#### PRESTRESSED CONCRETE STRUCTURES

Amlan K. Sengupta, PhD PE Department of Civil Engineering Indian Institute of Technology Madras

# Module - 5: Analysis and Design for Shear and Torsion

## Lecture-23: Analysis for Shear

Welcome back to prestressed concrete structures. This is the first lecture of Module 5 on analysis and design for shear and torsion.

(Refer Slide Time: 01:34)



In this lecture, first we shall study about the stresses in an uncracked beam. Then we shall learn about the types of cracks generated in reinforced concrete and in prestressed concrete beam. Then we shall learn about the components of shear resistance, the modes of failure due to shear, and the effect of prestressing force in a shear type of failure.

# (Refer Slide Time: 02:04)



The analysis of reinforced concrete and prestressed concrete members for shear is more difficult compared to the analysis for axial load or flexure.

(Refer Slide Time: 03:14)



The analysis for axial load and flexure are based on the following principles of mechanics: equilibrium of internal and external forces, compatibility of strains in concrete and steel, and constitutive relationships of materials. When we studied these two

actions earlier, we based our equations on these three principles and the behaviour of the member under each of this action was well defined. We were able to plot the complete load versus deformation behaviour for a member under axial load, or for a member under flexure. But for shear, this is relatively much more difficult.

(Refer Slide Time 03:13)



The conventional analysis for shear is based on equilibrium of forces by a simple equation. The compatibility of strains is not considered. The constitutive relationships (relating stress and strain) of the materials, concrete or steel are not used. The strength of each material corresponds to the ultimate strength.

The approach that we followed for axial load and flexure was more rational compared to the approach that we shall follow in shear. For shear, we shall consider only the equilibrium of forces. We are not considering the compatibility of strains in the concrete and steel; neither shall we use the constitutive relationships of the materials. Rather, we shall use only the ultimate strength of each material to get the shear capacity of a member.

# (Refer Slide Time 04:30)



The strength of concrete under shear although based on test results is empirical in nature. The equation that we shall use for the shear strength of concrete is not related to any stress–strain curve.

(Refer Slide Time: 05:05)



Shear stresses generate in beams due to bending or twisting. The two types of shear stress are called flexural shear stress and torsional shear stress, respectively. In this module, the

analysis for shear refers to flexural shear stress, or the shear stress due to flexure. The torsional shear stress will be covered under analysis for torsion.

(Refer Slide Time: 05:54)



To understand flexural shear stress, the behaviour of a simply supported beam under uniformly distributed load, without prestressing will be explained first. The presentation will be in the following sequence.

(Refer Slide Time: 06:19)



First, we shall study the stresses in an uncracked beam. An uncracked beam is considered as a homogeneous beam, where the effect of steel is negligible. Second, the type of cracks generated due to the combination of flexure and shear will be discussed. Next, we shall move on to components of shear resistance and the modes of failure due to shear. Finally, we shall see the effect of prestressing force on the shear resistance of the concrete member.

(Refer Slide Time: 06:51)



Here we are seeing a simply supported beam, subjected to uniformly distributed load. We are considering two points in the beam: one lies at the neutral axis and the other lies close to the tension face. For such a beam, from basic structural analysis, we know that the shear force varies along the span linearly. The moment also varies along the span along a parabolic curve.

If we look at the stresses at any point of the beam, the variation of normal stress along the depth is linear, it is compressive at the top and tensile at the bottom. The variation of shear stress at a section along the depth is parabolic in nature, where the shear stress is zero at the top and bottom, and maximum at the neutral axis. Thus the two types of stresses, the normal stress and the shear stress in a section of a beam under flexure have their maximum values at different locations.

(Refer Slide Time: 09:38)



Under a general loading, the shear force and the moment vary along the length. The normal stress and the shear stress vary along the length, as well as along the depth. The combination of the normal and shear stresses generate a two-dimensional stress field at a point.

(Refer Slide Time 10:11)



At any point in the beam, the state of two-dimensional stresses can be expressed in terms of the principal stresses. The principal stresses are the stresses in the planes where there is no shear stress. The Mohr's circle of stress is helpful to understand the state of stress. These concepts are covered in mechanics of materials; we shall review this again in the light of analysis for shear.

