

# PRESTRESSED CONCRETE STRUCTURES

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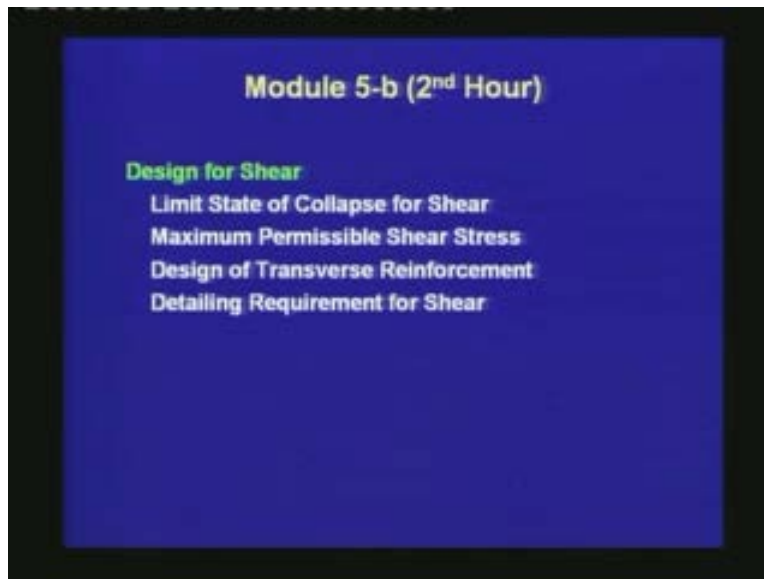
**Indian Institute of Technology Madras**

## **Module – 5: Analysis and Design for Shear and Torsion**

### **Lecture-24: Design for Shear (Part 1)**

Welcome back to prestressed concrete structures. This is the second lecture of Module 5 on analysis and design for shear and torsion. In this lecture, we shall study about the design for shear. First, we shall study about the limit state of collapse for shear; next, we shall move on to maximum permissible shear stress; then we shall move on to design of transverse reinforcement; finally, we shall look into detailing requirements for shear.

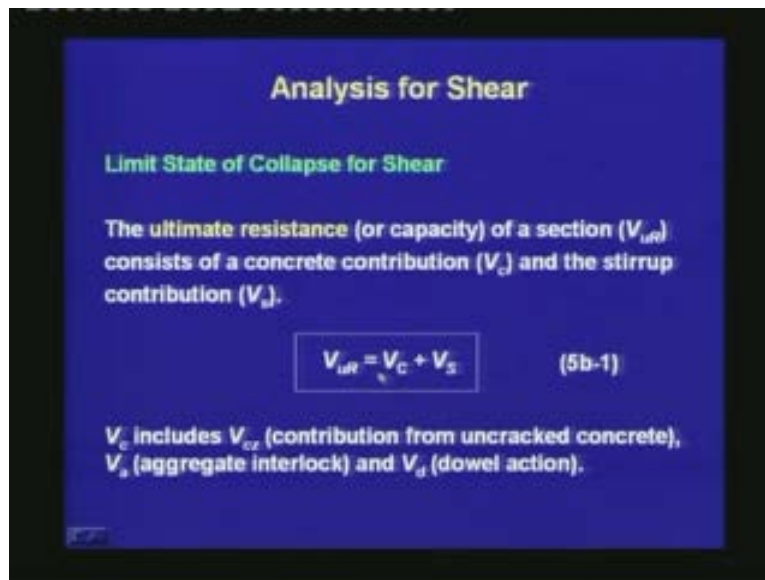
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First, limit state of collapse for shear. In the last lecture, I mentioned that when we study shear, we study the strength at ultimate. We do not use the principles of compatibility of strains or the constitutive relationships for the stress-strain behaviour of concrete or steel. We use an equilibrium equation that is based on the ultimate strength of the concrete

under flexural shear. The total shear capacity of the member is the summation of the capacity of concrete and that of steel.

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**Analysis for Shear**

**Limit State of Collapse for Shear**

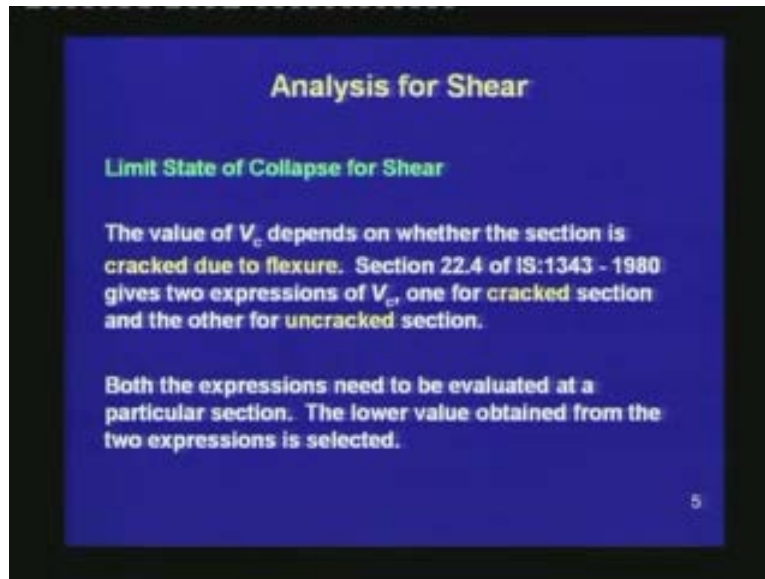
The ultimate resistance (or capacity) of a section ( $V_{uR}$ ) consists of a concrete contribution ( $V_c$ ) and the stirrup contribution ( $V_s$ ).

$$V_{uR} = V_c + V_s \quad (5b-1)$$

$V_c$  includes  $V_{cz}$  (contribution from uncracked concrete),  $V_a$  (aggregate interlock) and  $V_d$  (dowel action).

The ultimate resistance of a section, which is represented as  $V_{uR}$  consists of a concrete contribution  $V_c$  and the stirrup contribution  $V_s$ . Thus,  $V_{uR} = V_c + V_s$ . Here,  $V_c$  includes  $V_{cz}$  which is the contribution from uncracked concrete,  $V_a$  which is the aggregate interlock, and  $V_d$  which is the dowel action. Last time, when we studied the components of shear resistance, we found that for a prestressed concrete section, the components of shear resistance are  $V_{cz}$ ,  $V_a$ ,  $V_d$ , and  $V_s$ . If the tendon is inclined, then we get a vertical component of the prestressing force, which is represented as  $V_p$ . Three of these components are grouped together under the contribution of concrete, which is represented as  $V_c$ . Thus, now we are having three components: one is  $V_c$ , another is  $V_s$ , and when the tendon is inclined we include  $V_p$ .

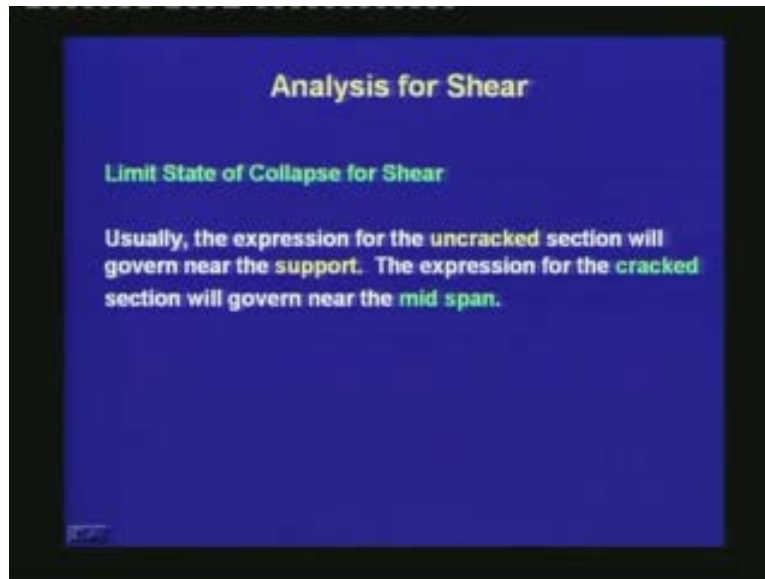
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The value of  $V_c$  depends on whether the section is cracked due to flexure or not. Section 22.4 of IS: 1343 - 1980, gives two expressions of  $V_c$ : one for cracked section, and the other for uncracked section. Last time, we had studied that when the concrete member is loaded, first the flexural cracks are initiated. Next, the flexural cracks tend to become flexure–shear cracks. As we move away from the mid-span, we may find some flexure–shear cracks developing.

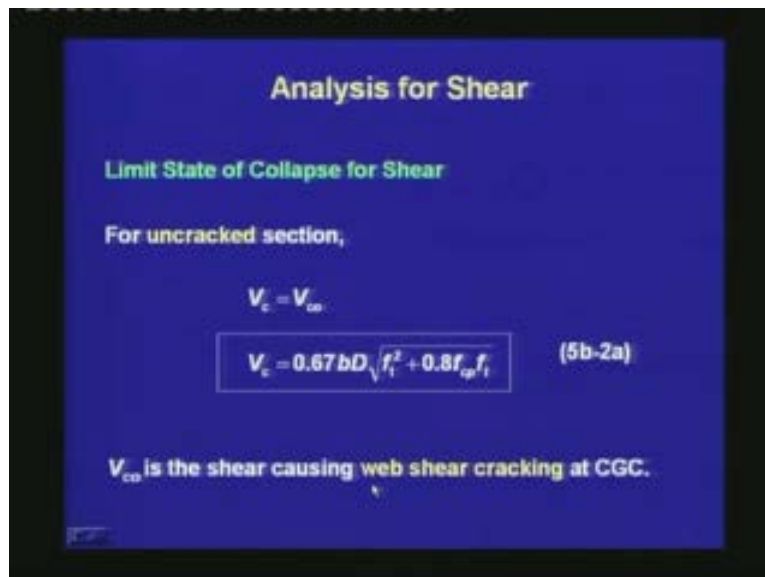
The shear strength of concrete depends on whether the section has cracked or not. The code says that we need to calculate the shear capacity of concrete from two expressions. One expression is given for the cracked section, and the other expression is given for an uncracked section. Both the expressions need to be evaluated at a particular section, and the lower value obtained from the two expressions is selected. Usually, the expression for the uncracked section will govern near the support. The expression for the cracked section will govern near the mid-span.

