PRESTRESSED CONCRETE STRUCTURES

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Module - 4: Design of Members

Lecture – 21: Magnel's Graphical Method

Welcome back to prestressed concrete structures. This is the fifth lecture of Module 4 of design of members. In this lecture, we shall study the design of sections for flexure by a special method called Magnel's graphical method.

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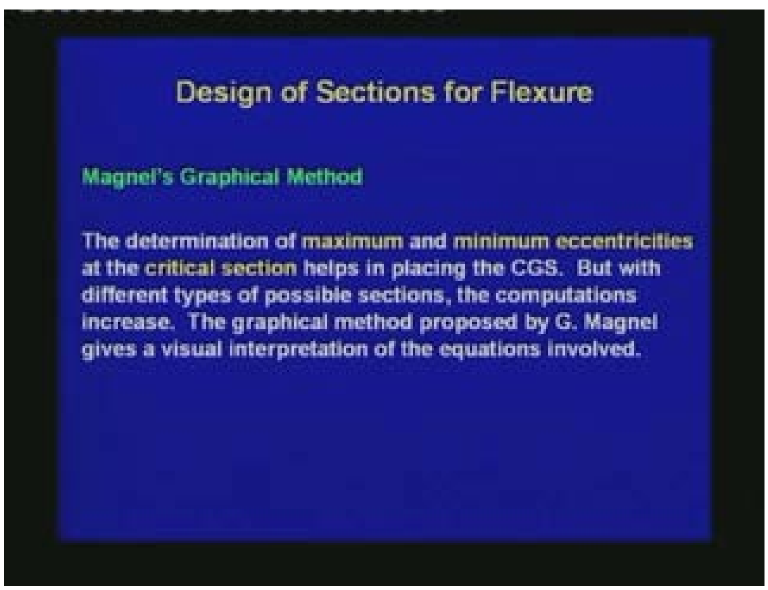

The determination of maximum and minimum eccentricities at the critical section helps in placing the CGS in the beam. But with different types of possible sections, the computations increase. The graphical method proposed by G. Magnel gives a visual interpretation of the equations involved. What we have studied till now is the computation of the maximum and minimum eccentricities at the critical sections of a beam; from there we can place the CGS and the tendons. We also learnt about the limiting zone of placing the CGS throughout the span of the beam. But if we are trying with several types of sections, then the computations increase. At that time, it is better that if we can adopt some faster method of computation. Magnel's graphical method is one such type, which can be implemented in a computer program. The sections can be verified for their possible application, and the combination of the prestressing force and the eccentricity can be selected quickly. Once this method is implemented in a computer program, the graphical method is faster than the hand computations.

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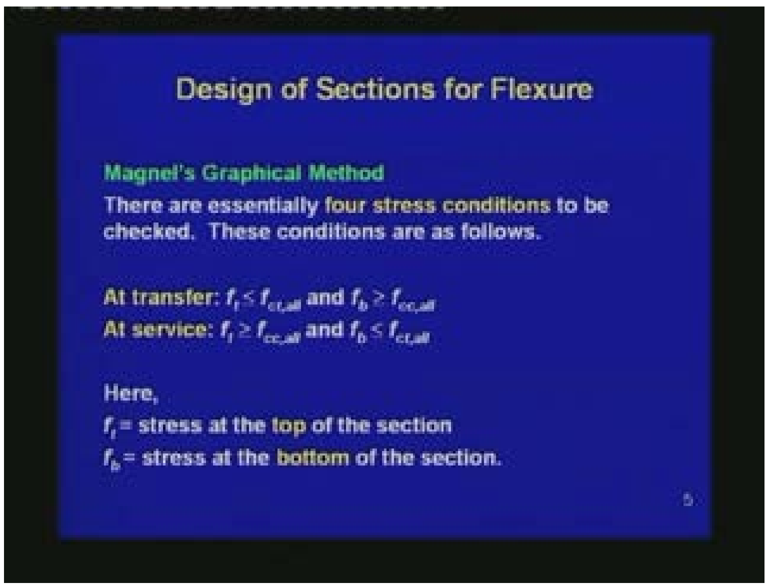

There are essentially four stress conditions to be checked at the critical section. These conditions are as follows: at transfer, we need to make sure that the tensile stress at the top of the section (f_t) is less than or equal to the allowable tensile stress for the concrete ($f_{ct,all}$), and the compressive stress at the bottom (f_b) should be greater than or equal to the allowable compressive stress for the concrete ($f_{cc,all}$). The equations are algebraic equations; a compressive stress is considered to be negative. Hence, the equation $f_b \geq f_{cc,all}$ means that the negative value of f_b is larger than or equal to the negative value of $f_{cc,all}$. In other words, the absolute value of f_b is smaller than or equal to the absolute value of $f_{cc,all}$.

At service, the compressive stress at the top (f_t) should be greater than or equal to the allowable compressive stress at service ($f_{cc,all}$). In other words, the absolute value of f_t has to be smaller than or equal to the absolute value of $f_{cc,all}$. The other stress condition is that the tensile stress at the bottom (f_b) should be less than or equal to the allowable tensile stress at service.

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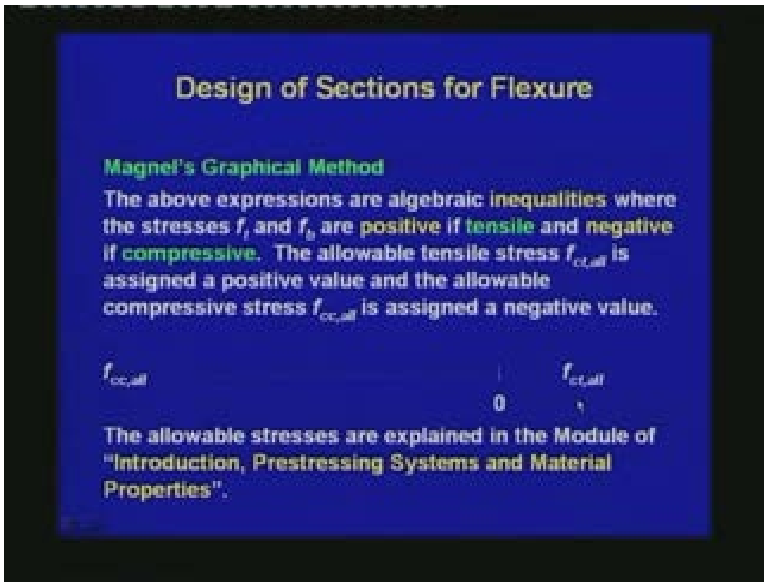

To repeat, the above expressions are algebraic inequalities, where the stresses f_t and f_b are positive if tensile, and negative if compressive. The allowable tensile stress $f_{ct,all}$ is assigned a positive value and the allowable compressive stress $f_{cc,all}$ is assigned a negative value. Thus, if we plot the stress values along a number axis, $f_{ct,all}$ falls on the right (positive) side of 0 and $f_{cc,all}$ falls on the left side of 0. These are the two limits of the stress conditions. If any tensile stress has to be less than $f_{ct,all}$, then the equation states that the stress is less than $f_{ct,all}$. Whereas, when a compressive stress has to be numerically less than $f_{cc,all}$, then the equation states that the stress is greater than $f_{cc,all}$, since as we move from left to right along the number axis, the values increase in an algebraic sense.