(Refer Slide Time: 11:10)



Before cracking, the stress carried by steel is negligible. Hence, we can neglect the presence of steel in the concrete beam. When the principal tensile stress exceeds the cracking stress, the concrete cracks and there is redistribution of stresses between concrete and steel.

It is easier to understand the state of stress before the cracking of the member, which we can derive by analyzing the beam by elastic analysis. After the cracking of the member, the state of stress is more complicated because there is redistribution of stresses between concrete and steel. With increasing load, the concrete enters into its non-linear behaviour. Also, the steel after yielding enters into its non-linear behaviour.

### (Refer Slide Time 11:55)



For a point at the neutral axis, which we are representing by Element 1, the shear stress is maximum and the normal stress is zero. From this state of stress if we draw the principal stresses, we observe that the shear stress generates a biaxial tensile–compressive stress field.  $\sigma_1$  is the principal tensile stress and  $\sigma_2$  is the principal compressive stress. They are inclined at 45° to the neutral axis.

Thus, if we try to understand the state of stress with the help of a Mohr's circle, we see that for the vertical and the horizontal surfaces, the states of stress are represented at the top and the bottom of the Mohr's circle, where there is shear stress v but there is no normal stress. At a surface which is inclined at 45°, we have a principal stress; in one side, there is the principal tensile stress and in the perpendicular side, we have the principal compressive stress. The magnitudes of  $\sigma_1$  and  $\sigma_2$  are same under pure shear.

# (Refer Slide Time: 13:56)



Since, the shear force is maximum near the supports, cracks due to shear occur around the neutral axis near the supports and perpendicular to  $\sigma_1$ . Hence, the cracking due to shear is expected to occur first near the supports and around the neutral axis. Thus, understanding of the inclination of the principal tensile stress helps us to understand the inclination of the cracks due to shear. Under pure shear since  $\sigma_1$  is inclined at 45° to the neutral axis, the cracking which occurs perpendicular to  $\sigma_1$  is also inclined at 45° to the neutral axis. (Refer Slide Time: 15:25)



For a point near the bottom edge, which is Element 2, the normal stress is maximum and the shear stress is close to zero. Below the neutral axis, the normal stress is tensile.

The principal stresses are as follows. Here, the inclination of  $\sigma_1$  is less than 45° to the neutral axis. The value of  $\sigma_1$  is much larger than the magnitude of  $\sigma_2$ , which is the principal compressive stress. If we plot the Mohr's circle we find that the state of stress in the horizontal and the vertical surfaces can be denoted by the following two points. For the vertical surface the state of stress is given as (f, v); it has both the normal stress and the shear stress. For the horizontal surface it has only the shear stress, and the point lies in the vertical axis. The principal tensile stress  $\sigma_1$  is almost parallel to the bottom edge, and the value of  $\alpha$  is much smaller than 45°.

# (Refer Slide Time: 17:17)



Since, the moment is maximum at the mid span, cracks due to flexure occur near mid span and perpendicular to  $\sigma_1$ . It starts from the bottom of the beam and it gradually goes up. If we see the state of stress, the inclination of the crack is perpendicular to  $\sigma_1$ . Since  $\sigma_1$  is almost parallel to the bottom edge, the cracks will be starting as perpendicular to the bottom edge.

In this figure, we see that the crack is almost perpendicular to the horizontal axis, and when we come to the bottom edge of the surface, this cracks becomes perpendicular to the bottom edge. With this understanding of the state of stress for an uncracked beam, we are now moving on to understand the types of crack that generates in a beam under flexure. (Refer Slide Time 18:26)



The types and formation of cracks depends on the span-to-depth ratio of the beam and the loading. In the following figures, the formation of cracks for a beam with large span-to-depth ratio and uniformly distributed loading is shown.

(Refer Slide Time: 18:46)

	Analysis for Shear
Types of	Cracks
a i i	
	6633
	a) Initiation of flexural cracks
	Fig. 5a-1 Formation of cracks

The formation of cracks in a reinforced concrete beam is inherently variant in nature. Wherever, there is a weak point in the concrete beam the crack propagates through that point. Instead of going into the detailed analysis of a crack formation, we are trying to have a visual representation of the growth of cracks, which will help us to understand, the mechanism of crack formation and finally the failure of concrete due to shear. For a long beam, the span to the depth ratio is large. For such a beam under a uniformly distributed load, the cracks start due to flexure and it starts from the bottom of the beam near the mid span.