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Usually, for a beam under a uniformly distributed load, cracks generate near mid-span and in that region the expression for the uncracked section will govern. For the portions near the support, cracks may not develop. Usually, the expression for uncracked section will govern in that region.

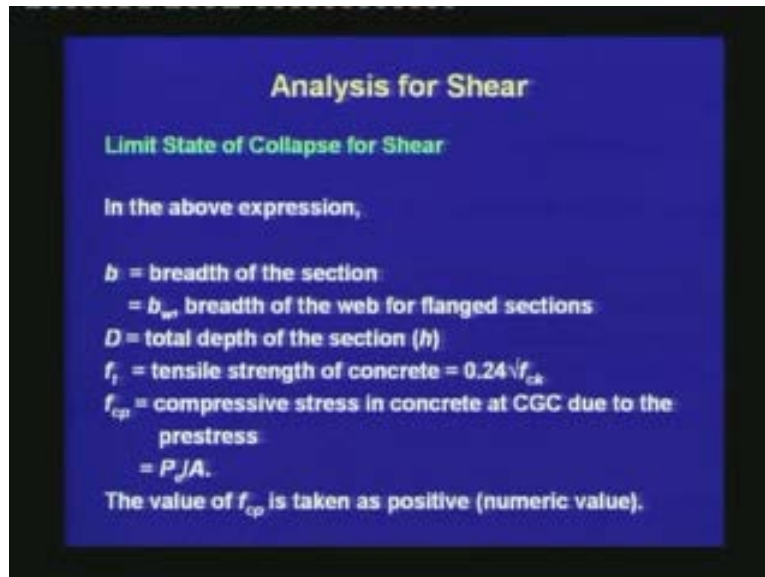
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Next, we are moving on to the two expressions for  $V_c$ . For uncracked section,  $V_c = V_{c0}$ , where  $V_{c0}$  is expressed as  $0.67bD \sqrt{f_t^2 + 0.8 f_{cp} f_t}$ .  $V_{c0}$  is the shear causing web shear

crack at CGC. Thus, the capacity of concrete for an uncracked section is the value of shear which causes a web shear crack near the support. This expression can be derived from the concept of Mohr's circle.

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**Analysis for Shear**

**Limit State of Collapse for Shear**

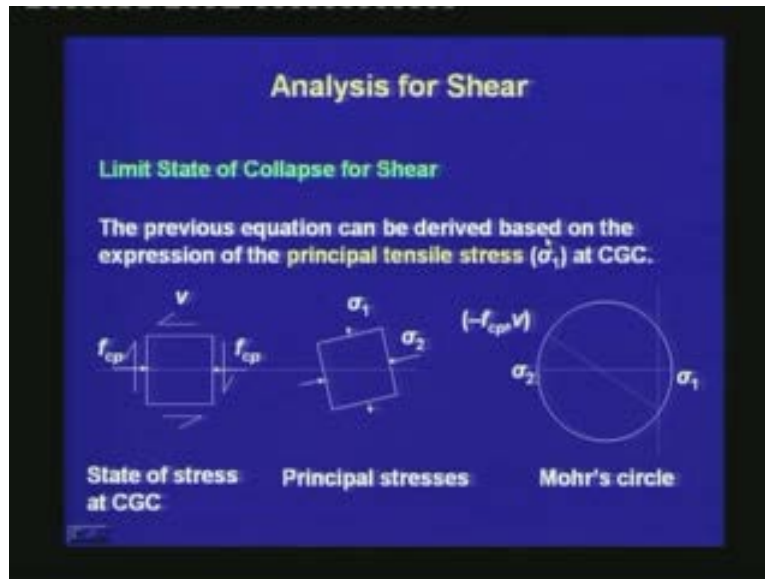
In the above expression,

- $b$  = breadth of the section  
=  $b_w$ , breadth of the web for flanged sections
- $D$  = total depth of the section ( $h$ )
- $f_t$  = tensile strength of concrete =  $0.24\sqrt{f_{ck}}$
- $f_{cp}$  = compressive stress in concrete at CGC due to the prestress  
=  $P/A$ .

The value of  $f_{cp}$  is taken as positive (numeric value).

In this expression,  $b$  is equal to breadth of the section, which is equal to  $b_w$ , the breadth of the web for flanged sections;  $D$  is the total depth of the section which we have represented as  $h$  in earlier occasions;  $f_t$  is the direct tensile strength of concrete which is  $0.24\sqrt{f_{ck}}$ ;  $f_{cp}$  is the compressive stress in concrete at CGC due to the prestress, which is equal to  $P_e/A$ . We are considering the numeric value of  $f_{cp}$ ; actually  $f_{cp}$  is compressive, but we are taking only the numeric value of  $f_{cp}$  in the expression.

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The previous equation can be derived based on the expression of the principal tensile stress  $\sigma_1$  at CGC. If we take an element at the CGC, then there is a shear stress ( $v$ ) and a compressive stress ( $f_{cp}$ ) at the vertical face. This state of two dimensional stresses can be transformed to a state of principal stresses, where  $\sigma_1$  is the principal tensile stress and  $\sigma_2$  is the principal compressive stress. If we plot the Mohr's circle, the point corresponding to the vertical face of the element has coordinates  $(-f_{cp}, v)$ . The point corresponding to the horizontal face is on the vertical axis, with no normal stress, but with a shear stress of  $v$ .

We observe that the value of  $\sigma_1$  is much smaller than the magnitude of  $\sigma_2$ . This is due to the prestressing force. The value of  $\sigma_1$  can be calculated if we are able to locate the origin of the circle. The origin of the circle is at  $(-f_{cp}/2, 0)$ . The distance of  $\sigma_1$  from the origin is the radius of the circle, which is equal to  $\sqrt{((-f_{cp}/2)^2 + v^2)}$ . Thus,  $\sigma_1 = -f_{cp}/2 + \sqrt{(f_{cp}^2/4 + v^2)}$ .

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**Analysis for Shear**

**Limit State of Collapse for Shear**

The principal tensile stress is equated to the direct tensile strength of concrete ( $f_t$ ).

$$\sigma_1 = -\frac{f_{cp}}{2} + \sqrt{\frac{f_{cp}^2}{4} + v^2}$$
$$= -\frac{f_{cp}}{2} + \sqrt{\frac{f_{cp}^2}{4} + \left(\frac{V_{cs}Q}{Ib}\right)^2}$$
$$= f_t$$

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At the generation of web shear crack at the CGC, the principal tensile stress ( $\sigma_1$ ) is equated to the direct tensile strength of concrete ( $f_t$ ). Also,  $v$  is expressed as  $v = V_{c0}Q/Ib$ , which is derived in mechanics of materials.

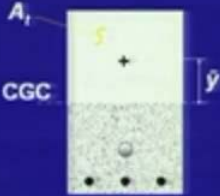
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**Analysis for Shear**

**Limit State of Collapse for Shear**

In the previous equation,

$I$  = gross moment of inertia  
 $Q = A_1 \bar{y}$   
 $A_1$  = area of section above CGC  
 $\bar{y}$  = vertical distance of centroid of  $A_1$  from CGC.



In the previous equation,  $I$  is the gross moment of inertia;  $Q$  is the product of the area above the CGC times the vertical distance of the centroid of the area from CGC, that is  $Q$

=  $A_t \hat{y}$ . We can transpose the terms to get the expression of the shear force, which initiates a web shear crack at the CGC.

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The slide is titled "Analysis for Shear" and discusses the "Limit State of Collapse for Shear". It shows the process of transposing terms in an equation to solve for the shear force  $V_{c0}$ . The equation shown is  $V_{c0} = \frac{Ib}{Q} \sqrt{f_t^2 + f_{cp}f_t}$ , which is then simplified to  $\rightarrow 0.67bD \sqrt{f_t^2 + 0.8f_{cp}f_t}$ . A note at the bottom states: "The term 0.67bD represents Ib/Q for the section. It is exact for a rectangular section and conservative for other sections."

Transposing the terms,  $V_{c0} = Ib/Q \times \sqrt{(f_t^2 + f_{cp}f_t)}$ . The expression of  $V_{c0}$  is in terms of the sectional properties of the section, which are  $I$ ,  $b$  and  $Q$ , the tensile strength of concrete  $f_t$ , and the amount of prestress  $f_{cp}$ . Here, we can note that if  $f_{cp}$  is increased, that means, if the prestress is increased, the shear corresponding to the initiation of web shear crack also increases. Thus, we are utilizing the benefit of the prestress in increasing the shear capacity of the section.

The above expression is substituted by a simpler expression  $V_{c0} = 0.67bD \sqrt{(f_t^2 + 0.8f_{cp}f_t)}$ . The term  $0.67bD$  represents  $Ib/Q$  for a section. It is an exact substitution for a rectangular section, but it is a conservative substitution for other types of sections. Also,  $f_{cp}$  is substituted by  $0.8f_{cp}$ . The reason is, to be conservative, only 80% of the prestress is considered in increasing the shear capacity.