The allowable stresses were explained in the module of "Introduction, Prestressing Systems and Material Properties".

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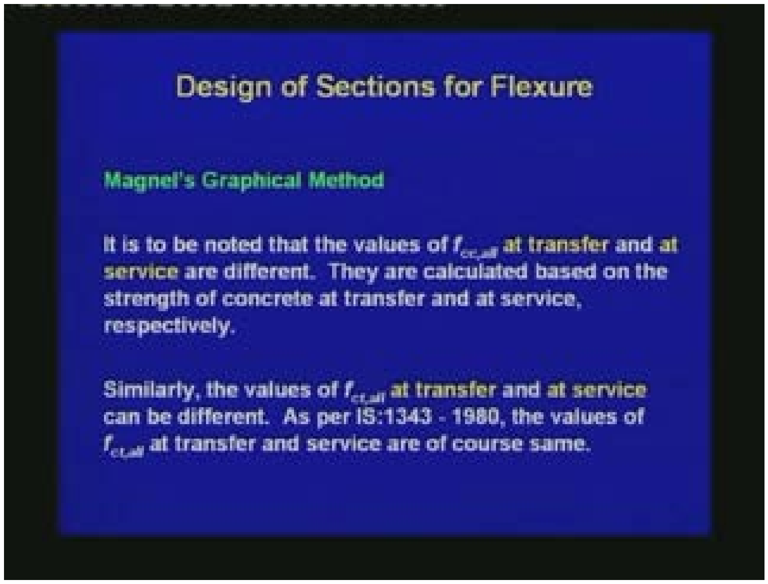

It is to be noted that the values of $f_{cc,all}$ at transfer and at service are different. They are calculated based on the strength of concrete at transfer and at service, respectively. Similarly, the values of $f_{ct,all}$ at transfer and at service can be different. As per IS:1343 – 1980, the values of $f_{ct,all}$ at transfer and at service have been made same based on the reason that the values are small.

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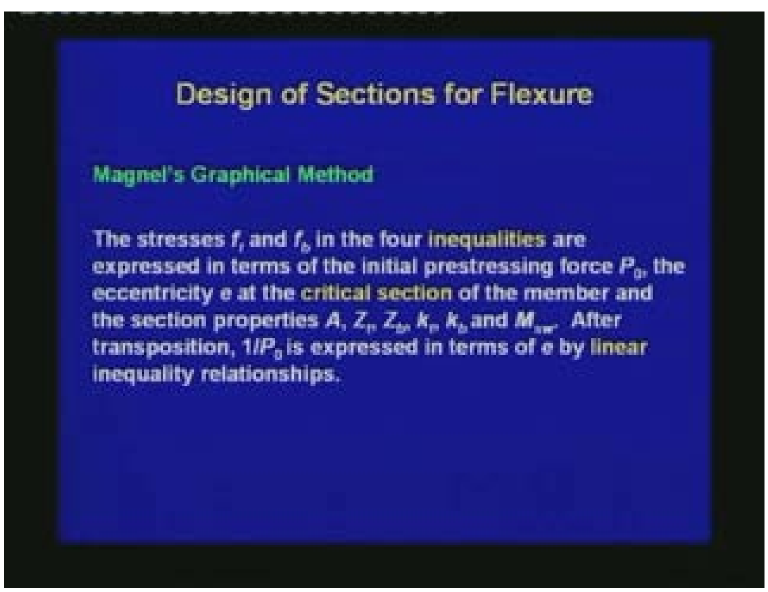

The stresses at the top and at the bottom in the four inequalities are expressed in terms of the initial prestressing force P_0, the eccentricity e at the critical section of the member and the section properties: area A, section moduli Z_t, Z_b; kern levels k_t, k_b; and self-weight moment M_{sw}. After transposition, $1/P_0$ (the inverse of the prestressing force at transfer) is expressed in terms of the eccentricity e by linear inequality relationships.

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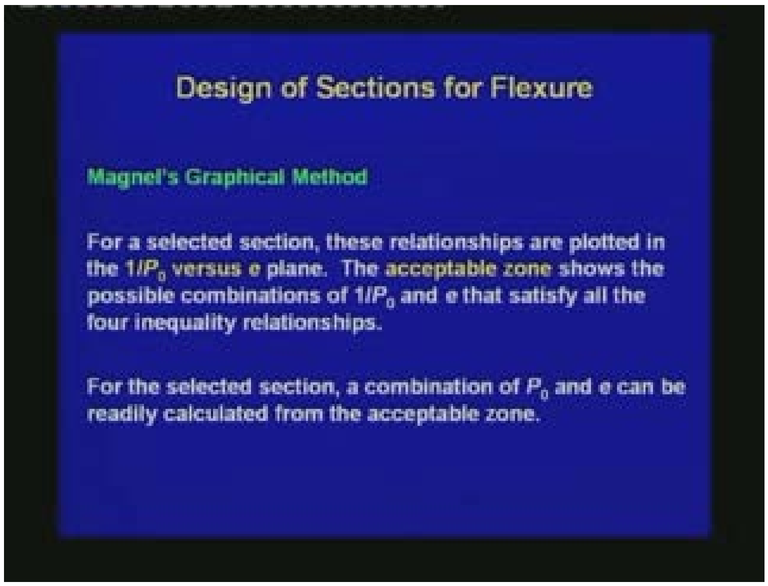

For a selected section, these relationships are plotted in the $1/P_0$ versus e plane. The acceptable zone shows the possible combinations of $1/P_0$ and e that satisfy all the four inequality relationships. For the selected section, a combination of P_0 and e can be readily calculated from the acceptable zone.

Let us now understand this method in a step-by-step fashion and try to have the concept cleared by the visual interpretation of the inequality relationships.

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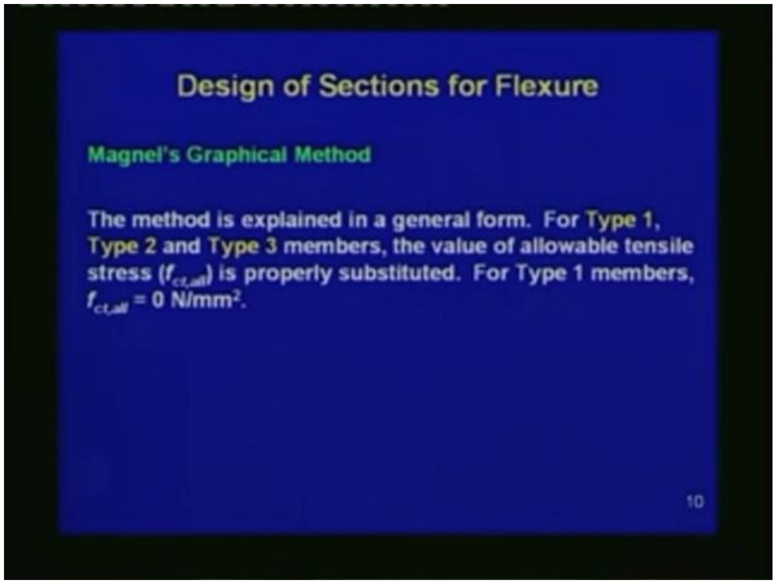

The method is explained in a general form. For Type 1, Type 2 and Type 3 members, the value of allowable tensile stress is properly substituted. For Type 1 member, the allowable tensile stress is 0. For Type 2 members, the allowable tensile stress is such that it is less than the cracking stress. For Type 3 members, the allowable tensile stress can be greater than the cracking stress, but the crack width should be limited, and hence there is an upper limit of the allowable tensile stress.

