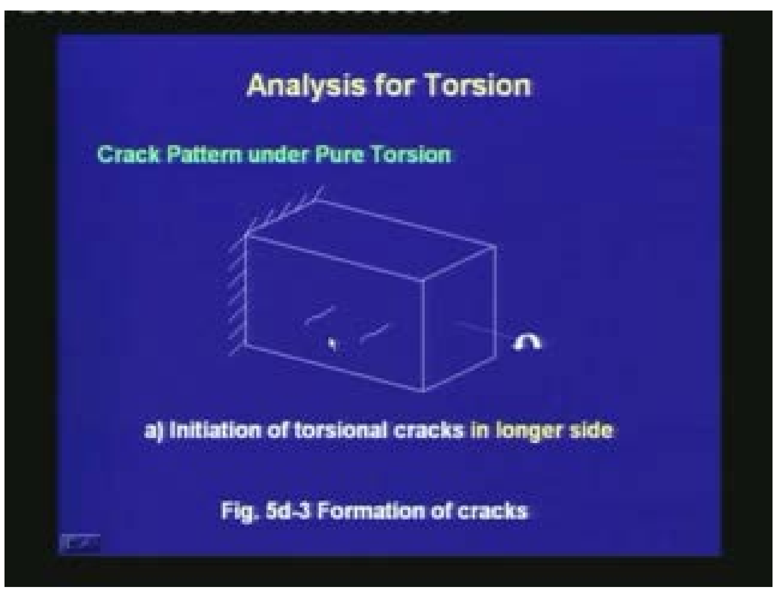

Let us try to understand with the help of the diagram. What we see here is that, the first crack occurs at the mid-depth of longer side. They are inclined at 45° to the axis of the beam. This is the first stage of initiation of torsional cracks in the longer side. Next, with increasing torque, we observe that more cracks form in the longer side, as well as cracks form in the shorter side.

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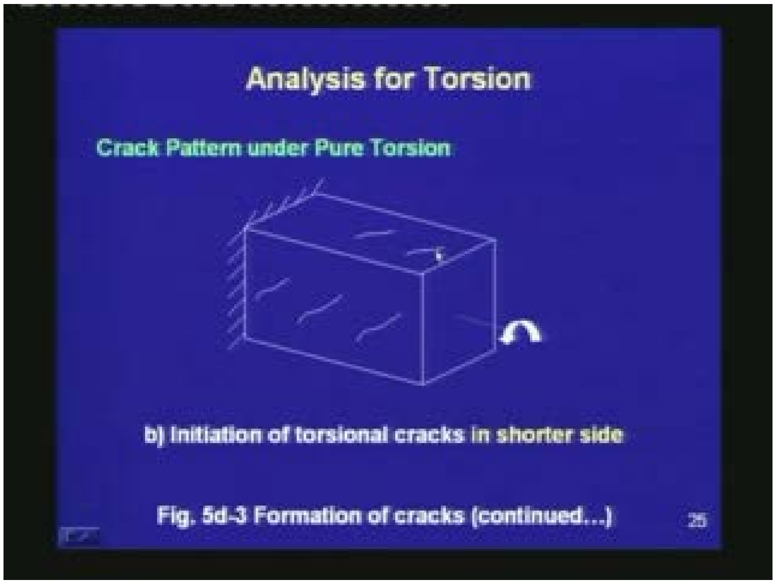

Even on the shorter side, the cracks are inclined at an angle of 45° to the axis of the beam. Finally, when the cracks have formed substantially, they merge together and form a spiral crack along the length of the beam. These cracks are circulatory in nature, and this is typical for beams under pure torsion.

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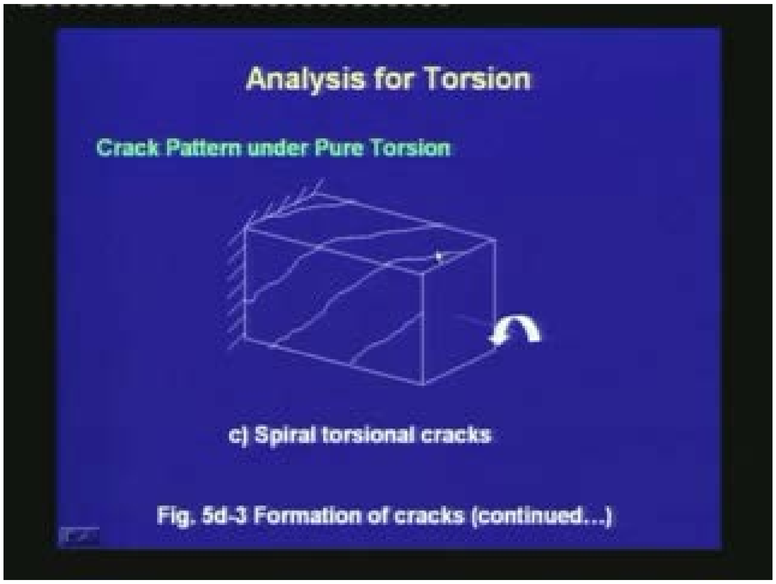

Let us observe the crack formation, once again. We are observing here that, at the beginning, there are some cracks at the mid-depth of the longer side. Next, with increasing torque, we find more cracks in the longer side and some cracks in the shorter side. With further increase in the torque, we find that the cracks have merged. Hence, they have formed a spiral pattern along the periphery of the beam.

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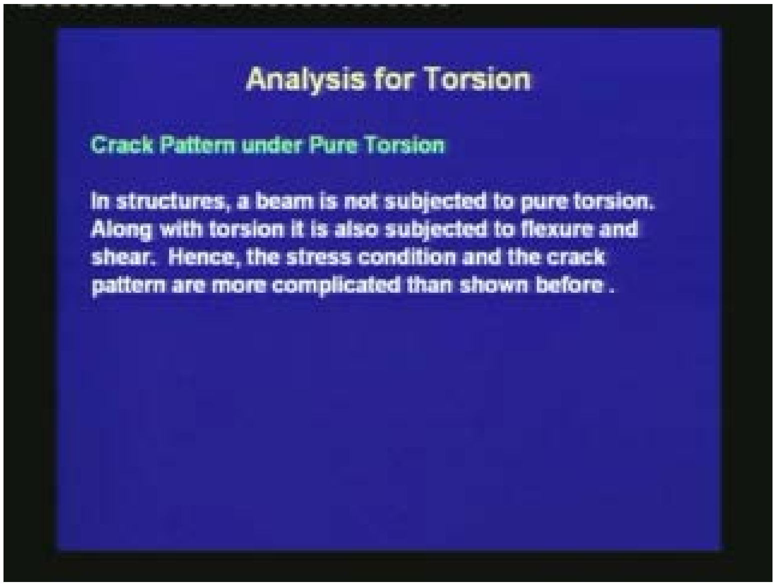

In structures, a beam is not subjected to pure torsion. Along with torsion, it is also subjected to flexure and shear. Hence, the stress condition and the crack patterns are more complicated than shown before.