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**Analysis for Shear**

**Limit State of Collapse for Shear**

To be conservative, only 80% of the prestressing force is considered in the term  $0.8f_{cp}$ .

$$V_{cb} = \frac{lb}{Q} \sqrt{f_t^2 + f_{cp} f_t}$$
$$\rightarrow 0.67bD \sqrt{f_t^2 + 0.8f_{cp} f_t}$$

The expression that we have derived is based on the stress at the level of CGC, being equal to the tensile strength of concrete.

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**Analysis for Shear**

**Limit State of Collapse for Shear**

For a flanged section, when the CGC is in the flange, the intersection of web and flange is considered to be the critical location. The expression of  $V_{cb}$  is modified by substituting  $0.8f_{cp}$  with  $0.8 \times$  (the stress in concrete at the level of the intersection of web and flange).

For a flanged section, when the CGC is in the flange, the intersection of web and flange is considered to be the critical location for the generation of web shear cracks. Hence, substitute  $0.8f_{cp}$  by the term 0.8 times the stress in concrete at the level of the intersection

of web and flange. This stress can be found out by the elastic analysis that we had studied under the analysis of sections for flexure.

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**Analysis for Shear**

**Limit State of Collapse for Shear**

In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force ( $V_p$ ) can be added to  $V_{c0}$ .

$$V_c \rightarrow V_{c0} + V_p$$

$$= 0.67bD \sqrt{f_t^2 + 0.8f_{cp}f_t} + V_p$$

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In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force  $V_p$  can be added to  $V_{c0}$ . Thus, the total strength can be considered as  $V_{c0} = 0.67bD \sqrt{(f_t^2 + 0.8f_{cp}f_t)} + V_p$ .

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**Analysis for Shear**

**Limit State of Collapse for Shear**

For cracked sections,

$$V_c = V_{cr}$$

$$V_c = \left(1 - 0.55 \frac{f_{pm}}{f_{pk}}\right) \tau_c b d + M_s \frac{V}{M} \quad (5b-2b)$$

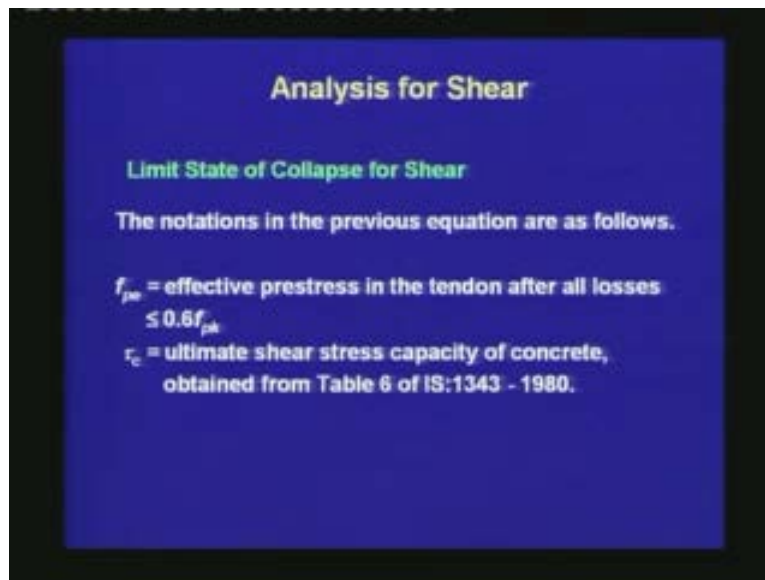
$$\geq 0.1bd \sqrt{f_{ck}}$$

$V_{cr}$  is the shear corresponding to flexure shear cracking. The term  $(1 - 0.55 f_{pm} / f_{pk}) \tau_c b d$  is the additional shear that changes a flexural crack to a flexure shear crack.

Next, we are studying the expression of  $V_c$  for a cracked section. This expression is more involved because the cracking of concrete is inherently variable in nature. The cracks are first generated due to flexure and then, they propagate due to shear. The expression of  $V_c$  is equated to  $V_{cr}$ , where  $V_{cr}$  is the shear corresponding to flexure–shear cracking. The earlier term  $V_{c0}$  was for web shear cracking, whereas  $V_{cr}$  is for flexure–shear cracking.  $V_{cr} = (1 - 0.55 f_{pe}/f_{pk})\tau_c bd + M_0 (V/M)$ .  $V_{cr}$  need not be less than  $0.1bd\sqrt{f_{ck}}$ .

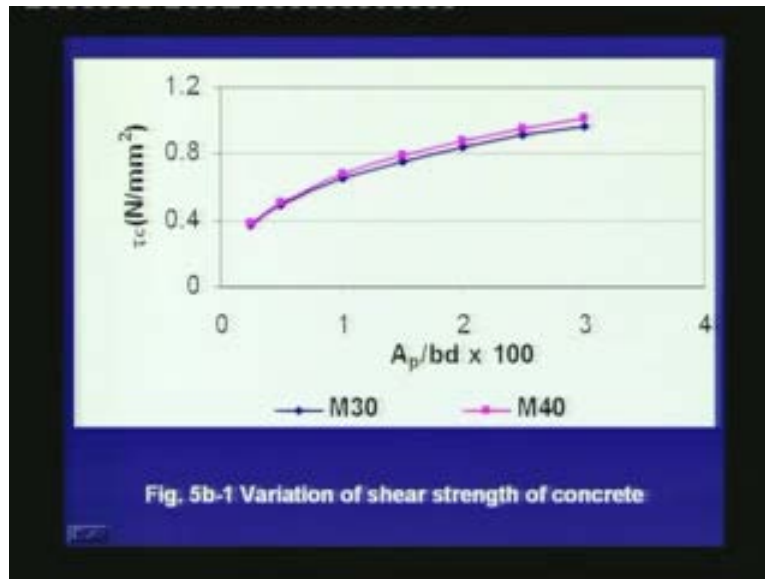
In the above expression there are two terms. The first term is the additional shear that changes a flexural crack to a flexure–shear crack. The second term  $M_0 (V/M) = V(M_0/M)$  is the shear corresponding to the generation of a flexural crack at the bottom of the beam.

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The first term is an empirical expression. The notations are as follows:  $f_{pe}$  is the effective prestress in the tendon after all losses and it should be less than or equal to  $0.6 f_{pk}$ ;  $\tau_c$  is the ultimate stress capacity of concrete which is obtained from Table 6 of IS: 1343 – 1980.

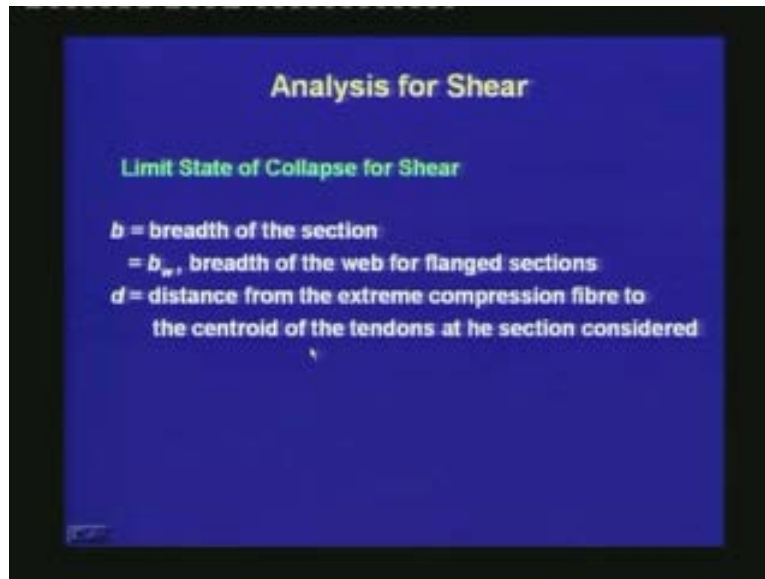
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The values of  $\tau_c$  are plotted in this graph. It is given in terms of the amount of prestressing steel. The graphs are given for two grades of concrete M30 and M40. We observe that as the amount of the prestressing steel is increasing, the strength of the concrete is also increasing; but, it gradually tapers off to a constant value. There is not much difference between the values for the two grades of concrete. From this we can infer that the strength of the concrete under shear even for a cracked section, increases with the amount of the prestressing steel.

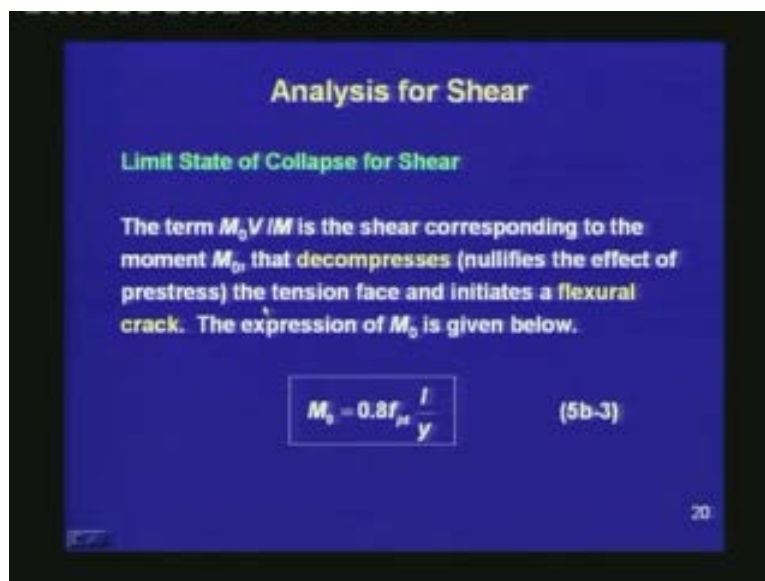
Even for the cracked region, since the prestressing steel applies a compression, the growth of crack is reduced, the depth of concrete under compression is increased, the aggregate interlock is increased, and the bond between the longitudinal steel and concrete is retained to generate dowel action. Hence, the strength of concrete under shear for a cracked section increases with the amount of prestressing steel.