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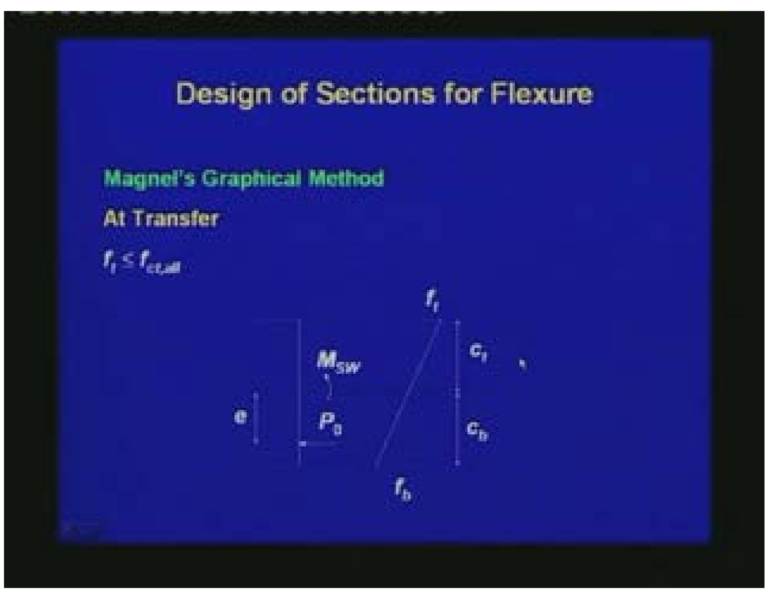

First, let us see the equations at transfer. The forces acting on the concrete section are the prestressing force P_0 below the CGC, and the self-weight moment M_{sw}. The stress diagram across the depth of the section has a value of f_t at the top, which can be tensile and a value of f_b at the bottom, which is compressive. The distances of the top and the bottom from the CGC are denoted as c_t and c_b, respectively.

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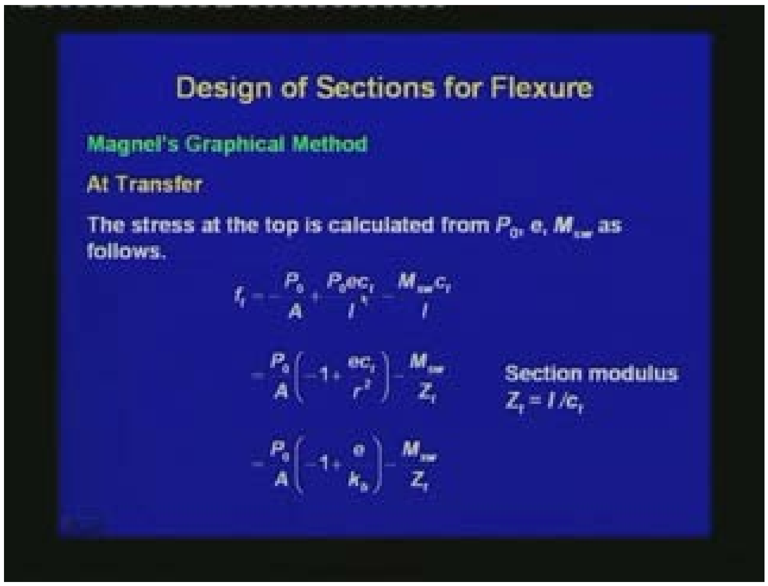

The stress at the top is calculated from P_0, e, M_{sw} as follows. We are using the stress concept for determining the value. The stress at the top consists of three components. First is the uniform component which is $-P_0/A$. Second is the component due to the eccentricity of the prestressing force P_0ec_t/I, which is tensile at the top. The third term which is due to the self-weight is compressive ($-M_{sw}c_t/I$). We are grouping the terms with P_0 together. For the term with the self-weight moment, we are substituting the section modulus Z_t which is equal to I/c_t.

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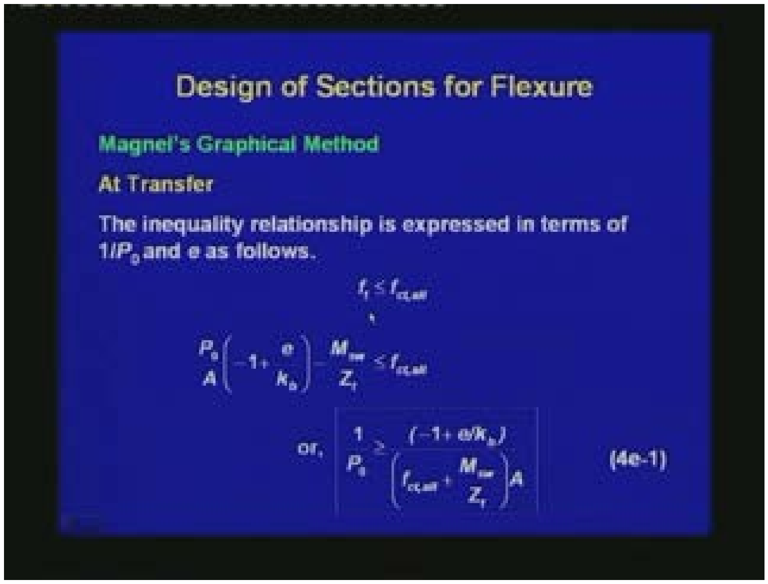

Next, the inequality relationship is $f_t \le f_{ct,all}$. Once, we substitute the expression of f_t in this inequality relationship, we get a relationship which relates P_0 with e. To get that relationship, we are transposing the terms such that on the left hand side of the inequality we are retaining $1/P_0$ and on the right hand side we are keeping the eccentricity e, the geometric variables k_b, Z_t and A, the self-weight moment M_{sw} and the allowable tensile stress $f_{ct,all}$. Thus, we have an equation from the first stress condition, which relates $1/P_0$ with e. This is an inequality relationship; that means to satisfy the stress condition, $1/P_0$ has to be greater than or equal to the value which is calculated from the right hand side.