Earlier, we had seen the behaviour of a beam under bending and shear due to bending. Here, we have seen a beam under pure torsion. Usually, a structure is subjected to combined bending, shear and torsion. In such a situation, the crack formation is more complicated. It depends upon the relative magnitudes of the flexural moment and the torsional moment.

Next, we move on to understand the components of resistance for pure torsion.

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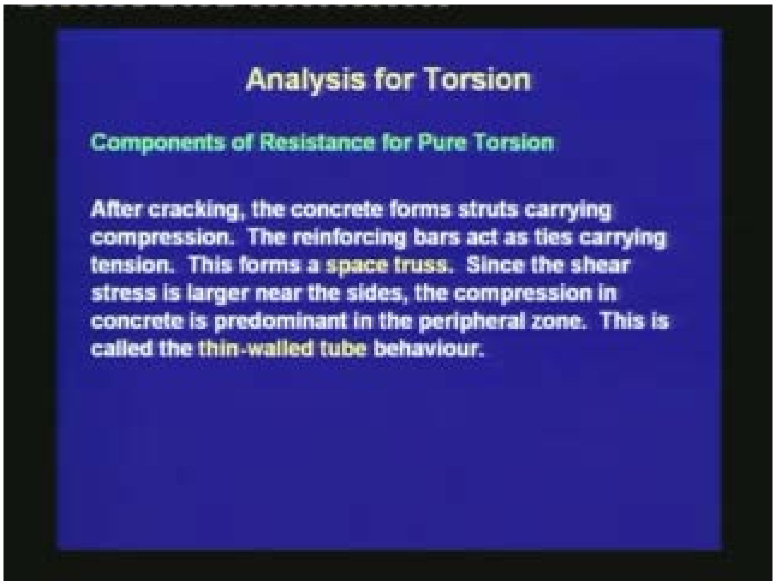

After cracking, the concrete forms struts carrying compression. The reinforcing bars act as ties carrying tension. This forms a space truss. Since the shear stress is larger near the sides, the compression in concrete is predominant in the peripheral zone. This is called the ‘thin-walled tube’ behaviour.

Here, we are introducing two concepts. The first is the truss behaviour under torsion after cracking of concrete. For shear, we have seen that after the concrete cracks, the concrete and the steel bars form a planar truss. The concrete forms diagonal compression struts and the compression chord at the top, whereas the steel bars form the ties. The longitudinal bars form the tensile chord at the bottom, and the stirrups form the vertical ties connecting the struts with the tensile chord.

For torsion also, there is truss behaviour after the cracking of concrete. Here, the concrete forms struts which are circulatory in nature. The steel bars, both the longitudinal and the transverse bars tie these struts together. These form the truss action which is called the ‘space truss analogy’ for a beam under pure torsion.

There is another concept which is called the thin-walled tube behaviour. Since the shear stress is more towards the periphery in a section, the resistance of concrete comes more at the periphery than at the interior region. Thus, the truss action actually is in the periphery of the beam. The interior of the beam does not carry substantial stress to resist the torsion. If we look into the truss mechanism, we can neglect the concrete which is inside. We can just consider the shell outside the interior region. This part of the member actually resists the torsion.

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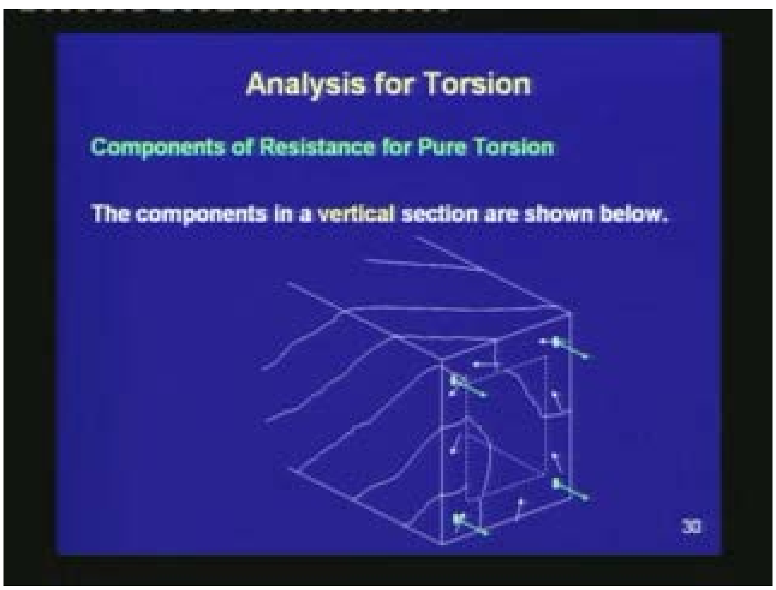

The concrete has formed struts, which carry compression. The steel reinforcement which is both longitudinal and transverse, they form ties. Here, the longitudinal bars are shown with actual tension, which is generated due to torsion. Thus, torsion generates stress in the longitudinal bars as well as the transverse bars. Here, the stresses are shown for a vertical section. If we consider a horizontal section, then we can observe that the transverse bars are also under tension. The concrete struts, the longitudinal bars and the transverse bars, they together form the space truss which resist the torsion after the cracking of concrete.

The components can be denoted as below: we have the first component, T_c which is the torsion resisted by concrete. Next, we have $T_{s,}$ which is the torsion resisted by the longitudinal and transverse reinforcing bars. The magnitude and the relative value of each component change with increasing torque.

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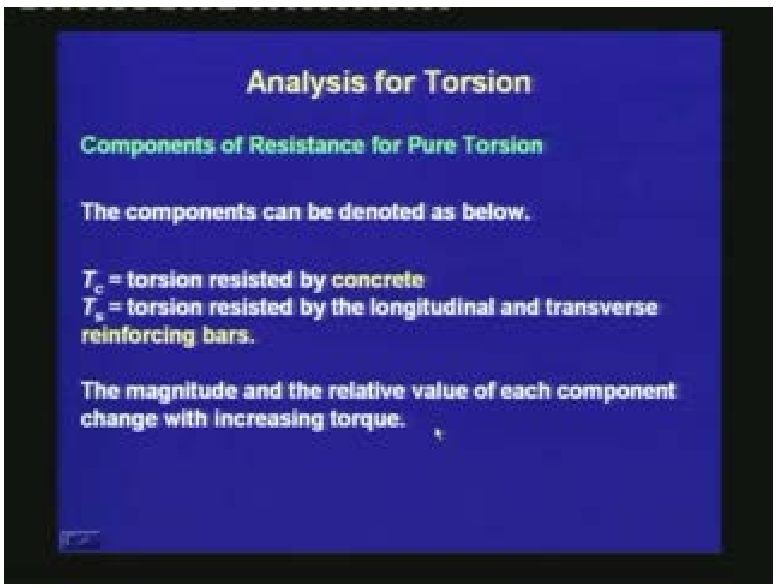

That means, when a beam is resisting the torsion, both the concrete and steel are taking part in resisting the torsion. The concrete takes the component which is denoted as T_c. The steel reinforcing bars take the component, which is denoted as T_s. In the space stress, both the concrete struts and the steel ties perform together to resist the torsion.