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In the expression,  $b$  is the breadth of the section, which is equal to  $b_w$  the breadth of the web for a flanged section, and  $d$  is the distance from the extreme compression fibre to the centroid of the tendons at the section considered.

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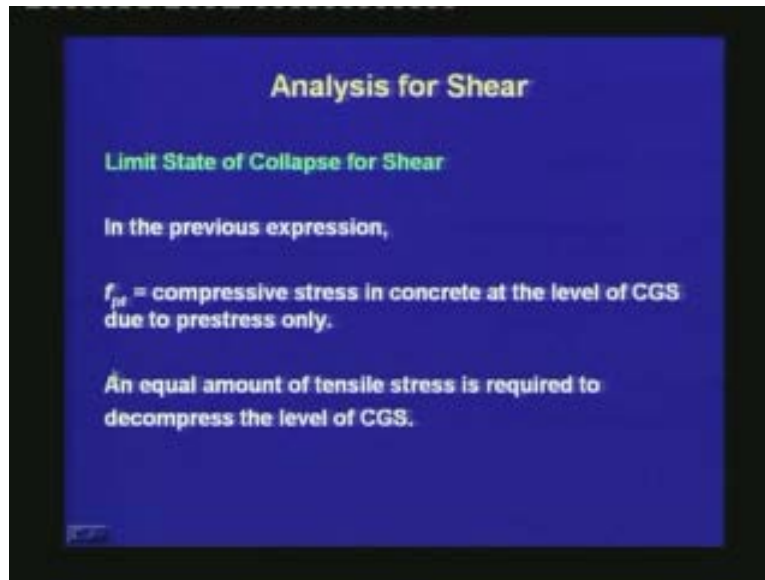


The second term  $M_0 (V/M) = V(M_0/M)$  is the shear corresponding to the moment  $M_0$  that decompresses, or nullifies the effect of prestress at the tension face, and initiates a flexural crack. In this expression,  $M_0/M$  is a ratio of moments. Hence, it is an expression

without any units.  $V$  is the shear corresponding to  $M$ , the moment at the section of investigation due to external loading.

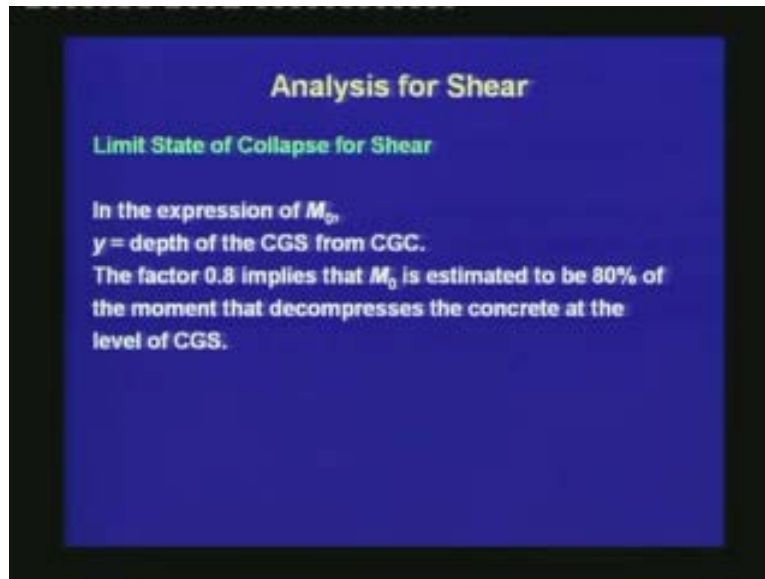
The expression of  $M_0$  is given as  $0.8f_{pt} I/y$ .

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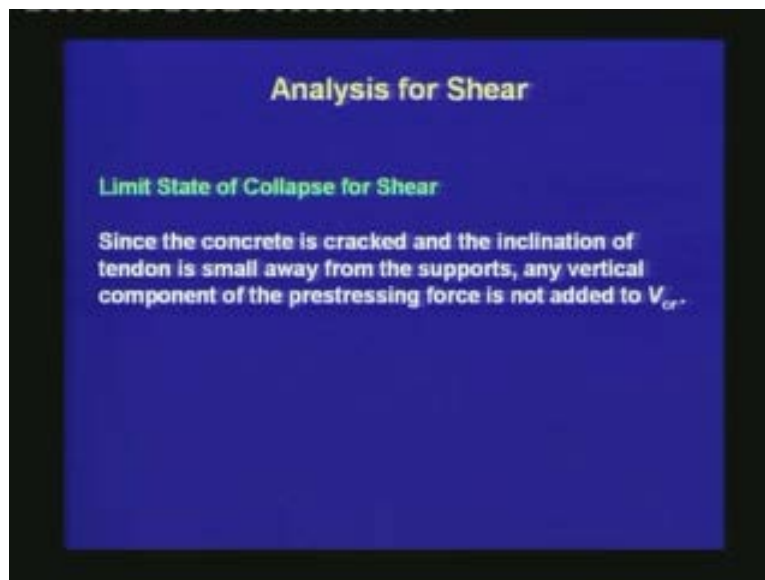
In the previous expression,  $f_{pt}$  is the compressive stress in concrete at the level of CGS due to prestress only. An equal amount of tensile stress is required to decompress the level of CGS.

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In the expression of  $M_0$ ,  $y$  is the depth of the CGS from CGC. The expression is actually 80% of  $f_{pt} I/y$ , the moment that causes decompression of the concrete at the level of CGS. To recollect,  $M_0$  is the moment corresponding to the decompression of the tension face, which occurs earlier. Hence,  $M_0$  is taken as 80% of that moment which causes decompression at the level of CGS.

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Since the concrete is cracked and the inclination of tendon is small away from the supports, any vertical component of the prestressing force is not added to  $V_{cr}$ . Earlier we had seen, that the code allows us to incorporate  $V_p$  along with  $V_{c0}$ , which is the shear causing web shear cracking. But here, we find that the code does not allow us to include  $V_p$  with  $V_{cr}$ , which is the shear corresponding to flexure–shear cracking.

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The slide is titled "Analysis for Shear" and contains the following text:

**Maximum Permissible Shear Stress**

To check the crushing of concrete in shear compression failure, the shear stress is limited to a maximum value ( $\tau_{c,max}$ ). The value of  $\tau_{c,max}$  depends on the grade of concrete and is given in Table 7 of IS:1343 - 1980.

$$\frac{V_u}{bd_t} \leq \tau_{c,max} \quad (5b-4)$$

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Next, let us learn the maximum permissible shear stress in concrete. As mentioned in the last lecture, there are several modes of shear failure. The approach in shear design is that, we design the reinforcement as per the requirement based on the capacity of concrete to take shear. We also do some other checks and satisfy detailing requirements to avoid other modes of failure. The maximum permissible shear stress in concrete is one way to check the crushing of concrete in shear compression failure.

The shear stress is limited to a maximum value which is represented as  $\tau_{c,max}$ . The value of  $\tau_{c,max}$  depends on the grade of concrete and is given in Table 7 of IS: 1343-1980. The nominal shear stress,  $V_u/bd_t$  should be less than  $\tau_{c,max}$ . This is to avoid a brittle shear compression failure.



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### Analysis for Shear

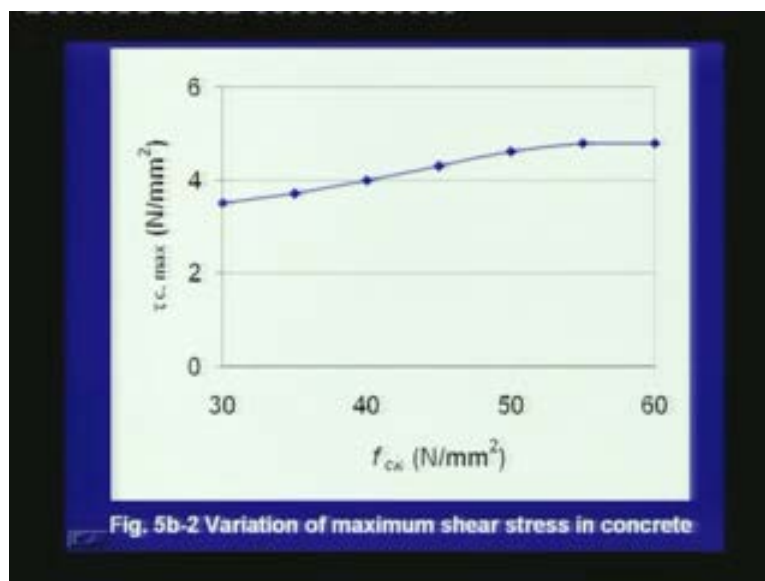
**Maximum Permissible Shear Stress**

In the previous expression,

$d_t$  = greater of  $d_p$  or  $d_s$   
 $d_p$  = depth of CGS from the extreme compression fiber  
 $d_s$  = depth of centroid of regular steel  
 $V_u$  = shear force at a section due to ultimate loads.

In the previous expression,  $b$  is the breadth of the section,  $d_t$  is the greater of  $d_p$  or  $d_s$ , where  $d_p$  is the depth of the CGS from the extreme compression fiber,  $d_s$  is depth of centroid of regular steel,  $V_u$  is the shear force at a section due to ultimate loads. Thus, whenever we are doing this shear check, we are calculating the average shear stress based on the depth, which is larger of the depth of the CGS and the depth of the non-prestressed steel. This larger value is denoted as  $d_t$ .