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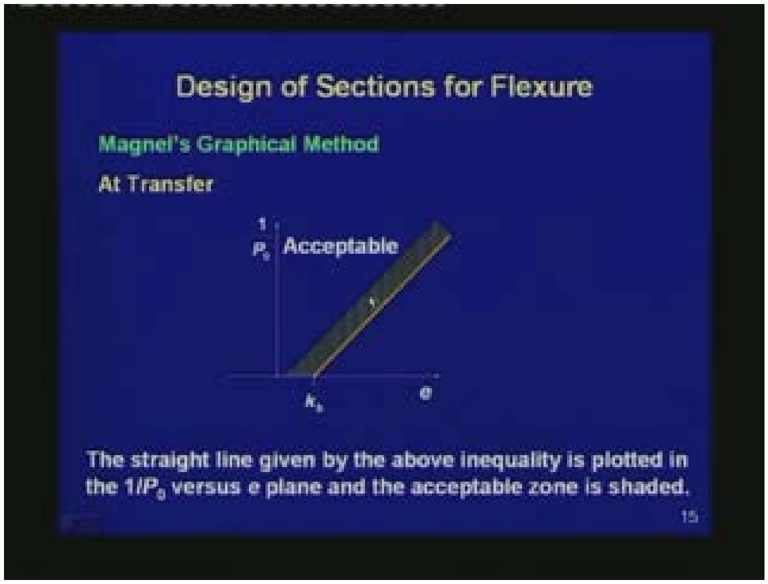

The straight line given by the above inequality is plotted in the $1/P_0$ versus e plane, and the acceptable zone is shaded. If we look back into the equation we see that, if $e = k_b$ on the right hand side, then the right hand side becomes 0. Hence, we can understand that the straight line passes through the point k_b and at that instant, $1/P_0 = 0$. Since, in the inequality relationship $1/P_0$ has to be greater than the value on the right hand side, the acceptable zone is the zone above the line. Any combination of $1/P_0$ and e which falls above the line will satisfy the stress condition that $f_t \leq f_{ct,all}$ at transfer.

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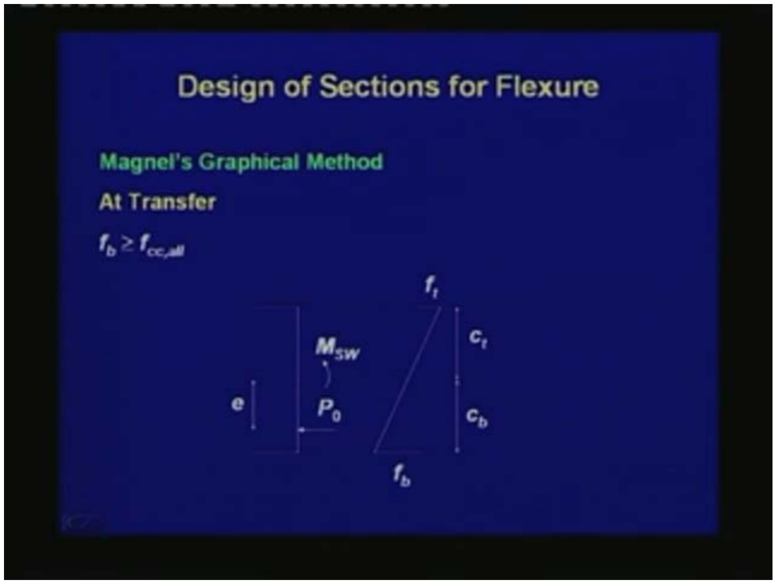

The second stress condition at transfer is that $f_b \geq f_{cc,all}$ in an algebraic sense, because both these values are negative.

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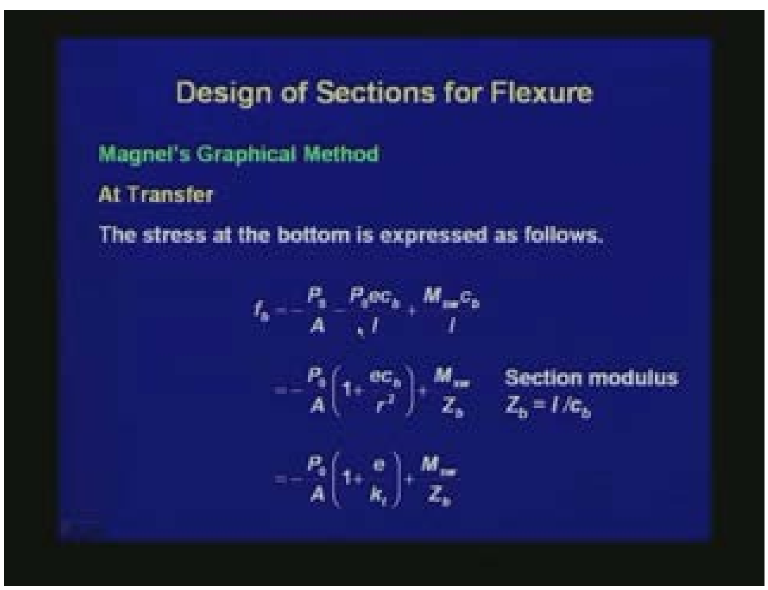

We are calculating the stress f_b just as before. It consists of the uniform component, which is $-P_0/A$. Then, it has the varying component ($-P_0ec_b/I$) which is due to the eccentricity of the prestressing force. Since the stress is compressive, we are placing a negative sign. The component due to the self-weight ($M_{sw}c_b/I$) is tensile, and hence we are placing a positive value. Again, we are grouping the terms of P_0 together. For the term with M_{sw} we are substituting the section modulus $Z_b = I/c_b$. We get an expression of the stress at the bottom in terms of P_0, e, the section variables and the self-weight moment.

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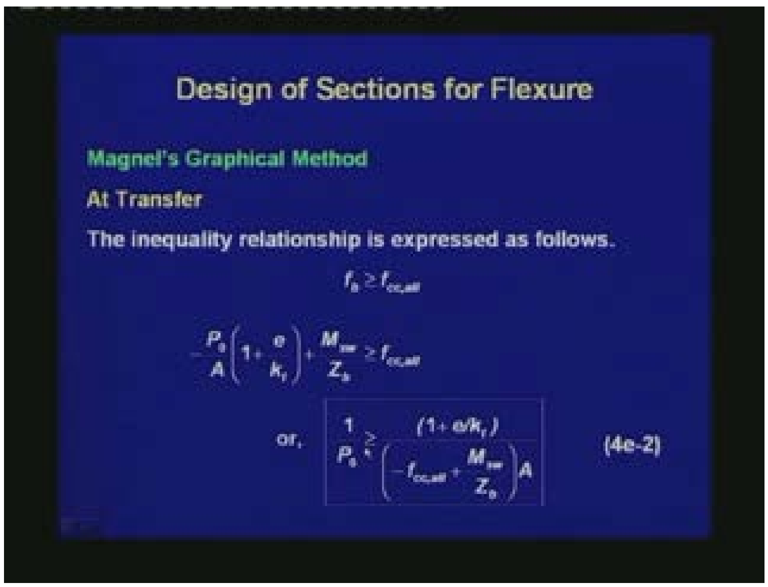

We have to satisfy the inequality relationship that $f_b \geq f_{cc,all}$. Once we substitute the expression of f_b and we transpose the terms in such a way that on the left hand side we have $1/P_0$, and on the right hand side we have all other variables which includes the eccentricity, the section properties and the self-weight moment, we get the second inequality relation. If $e = -k_t$, that is if the CGS is located above CGC at the top kern point, then $1/P_0 = 0$.