In this sketch we find that the first cracks that are initiated are near the middle of the beam and it starts from the bottom. Since the moment is maximum near the middle and the flexural stresses are tensile and maximum at the bottom, the cracks generate from the bottom surface. These cracks are called flexural cracks because they have generated due to the maximum value of the moment.

(Refer Slide Time: 21:00)



As we go on increasing the load, we find more number of flexural cracks and those cracks have increased in size. We also find the formation of some flexure–shear cracks; cracks are generating due to flexure, but then they are propagating due to the effect of shear. Such types of cracks are initially perpendicular to the bottom edge, but as they increase, they get inclined to the neutral axis. We also observe some web shear cracks, which occur due to the large value of shear near the supports. Near the supports, the

moment is very small and these cracks form at the neutral axis because the shear stress is maximum at the neutral axis. There is no flexural stress at the neutral axis. Moreover the flexural stress near the supports is very small because the moment is very small. Hence the cracks that are initiated by shear alone are near the supports, and they are inclined to the neutral axis at an angle of 45°. If we load the beam further then we find a complete picture of the types of crack that is generated in the beams.

(Refer Slide Time 22:15)



In the middle region, we have the flexural cracks which are perpendicular to the axis of the beam, and they start from the bottom and propagate upwards. On the two sides of the flexural cracks, we have flexure shear cracks; they start at the bottom, initially they are perpendicular but then they gradually get inclined due to the effect of shear. These types of cracks are at transition between the web shear cracks and the flexural cracks. Near the supports of the beam we find web shear cracks, they generate near the neutral axis of the beam and are inclined to the neutral axis. These are the three types of cracks that are generated in a long span beam under uniformly distributed load.

# (Refer Slide Time 23:23)



The crack pattern can be predicted from the principal stress trajectories. In this figure, the solid lines give the trajectories of the compressive stress and the dotted lines give the trajectories of the tensile stress. As we know that the cracks form perpendicular to the tensile stress, or they are parallel to the compressive stress. Hence, the formation of the cracks follows the stress trajectories of the compressive stress. We find that near the middle span the cracks are perpendicular, whereas when we go closer to the support the cracks at the neutral axis are inclined. Thus, the crack pattern in a reinforced concrete beam can be explained by the help of the trajectories of the principal stresses in a homogeneous beam.

(Refer Slide Time: 24:52)



For a simply supported beam, under a uniformly distributed load without prestressing, three types of cracks are identified.

- 1) Flexural cracks: these cracks form at the bottom near the mid span and propagate upwards.
- 2) Web shear cracks: these cracks form near the neutral axis close to the support and propagate inclined to the beam axis.
- 3) Flexure–shear cracks: these cracks form at the bottom due to flexure and propagate due to both flexure and shear.

Next, we move on to understand the components of shear resistance in a reinforced concrete beam. This is studied based on the internal forces at a flexure shear crack. The components are as follows.

# (Refer Slide Time: 25:05)



Thus, we are seeing the free body of a section of reinforced concrete beam. This section has been drawn as per the direction of a flexural shear crack. It is perpendicular to the bottom edge, but as we go up this crack is getting inclined to the neutral axis. What we observe is that the shear stress is resisted by several components in the beam. The first component of shear resistance is due to the concrete under compression.

(Refer Slide Time: 26:04)



Above the tip of the crack, the concrete is uncracked and this part of the concrete can carry shear just like a homogeneous material. Next, since the crack traverses the stirrups, the stirrups carry shear after the cracking of concrete and that is represented by  $V_s$ . The crack surface is not smooth, it is an irregular jagged surface. There due to the interlocking of the aggregates we have a component called aggregate interlock and that is denoted as  $V_a$ .

Next, there is the dowel action; that means, whenever a longitudinal bar is subjected to deformation along a direction perpendicular to its axis, then it generates some resistance which is called the dowel action. This action generates when the bar is properly inserted within the concrete, and the bond has not deteriorated. The dowel action also provides some resistance to shear.