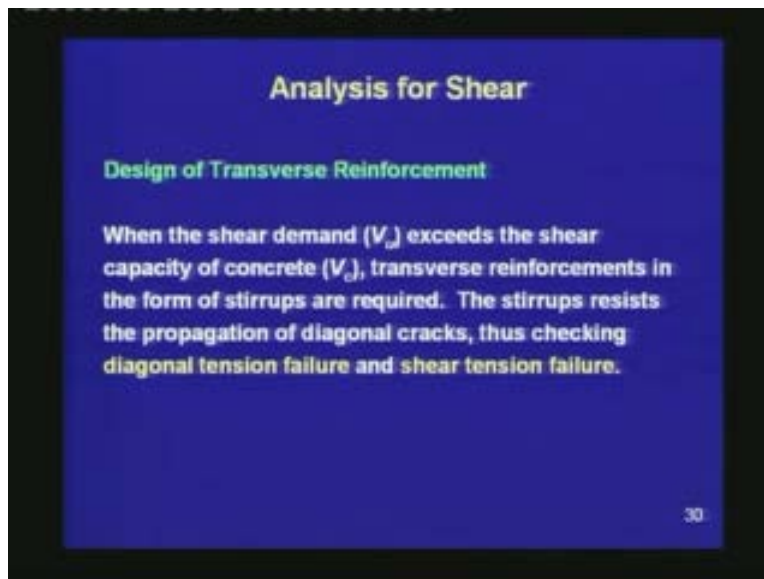
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When the values of  $\tau_{c,max}$  from Table 7 are plotted, their variation is as shown in the graph. For different grades of concrete,  $\tau_{c,max}$  slightly increases and more or less it stabilizes when we reach a concrete strength of  $60 \text{ N/mm}^2$ .

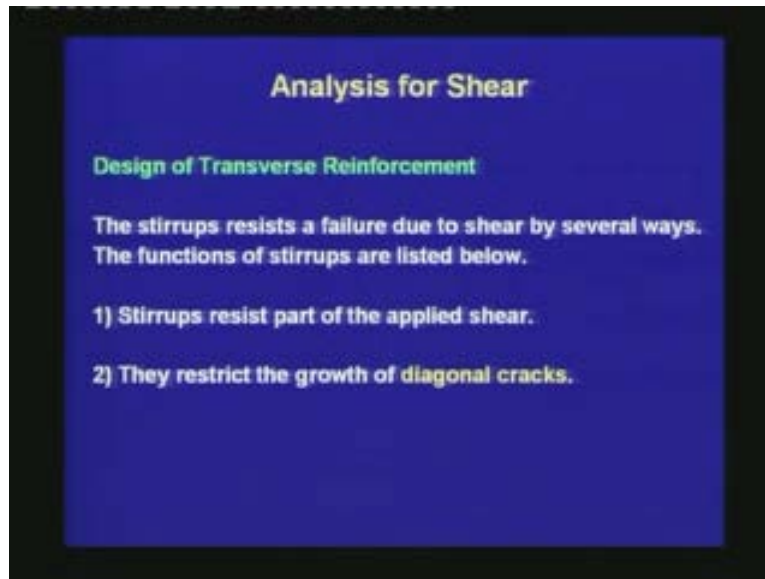
To check web crushing failure, the Indian Roads Congress code IRC 18-2000 specifies minimum thickness of the web for a T-section in Clause 9.3.1.1. The minimum thickness is 200 mm plus diameter of the duct hole. This provision is not there in IS: 1343-1980. But, it is advisable to maintain a minimum width of the web to avoid web crushing failure for a flanged section.

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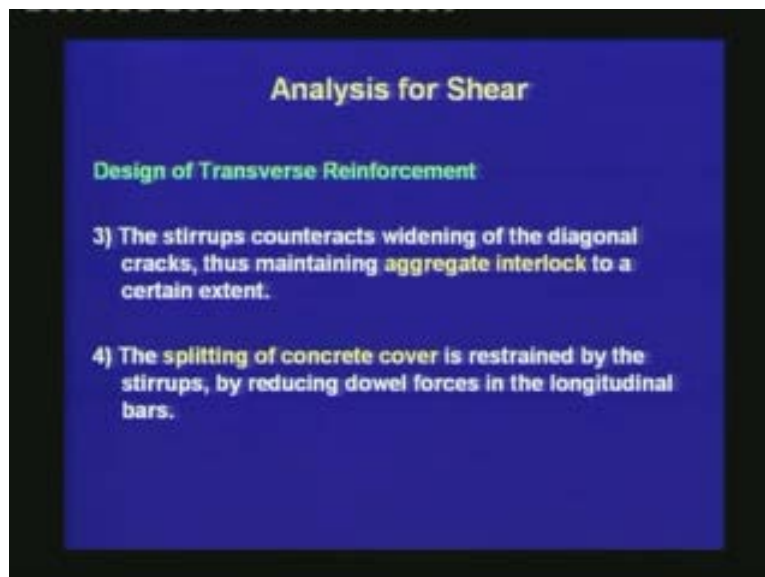
Next, let us look into the design of transverse reinforcement in more details. When the shear demand  $V_u$  exceeds the shear capacity of concrete which is denoted as  $V_c$ , transverse reinforcements in the form of stirrups are required. The stirrups resist the propagation of diagonal cracks, checking both the diagonal tension failure and shear tension failure. The diagonal tension failure occurs when a diagonal crack propagates rapidly through the depth of the section.

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In a shear tension failure, the diagonal crack merges with the horizontal crack at the level of the bottom longitudinal bars. The crack propagates towards the support and the longitudinal bars lose their anchorage. To resist both these types of failures, we need shear reinforcement which will traverse the diagonal cracks. The stirrups resist a failure due to shear by several ways.

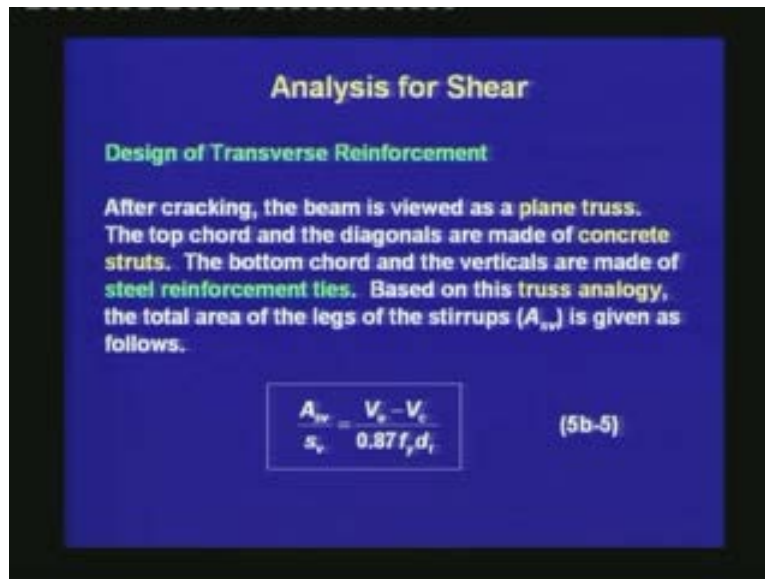
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The functions of stirrups are listed below: first, stirrups resist part of the applied shear which is denoted as  $V_s$ ; second, they restrict the growth of diagonal cracks; third, the stirrups counteract the widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent; fourth, the splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

Thus, the contribution of the stirrups is not only in  $V_s$ , but it has a contribution in  $V_c$  as well. For the different reasons, stirrups are mandatory for a reinforced concrete or a prestressed concrete beam.

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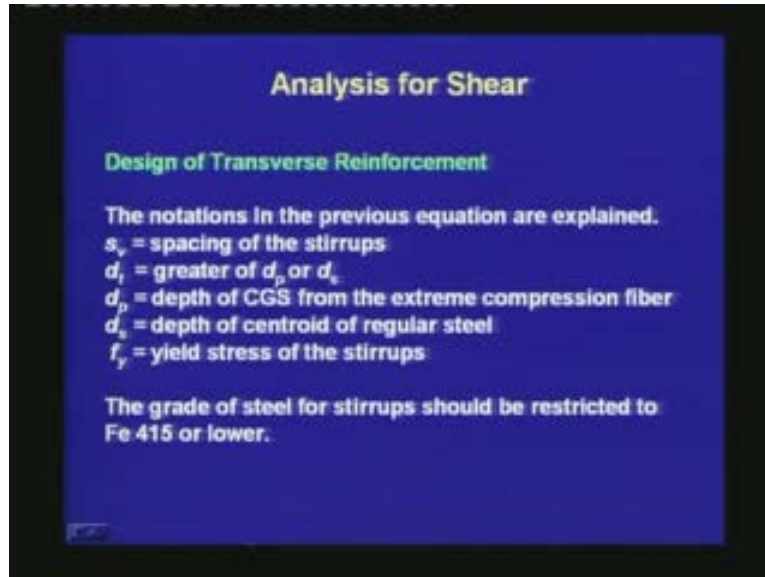
After cracking, the beam is viewed as a plane truss. The top chord and the diagonals are made of concrete struts and the bottom chord and the verticals are made of steel reinforcement ties. Based on this truss analogy, the total area of the legs of the stirrups which is represented as  $A_{sv}$  is given as follows.

$$A_{sv} / s_v = (V_u - V_c) / 0.87f_y d_t$$

$V_u$  is the shear demand. From this, we are subtracting the concrete contribution  $V_c$ .  $V_c$  can be calculated for a particular section from the expressions of  $V_{cr}$  or  $V_{c0}$ , whichever gives the lower value. Subtracting  $V_c$  from  $V_u$ , we get the required value of  $V_s$ . Then, from the truss analogy, we can write the expression of  $A_{sv} / s_v$ . We are not going into the

details of the truss analogy here. The angle of inclination of the cracks is considered to be  $45^\circ$ .