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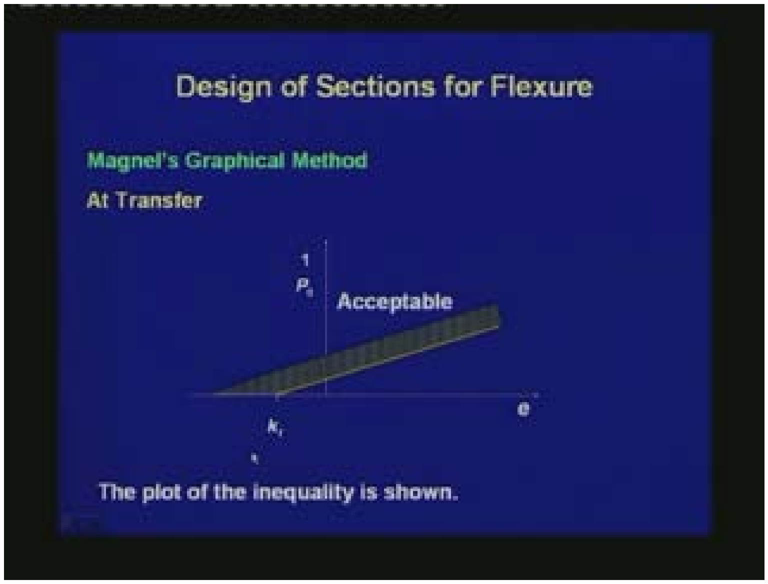

Hence, when we plot the corresponding straight line, it passes through the point $e = -k_t$. Since, in the equation $1/P_0$ has to be greater than the value on the right hand side, any value above the straight line is acceptable.

Next, let us move on to the computations at service.

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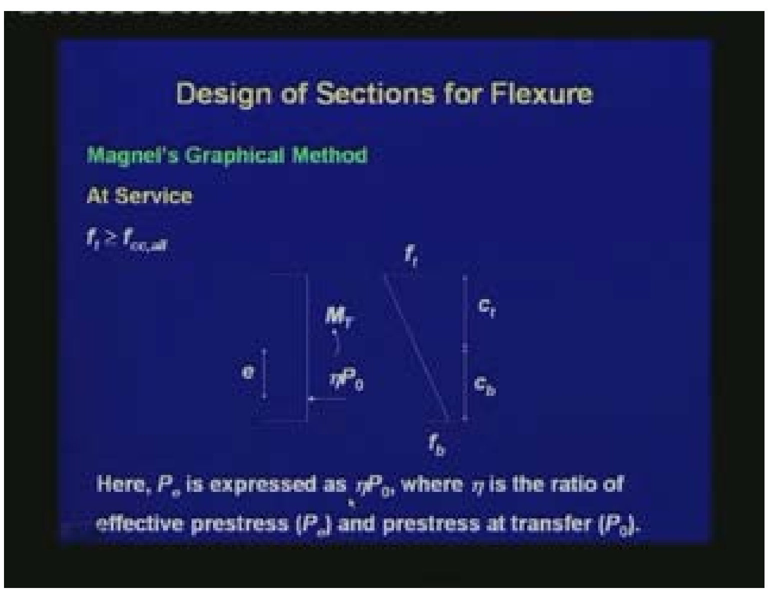

At service, we have to satisfy $f_t \geq f_{cc,all}$; both these values are compressive and hence negative, and this equation is in an algebraic sense. At service, the prestressing force is denoted as ηP_0, where η is a fraction of the initial prestressing force. The prestressing force has reduced due to the long-term losses. M_T is the total moment due to the dead load and live load.

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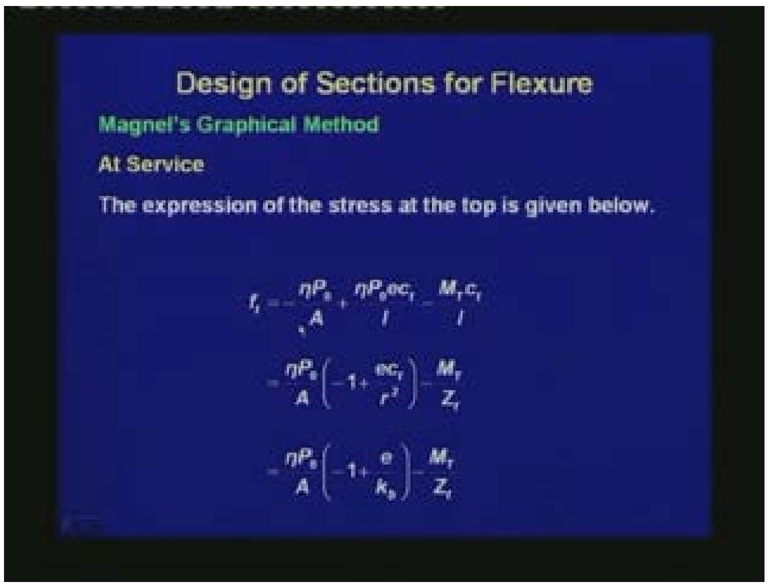

First, we are writing the expression of the compressive stress at the top. The first term is the uniform component, which is $-\eta P_0/A$. The second is the tensile component due to the eccentricity of the prestressing force, which is $\eta P_0 e c_t/I$. The third component is the compressive component due to the total moment, which is $-M_T c_t/I$. We are grouping the terms with P_0 together, and for the term with M_T, we are substituting the section modulus $Z_t = I/c_t$. Once, we have done the substitutions $I = Ar^2$ and $r^2/c_t = k_b$, we get an expression of the stress at the top f_t in terms of the prestressing variables, the section properties and the total moment.

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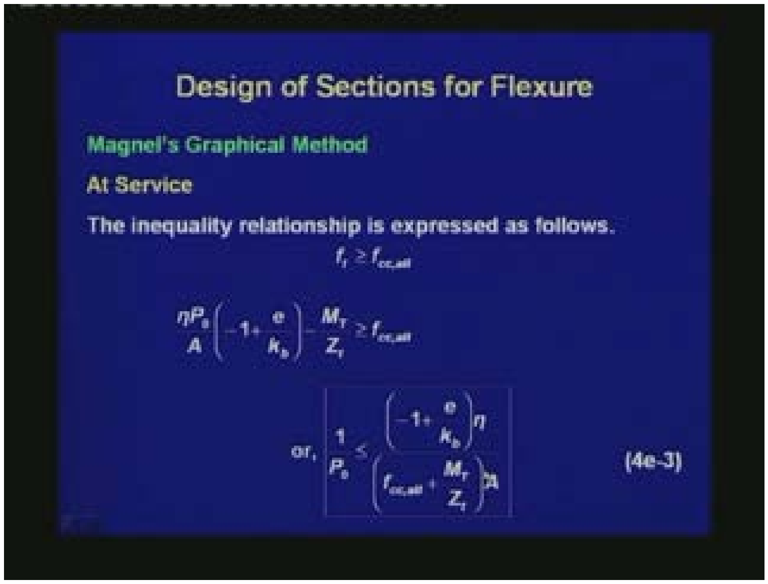

In the inequality relationship $f_t \geq f_{cc,all}$, we are substituting the expression of f_t and then we are transposing the terms such that we have $1/P_0$ on the left and all the other variables on the right side. We arrive at an equation which is the third inequality relationship related with the stress condition at service.