For a prestressed concrete beam, we have another component, which is the vertical component of the prestressing force for an inclined tendon, and that is represented as  $V_p$ . Thus, the value of  $V_p$  depends on the inclination of the tendons.

(Refer Slide Time 28:47)



To summarize, the components of the shear resistance are as follows:  $V_{cz}$  is the shear carried by uncracked concrete,  $V_a$  is the shear resistance due to aggregate interlock,  $V_d$  is the shear resistance due to dowel action,  $V_s$  is the shear carried by stirrups, and  $V_p$  is the

vertical component of prestressing force in inclined tendons. Once we have identified these components, we have to understand their individual contribution in the shear resistance of the member.

The magnitude and the relative value of each component change with increasing load. In the initial stage after the cracking, the aggregate interlock and the dowel actions are high. But as the load is increased, as the cracks open up, the aggregate interlock gets reduced. As the bond between the longitudinal bars and the concrete gets disrupted near the cracks, the dowel action gets reduced. The zone of concrete under compression gets reduced with the propagation of the cracks, thus  $V_{cz}$  gets reduced. Finally, it is the  $V_s$  which increases and resists the external shear.

Next, we are studying the modes of failure due to shear.

(Refer Slide Time 30:04)



For beams with low span-to-depth ratio or inadequate shear reinforcement, the failure can be due to shear. A failure due to shear is sudden as compared to a failure due to flexure. Earlier, when we studied the behaviour of a beam under flexure, we had studied the moment versus curvature curve, and we had studied ductility. After the steel yields, the section is still able to resist the moment with some large deformation. That is called ductility in the moment versus curvature behaviour of a beam. Unlike the behaviour under flexure, the failure under shear is quite sudden, it is a brittle failure. The cracks open up suddenly and the beam tends to fail in a brittle mode.

(Refer Slide Time 31:07)



Five modes of failure due to shear are identified. The occurrence of a mode of failure depends on the span-to-depth ratio, loading, cross-section of the beam, amount and anchorage of reinforcement. The modes of failure are explained next.

(Refer Slide Time 31:27)



The first one is the diagonal tension failure. In this mode, an inclined crack propagates rapidly due to inadequate shear reinforcement. That means, if the shear reinforcement is not adequate enough, the cracks that have formed due to shear or flexural shear will propagate through the depth of the beam quickly. We see that the right part of the beam has got literally separated from left part. This is an instance of a failure due to shear, and this type of failure is called the diagonal tension failure.

(Refer Slide Time 32:14)



The second one is the shear compression failure: There is crushing of the concrete near the compression flange, above the tip of the inclined crack. If the concrete near the top flange is not strong enough, then as the inclined crack propagates there will be crushing of the concrete at the top of this crack. This will also be affected by the amount of moment at that particular section. When there is crushing of the concrete at the top of a flexural shear crack, that type of failure is called a shear compression failure.

(Refer Slide Time 32:58)



The third type of failure is the shear tension failure. Due to inadequate anchorage of the longitudinal bars, the diagonal crack propagates horizontally along the bars. Once the crack hits the level of the bars, it propagates horizontally towards the supports and the bond between the concrete and steel gets disrupted. It leads to an anchorage failure. This type of failure is called the shear tension failure, which leads to a separation of the longitudinal bars with the concrete.

(Refer Slide Time 33:57)



The fourth type of failure is the web crushing failure: the concrete in the web crushes due to inadequate web thickness. Thus, this type of failure can occur in an I-girder if the width of the web is small. Since, there is a principal compression in an inclined direction, if the web is not strong enough then it will crush due to the principal compression.

(Refer Slide Time 35:00)



The fifth type of failure, the arch rib failure is observed in deep beams, where the spanto-depth ratio is small, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing. This type of failure due to the 'arch action' within the deep beam is called an arch rib failure.

## (Refer Slide Time 35:28)



The objective of design for shear is to avoid shear failure. The beam should fail in flexure at its ultimate flexural strength. Hence, each mode of failure is addressed in the design for shear.

The design involves not only the design of the stirrups but also limiting the average shear stress in concrete, providing adequate thickness of the web and adequate development length of the longitudinal bars.