Let us study the modes of failure.

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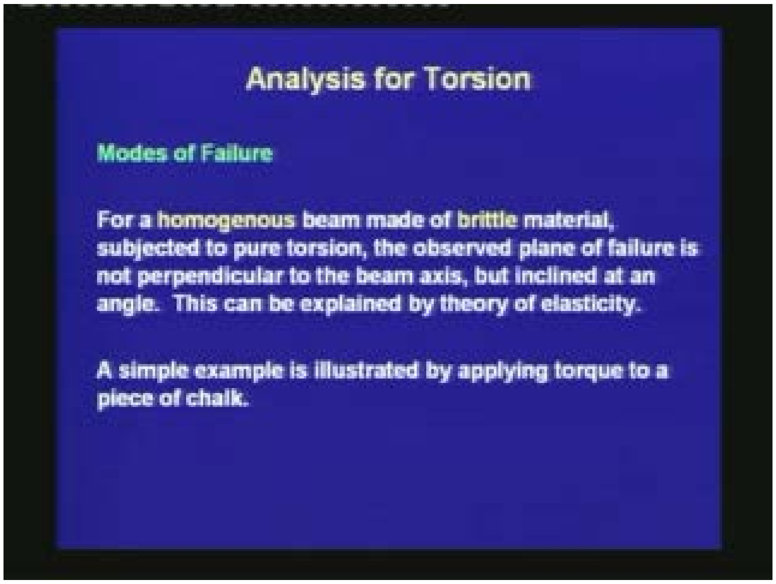

For a homogeneous beam made of brittle material, subjected to pure torsion, the observed plane of failure is not perpendicular to the beam axis, but inclined at an angle. This can be explained by the theory of elasticity. If we take a piece of chalk and try to apply a torque in that chalk, then we can observe the failure plane in the chalk piece. This failure plane is not a planar surface and it is not vertical to the axis either. It is a curved surface, which is inclined to the axis of the beam.

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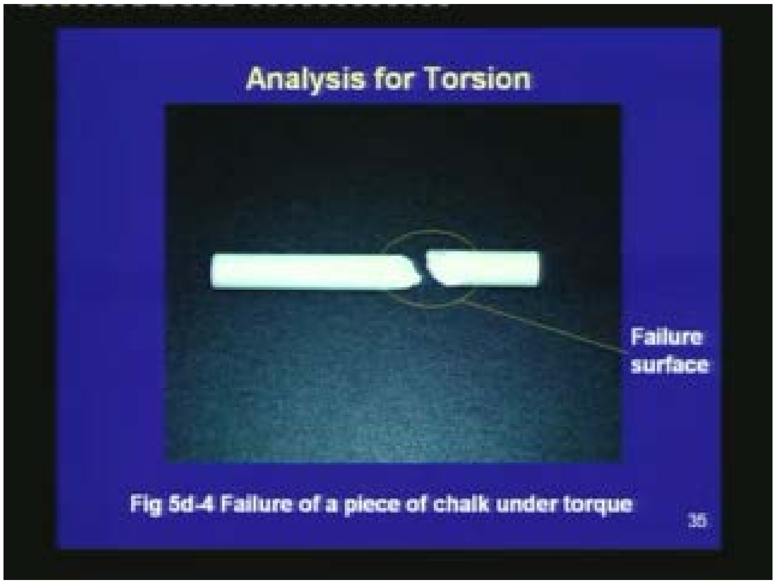

Let us see this, in this photograph. Here, a piece of chalk, which is a brittle material, has been subjected to a pure torque. We can observe that the failure surface is curved and it is inclined to the axis of the beam. If we take a rectangular piece of chalk, then also we shall observe the same behaviour.

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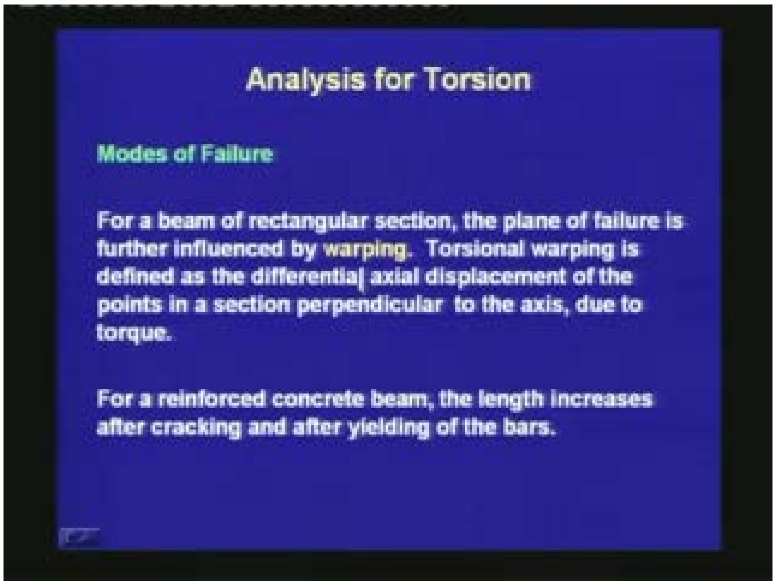

But in a rectangular section, the plane of failure is further influenced by warping. Torsional warping is defined as the differential axial displacement of the points in a section perpendicular to the axis, due to torque. That means, when a pure torque is applied the points in a particular section do not remain as a vertical section. The points have an axial displacement and that is called warping of the section.

In reinforced concrete, the concrete is tied with the steel bars. Once the concrete cracks, it tries to expand, but the steel ties hold them together. But as the torque is increased, the steel ties and the longitudinal bars may yield and the expansion will increase. Thus, for a reinforced concrete beam, the length increases after cracking and after yielding of the bars. Finally, the concrete will crush along the diagonal compressive struts. If the steel does not yield, then the concrete will crush and that will be a brittle failure. But if the steel yields, then we shall have some ductility in the behaviour of the beam under pure torsion.