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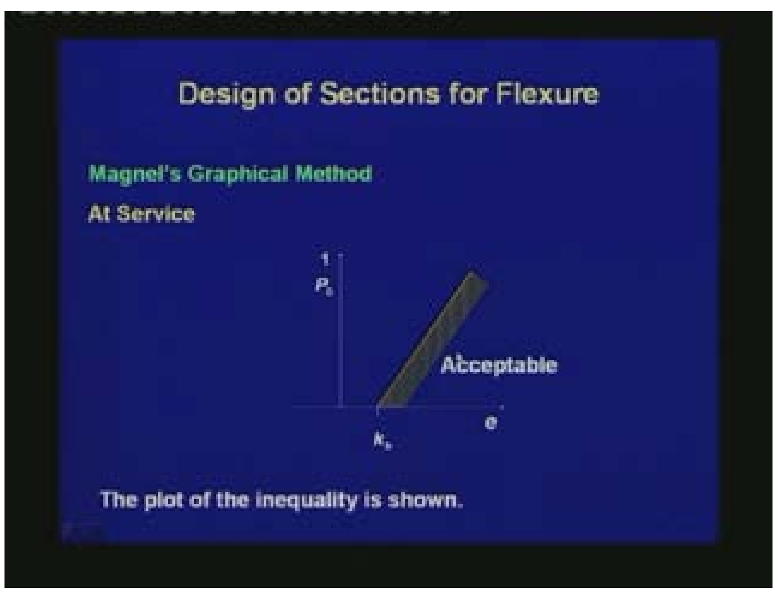

Here, we find that if $e = k_b$, then the numerator on the right hand side becomes 0 and hence $1/P_0 = 0$. Thus, when we plot the straight line, it will pass through the point $e = k_b$. Note that, in this inequality relationship, $1/P_0$ has to be less than or equal to the value on the right hand side. Unlike the equations at transfer, where we had $1/P_0$ to be greater than the value on the right hand side, here we have $1/P_0$ to be less than or equal to the value on the right hand side. Thus, the zone below the straight line is the acceptable region.

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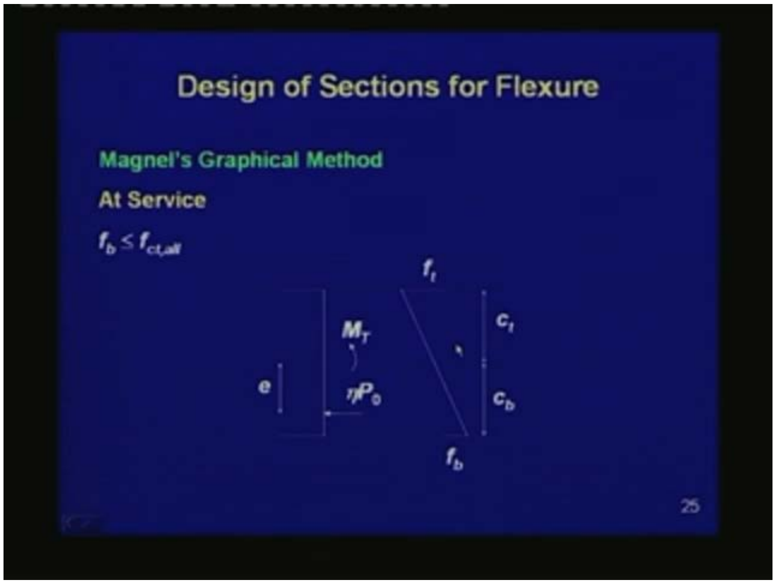

The fourth stress condition is that at service, the bottom stress $f_b \le f_{ct,all}$.

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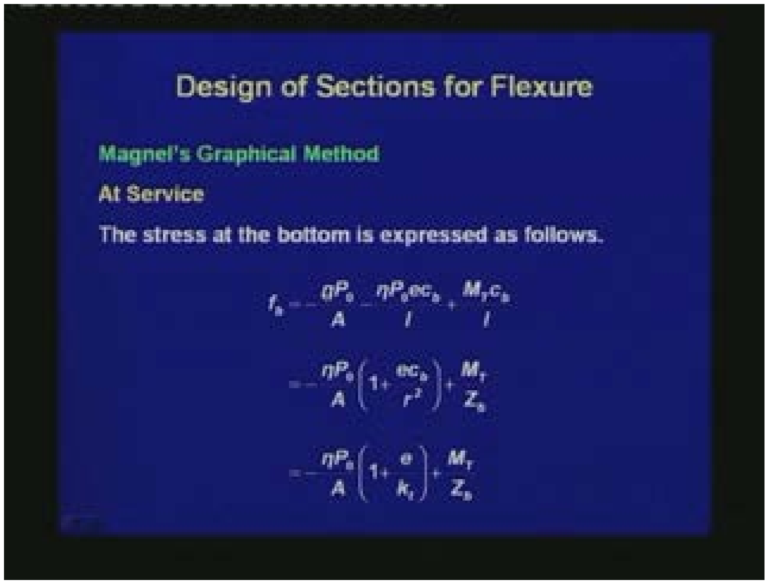

The first term of the bottom stress is the uniform component $-\eta P_0/A$. The second term is $-\eta P_0 e c_b/I$; this is also negative because the eccentric prestressing force causes compression at the bottom. Then, the tensile stress due to the external moment is $M_T c_b/I$. We are grouping the terms with P_0. For the term with M_T, we are substituting $Z_b = I/c_b$. Once we have substituted $I = Ar^2$ and $r^2/c_b = k_t$, we get an expression of the stress in terms of P_0, e, the section properties and the total moment M_T.

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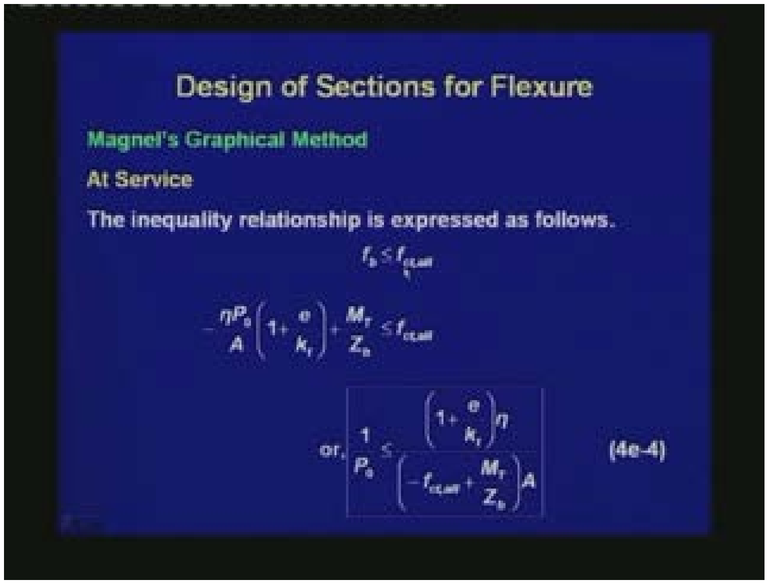

In the inequality relationship $f_b \leq f_{ct,all}$, we are substituting the expression of f_b and again we are transposing the terms such that, we have $1/P_0$ on the left hand side and all the other terms on the right hand side.

