

PRESTRESSED CONCRETE STRUCTURES

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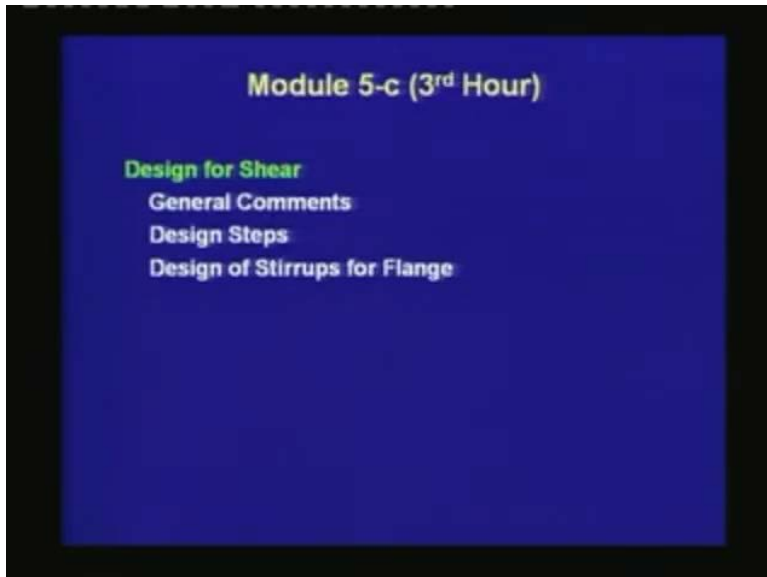
Indian Institute of Technology Madras

Module – 5: Analysis and Design for Shear and Torsion

Lecture-25: Design for Shear (Part 2)

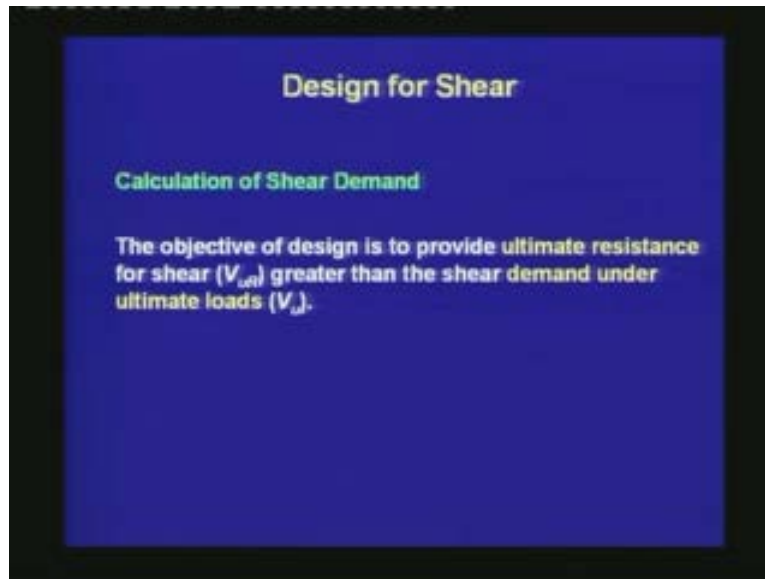
Welcome back to prestressed concrete structures. This is the third lecture of Module 5 on analysis and design for shear and torsion.

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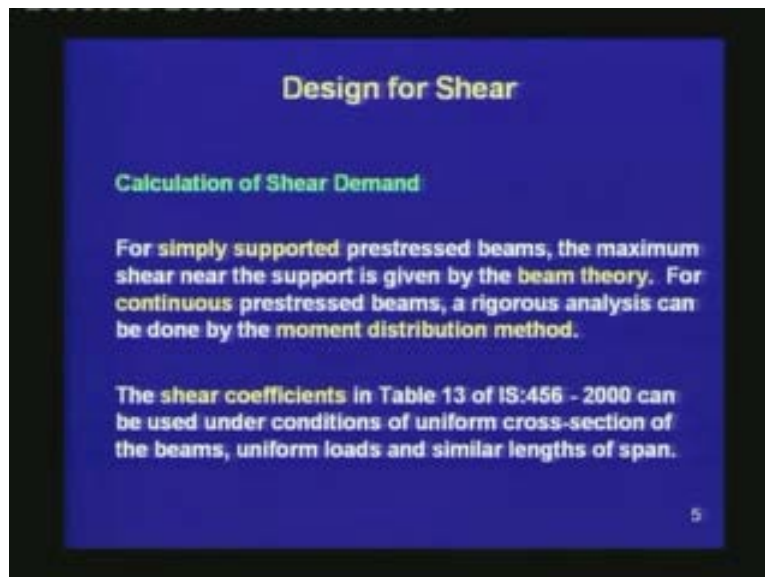
In this lecture, we shall study the design for shear. First, we shall study some general comments. Next, we shall move on to the design steps. Then, we shall study the design of stirrups for flange.

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General comments: The objective of design is to provide ultimate resistance for shear, which we shall denote as V_{uR} , greater than the selected shear demand under the ultimate loads, which we shall denote as V_u .

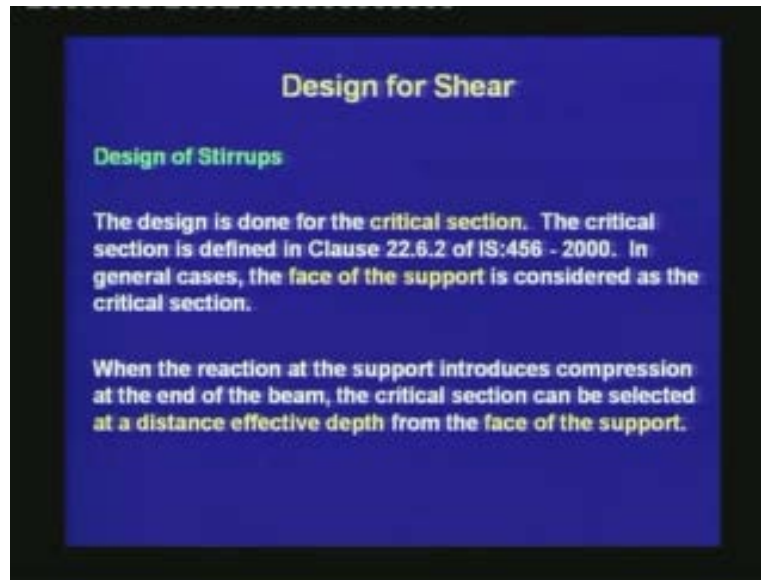
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Thus, first we need to know the shear demand V_u . For simply supported prestressed beams, the maximum shear near the support is given by the beam theory. For continuous

prestressed beams, a rigorous analysis can be done by the moment distribution method. Or else, the shear coefficients of Table 13 of IS: 456-2000 can be used under conditions of uniform cross-section of the beams, uniform loads, and similar lengths of span. Thus, at the beginning, we have to do a structural analysis to get the design ultimate shear.

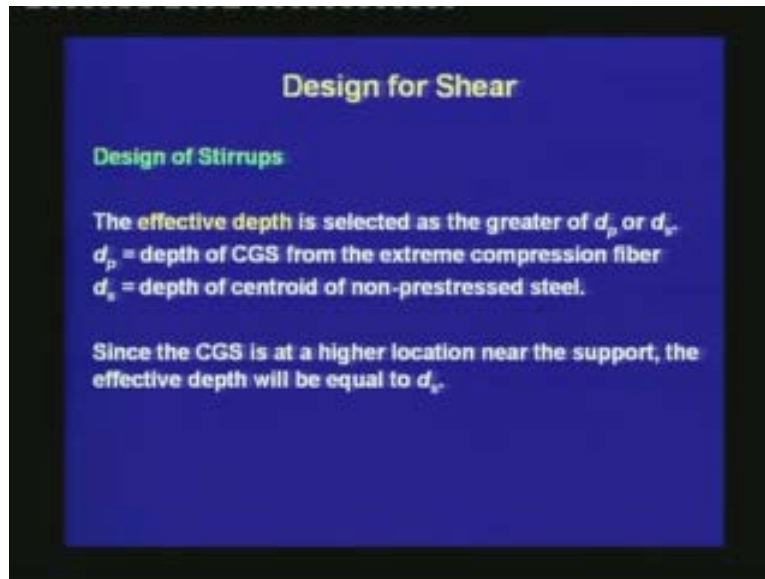
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The design is done for the critical section. The critical section is defined in Clause 22.6.2 of IS: 456-2000. In general cases, the face of the support is considered as the critical section. Once we traverse from the middle of the beam towards the support, for a uniformly distributed load, the shear increases. Now, the shear at the face of the support can be used for the design, because once we enter the support, then the depth of the section is large. Hence, we do not expect a shear failure.

When the reaction at the support introduces compression at the end of the beam, the critical section can be selected at a distance effective depth from the face of the support. The reason is, when the concrete is under compression the strength of concrete to resist shear increases. A simple way to consider the increase in the shear capacity of concrete is to design for a section, which is at a distance effective depth from the face of the support.

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The effective depth is selected as the greater of d_p or d_s , where d_p is the depth of CGS from the extreme compression fiber, and d_s is depth of centroid of non-prestressed steel. Since the CGS is at a higher location near the support, the effective depth will be equal to d_s .

We have two depths for the two types of steel. One is for the prestressing steel and another is for the non-prestressed steel. Usually, at the supports, the depth of the non-prestressed steel is larger than the depth of the prestressing steel. Hence, the effective depth can be considered as the depth of the non-prestressed steel. To vary the spacing of stirrups along the span, other sections may be selected for design.