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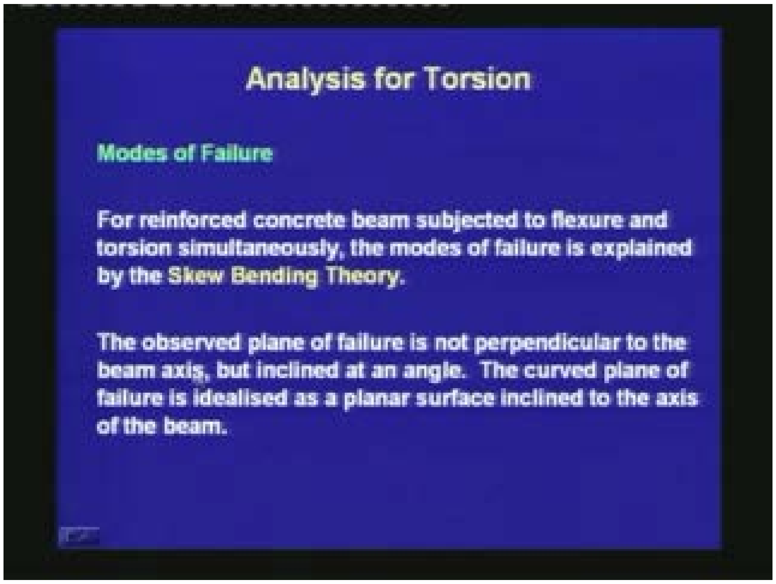

For a reinforced concrete beam under combined flexure and torsion, the modes of failure are explained by the 'Skew Bending Theory'. The observed plane of failure is not perpendicular to the beam axis, but inclined at an angle. The curved plane of failure is idealised as a planar surface inclined to the axis of the beam.

We have understood the complicacy of the analysis for a beam under pure torsion. If we are analyzing a beam under combined torsion and flexure, then the analysis is even more difficult. The analysis is based on the skew bending theory, where the surface of failure is idealised as a planar surface, which is inclined to axis of the beam. Skew bending means the bending is occurring not along the transverse axis of the beam, but the axis of bending is inclined to the longitudinal axis of the beam.

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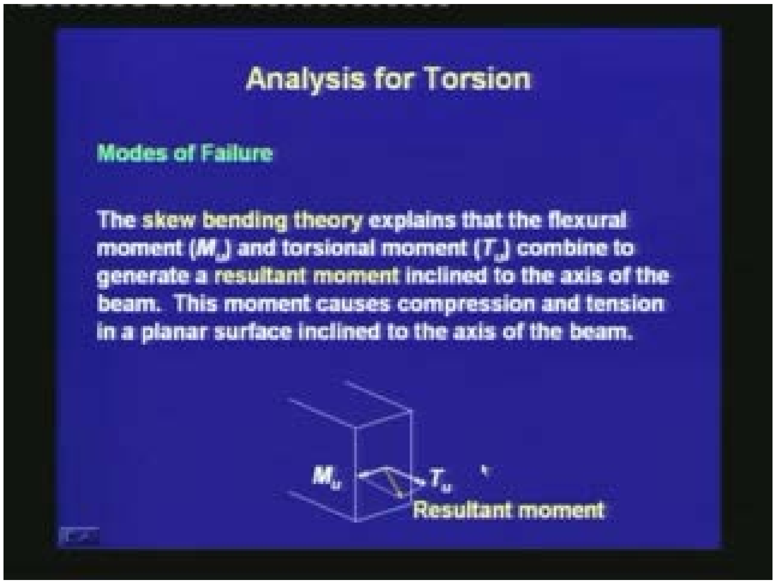

Let us understand this with the help of the diagram. The skew bending theory explains that the flexural moment, which is represented as M_u and torsional moment, which is represented as T_u combine to generate a resultant moment, inclined to the axis of the beam. This moment causes compression and tension in a planar surface, inclined to the axis of the beam.

For a beam under flexure, a plane section perpendicular to the axis remains plane till failure, as per the Bernoulli's hypothesis. For a beam under pure torsion, we have seen that the failure surface is not plane anymore. The failure surface is curved and it is inclined to the axis of the beam.

Under combined moment and torsion, we identify a resultant moment which causes the failure of the beam and the failure surface is inclined to the axis of the beam. The curved failure surface is idealised as a planar surface for the purpose of analysis. In the planar surface, one part is in compression and the other part is in tension. There the action of the reinforcing bars comes into play. Thus, the skew bending theory analyses a beam under combined flexure and torsion, by considering a resultant moment which causes failure in a surface which is inclined to the axis of a beam, and part of the surface is in compression and the tension is taken by the steel ties.

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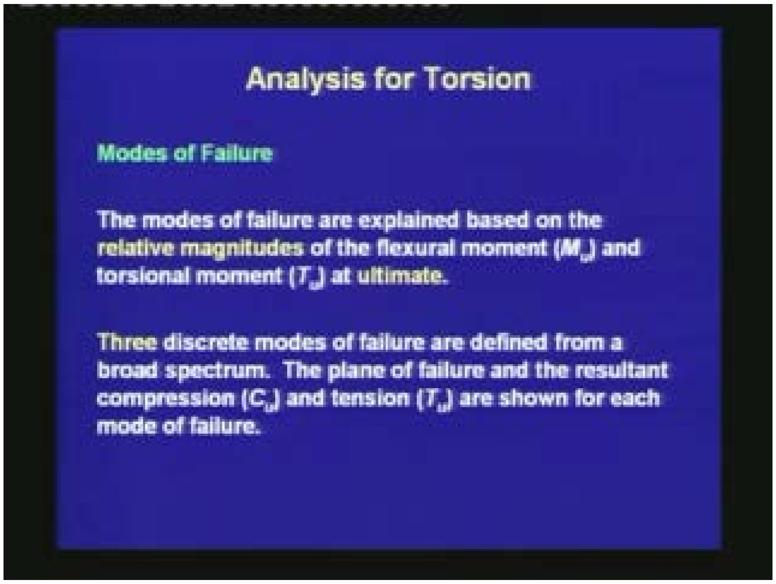

The modes of failure are explained based on the relative magnitudes of the flexural moment and torsional moment at ultimate. We can appreciate that two moments are acting in the beam; the flexural moment and the torsional moment. The failure depends upon the relative magnitudes of these two moments. The modes of failure are identified based on these relative magnitudes. Three discrete modes of failures are defined from a broad spectrum. The plane of failure, and the resultant compression and tension are shown for each mode of failure.