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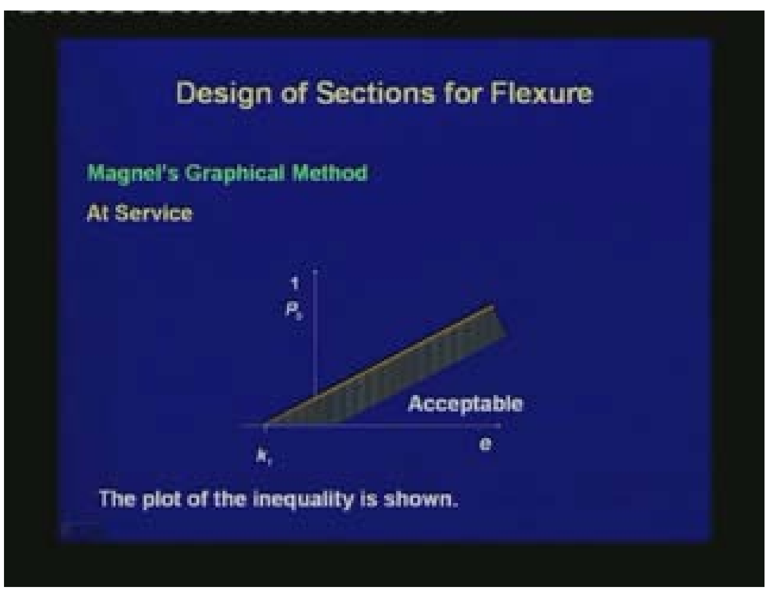

For this equation, we see that if $e = -k_t$ then the numerator will be 0 and $1/P_0 = 0$. Hence, the straight line will pass through the point $e = -k_t$. The region below the straight line is the acceptable region for the combination of $1/P_0$ and e. Thus, we have the fourth inequality relationship and the corresponding straight line in the $1/P_0$ versus e plane. At this stage, we are having four straight lines for the four stress conditions, and now we are superposing those straight lines in the $1/P_0$ versus e plane to determine the common acceptable zone which satisfies all the four stress conditions.

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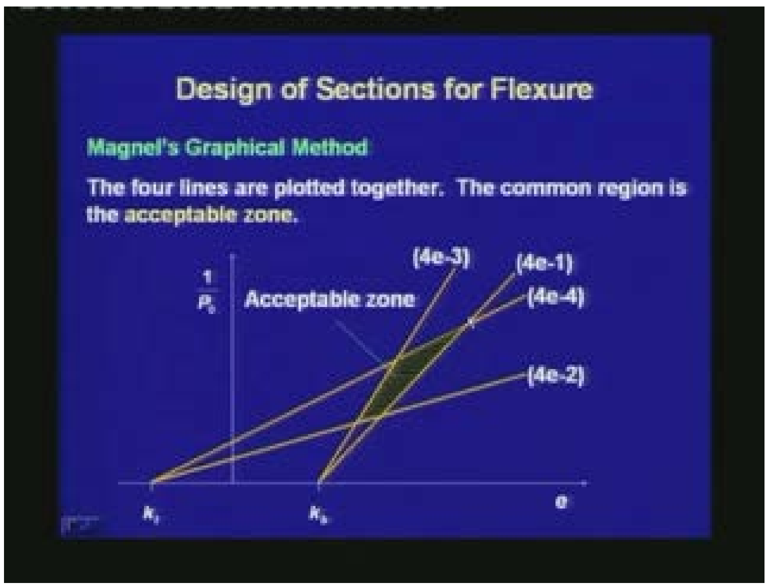

The four lines are plotted together. The common region is the acceptable zone. For the two equations for the stress conditions at transfer, Eqns. 4e-1 and 4e-2, any point lying above the straight lines is acceptable; whereas, for the stress conditions at service, which are represented by Eqns. 4e-3 and 4e-4, any combination which lies below the straight lines is acceptable. The common region which is acceptable for all the four stress conditions is the area which lies in between these four straight lines, and that region is called the acceptable zone.

Thus, for a particular section, we may have an acceptable zone and we have the freedom of selecting values of P_0 and e within that acceptable zone. If there is no acceptable zone for a section, then it implies that, that section is not suitable for the given loading conditions. If the acceptable zone for a section is small, then it implies that we do not have much freedom in the combinations of P_0 and e, and we have to select a value which lies in that acceptable zone. Thus, the graphical method gives a visual interpretation of the stress conditions that we have to satisfy, and it helps us to select the combination of the prestress at transfer (P_0) and the eccentricity (e) from the acceptable zone.

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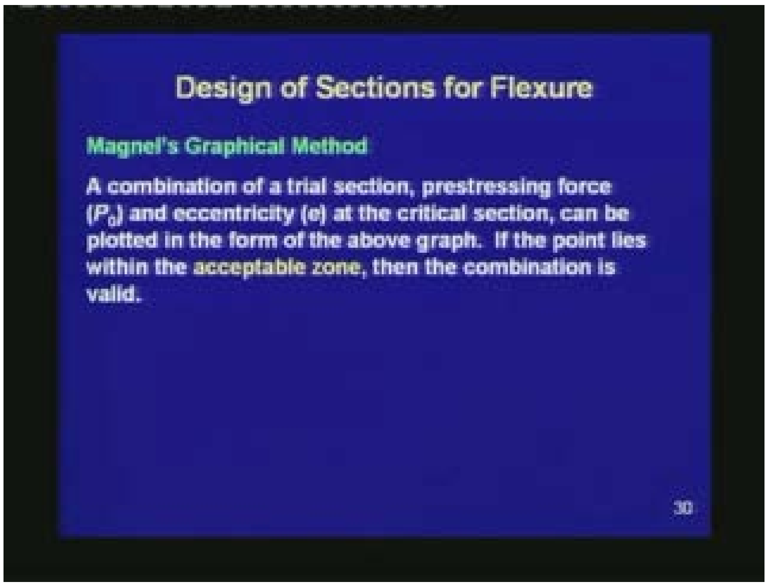

A combination for a trial section, the prestressing force (P_0) and eccentricity (e) at the critical section can be plotted in the form of the above graph. If the point lies within the acceptable zone then the combination is valid, and it will satisfy all the four stress conditions.

Let us understand the Magnel's graphical method by an example problem.

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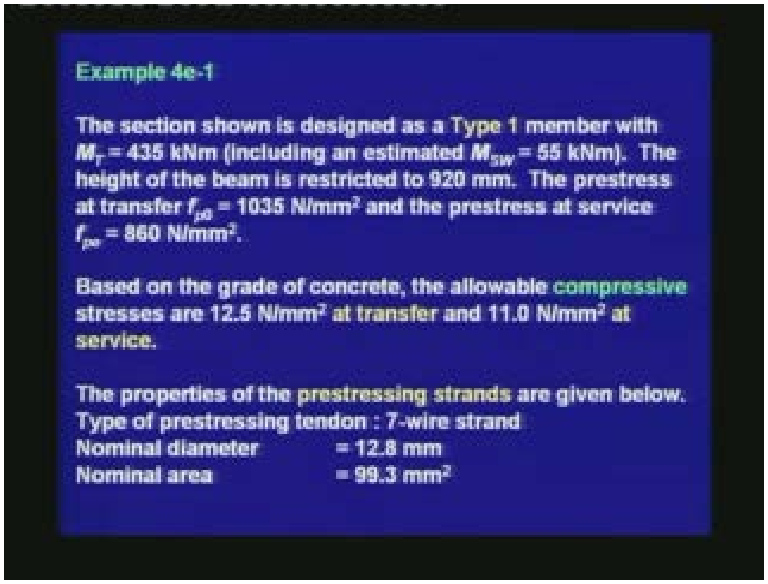

The section shown in the next slide, is designed as a Type 1 member with the total moment M_T = 435 kNm, which includes an estimated self-weight of M_{sw} = 55 kNm. The height of the beam is restricted to 920 mm. The prestress at transfer f_{p0} = 1035 N/mm^2 and the prestress at service f_{pe} = 860 N/mm^2. Based on the grade of concrete, the allowable compressive stresses are 12.5 N/mm^2 at transfer and 11 N/mm^2 at service.