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Design for Shear

Design of Stirrups

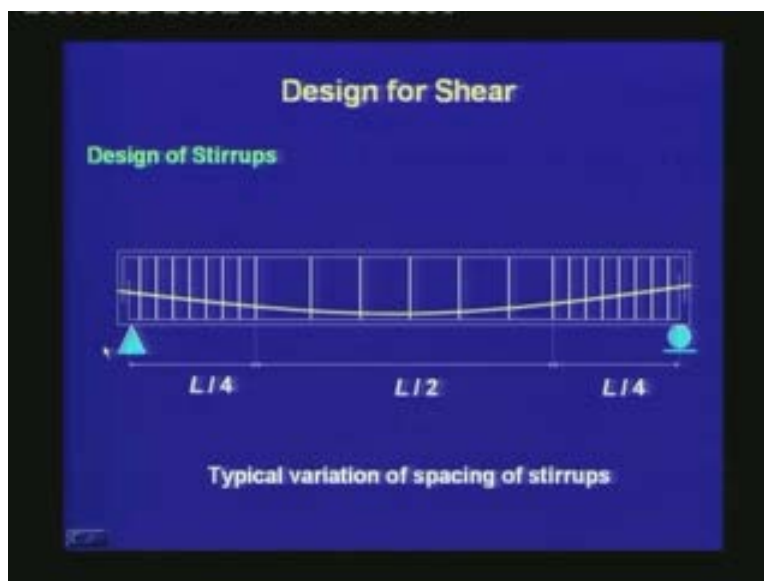
To vary the spacing of stirrups along the span, other sections may be selected for design. Usually the following scheme is selected for beams under uniform load.

- 1) Close spacing for quarter of the span adjacent to the supports.
- 2) Wide spacing for half of the span at the middle.

For large beams, more variation of spacing may be selected.

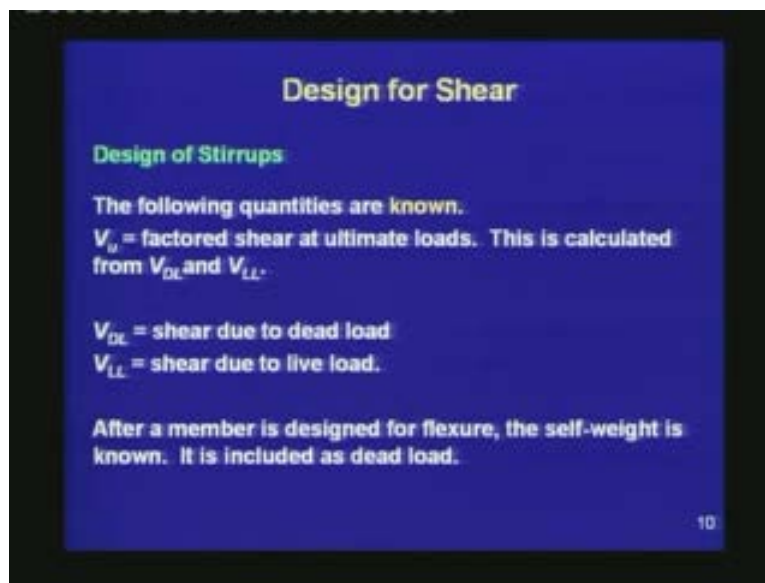
Usually, the following scheme is selected for beams under uniform load: close spacing for quarter of the span adjacent to the supports, wide spacing for half of the span at the middle. For large beams, more variation of spacing may be selected. Let us understand this from the following sketch.

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Under a distributed load, the shear varies along the length of the span. We may vary the spacing of stirrups to economize the amount of steel used for stirrups. For a beam with a uniformly distributed load, the convention is that to provide a close spacing of the stirrups in the two ends of the beam; the lengths of the two ends are about the quarter of the span. For the middle half, the spacing of the stirrups is wider than the spacing at the two ends. In the above sketch, the two ends have closer spacing of stirrups; whereas, the middle half of the beam has wider spacing of stirrups. Of course the spacing should satisfy the maximum spacing requirement. When we design the stirrups in this fashion, we have two design sections; one at a distance d from the face of the support, and another at a distance $L/4$ from the centre line of the support, where L is the span of the beam.

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Design for Shear

Design of Stirrups

The following quantities are known.

V_u = factored shear at ultimate loads. This is calculated from V_{DL} and V_{LL} .

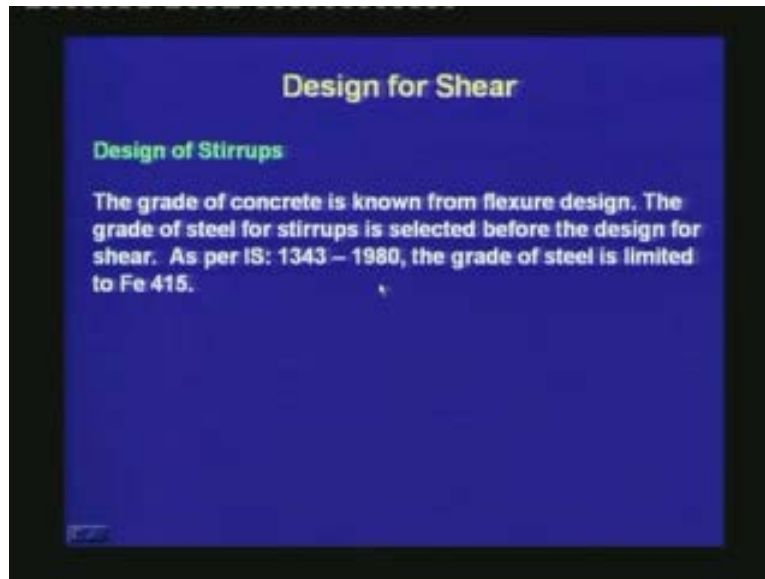
V_{DL} = shear due to dead load
 V_{LL} = shear due to live load.

After a member is designed for flexure, the self-weight is known. It is included as dead load.

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For the design of stirrups, the following quantities are known: first, V_u which is the factored shear at ultimate loads. This is calculated from V_{DL} and V_{LL} . V_{DL} is the shear due to the dead load and V_{LL} is the shear due to live load. After a member is designed for flexure, the self-weight is known and it is included as dead load. Thus, for gravity load design, once we know the shear due to dead load and live load, we can calculate the ultimate shear due to the factored loads by placing the suitable load factors.

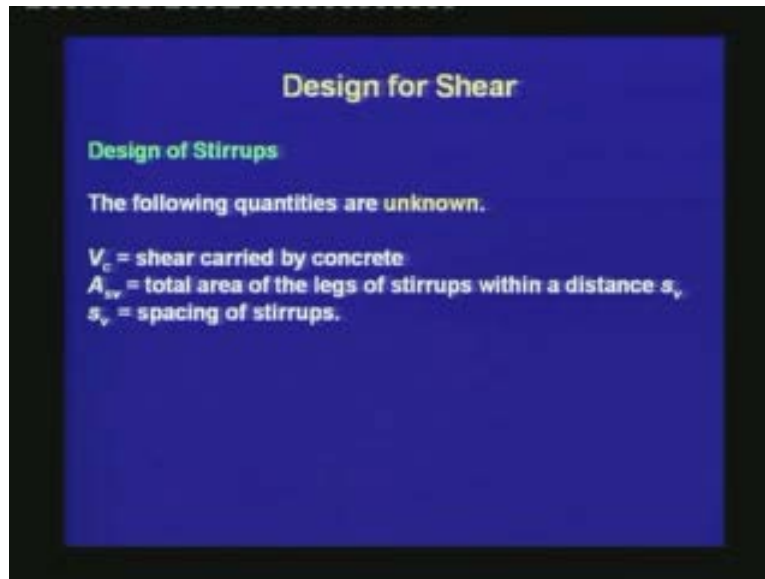
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If we have other types of loads such as lateral loads, then we have to calculate V_u as per the load combinations specified in IS: 456-2000. The grade of concrete is known from flexure design. The grade of steel for stirrups is selected before the design for shear. As per IS: 1343-1980, the grade of steel is limited to Fe 415.

Thus, before we embark into the shear design, we know the material properties. The grade of steel is limited such that there is adequate ductility in the stirrups. With this, we start to calculate the unknown quantities.

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Design for Shear

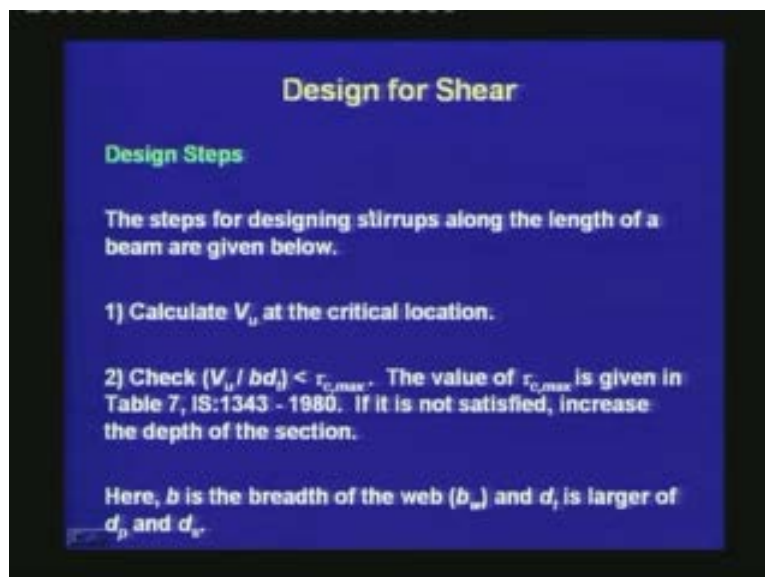
Design of Stirrups

The following quantities are unknown.

V_c = shear carried by concrete
 A_{sv} = total area of the legs of stirrups within a distance s_v
 s_v = spacing of stirrups.

The following quantities are unknown: V_c which is the shear carried by concrete, A_{sv} is the total area of the legs of stirrups within a distance s_v , and s_v is the spacing of the stirrups. Next, we are moving on to the steps for shear design of a member.

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Design for Shear

Design Steps

The steps for designing stirrups along the length of a beam are given below.