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The notations in the previous equation are as follows:  $s_v$  is the spacing of the stirrups,  $d_t$  is the effective depth which is greater of  $d_p$  or  $d_s$  where  $d_p$  is the depth of CGS from the extreme compression fibre and  $d_s$  is the depth of centroid of the longitudinal steel,  $f_y$  is the characteristic yield strength of the stirrups. The maximum stress in the stirrups is limited to  $0.87f_y$ . The grade of steel for stirrups should be restricted to Fe 415 or lower. That means, the code does not allow a grade of steel greater than Fe 415 because traditionally such a grade has less yielding.

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The slide is titled "Analysis for Shear" and contains the following text:

**Design of Transverse Reinforcement**

For  $V_u < V_c$ , minimum amount of transverse reinforcement is provided based on the following equation.

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87f_y} \quad (5b-6)$$

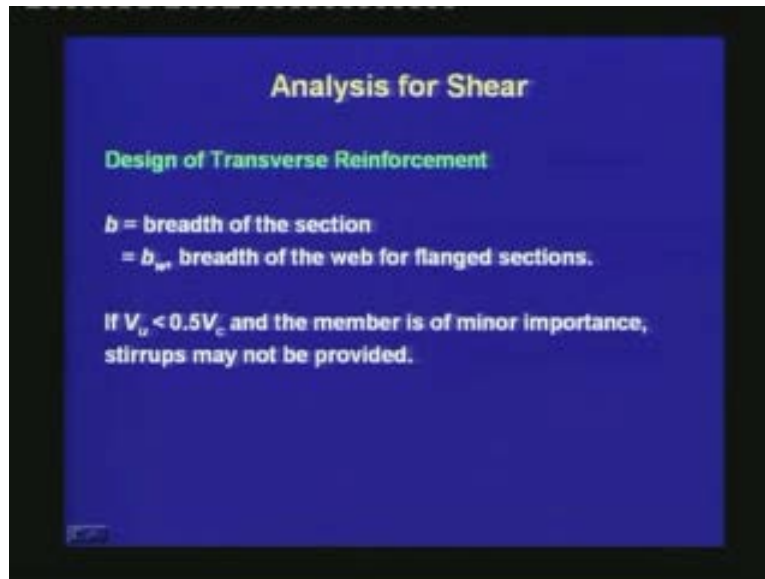
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For design, if  $V_u$  is less than  $V_c$ , theoretically we do not need stirrups. But the code says that we have to provide a minimum amount of transverse reinforcement which is based on the following equation.

$$A_{sv} / bs_v = 0.4/0.87f_y$$

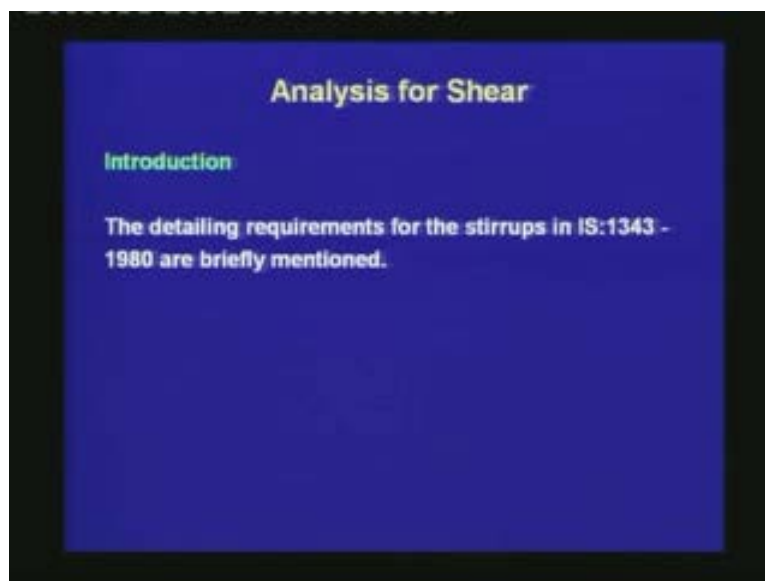
This equation is derived by assuming that the stirrups should have a minimum stress of  $0.4 \text{ N/mm}^2$ .

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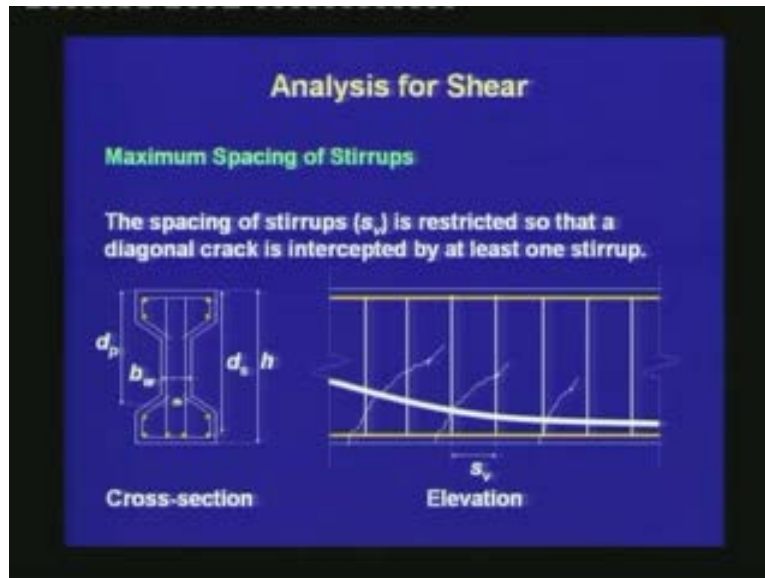
Here,  $b$  is the breadth of the section which is equal to  $b_w$  the breadth of the web for flanged sections. If  $V_u$  is less than  $0.5 V_c$  and the member is of minor importance, stirrups may not be provided. That means, the code gives us some freedom that if the shear demand is half of the shear capacity of concrete and the member is of minor importance, in that situation we may not provide stirrups. Otherwise we have to provide stirrups. If  $V_u$  is less than  $V_c$ , we have to provide the minimum amount of stirrups.

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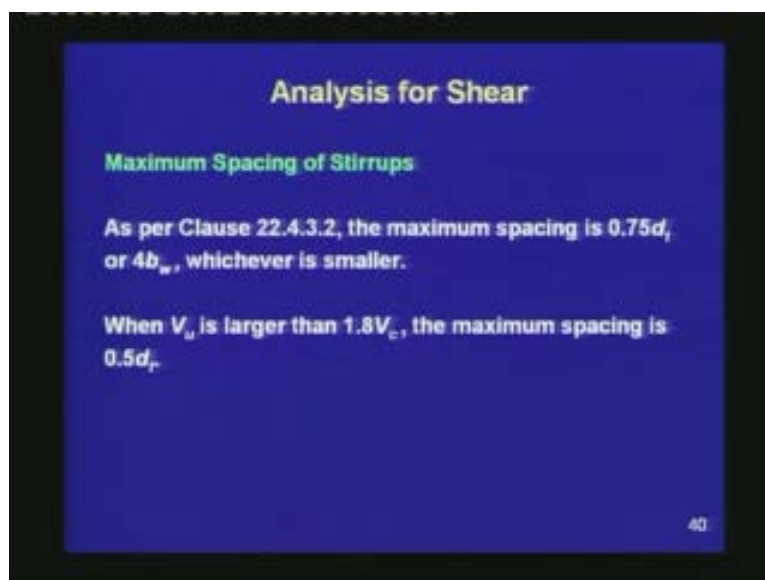
Next, let us look at the detailing requirements. The detailing requirements for the stirrups in IS: 1343 -1980 are briefly mentioned.

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The maximum spacing of stirrups is given. The spacing of stirrups is restricted so that a diagonal crack is intercepted by at least one stirrup. In the sketch of the cross-section, the notations are:  $d_p$  is the depth of the CGS,  $d_s$  is the depth of the longitudinal steel,  $b_w$  is the breadth of the web, and  $h$  is the total depth.