Once we have understood the modes of failure, we can understand the objectives of shear design. The shear design includes not only the design of stirrups, but it includes other detailing requirements which check the different modes of shear failure. We have to have adequate thickness of the web, we have to have adequate anchorage of the longitudinal bars, and we have to have the stress in the concrete limited, because if the concrete crushes then it will lead to a sudden shear failure. As the shear failure is brittle, we do not allow shear failure before the concrete reaches its flexural strength. The objective of the design is that a beam should fail in flexure, and it should not fail in shear before it reaches its ultimate strength.

(Refer Slide Time 37:13)



Next, we are studying about the effect of prestressing force. In presence of prestressing force, the flexural cracking occurs at a higher load. For Type 1 and Type 2 sections, there is no flexural crack under service loads. This is evident from the typical moment versus curvature curve for a prestressed section.

(Refer Slide Time 37:44)



We had seen these curves earlier; for a reinforced concrete beam, the moment versus curvature curve passes through the origin. After cracking, there is redistribution of stresses which increases the curvature. Then we observe non-linearity in the behaviour due to the non-linearity of concrete. Once the steel yields, then we observe the non-linearity of the steel as well and gradually the curve tapers off to the ultimate strength.

If we have prestressing in the beam, the curve gets shifted from the origin, which means that for zero curvature we need to have an external moment. In other words, in absence of any moment there is a negative curvature which is the cambering effect due to prestressing. As we increase the load, the linear behaviour is up to a much higher level. Usually the prestressed concrete members are designed such that it does not crack under service loads. This type of sections has a cracking load which is higher than the service load. After cracking, we observe the non-linearity in the behaviour and the curve tapers off to the ultimate strength. These two curves have been drawn for a reinforced concrete section and a prestressed concrete section which have equal flexural strength.

(Refer Slide Time 39:35)



In presence of prestressing force, the web shear cracks also generate under higher load. Thus, whenever there is a prestressing force, both the flexural and the web shear cracks occur at higher loads. With increase in the load beyond the cracking load, the cracks generate in a similar sequence; for a long beam, we will observe flexural cracks to occur first, then we shall observe some flexural cracks to change to flexure–shear cracks, then we may observe web shear cracks near the supports. The sequence is similar but the inclination of the flexure–shear and the web shear cracks are reduced, depending on the amount of prestressing and the profile of the tendon.

(Refer Slide Time 40:36)



Let us understand the effect of prestressing for a beam with a concentric prestressing force. This is a simply supported beam with a uniformly distributed load subjected to a concentric prestressing force  $P_e$ . We shall observe the state of stress for an element which is at the neutral axis. For a point at the neutral axis which is represented by Element 1, there is normal stress due to the prestressing force which is compressive and denoted as  $-f_{pe}$ . This is the main difference with reinforced concrete, where at the neutral axis there is no normal stress, it is a case of pure shear stress. For a prestressed concrete beam, we are not only having a shear stress at the neutral axis, but we are also having a compressive stress due to the prestressing force.

#### (Refer Slide Time 41:41)



The state of stress is such that the principal tensile stress  $\sigma_1$  is inclined to the neutral axis at an angle greater than 45°. The principal tensile stress  $\sigma_1$  is much smaller compared to the principal compressive stress  $\sigma_2$ . If we plot the Mohr's circle for this state of stress, then the point corresponding to the vertical surface has the shear stress (v) and also the normal stress ( $-f_{pe}$ ). The point lying in the vertical axis is for the horizontal surface, which has only the shear stress. We find that the magnitude of  $\sigma_1$  is much smaller than the magnitude of  $\sigma_2$ . Since  $\sigma_1$  is inclined at an angle greater than 45°, the crack which occurs perpendicular to  $\sigma_1$  (or parallel to  $\sigma_2$ ) is inclined at an angle which is lower than 45° in presence of the prestressing force.

# (Refer Slide Time 43:06)



Thus, for a prestressed concrete beam also, the formation of the cracks can be explained based on the state of stress. If the prestressing force is eccentric, and if the profile of the tendon is parabolic, then the analysis gets more involved, but the basic concept remains the same, that due to the effect of prestressing force the inclination of the cracks due to shear gets reduced. The principal tensile stress is much smaller than the principal compressive stress. Hence, the cracking occurs at a much higher level as compared to a reinforced concrete beam.