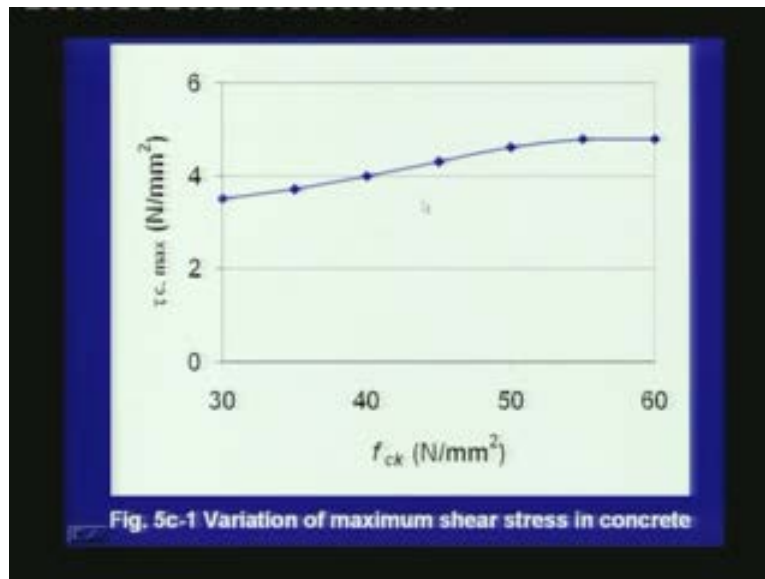
- 1) Calculate V_u at the critical location.
- 2) Check $(V_u / bd) < r_{c,max}$. The value of $r_{c,max}$ is given in Table 7, IS:1343 - 1980. If it is not satisfied, increase the depth of the section.

Here, b is the breadth of the web (b_w) and d is larger of d_p and d_v .

The steps for designing stirrups along the length of a beam are given below: first, calculate V_u at the critical location. This can be calculated from the structural analysis or

by using coefficients given in IS: 456. Second, check V_u/bd_t to be less than $\tau_{c,max}$. The value of $\tau_{c,max}$ is given in Table 7, IS: 1343-1980. If it is not satisfied, increase the depth of the section. Here, b is the breadth of the web which we had earlier denoted as b_w , and d_t is larger of d_p and d_s , which are the depths for the prestressing steel and the non-prestressed steel, respectively.

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As was mentioned earlier, the design for shear involves not only the design of stirrups, but also makes sure that the different modes of shear failure are restricted. The purpose of checking the maximum shear stress to be within a certain value is to ensure that the concrete does not crush under the shear compression failure. Hence, we need to make sure that the average shear stress at the critical location, which is given as V_u/bd_t is less than the maximum permissible shear stress, which is $\tau_{c,max}$ given in the code. The values of $\tau_{c,max}$ are given for different grades of concrete. If we plot the values, then we observe that there is an increase of $\tau_{c,max}$ from Grade 30 to Grade 60 concrete, but roughly, the value of $\tau_{c,max}$ is around 4 N/mm².

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Design Steps

3) Calculate V_c from the lower of V_{c0} and V_{cr} .

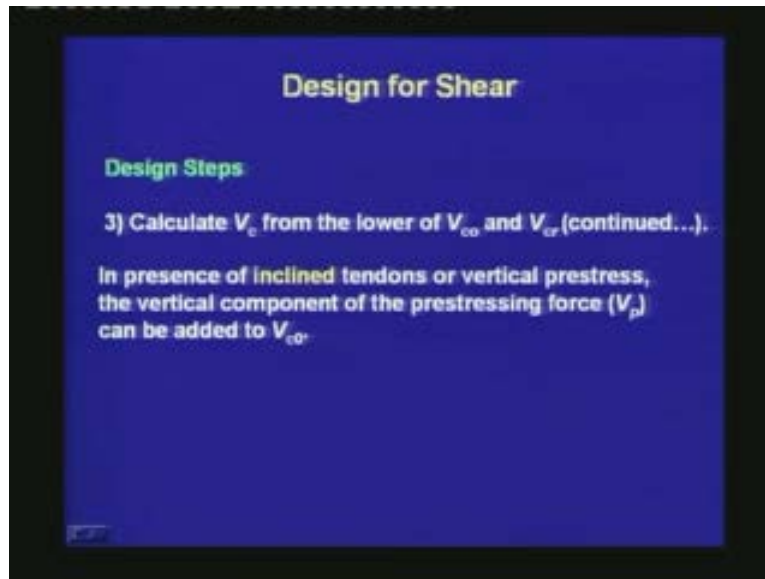
$$V_{c0} = 0.67 bD \sqrt{f_t^2 + 0.8 f_{cp} f_t}$$
$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pk}} \right) \tau_c b d + M_c \frac{V_u}{M_u}$$
$$\geq 0.1 b d \sqrt{f_{ck}}$$

The third step is to calculate V_c from the lower of V_{c0} and V_{cr} . In our previous lecture, we studied that there are two expressions of capacity of concrete to resist shear. One is for uncracked section, which is denoted as V_{c0} and another is for a cracked section, which is denoted as V_{cr} . Usually, near the supports V_{c0} will govern the value of V_c whereas near the middle of the span, V_{cr} will govern the value of V_c . Whichever is lower, out of V_{c0} and V_{cr} that we are selecting as V_c .

V_{c0} is given as $0.67bD \sqrt{(f_t^2 + 0.8 f_{cp}f_t)}$, where D is the total depth, f_t is the direct tensile strength of concrete, and f_{cp} is the prestress at the level of CGC. We have 0.8 times the prestress to be on the conservative side.

V_{cr} has two terms: the first term on the left represents the shear required to change a flexural crack to a flexure shear crack. It is dependent on the value of τ_c and the amount of prestressing force. The second term is the shear that corresponds to the moment causing a flexural crack at that critical location. The value of V_{cr} need not be less than $0.1bd\sqrt{f_{ck}}$.

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In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force, which is denoted as V_p can be added to V_{c0} . Thus, the code allows us to take advantage of the curved profile of a tendon in a post-tensioned beam because the vertical component of the prestressing force adds up to the shear resistance and it can be added to the contribution from concrete. The code says that V_p can be added to V_{c0} , which is expression for uncracked concrete, but V_p cannot be added to V_{cr} , which is an expression for cracked concrete. This is because after cracking, the effect of prestressing is not significant.

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Design Steps

4) Calculate A_{sv} / s_v .

If $V_c / 2 < V_u < V_c$, minimum amount of stirrups is satisfactory. The minimum amount of stirrups is given by the following equation.

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87f_y}$$

The fourth step is to calculate A_{sv}/s_v , where A_{sv} is the area of the stirrups and s_v is the spacing. If V_u is less than V_c but greater than $V_c/2$, that is the shear demand is less than the shear capacity of concrete, but is greater than half the capacity of concrete, then, the minimum amount of stirrups needs to be provided. The minimum amount of stirrups is given by the following equation.

$$A_{sv}/bs_v = 0.4/0.87f_y$$

This equation is based on the consideration that the stirrups must have a minimum stress of 0.4 N/mm^2 .

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Design for Shear

Design Steps

Another provision for minimum amount of stirrups ($A_{sv,min}$) is as follows.

In presence of dynamic load,

$$A_{sv,min} = 0.3\% A_{wh}$$
$$= 0.2\% A_{wh}, \text{ when } h \leq 4b_w$$

With high strength bars,

$$A_{sv,min} = 0.2\% A_{wh}$$
$$= 0.15\% A_{wh}, \text{ when } h \leq 4b_w$$

Another provision for minimum amount of stirrups, which we shall denote as $A_{sv,min}$ is as follows: in presence of dynamic load, $A_{sv,min} = 0.3\% A_{wh}$ or $A_{sv,min} = 0.2\% A_{wh}$ when the total height h is less than or equal to $4b_w$. With high strength bars, $A_{sv,min} = 0.2\% A_{wh}$ or $A_{sv,min} = 0.15\% A_{wh}$, when h is less than or equal to $4b_w$.

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Design for Shear

Design Steps

In absence of dynamic load, when $h > 4b_w$

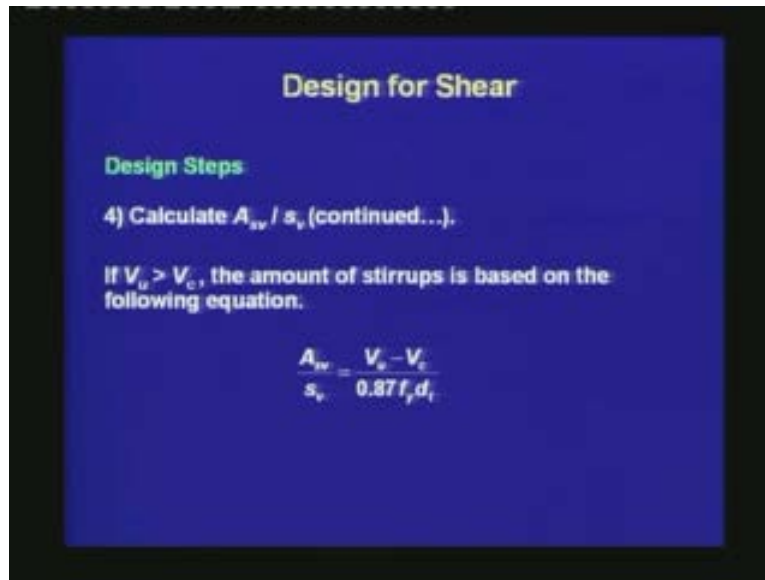
$$A_{sv,min} = 0.1\% A_{wh}$$

There is no specification for $A_{sv,min}$ when $h \leq 4b_w$.

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In absence of dynamic load, when h is greater than $4b_w$, $A_{sv,min} = 0.1\% A_{wh}$. There is no specification for $A_{sv,min}$, when h is less than or equal to $4b_w$. Thus, the second requirement of minimum amount of stirrups is based on the horizontal area of the web. The expression for the minimum amount of stirrups is given as a certain percentage of that area.