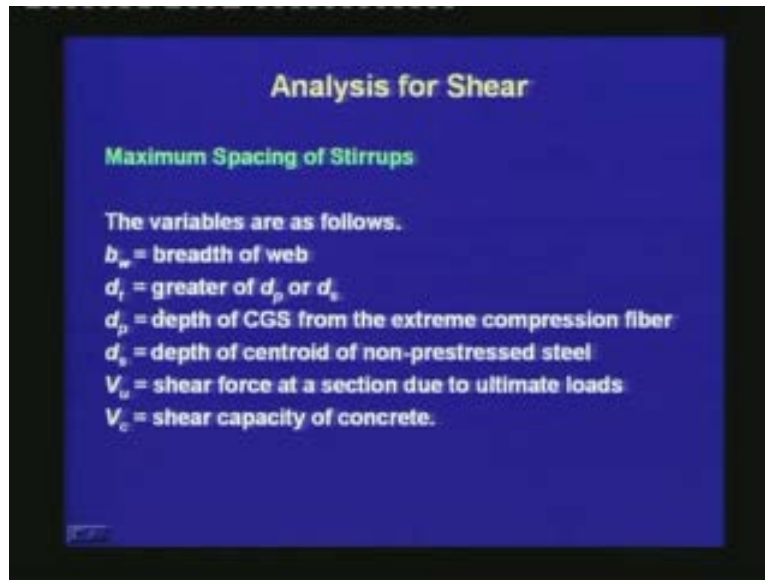
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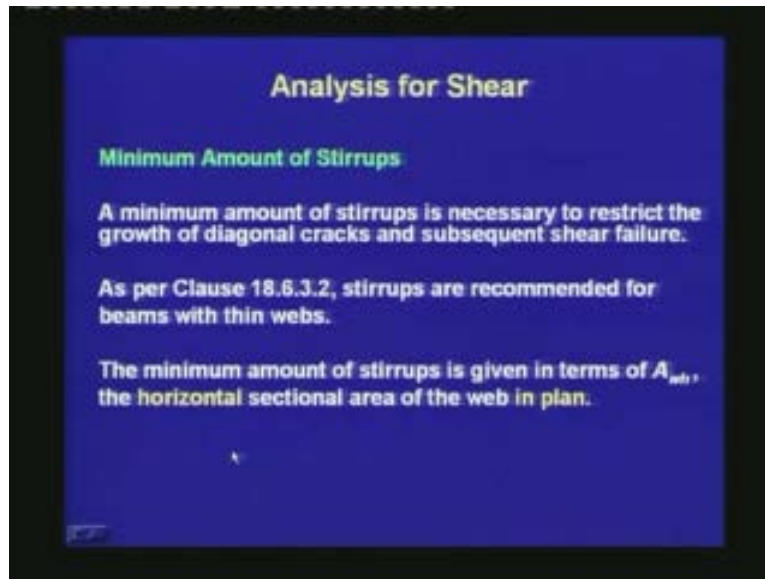
The maximum spacing is given in Clause 22.4.3.2. It is equal to  $0.75d_t$  or  $4b_w$  whichever is smaller. Thus, the maximum spacing is the smaller value of 75% of the value of  $d_t$  or 4 times the breadth of the web. If the shear demand  $V_u$  is high, if it crosses 1.8 times  $V_c$ , then the maximum spacing is reduced to 0.5 times  $d_t$ . Thus, for higher values of  $V_u$  the stirrup spacing is even restricted.

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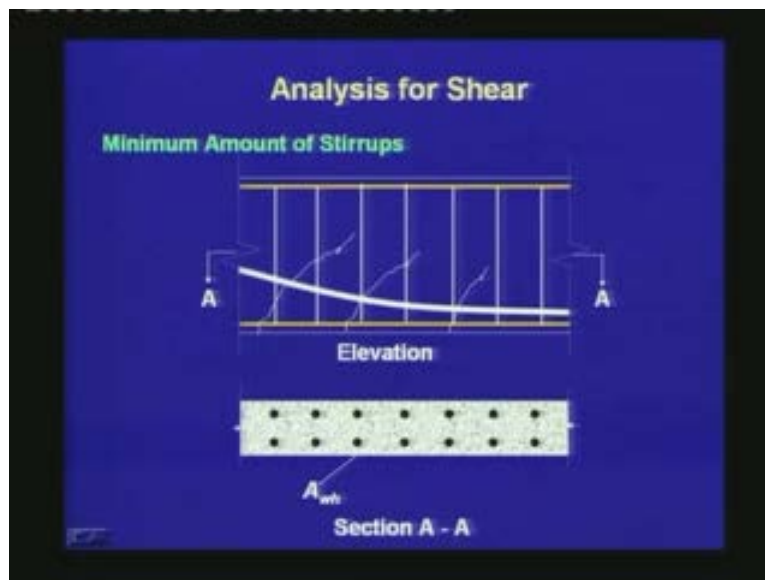
The variables are:  $b_w$  is the breadth of the web,  $d_t$  is greater of  $d_p$  or  $d_s$ ,  $d_p$  is depth of CGS from extreme compression fiber,  $d_s$  is depth of centroid of non-prestressed steel,  $V_u$  is the shear force at a section due to ultimate loads,  $V_c$  is the shear capacity of concrete which is determined from  $V_{c0}$  or  $V_{cr}$ .

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A minimum amount of stirrups is necessary to restrict the growth of diagonal cracks and subsequent shear failure. As per Clause 18.6.3.2, stirrups are recommended for beams with thin webs. The minimum amount of stirrups is given in terms of  $A_{wh}$ , the horizontal sectional area of the web in plan.

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The area is shown in this figure. If we take a horizontal section, then  $A_{wh}$  is the area of the web for this horizontal section.

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**Analysis for Shear**

**Minimum Amount of Stirrups ( $A_{sv,min}$ )**

In presence of dynamic load,

$$A_{sv,min} = 0.3\% A_{wh}$$
$$= 0.2\% A_{wh}, \text{ when } h \leq 4b_w$$

With high strength bars,

$$A_{sv,min} = 0.2\% A_{wh}$$
$$= 0.15\% A_{wh}, \text{ when } h \leq 4b_w$$

In the presence of dynamic load, minimum amount of  $A_{sv}$  ( $A_{sv,min}$ ) is equal to 0.3% of  $A_{wh}$ , which can be lowered to 0.2%  $A_{wh}$ , when  $h$  is less than or equal to  $4b_w$ . With high strength bars,  $A_{sv,min}$  is equal to 0.2%  $A_{wh}$  which can be lowered to 0.15%  $A_{wh}$ , when  $h$  is less than or equal to  $4b_w$ .

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**Analysis for Shear**

**Minimum Amount of Stirrups ( $A_{sv,min}$ )**

In absence of dynamic load, when  $h > 4b_w$

$$A_{sv,min} = 0.1\% A_{wh}$$

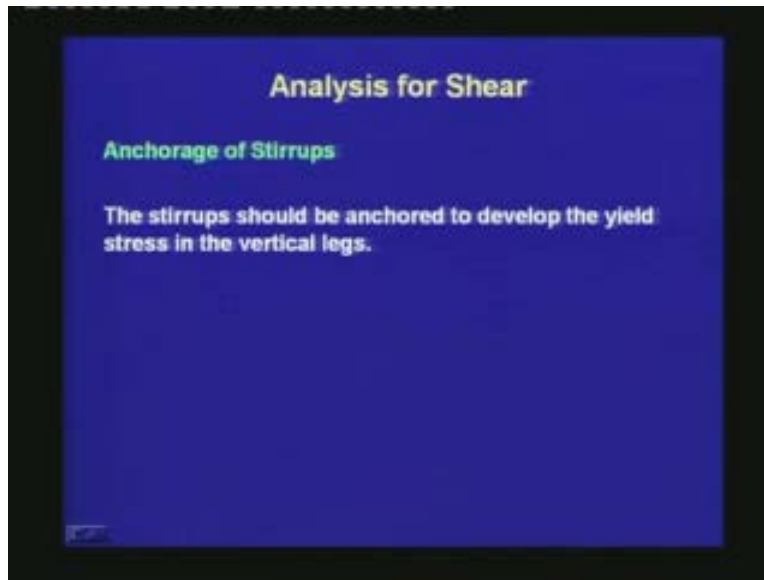
There is no specification for  $A_{sv,min}$  when  $h \leq 4b_w$ .

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In the absence of dynamic load, when  $h$  is greater than  $4b_w$ , then  $A_{sv,min}$  is equal to 0.1% of  $A_{wh}$ . There is no specification of  $A_{sv,min}$  when  $h$  is less than or equal to  $4b_w$ . Thus, we

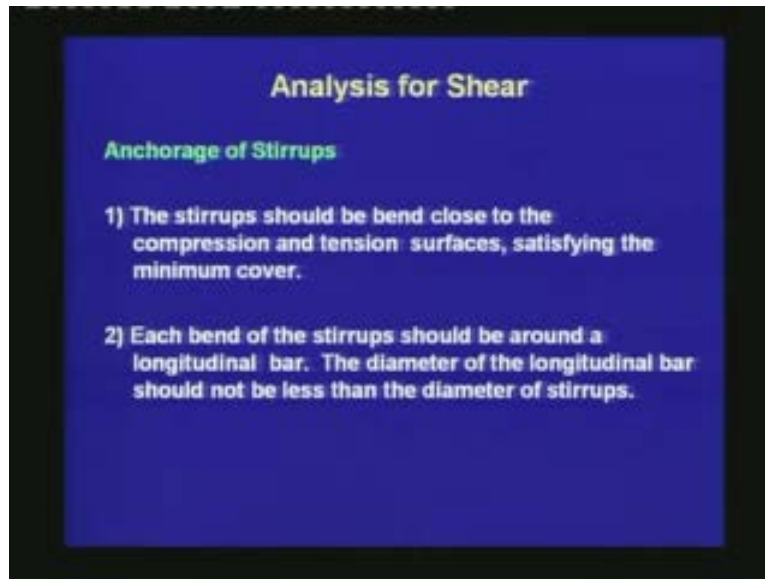
find that there are two requirements of minimum amount of stirrup. One is based on the truss concept that the stirrups have to take a minimum amount of shear stress. The other expression is for beams with thin webs, given in terms of the horizontal section of the web.

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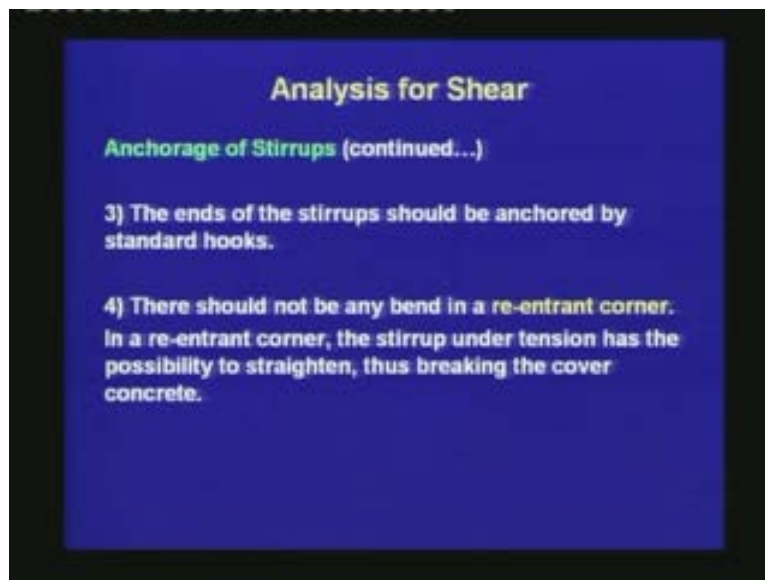
The stirrups should be anchored to develop the yield stress in the vertical legs. This is very important that we anchor the stirrups properly, so that it can develop the yield stress at ultimate.