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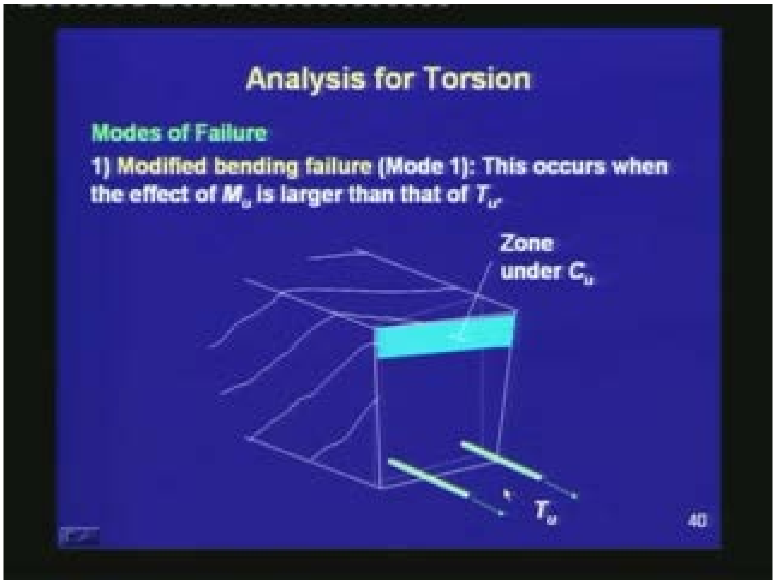

The first mode of failure is called the modified bending failure, and it is denoted as Mode 1. This occurs when the effect of M_u is larger than that of T_u. Thus, the beam is primarily under flexure, but there has been some amount of torsion which has inclined the plane of failure. The plane of failure is not perpendicular to the axis anymore. We find that in this plane, there is a zone which is under compression which is shaded by blue and the tension is carried by the bottom steel. Here, the beam is under predominant flexural moment. The bottom steel is taking the tension; the top of the beam is under compression. But the failure surface is not perpendicular to the axis anymore. That is the effect of torsion, which has modified the plane of failure from that of flexure.

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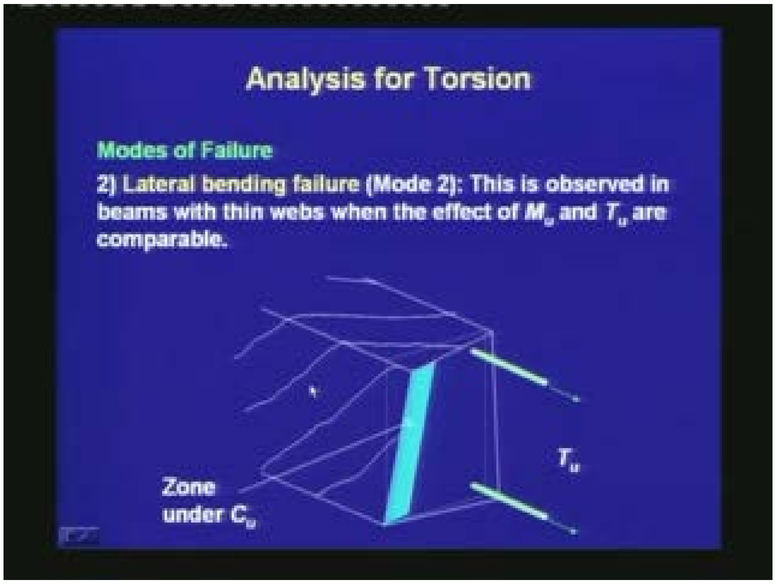

The second mode of failure is called the lateral bending failure, which is denoted as Mode 2. This is observed in beams with thin webs, when the effects of M_u und T_u are comparable. Here, the zone under compression is along the lateral side. We can observe that, the longitudinal reinforcement in the opposite side to the compression is in tension. The effect of torsion is substantially high. It has changed the mode of failure from compression at the top to compression at the lateral face. The crushing is at the side of the beam and the yielding of the bars occurs at the other side of the beam.

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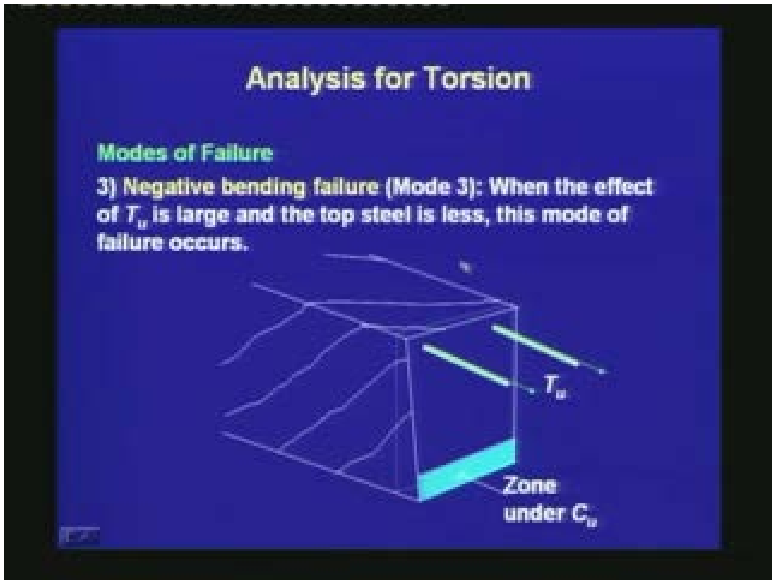

The third mode of failure, which is the negative bending failure, is denoted as Mode 3. When the effect of torsion T_u is large and the top steel is less, this mode of failure occurs. In this case, the magnitude of torsion is substantially high than that of the flexure. If the top steel is less, then the top steel will yield and the crushing will occur at the bottom. Remember that under pure torsion, the crushing can occur at any side of the beam depending on how much reinforcement has been provided.

The modes of failures that have been identified here are three distinct modes from a broad spectrum. The design is based on these three modes of failure. Next, let us understand the effect of prestressing force.