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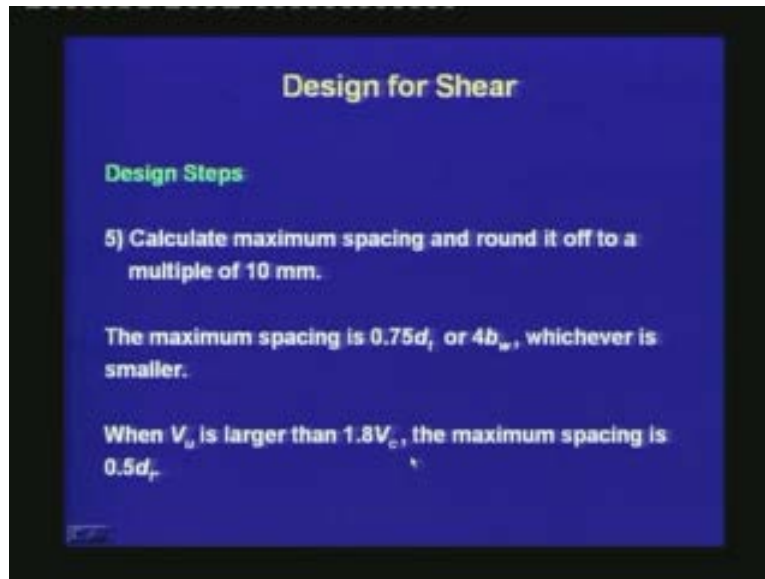


If V_u is greater than V_c , that means the shear demand exceeds the shear capacity of concrete, the amount of stirrups is based on the following equation.

$$A_{sv}/s_v = (V_u - V_c)/0.87f_yd_t$$

Thus, this expression is used to calculate the amount of stirrups when the shear demand exceeds the shear capacity of concrete. This expression is based on the truss analogy for the study of shear.

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The fifth step is to calculate maximum spacing and round it off to the multiple of 10 mm. The maximum spacing is $0.75d_t$ or $4b_w$, whichever is smaller. When V_u is larger than $1.8V_c$, the maximum spacing is $0.5d_t$.

The purpose of having a maximum spacing is that, a diagonal crack should be intercepted by at least one stirrup. If the shear demand is high, then the maximum spacing is further restricted.

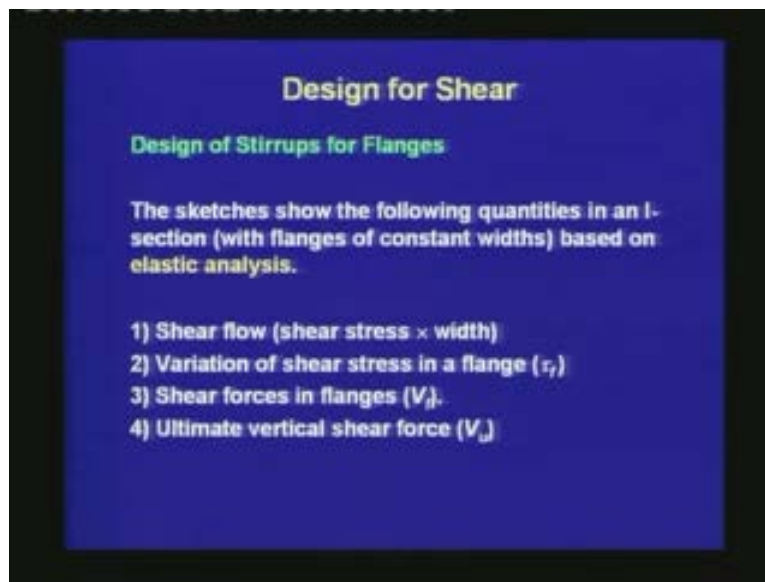
Sixth is to calculate the size and number of legs of the stirrups based on the amount required. We have to do the proper detailing of the stirrups. We have to provide standard hooks at the ends. We have to make sure that there are longitudinal bars at their bends. This will complete the shear design at the critical location.

Repeat the calculations for other locations of the beam, if the spacing of the stirrups needs to be varied. As we had mentioned earlier, to economize the amount of stirrups, we may vary the spacing along the length of the beam. In such a situation, we need to repeat design steps 1 to 6 for the other selected locations. It is left up to the designer to select the number of stirrup spacings along a beam. A simple convention has been suggested here, to have a close spacing at the two ends and a wider spacing at the central half of the beam.

There is another consideration, which is the design of stirrups for flanges. For a flanged section, although the web carries the vertical shear stress, there is shear stress in the flanges due to the effect of shear lag. Horizontal reinforcement in the form of a single leg, or closed stirrups are provided in the flanges. The effect of shear lag can be explained based on theory of elasticity. When a beam bends, the total flange does not bend equally throughout the width of the flange. There is a shear lag effect, that means portions of the flange which are closer to the web have higher flexural stress; whereas, the portions away from the web have lower stress. This variation of the normal stress in the flange generates an in-plane shear stress within the flange, which is called the 'shear-lag effect'.

Stirrups need to be provided in the flanges for the shear lag effect. In conventional reinforced concrete design, the transverse reinforcement in the slabs is adequate enough for the shear-lag effect. But for prestressed concrete beams, we need to provide separate stirrups in the flanges to take care of the shear-lag effect.

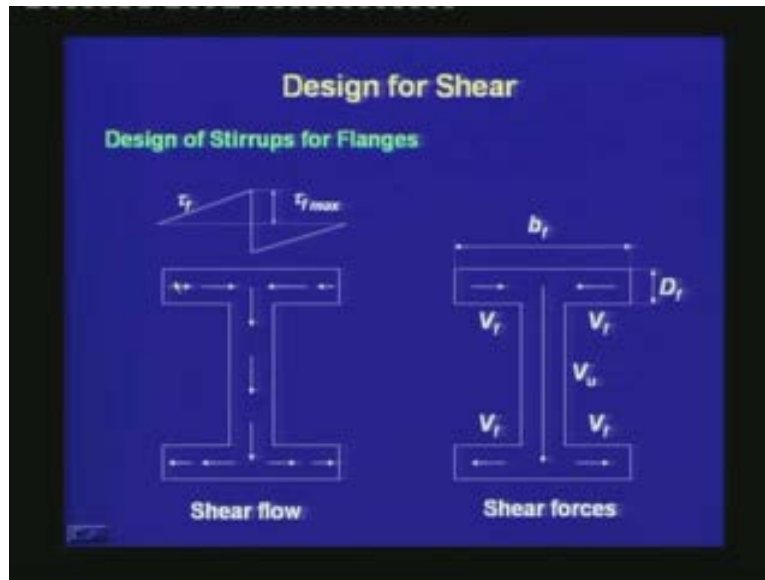
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The sketches below show the following quantities in an I-section (with flanges of constant widths) based on elastic analysis. We shall see the shear flow diagram. Shear flow refers to the product of the shear stress times the width of the flange or the web. We shall see the variation of shear stress in the flange which will be represented as τ_f . We

shall see the shear force in the flanges, which will be represented as V_f . We shall calculate the ultimate vertical shear force, which is represented as V_u . In this sketch, on the left hand side, we see the shear flow diagram in an I-section with constant width of the web and constant depth of the flange.

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The shear flow increases from the end and it becomes maximum at the centre of the flange. Correspondingly, the shear stress which is the shear flow divided by the width of the flange increases from zero to a maximum value at the centre of the flange. On the other side of the flange, the variation is similar but the sign is in opposite direction. Then, the shear flow goes down vertically and it spreads out in the bottom flange in the same way as we have seen in the top flange. V_f is the shear force acting in the flange, V_u is the vertical shear which is acting in the section. We shall calculate V_f from the linear variation of the shear stress in the flange. Here, b_f is the breadth of the flange and D_f is the depth of the flange.

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Design for Shear

Design of Stirrups for Flanges

The design shear force in a flange is given as follows.

$$V_f = \frac{\tau_{f,max}}{2} \frac{b_f}{2} D_f \quad (5c-1)$$

Here,
 b_f = breadth of the flange
 D_f = depth of the flange
 $\tau_{f,max}$ = maximum shear stress in the flange.

The design shear force in a flange is given as follows.

$$V_f = (\tau_{f,max}/2) (b_f D_f/2)$$

This expression is obtained from the average value of τ_f and half the area of the flange.

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Design for Shear

Design of Stirrups for Flanges

The maximum shear stress in the flange is given as follows.

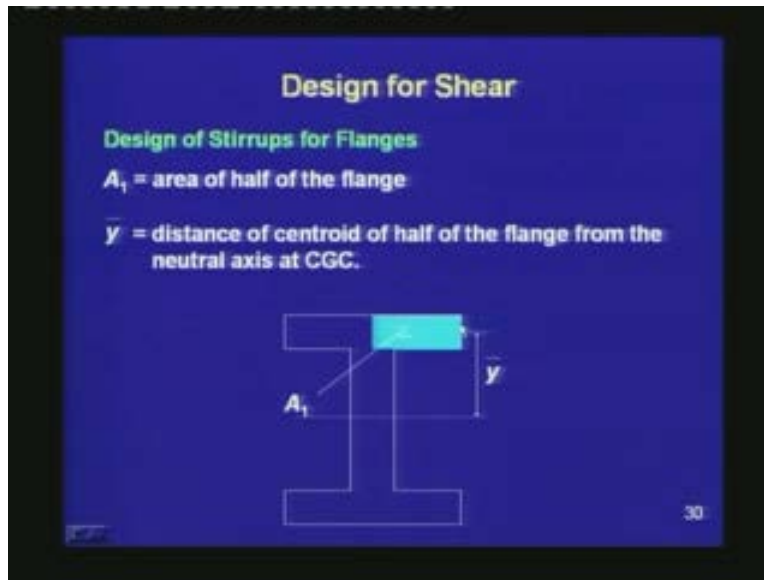
$$\tau_{f,max} = \frac{V_u A_f \bar{y}}{I D_f} \quad (5c-2)$$

Here,
 V_u = ultimate vertical shear force
 I = moment of inertia of the section.

Next, we have the expression of the shear stress. Based on elastic analysis, the maximum shear stress in the flange is given as follows.

$$\tau_{f,\max} = V_u A_1 \bar{Y} / I D_f.$$

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Here, V_u is the ultimate vertical shear force, I is the moment of inertia of the section, A_1 is area of half of the flange, \bar{Y} is the distance of centroid of half of the flange from the neutral axis at CGC. The above expression is analogous to the expression of shear stress at a level of the web. Thus, we calculate $\tau_{f,\max}$ from V_u . From $\tau_{f,\max}$ we can calculate V_f , and we can design the stirrups in the flange for V_f .