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The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover. That means the stirrups should run throughout the full depth as much as possible. The stirrups should be bent around the top steel and the bottom steel. Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of the stirrups.

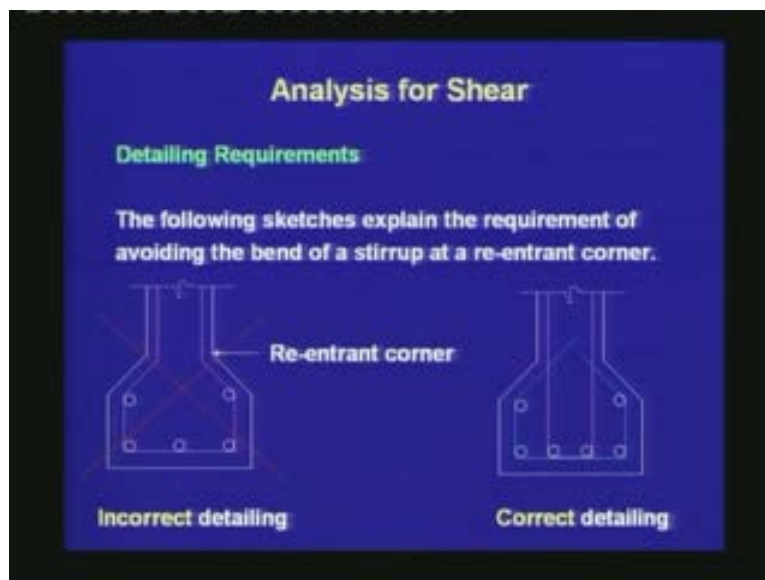
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In order to maintain the bend in the stirrups, the diameter of the longitudinal bar has to be at least equal to the diameter of the stirrup. The end of the stirrups should be anchored by standard hooks. The specifications for the hooks are given in handbook on detailing.

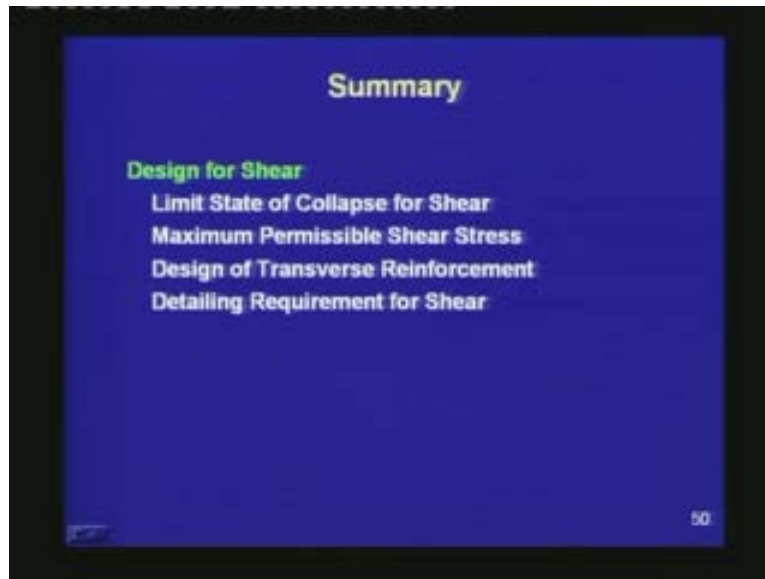
There should not be any bend in a re-entrant corner. In a re-entrant corner, the stirrup under tension has the possibility to straighten, thus, breaking the cover concrete. Let us understand this with the help of a sketch. The following sketches explain the requirement of avoiding the bend of a stirrup at a re-entrant corner.

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In a re-entrant corner, if we provide a single stirrup, then the stirrup tends to straighten out. It tends to break the concrete near the re-entrant corner. This detailing is incorrect. The correct detailing is that, we have to provide two pieces of stirrups. None of them has any bend near the re-entrant corners and hence, does not pose a threat of breaking the concrete near the corners.

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Today, we studied the design for shear. First, we studied the limit state for shear. As we had said earlier, that in shear analysis and design, we do not consider the complete behaviour. The design is based on the ultimate strength of the section under shear. This strength, which is represented as  $V_{uR}$  is a summation of  $V_c$  and  $V_s$ , where  $V_c$  is considered to be the contribution of the concrete, and  $V_s$  is the contribution of the stirrups. In  $V_c$ , we are placing all the three individual components: the contribution of uncracked concrete under compression is represented as  $V_{cz}$ , the aggregate interlock is represented as  $V_a$ , and the dowel action is  $V_d$ . All these three are clubbed together in the expression of  $V_c$ . The code gives us two expressions of  $V_c$ : one for uncracked section and the other for a cracked section. The uncracked section expression usually governs the region near the supports, and the cracked section expression usually governs the region near the mid-span.

When a beam is loaded, first, we observe flexural cracks near the mid-span. Then, the flexural cracks away from the mid-span tend to bend due to the effect of shear. These types of cracks are called flexure shear cracks. Then, with further increase in the load, we may have cracks near the centroid of the section close to the supports. Those cracks are inclined and are called web shear cracks.

The shear strength of uncracked concrete is given as the shear force which initiates a web shear crack at the level of CGC. The expression of  $V_c = V_{c0}$  is derived from the concept of Mohr's circle of stress at the level of CGC.

For a point at the level of CGC, there is a shear stress. Along with that, for a prestressed concrete beam there is a normal compressive stress which we are denoting as  $f_{cp}$ . From the values of  $V$  and  $f_{cp}$ , we can write the expression of the principal tensile stress  $\sigma_1$  based on the Mohr's circle. Then we equate  $\sigma_1$  to the direct tensile strength of concrete which is denoted as  $f_t$ .

Once we write this expression, we can substitute the shear stress  $v = V_{c0}Q/Ib$ . After transposing the terms, we get an expression of  $V_{c0}$ . In this expression,  $f_{cp}$  is reduced to 80% to stay conservative. If we have a flanged section, then the code recommends that the crack may initiate at the level of the intersection of the web and the flange. Hence,  $0.8f_{cp}$  should be substituted by 0.8 times the stress in the concrete at the level of intersection of web and the flange.

The second expression for  $V_c$  is based on a cracked section, where  $V_c = V_{cr}$  consists of two terms. The first term is the shear that propagates flexural crack to a flexure shear crack. This expression is in terms of the prestressing force  $f_{pe}$  and the capacity of concrete, which is given as  $\tau_c$ . The capacity of concrete  $\tau_c$  increases with the amount of prestressing steel, because the more prestressing steel we have, we have a reduced length of the flexure shear cracks, we have more concrete under compression, there will be more aggregate interlock, and a better dowel action. Hence, the capacity of concrete is increased with the amount of prestressing steel.

The second term of  $V_{cr}$  is the shear corresponding to the generation of a flexural crack at the bottom of the beam. This is based on the ratio of the moments  $M_0/M$ , where  $M_0$  is the moment required to generate a flexural crack at the bottom and  $M$  is the moment demand based on service loads.

We can have both  $V$  and  $M$  as the demands based on ultimate loads, because the load factors anyhow will cancel out from the numerator and the denominator.  $M_0V/M$  denotes



the shear, which corresponds to the generation of a flexural crack at the bottom of the beam. Thus,  $V_{cr}$  consists of two terms: the first term is a shear which extends a flexural crack to a flexure shear crack. The second one is the shear which is needed for a generation of a flexural crack at the bottom of the beam.

Once we have learned these two expressions of  $V_{c0}$  and  $V_{cr}$ , we can calculate  $V_c$  as the lower of these two values at any particular section. We can add  $V_p$  to  $V_c$ , if the tendon is inclined and this is allowed only with the expression of  $V_{c0}$ . Once we have calculated  $V_c$ , we can subtract this from  $V_u$ , the shear demand and get the required value of  $V_s$ . The amount of steel and the spacing of the stirrups are given by an expression which is based on the truss analogy for shear. This is in terms of  $V_s$  and the maximum stress that can be generated in the stirrups equal to  $0.87f_y$ . The code recommends to provide a minimum amount of stirrups even if  $V_u$  is less than  $V_c$ . That is based on a certain minimum stress to generate within the stirrups. The code also gives another expression of minimum amount of stirrups which is applicable for beams with thin webs. The values are given in the presence of dynamic load and in the absence of dynamic load.

Next, we studied about the detailing of the stirrups. The stirrups should be properly anchored at the top steel and at the bottom steel. They should traverse the depth as much as possible, satisfying the cover requirements. The spacing of the stirrups is restricted because a diagonal crack should be intercepted by at least one stirrup. The stirrups should have standard hooks at the ends. The functions of the stirrups are multiple. They not only provide shear capacity  $V_s$  but also influence  $V_c$ , by increasing the dowel action, the aggregate interlock and checking the growth of the crack.

There are other detailing requirements which check the other modes of failure such as the shear compression failure and the web crushing failure. We have to limit the shear stress to a maximum value, so that, we do not have any sudden shear failure. In the next class, we shall move on to the design steps for shear.

Thank you.