The crack pattern that occurs in a prestressed concrete beam is shown in this figure. Here you can see that the lengths of the flexural cracks are much smaller as compared to those in a reinforced concrete beam. The number of cracks is also small. The flexure–shear cracks are more inclined. The web shear cracks are also small, they are more inclined to the neutral axis as compared to the cracks in a reinforced concrete beam.

# (Refer Slide Time 44:45)



If we compare the cracking pattern in the two types of beams, what we observe is that in a reinforced concrete beam, the flexural cracks are much more compared to a prestressed concrete beam. The growth of the flexural cracks is more than in a prestressed concrete beam for the same load. The flexure–shear cracks are more inclined in a prestressed concrete beam. The growth of these cracks is also less. The web shear cracks are at an angle of  $45^{\circ}$  in the reinforced concrete beams, whereas for prestressed concrete beams the inclination is smaller.

Thus, the amount of cracking in a prestressed concrete beam is much lower as compared to a reinforced concrete beam. This is the big benefit of prestressing. (Refer Slide Time 45:55)



After cracking, in presence of prestressing force, the length and crack width of a diagonal crack are low. Thus, the aggregate interlock and zone of concrete under compression are larger as compared to a non-prestressed beam under the same load. Hence, the shear strength of concrete which is represented as  $V_c$ , increases in presence of prestressing force. This is accounted for in the expression of  $V_c$ .

In addition, there is also the vertical component of the prestressing force if the tendon is inclined. Thus, the shear capacity of a prestressed concrete beam is larger than a reinforced concrete beam. This helps to check the different modes of failure under shear, provided the detailing is proper. The beam is expected to reach its ultimate flexural capacity before it fails under shear.

(Refer Slide Time 48:33)



In today's lecture, we studied the analysis for shear. First, we studied the stress condition in an uncracked beam based on the basic structural analysis for a simply supported beam under uniformly distributed load. The shear force is maximum near the supports and minimum near the mid span, whereas the moment is minimum near the supports and maximum at the mid span. Regarding the variations of stresses along the depth of a section, the flexural normal stresses are maximum at the top and at the bottom, and minimum at the neutral axis. The shear stress is minimum at the top and the bottom, and maximum at the neutral axis.

Around the mid span, since the moment is high, the flexural stress at the bottom leads to cracking. The flexural cracks start from the bottom of the beam and goes up. Near the supports, the shear stress is high near the neutral axis. Hence, the cracking due to shear occurs near the neutral axis and inclined at an angle of 45° to the axis. In between the web shear cracks and the flexural cracks, there is a transition region where we observe flexure–shear cracks. After the concrete cracks, there is redistribution of stresses. We understood the components of shear resistance from the free body diagram of a concrete beam adjacent to a flexure–shear crack.

From the components of shear resistance, we moved on to the modes of failure due to shear. First, in the diagonal tension failure, if there is inadequate amount of stirrups, the diagonal cracks propagate through the depth of the beam quite rapidly, and this will lead to a sudden failure of the beam. Second, in the shear compression failure, the concrete under compression can crush at the tip of a diagonal shear crack. Third, in the shear tension failure, if the longitudinal bars are not properly anchored near the support, then a diagonal crack after reaching the level of bars, propagates horizontally. In the fourth type of failure, web crushing occurs for beams with thin webs. The fifth type of failure is observed for deep beams where the span-to-depth ratio is small. Due to the arch action, there is crushing of the concrete in the web and near the bearing. It can also have an anchorage failure.

Next, we moved on to the effect of prestressing force. The additional compressive normal stress influences the principal stresses and their inclination. The principal tensile stress is much smaller in magnitude than the principal compressive stress. The inclination of the cracking is lower than 45°. Cracks are much less in a prestressed concrete beam as compared to a reinforced concrete beam. This has a direct benefit in the shear resistance of the section. The aggregate interlock is sustained, the zone of concrete under compression is higher and the dowel action is also retained. Thus, the shear capacity of a concrete beam increases in presence of prestressing force.

The objective of shear design is to check the modes of shear failure, such that the beam attains its flexural capacity before it fails in shear. It involves not only the design of stirrups, but also other detailing to check the different modes of failure.

In our next class, we shall move on to the formulation of the shear resistance, from which we are able to design for shear in a prestressed concrete beam.

Thank you.