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Design for Shear

Design of Stirrups for Flanges

The amount of horizontal reinforcement in the flange (A_{svf}) is calculated from V_f .

$$A_{svf} = \frac{V_f}{0.87 f_y} \quad (5c-3)$$

The yield stress of the reinforcement is denoted as f_y .

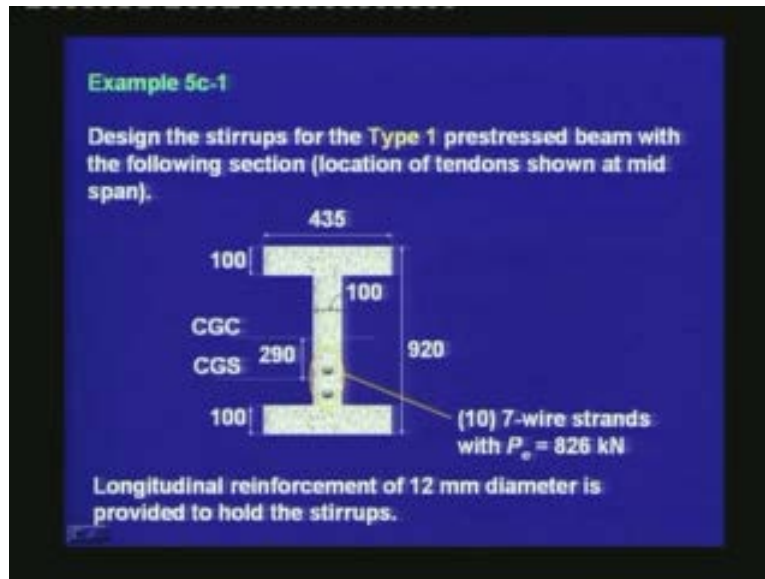
The amount of horizontal reinforcement in the flange, which we shall denote as A_{svf} , is calculated from V_f as follows.

$$A_{svf} = V_f / 0.87 f_y$$

Here, we are not considering any contribution of the concrete. Given the maximum permissible stress in the stirrups, we are able to find out the amount of stirrups that is needed in the flanges. The yield stress of the reinforcement is denoted as f_y .

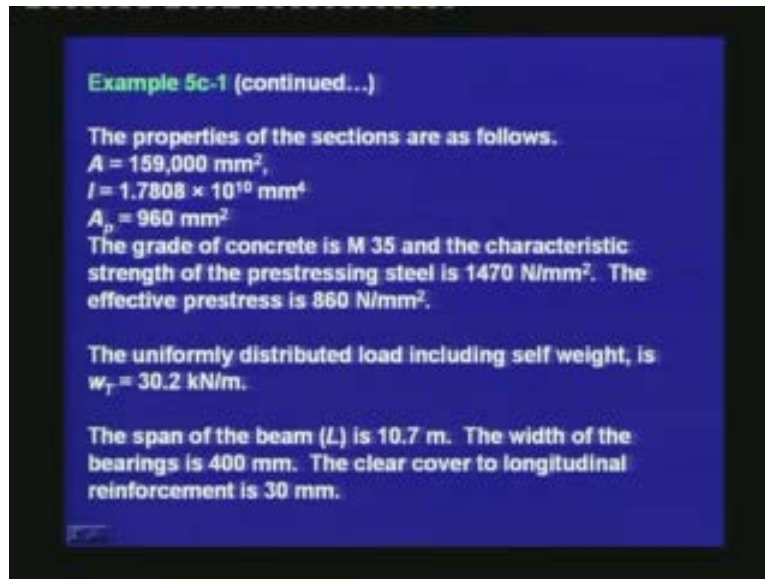
Next, let us understand the design of stirrups with the help of an example.

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Design the stirrups for the Type 1 prestressed beam with the following section. The location of tendons is shown at the mid-span of the beam. This is a symmetric I section; the breadth of the flange is 435 mm, the depth is 100 mm, the width of the web is 100 mm, the prestressing is done by 10 number of 7-wire strands with an effective prestress of $P_e = 826$ kN. They are provided in two ducts with the CGS located at 290 mm below the CGC. This is the section we had designed earlier, under the design of Type 1 section. Longitudinal reinforcement of 12 mm diameter bars is provided to hold the stirrups.

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The properties of the sections are as follows:

Area $A = 159,000 \text{ mm}^2$, moment of inertia $I = 1.7808 \times 10^{10} \text{ mm}^4$, area of the prestressing steel $A_p = 960 \text{ mm}^2$, the grade of concrete is M 35 and the characteristic strength of the prestressing steel $f_{pk} = 1470 \text{ N/mm}^2$. The effective prestress $f_{pe} = 860 \text{ N/mm}^2$. The uniformly distributed load including the self-weight is $w_T = 30.2 \text{ kN/m}$. The span of the beam $L = 10.7 \text{ m}$, the width of the bearings is 400 mm . The clear cover to longitudinal reinforcement is 30 mm .

Thus, the variables after the flexure design of the beam are given, and now we are designing the stirrups for this beam.

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Solution

1) Calculate V_u at the face of the support (neglecting the effect of compression in concrete).

$$V_u = 1.5 \times w_T \left(\frac{L}{2} - x \right)$$
$$= 1.5 \times 30.2 \times \left(\frac{10.7}{2} - 0.2 \right)$$
$$= 233.3 \text{ kN}$$

Here, x denotes half of the width of bearing. $x = 200 \text{ mm}$.

The first step is to calculate the shear demand V_u at the critical section. Here, neglecting the effect of compression in concrete, we are calculating V_u at the face of the support. The expression of V_u is equal to $1.5 \times w_T (L/2 - x)$, where x is the distance of the face of the support from the centre-line of the support. Since the supports are 400 mm wide, the value of x is half of 400 mm, which is 200 mm.

Once we substitute the values of w_T , L , and x in the expression, we get $V_u = 1.5 \times 30.2 \times (10.7/2 - 0.2) = 233.3 \text{ kN}$. Thus, the shear demand for this beam is 233.3 kN at the critical section.

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Solution

2) Check $(V_u / bd) < \tau_{c,max}$.

Effective depth d_t = total depth – cover – diameter of stirrups – $\frac{1}{2}$ diameter of longitudinal bar.

Assume the diameter of stirrups to be 8 mm.

$$\therefore d_t = \left(920 - 30 - 8 - \frac{1}{2} \times 12 \right)$$
$$= 876 \text{ mm}$$

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Next, we are checking the average shear stress to be within the maximum permissible value. To do that, we need to calculate the effective depth d_t which will be governed by the depth of the non-prestressed steel. We are estimating the effective depth equal to the total depth minus cover, minus diameter of the stirrups, minus half the diameter of the longitudinal bars. The cover is 30mm. We are assuming the diameter of the stirrups to be 8 mm. The diameter of the longitudinal bars is given as 12 mm. Thus, $d_t = 920 - 30 - 8 - 12/2 = 876$ mm.

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Solution

2) Check $(V_u / bd) < \tau_{c,max}$ (continued...).

$$\frac{V_u}{b_w d_t} = \frac{233.3 \times 10^3}{100 \times 876}$$
$$= 2.7 \text{ N/mm}^2$$

$\tau_{c,max}$ for M 35 is 3.7 N/mm^2 . Hence, $(V_u / bd) < \tau_{c,max}$.

For calculating the average shear stress at the critical section, we are substituting $b = b_w = 100 \text{ mm}$. Thus, $V_u/b_w d_t = 233.3 \times 10^3 / 100 \times 876 = 2.7 \text{ N/mm}^2$. For M 35, $\tau_{c,max} = 3.7 \text{ N/mm}^2$, which is available from Table 7 in the Code. Thus, the average shear stress is less than $\tau_{c,max}$. Hence, the depth of the section is adequate.

(Refer Slide Time 37:18)

Solution

3) Calculate V_c from the lower of V_{co} and V_{cp} .

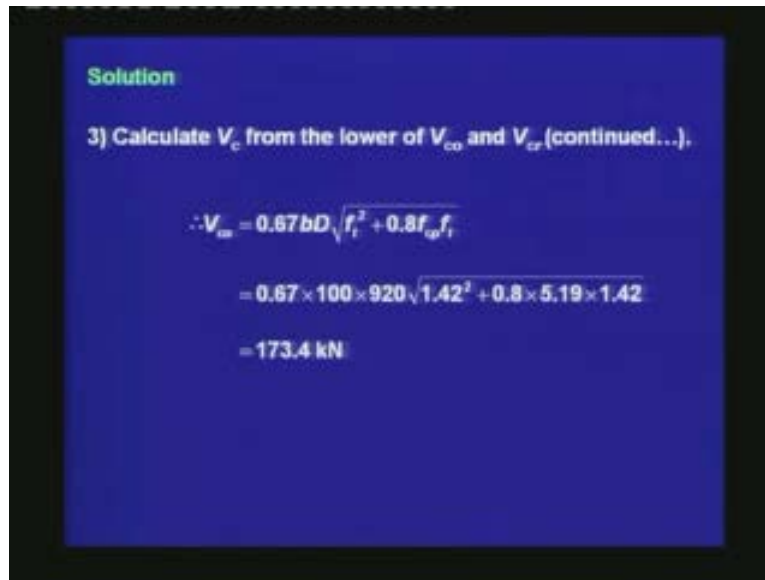
$$V_{co} = 0.67 b D \sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

Here,

$$f_t = 0.24 \sqrt{35}$$
$$= 1.42 \text{ N/mm}^2$$
$$f_{cp} = \frac{P_r}{A}$$
$$= \frac{826 \times 10^3}{159,000}$$
$$= 5.19 \text{ N/mm}^2$$

Third, calculate V_c from the lower of V_{c0} and V_{cr} . First, we estimate the cracking stress $f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{35} = 1.42 \text{ N/mm}^2$. This is the direct tensile strength of the concrete. The other term we are estimating is f_{cp} , which is the compressive stress in concrete at the level of the CGC. We are considering the numeric value, $f_{cp} = P_e/A = 826 \times 10^3 / 159,000 = 5.19 \text{ N/mm}^2$ at the level of CGC.