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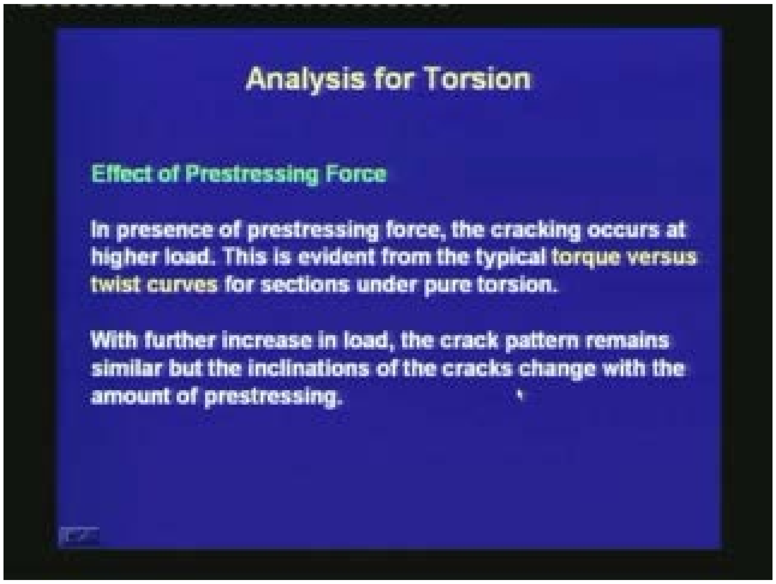

In presence of prestressing force, the cracking occurs at higher load. This is evident from the typical torque versus twist curves for a beam under pure torsion. With further increase in load, the crack pattern remains similar but the inclinations of the cracks change with the amount of prestressing. If we compare the torque versus twist curves for a non-prestressed section with that of prestressed section, then we will be able to understand the effect of prestressing in the behaviour of the two beams.

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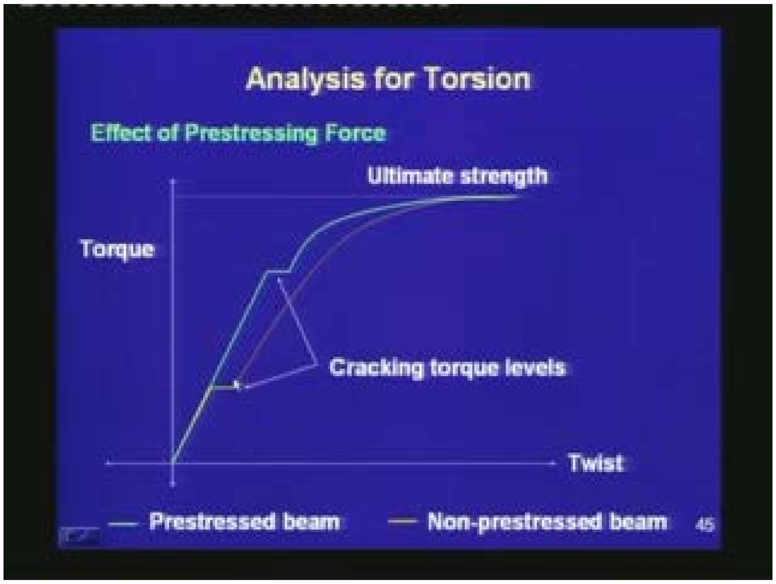

In this figure, we are comparing two beams with equal ultimate strength. For the reinforced concrete beam, the beam cracks at a certain level. Then, after cracking, the twist increases. After that, we see the non-linear curve due to the non-linear behaviour of concrete. Either of the steel, the longitudinal or the transverse can yield first, depending upon their amount and then, the other steel may yield and finally the beam will attain its ultimate strength. Whether any of the two steel will yield first or the concrete will reach its ultimate capacity, depends on the design. If it is an under-reinforced design, then a steel will yield before the concrete crushes.

For prestressed concrete beam, the cracking is at a higher load.

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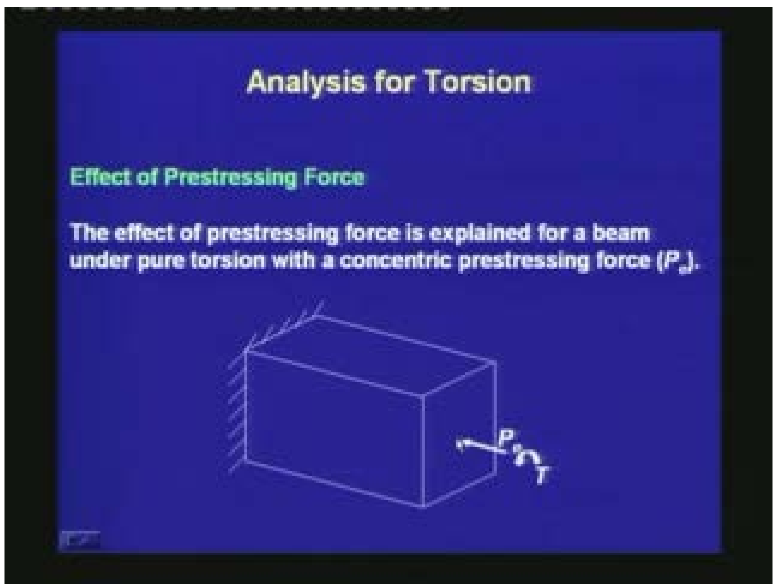

The effect of prestressing force is explained for a beam under pure torsion, with a concentric prestressing force P_e. This concentric force will have uniform compression throughout the section and that will lead to an increase in the cracking load.

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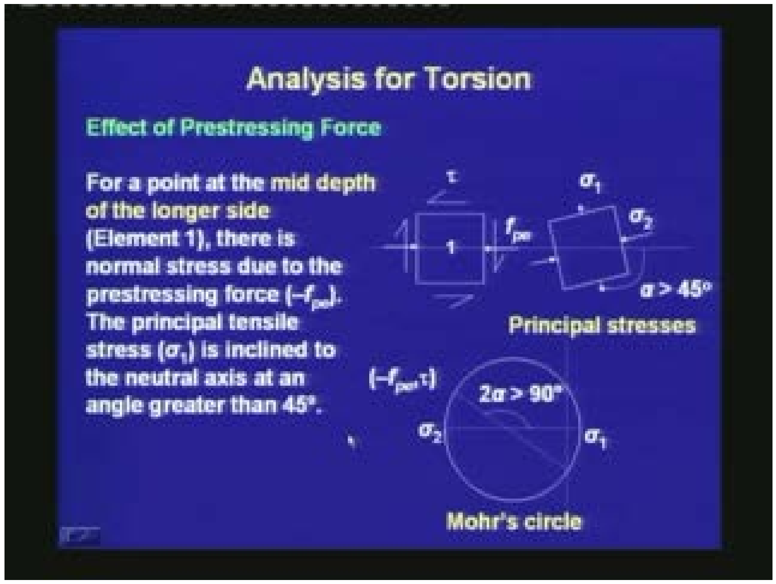

For a point at mid-depth of the longer side (which is denoted as Element 1), there is normal stress due to the prestressing force which is denoted as f_{pe}. Thus, the difference between the previous case of pure torsion, and pure torsion along with prestressing force is that, the element at the mid-depth of the long side has not only the pure shear due to the torsion but it also has the axial compression due to the prestressing force. This will change the inclination of the principal stresses. Here, the inclination of the principal tensile stress σ_1 is larger than 45°. For the principal compressive stress, the inclination is lower than 45°. Since the crack is perpendicular to σ_1 and parallel to σ_2, the inclination of the cracks will be lower than 45°. The inclination depends on the amount of prestressing force.