(Refer Slide Time 38:20)



Solution

3) Calculate V_c from the lower of V_{c0} and V_{cr} (continued...).

$$\therefore V_{c0} = 0.67bD\sqrt{f_t^2 + 0.8f_{cp}f_t}$$
$$= 0.67 \times 100 \times 920 \sqrt{1.42^2 + 0.8 \times 5.19 \times 1.42}$$
$$= 173.4 \text{ kN}$$

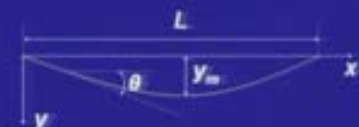
Substituting the values $b = 100 \text{ mm}$, $D = 920 \text{ mm}$, we get $V_{c0} = 173.4 \text{ kN}$. We can add the vertical component of the prestressing force to V_{c0} .

(Refer Slide Time 38:55)

Solution

3) Calculate V_c from the lower of V_{co} and V_{cr} (continued...).

The vertical component of the prestressing force can be found out from the equation of the parabolic tendon.

$$y = \frac{4y_m}{L^2} x (L - x)$$


Since this is a post-tensioned beam with a parabolic tendon, we are taking advantage of the vertical component of the prestressing force. The vertical component of the prestressing force can be found out from the equation of the parabolic tendon.

$$y = (4y_m / L^2) x (L - x).$$

Here, y_m is the vertical displacement of the CGS at mid-span, from the ends. To know the vertical component of the prestressing force, we have to find out the slope of the parabolic profile. The following is the expression of the slope at a point.

$$\tan \theta = dy/dx = (4y_m / L^2) (L - 2x)$$

(Refer Slide Time 40:02)

Solution

3) Calculate V_c from the lower of V_{c0} and V_{cr} (continued...).

At $x = 0.2$ m, $y = 20$ mm, $dy/dx = 0.105$ and $\theta = 6.0^\circ$.

$$\therefore V_p = P_e \sin \theta$$
$$= 826 \times 0.104$$
$$= 86.0 \text{ kN}$$
$$V_{c0} + V_p = 173.4 + 86.0$$
$$= \boxed{259.4 \text{ kN}}$$

At $x = 0.2$ m, $y = 20$ mm, $dy/dx = 0.105$ and $\theta = 6^\circ$. The vertical component of the prestressing force $V_p = P_e \sin \theta = 826 \times 0.104 = 86.0$ kN. Next, $V_{c0} + V_p = 173.4 + 86.0 = 259.4$ kN. Thus, we have found that the shear capacity of concrete is equal to 259.4 kN.

(Refer Slide Time 41:21)

Solution

3) Calculate V_c from the lower of V_{c0} and V_{cr} (continued...).

$$V_{cr} = \left(1 - 0.55 \frac{f_{pe}}{f_{pk}} \right) \tau_c b d_f + M_g \frac{V_u}{M_u}$$

Here,

$$\frac{f_{pe}}{f_{pk}} = \frac{860}{1470} = 0.58$$

Second, we are calculating V_{cr} , which is the shear to change a flexural crack to a flexure shear crack. In the expression of V_{cr} , we are first calculating f_{pe}/f_{pk} , the ratio of the

effective prestress divided by the characteristic strength of the steel, which is equal to $860/1470 = 0.58$.

(Refer Slide Time 42:57)

Solution

3) Calculate V_c from the lower of V_{co} and V_{cr} (continued...).

Here, $\frac{100A_p}{bd} = \frac{100 \times 960}{100 \times 480} = 2.0$ $d = 460 + y$
 $= 460 + 20$
 $= 480 \text{ mm}$

From Table 6, for M 35 concrete, $\tau_c = 0.86 \text{ N/mm}^2$.

To calculate τ_c , we need to know the percentage of prestressing steel provided, which is given as $100A_p/bd$. The depth of the CGS at the critical location (d) is calculated as the depth of the CGC, which is 460, plus the displacement of CGS from the CGC at the critical location, which is given as y . Thus, $d = 460 + 20 = 480 \text{ mm}$. The percentage of the prestressing steel is $100 \times 960 / (100 \times 480) = 2.0$. From Table 6 for M 35 concrete, we can find out $\tau_c = 0.86 \text{ N/mm}^2$.

(Refer Slide Time 44:23)

Solution

3) Calculate V_c from the lower of V_{co} and V_{cr} (continued...).

$$M_s = 0.8f_{pt} \frac{I}{y}$$

Here,

$$f_{pt} = -\frac{P_c}{A} - \frac{P_c y}{I}$$
$$= -\frac{826 \times 10^3}{159,000} - \frac{826 \times 10^3 \times 20}{1.7808 \times 10^{10}}$$
$$= -5.19 - 0.02$$
$$= -5.21 \text{ N/mm}^2$$

Next, we are calculating the term M_0 , which is the moment that generates a flexural crack at the location of the design for shear. $M_0 = 0.8f_{pt}I/y$; this is equal to 80% of the moment that decompresses the CGS at the location of our study. The value of f_{pt} is the compressive stress in the concrete at the level of the CGS, which is given as $-P_c/A - (P_c y/I)y$. Here, y is the eccentricity of the CGS with respect to CGC. $P_c = 826 \text{ kN}$, $A = 159,000 \text{ mm}^2$, $y = 20 \text{ mm}$ and $I = 1.7808 \times 10^{10} \text{ mm}^4$. Once we substitute the values, we find $f_{pt} = -5.21 \text{ N/mm}^2$.

(Refer Slide Time 46:00)

Solution

3) Calculate V_c from the lower of V_{co} and V_{cr} (continued...).

$$M_0 = 0.8 \times 5.21 \times \frac{1.7808 \times 10^{10}}{20}$$
$$= 3711.2 \times 10^3 \text{ Nmm}$$
$$= 3711.2 \text{ kNm}$$

45

From this, we calculate $M_0 = 0.8 \times 5.21 \times 1.7808 \times 10^{10} / 20 = 3711.2 \text{ kNm}$.

(Refer Slide Time 46:45)

Solution

3) Calculate V_c from the lower of V_{co} and V_{cr} (continued...).

At the critical section,

$$M_u = 1.5w_t \frac{x}{2} (L - x)$$
$$= 1.5 \times 30.2 \times \frac{0.2}{2} (10.7 - 0.2)$$
$$= 47.6 \text{ kNm}$$

We have to also calculate M_u at the critical section. M_u at the critical section is given as $1.5w_t (x/2)(L - x)$. Once we substitute the values of w_t , x , and L , we get $M_u = 47.6 \text{ kNm}$.

(Refer Slide Time 47:40)

Solution

3) Calculate V_c from the lower of V_{c0} and V_{cr} (continued...).

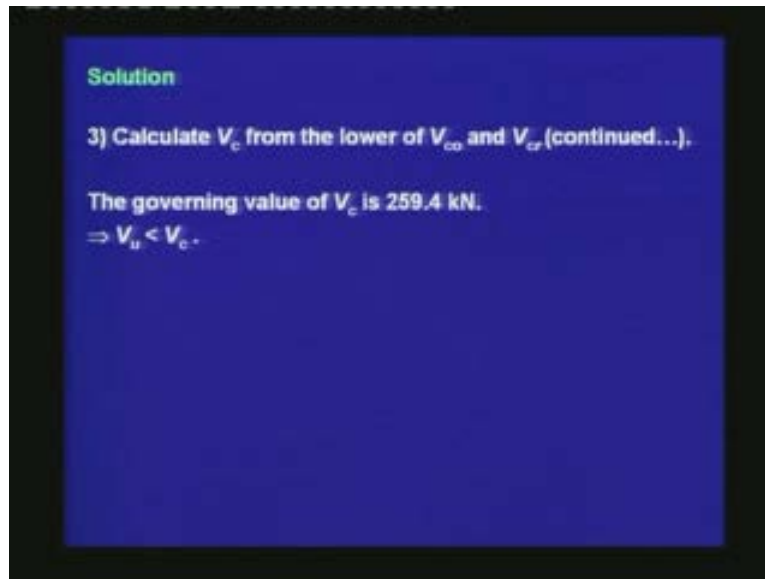
$$V_{cr} = \left(1 - 0.55 \frac{f_{pc}}{f_{pb}}\right) f_c b d + M_u \frac{V_u}{M_u}$$
$$V_{cr} = (1 - 0.55 \times 0.58) \times \frac{0.86}{10^3} \times 100 \times 876 + 3711.2 \times \frac{233.3}{47.6}$$
$$= 51.3 + 18204.8$$
$$= 18256.0 \text{ kN}$$

We are substituting the values of the calculated variables.

$$V_{cr} = (1 - 0.55 \times 0.58) \times (0.86/10^3) \times 100 \times 876 + 3711.2 \times 233.3/47.6 = 18256 \text{ kN.}$$

Here, $V_u = 233.3 \text{ kN}$ and $M_u = 47.6 \text{ kNm}$ at the critical location. V_{cr} is substantially high because of the second term. This means, the shear corresponding to a moment that will generate a flexural crack at the critical section, which is at the face of the support, is extremely high. We do not expect to have a flexural crack near the face of the support. Since V_c is the lower of V_{c0} and V_{cr} , we select $V_c = V_{c0} = 259.4 \text{ kN}$.

(Refer Slide Time 48:24)



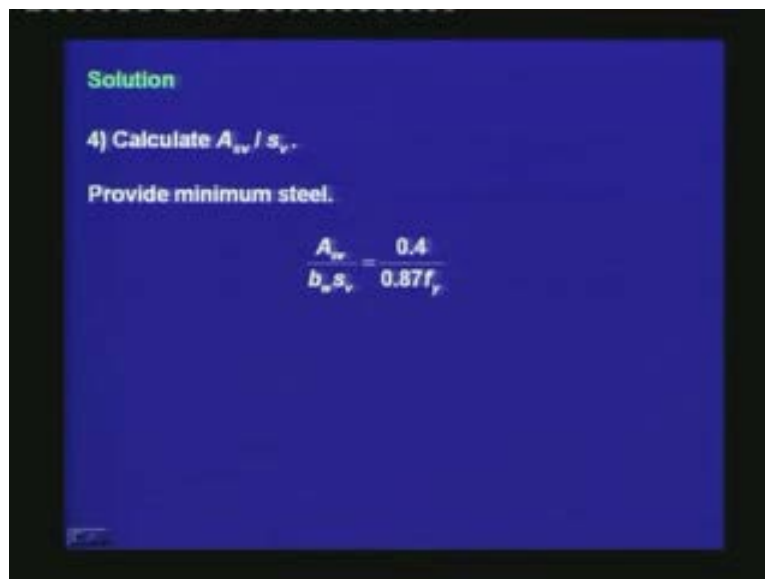
Solution

3) Calculate V_c from the lower of V_{c0} and V_{cr} (continued...).

The governing value of V_c is 259.4 kN.
 $\Rightarrow V_u < V_c$.

Thus we observe that at the face of the support, V_{c0} which is the shear required to generate a web shear crack is much lower than the shear required to generate a flexural crack and then to change that to a flexure shear crack. Thus near the support, the capacity of concrete is governed by the web shear cracking and is given as 259.4 kN. We observe that for this particular section, the shear demand V_u , given as 232 kN is in fact less than V_c .

(Refer Slide Time 49:22)



Solution

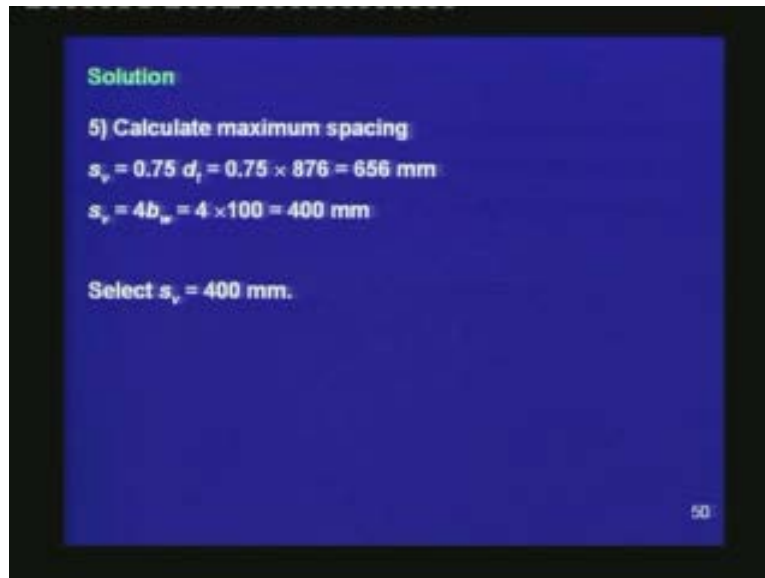
4) Calculate A_{st} / s_v .

Provide minimum steel.

$$\frac{A_{st}}{b_v s_v} = \frac{0.4}{0.87 f_y}$$

The fourth step is to calculate A_{sv}/s_v . Since $V_u < V_c$, we are providing minimum amount of stirrups, which is given by the expression $A_{sv}/b_w s_v = 0.4/0.87f_y$.

(Refer Slide Time 49:50)



Next, we are calculating the maximum spacing. There are two expressions of maximum spacing. When V_u is not substantially high, $s_v = 75\%$ of $d_t = 0.75 \times 876 = 656 \text{ mm}$, or $s_v = 4b_w = 4 \times 100 = 400 \text{ mm}$. Out of this, we are selecting the lower one. Hence, $s_v = 400 \text{ mm}$.

(Refer Slide Time 50:30)

Solution

6) Calculate the size and number of legs of the stirrups

Select $f_y = 250 \text{ N/mm}^2$.

$$A_{sv} = 100 \times 400 \times \frac{0.4}{0.87 \times 250}$$
$$= 73.6 \text{ mm}^2$$

Provide 2 legged stirrups of diameter 8 mm.

$$A_{sv, \text{ provided}} = 2 \times 50.3$$
$$= 100.6 \text{ mm}^2$$

The sixth step is to calculate the size and number of legs of the stirrups. Select $f_y = 250 \text{ N/mm}^2$. $A_{sv} = b_w s_v \times 0.4 / 0.87 f_y = 100 \times 400 \times 0.4 / (0.87 \times 250) = 73.6 \text{ mm}^2$. Provide two legged stirrups of diameter 8 mm. Thus, we are providing $A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$.

(Refer Slide Time 51:32)

Solution

Check minimum amount of stirrups.

$$A_{sv, \text{ min}} = 0.1\% A_{wh}$$
$$= \frac{0.1}{100} \times 100 \times 400$$
$$= 40 \text{ mm}^2$$

Provided amount of stirrups is larger. OK.

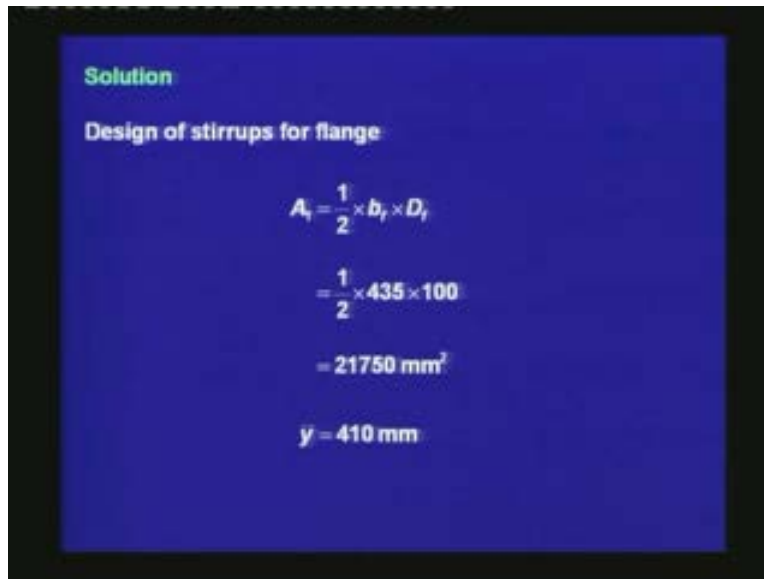
Provide same spacing of stirrups throughout the span.

There is another provision of minimum amount of stirrups and we are checking that. In absence of dynamic loads, $A_{sv\text{min}} = 0.1\%$ of A_{wh} , where A_{wh} is the area of the horizontal

section of the web in a distance of s_v . The area is given as $100 \times 400 \text{ mm}^2$. From this we get $A_{svmin} = 40 \text{ mm}^2$. The provided amount of stirrups, which is 100.6 mm^2 , is larger than A_{svmin} . Hence, it is okay.

Now, in this case, the amount of stirrups that is provided near the support is itself minimum. Hence, the spacing of the stirrups along the length of the beam is not changed.

(Refer Slide Time 52:54)



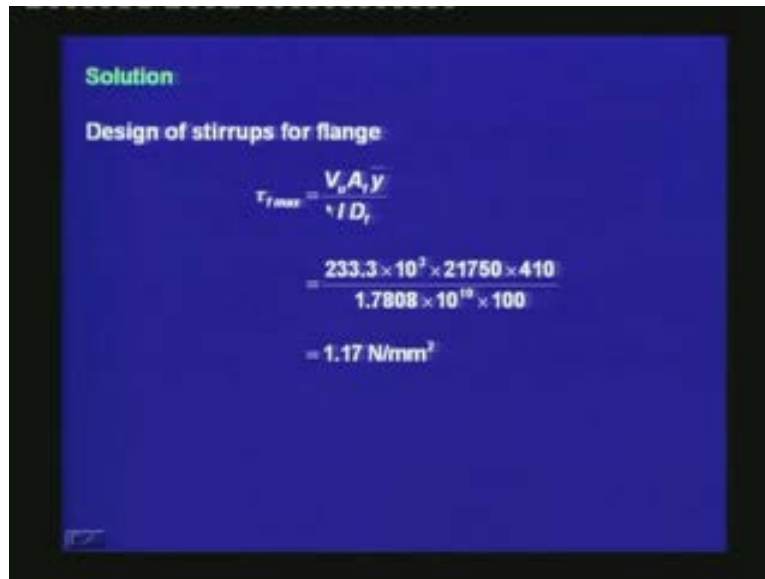
Solution

Design of stirrups for flange

$$A_1 = \frac{1}{2} \times b_f \times D_f$$
$$= \frac{1}{2} \times 435 \times 100$$
$$= 21750 \text{ mm}^2$$
$$\bar{y} = 410 \text{ mm}$$

Next, we are designing the stirrups for each flange. First, we are calculating the area of half of the flange. $A_1 = \frac{1}{2} b_f D_f = \frac{1}{2} \times 435 \times 100 = 21750 \text{ mm}^2$. The distance of the centroid of A_1 from CGC is equal to $\bar{Y} = 410 \text{ mm}$.

(Refer Slide Time 53:16)



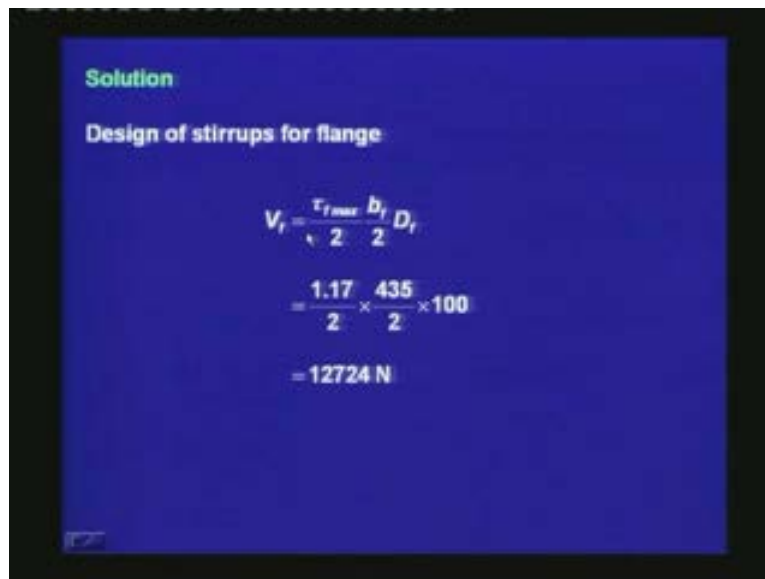
Solution

Design of stirrups for flange

$$\tau_{fmax} = \frac{V_u A_1 \bar{Y}}{I D_f}$$
$$= \frac{233.3 \times 10^3 \times 21750 \times 410}{1.7808 \times 10^{10} \times 100}$$
$$= 1.17 \text{ N/mm}^2$$

The expression for the maximum stress in the flange is $\tau_{fmax} = V_u A_1 \bar{Y} / I D_f = 233.3 \times 10^3 \times 21750 \times 410 / 1.7808 \times 10^{10} \times 100 = 1.17 \text{ N/mm}^2$.