If we draw the Mohr's circle for that element, we find that the vertical face will be denoted by a point whose coordinates are $(-f_{pe}, \tau)$. The horizontal surface will be denoted by a point on the vertical axis, which has only the shear stress τ. The magnitudes of the principal stresses are now different. σ_1 is much lower than σ_2 and this is the benefit of prestressing, that the principal tensile stress is reduced for a certain torque. Hence, the cracking torque is higher as compared to reinforced concrete beam. How much will be the cracking torque, depends on the magnitude of the prestressing force. The inclination of the cracks will be less than 45°.

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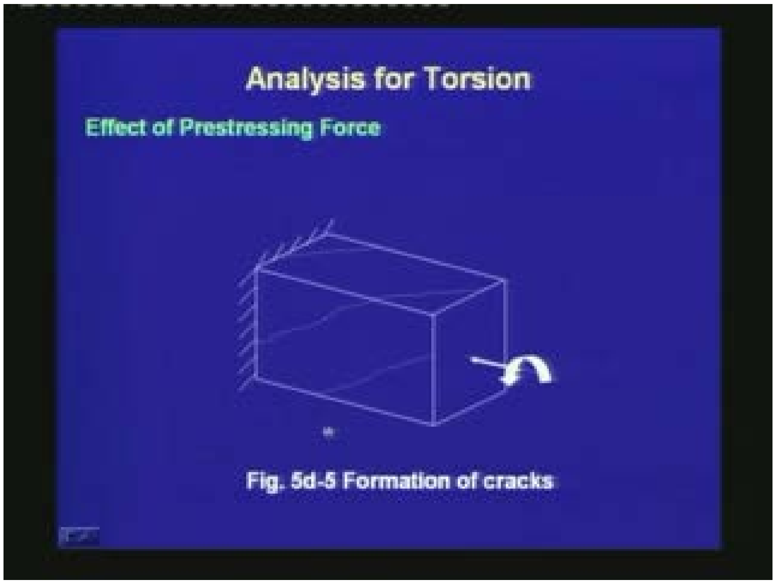

The formation of cracks is as follows. With the increase in torque, we will observe cracks in the beam, which are inclined at an angle less than 45° to the axis of the member. The crack width will be smaller and the expansion of the beam will be lower. Thus, the behaviour of the beam will be better in presence of the prestressing force.

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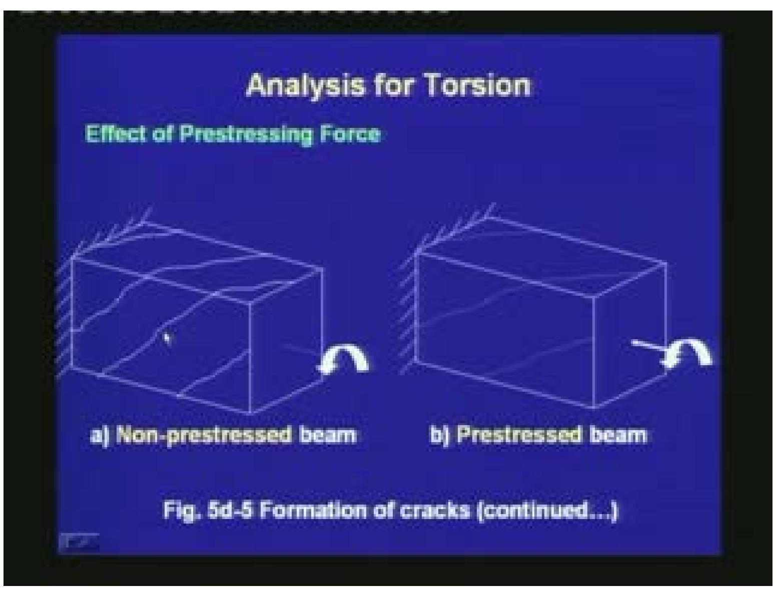

If we compare the formations of cracks for a reinforced concrete beam and a prestressed concrete beam, we shall observe that for the reinforced concrete beam, the first crack forms at the mid-depth of the long side. Then, we observe cracks at the mid-depth of the shorter side. As the torque is increased, the cracks can get connected and they form a spiral pattern circulating the periphery of the beam. As the torque is increased the cracks open up, the steel bars take stress. After the yielding of some steel bars, the crack width further increases, and the length of the beam gets increased.

When we apply a compressive force in the member, the cracking load is much higher. The angle of the inclination of the crack is now smaller than 45°. The cracks do not open up and the number of cracks is also less as compared to the non-prestressed beam. The figure shows the benefit of prestressing a beam which is subjected to torsion.

When a beam is subjected to simultaneous flexure and torsion, the benefits of prestressing are observed. When the prestressing is applied such that it is within the allowable stresses of concrete at transfer, then the behaviour under service load gets better in presence of prestressing force. The cracking load is higher and under service condition, the beam may not have any crack. Thus, the durability of the beam becomes better in absence of cracks under service loads.

After cracking, the crack width is low. Thus, the aggregate interlock is larger as compared to a non-prestressed beam under the same torque. Hence, the torsional strength of concrete increases in the presence of prestressing force. This is accounted for in the expression of T_c.

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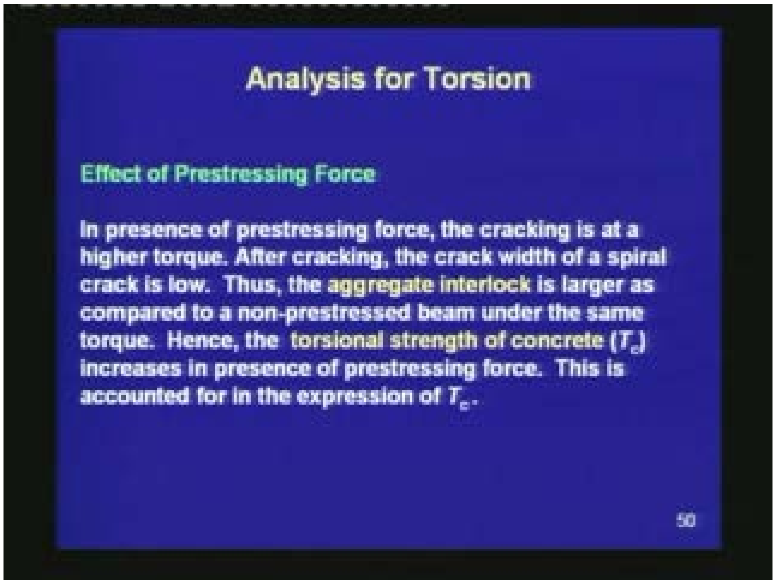

In the presence of prestressing force, the cracking in the concrete occurs at a higher torque. That itself shows that the concrete will have a higher resistance to torsion in presence of the prestressing force. Then, even after cracking since the prestressing force is reducing the crack width, there is better aggregate interlock at the interface of the cracks and the resistance of the concrete is sustained. Hence, the behaviour is better; the twist is smaller even after the cracking of the concrete. Whereas, in a reinforced concrete member, if the torque is increased then the member tends to have more number of cracks and it has larger crack width; the aggregate interlock is gradually lost and the capacity of concrete to resist the torsion gets limited. Thus, the benefit of prestressing is that it checks the growth of the cracks in the concrete member. Hence, it increases the capacity of the concrete, which will be reflected in the expression of T_c. The expression of T_c will involve the prestressing force that is applied in the member.