(Refer Slide Time 54:08)



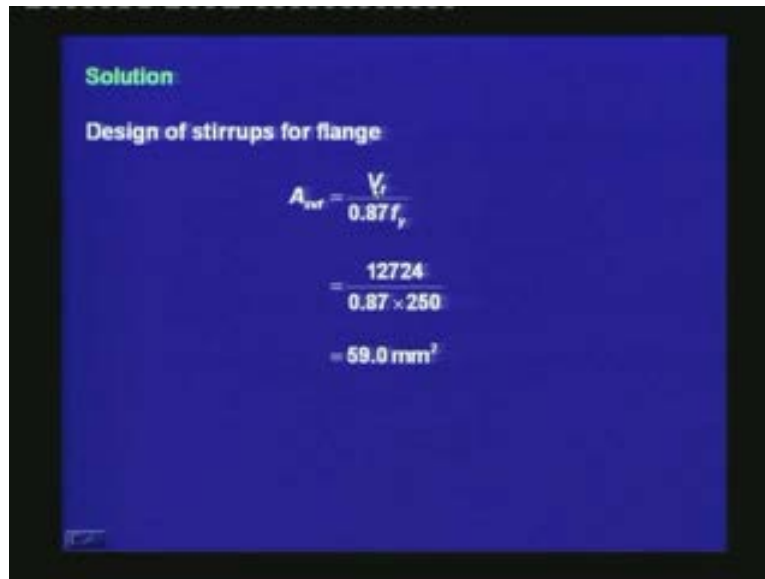
Solution

Design of stirrups for flange

$$V_f = \frac{\tau_{fmax} b_f D_f}{2}$$
$$= \frac{1.17 \times 435}{2} \times 100$$
$$= 12724 \text{ N}$$

The shear in the flange, $V_f = (\tau_{fmax}/2) \times (b_f/2) \times D_f = (1.17/2) \times (435/2) \times 100 = 12724 \text{ N}$.

(Refer Slide Time 54:35)



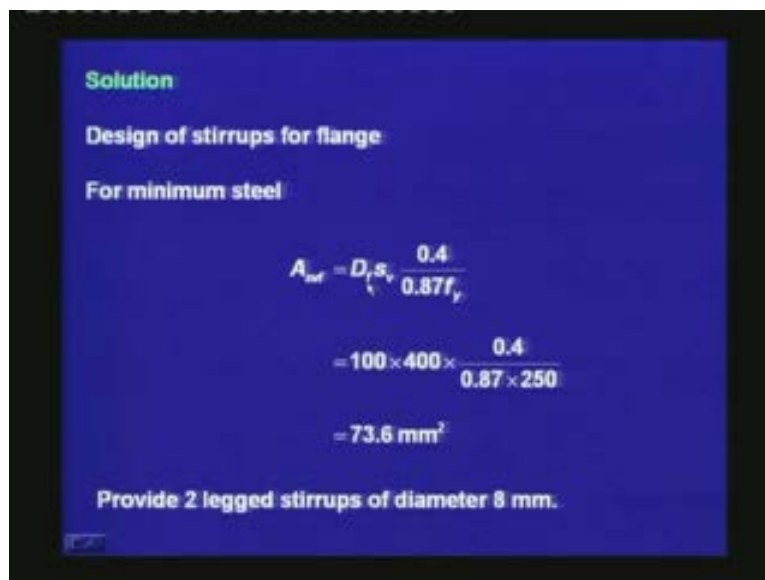
Solution

Design of stirrups for flange

$$A_{svf} = \frac{V_f}{0.87 f_y}$$
$$= \frac{12724}{0.87 \times 250}$$
$$= 59.0 \text{ mm}^2$$

The area of the stirrups in the flanges is calculated as $A_{svf} = V_f/0.87f_y = 12724/(0.87 \times 250) = 59.0 \text{ mm}^2$.

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Solution

Design of stirrups for flange

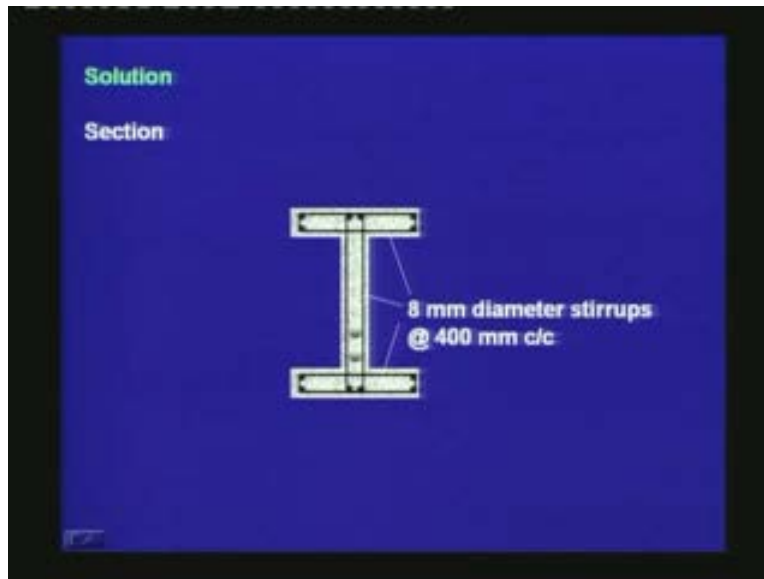
For minimum steel

$$A_{svf} = D_f s_v \frac{0.4}{0.87 f_y}$$
$$= 100 \times 400 \times \frac{0.4}{0.87 \times 250}$$
$$= 73.6 \text{ mm}^2$$

Provide 2 legged stirrups of diameter 8 mm.

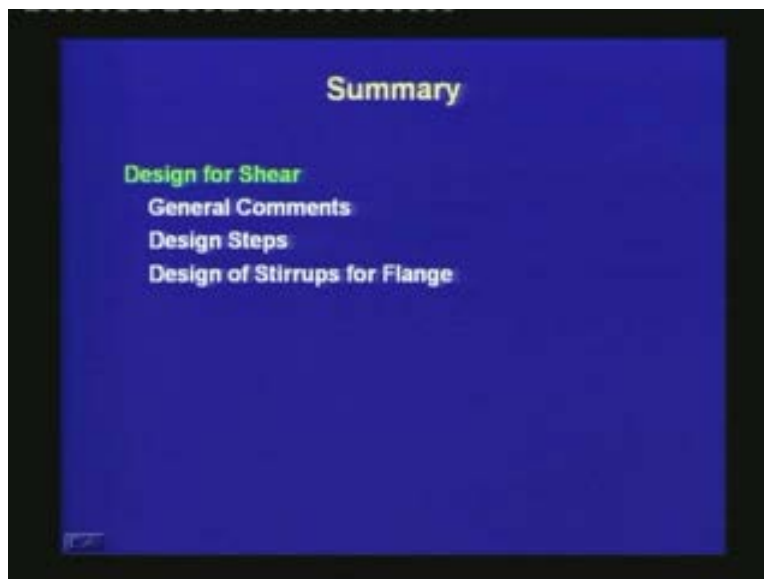
For minimum steel, $A_{svf} = D_f s_v \times 0.4/0.87 f_y$. When we substitute the values of D_f , s_v and f_y , we get $A_{svf} = 73.6 \text{ mm}^2$. Thus, the minimum value is governing. Hence, we can provide 2-legged stirrups of 8 mm diameter in each flange.

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The designed stirrups are shown in the sketch. We are providing the same amount of stirrup in the web and in the flanges. It is 8 millimeter diameter stirrups at the maximum spacing, which is 400 mm center-to-center. Note, that for each of the stirrups, we have provided longitudinal bars at the bends. The selected size of the longitudinal bars is 12 mm, which is greater than 8 mm.

(Refer Slide Time 56:14)



Thus, in today's lecture, we studied the design for shear. We first studied some general comments. We also studied how to calculate the shear demand. For simply supported beams, we can use the beam theory. For continuous beams, we can use the shear coefficients. Then, we moved on to the design steps.

First, we are selecting the critical location and calculating V_u , the shear demand at the critical location. Next, we are making sure that the average shear stress in the critical location is less than the maximum permissible value, this is to check shear compression failure. Once we have done that, we are calculating the shear capacity of the concrete V_c . The shear capacity of concrete is given by two expressions: one for the uncracked section and another for the cracked section. The code recommends evaluating both the expressions at every location. Once we have evaluated the two expressions, we are picking up the smaller value of the two as the value for V_c . If V_u is less than V_c , then we should provide minimum amount of stirrups. We have seen the expression for the minimum amount of stirrups. There is also an additional requirement of minimum amount of stirrups, which is in terms of the horizontal area of the web.

If V_u is greater than V_c , then we have to design the stirrups. The design is based on the amount of stress carried by the stirrups. We also learnt about the design of stirrups for flanges. The flanges are subjected to shear due to the shear lag effect. We saw the expressions for designing the stirrups in the flanges. We saw the design procedure with the help of an example. With this, we are ending the analysis and design for shear.

In our next lecture, we shall move on to the analysis and design for torsion.

Thank you.