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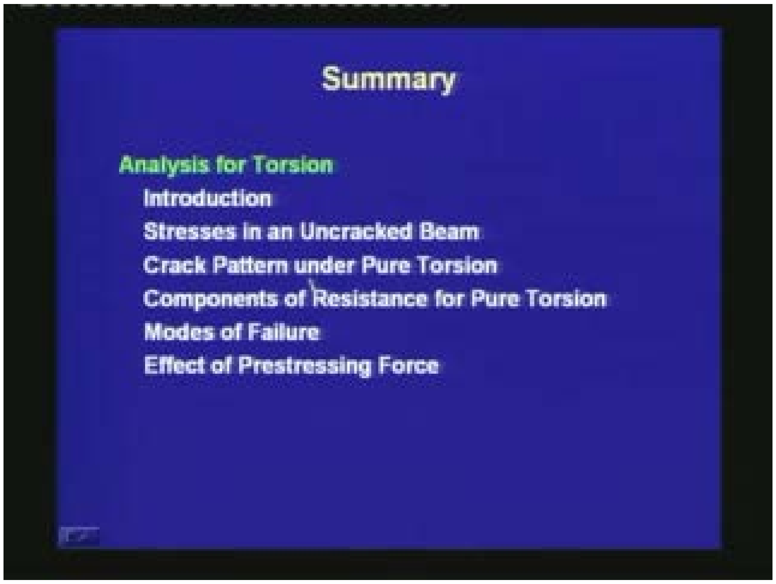

In today's lecture, we first studied the two types of torsion that comes in civil engineering structures. One is the equilibrium torsion and another is the compatibility torsion. In equilibrium torsion, a statically determinant member is subjected to torsion due to eccentric loading, and this member can be analyzed based on the equilibrium equations only. Since there is no scope of redistribution, these members have to be designed for torsion.

The second type of torsion, the compatibility torsion occurs in an indeterminate system like a grid of beams, where a secondary beam applies some twist in the primary beam. Now, if the primary beam cracks due to the twist, then its torsional resistance is lowered and the secondary beam is considered to be simply supported in between the primary beams. In case of compatibility torsion, we may not design the primary beams for torsion if we are designing the secondary beams as simply supported beams. The code allows us to neglect the effect of torsion, in case of compatibility torsion.

The analysis and design for torsion is simplified in the code. It is based on equilibrium forces only and that too it is only at the ultimate. We do not study the compatibility of strains between concrete and steel or the constitutive relationships of either of concrete or steel. We consider only the ultimate strength of each material at failure.

Next, we studied the stresses in an uncracked beam. To understand the behaviour of reinforced concrete beam, first we studied the behaviour of uncracked beam which can be considered to be homogeneous. The steel can be neglected before cracking. For an uncracked beam we found that the shear stress that is generated due to pure torsion is maximum at the mid-depth of the longer side. The principal tensile stress is inclined to the axis of the beam and the crack that forms perpendicular to the principal tensile stress will be inclined at an angle of 45° to the axis of the beam. The stress condition in an uncracked beam helps us to understand the formation of cracks of a beam under pure torsion.

We moved on to study the crack pattern under pure torsion. We observed that in a rectangular section, the cracks first occur at the mid-depth of longer side. With increasing torque, there will be cracks at mid-depth of the shorter side. With further increase in the torque, the cracks can get connected, and they form a circulatory pattern around the beam. The concrete forms struts carrying compression and the steel bars act as ties, which hold these struts. The struts and ties form a space truss, which resist the torsion. We also observed that the resistance comes mostly in the peripheral region of the beam. The region which is inside can be considered to be inactive in resisting torsion. Another concept we studied is the thin walled tube, which comes from this observation of the resistance of torsion in the peripheral region of the member.

The components of the resistance for pure torsion were discussed. The torsion is resisted both by the concrete and the steel bars. The components can be identified in two groups. One is T_c which is the contribution of concrete to resist the torsion and the other is $T_{s,}$ which is the contribution of steel to resist the torsion. In our design, we shall find expressions of each of these quantities for the ultimate state.

For a beam under pure torsion after the formation of the struts, if the beam is under-reinforced then some steel bars will yield. It will lead to expansion along the length of the beam and finally, the concrete will crush. The failure surface is not perpendicular to the axis of the beam. It is a curved surface, which is inclined to the axis of the beam. When a beam is subjected to combined flexure and torsion, then the analysis gets even more complicated. The skew bending theory is used to analyze a beam which is subjected to combine flexure and torsion.

In this theory, the plane of failure is considered to be a planar surface, which is inclined to the axis of the beam. This planar surface generates due to a resultant moment, which is the resultant of the flexure moment and the torsional moment. Three modes of failure are defined which are the discrete modes from a broad spectrum. The first mode is the modified bending failure, where the beam has substantial flexural moment and small torsional moment. The concrete at the top crushes and the steel at the bottom is under tension. The failure plane is inclined to the axis of the beam. The second mode of failure, which is the lateral bending failure, is observed in beams with thin webs and when the torsion is substantially high. The crushing of concrete occurs at the side of the beam and the other side, the steel will be under tension.

In the third mode of failure we observed that the effect of flexure is very small and the torsion is substantially high. The crushing can occur at any side of the beam. If the top steel is small, then the crushing will occur at the bottom, which is actually the opposite of flexure. This is called a negative bending failure because the crushing is occurring at the bottom and the steel at the top is yielding. In the design, these three modes of failure will be addressed.

Finally, the effect of prestressing force was studied. Due to the prestressing force, the principal tensile stress gets reduced. The angle of inclination of the cracks is smaller than 45°. The cracking torque is higher and after cracking, the crack width is not large. The aggregate interlock is retained. The capacity of concrete under prestressing force is larger and this is the direct benefit of applying prestressing to a member under torsion.

In our next class, we shall move on to the analysis of the members at ultimate, and we shall also get familiar with the design equations.

Thank you.