The properties of the prestressing strands are given below. Type of prestressing strand is 7-wire strands; the nominal diameter of each strand is 12.8 mm and the nominal area is 99.3 mm^2.

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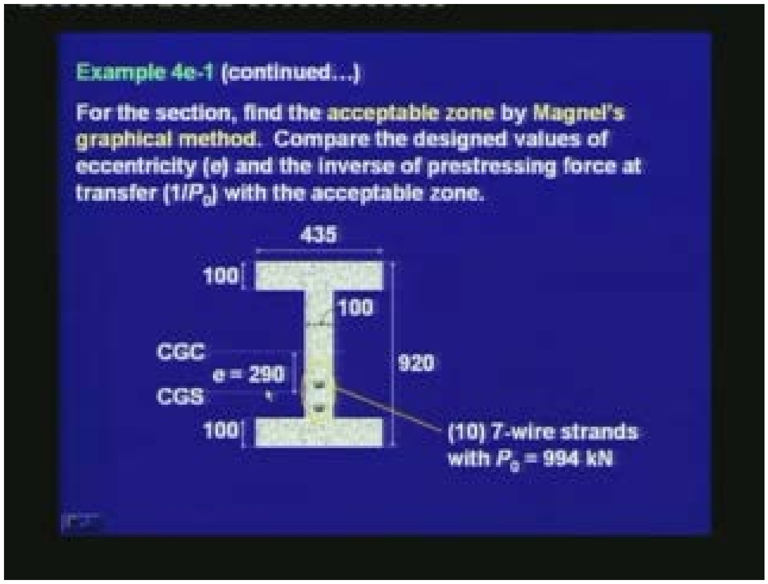

For the section, find the acceptable zone by Magnel's graphical method. Compare the designed values of the eccentricity e and the inverse of prestressing force at transfer (which is $1/P_0$) with the acceptable zone.

This problem we had solved earlier (Lecture 18) and from the preliminary design, we had calculated the type of the section. Then from the final design, we had selected the final dimensions. The flange width was 435 mm. It is a section symmetric about the horizontal axis and we have satisfied the requirement of the height which is 920 mm. The thickness of the web and the flange both are 100 mm. The calculated eccentricity has been rounded off to a value of 290 mm. That means the CGS is located at 290 mm below the CGC. We had selected ten 7-wire strands with P_0 = 994 kN.

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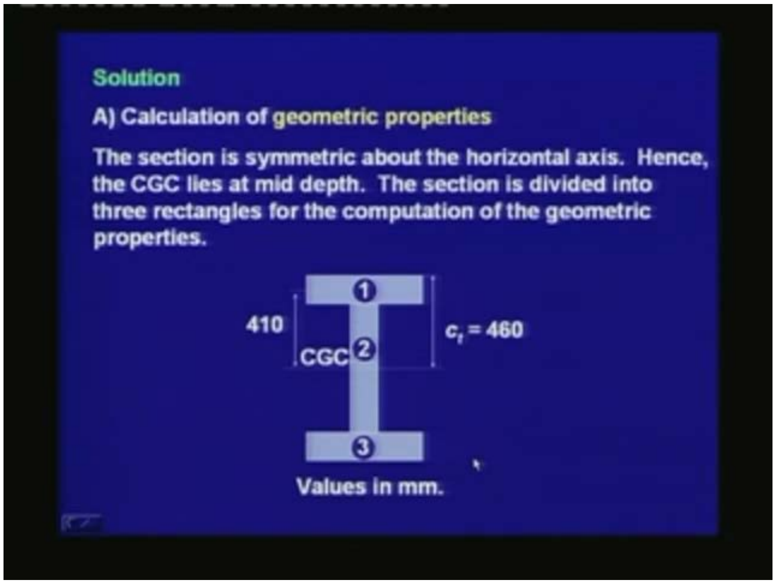

Here, we are checking the section based on the Magnel's method. First, we need to calculate the geometric properties of the section to plot the four lines in the $1/P_0$ versus e plane.

The section is symmetric about the horizontal axis; hence, CGC lies at mid depth. The section is divided into three rectangles for the computation of the geometric properties. From the CGC, the centroid of the top rectangle is at a distance 410 mm. The distances of the top and bottom fibres are 460 mm from the CGC.

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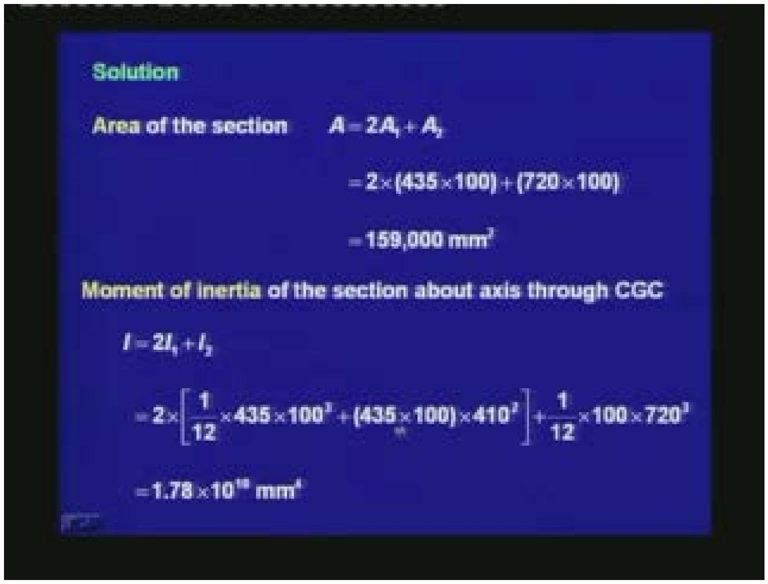

We are calculating the area of the section. Since, A_1 is equal to A_3, the total area is given as $2A_1 + A_2$; A_1 is the flange width times the depth of the flange, which is 435×100; A_2 is equal to the remaining part of the depth of the web times the width of the web, which is 720×100. Once, we substitute the values the area is $A = 159{,}000 \text{ mm}^2$.

Next, we are calculating the moment of inertia of the section about the axis through CGC. Here also, $I = 2I_1 + I_2$. To calculate I_1, we are using the parallel axes theorem. It states that the moment of inertia of the top rectangle about the axis through CGC is equal to the moment of inertia through the centroidal axis of the rectangle, plus the area of the rectangle times the distance squared, where the distance is between the CGC and the centroidal axis of the rectangle.

Thus, $I_1 = 1/12 \times 435 \times 100^3 + (435 \times 100) \times 410^2$. For I_2, the centroidal axis is same as the axis through CGC and hence, $I_2 = 1/12 \times 100 \times 720^3$. After substitution, $I = 1.78 \times 10^{10} \text{ mm}^4$.

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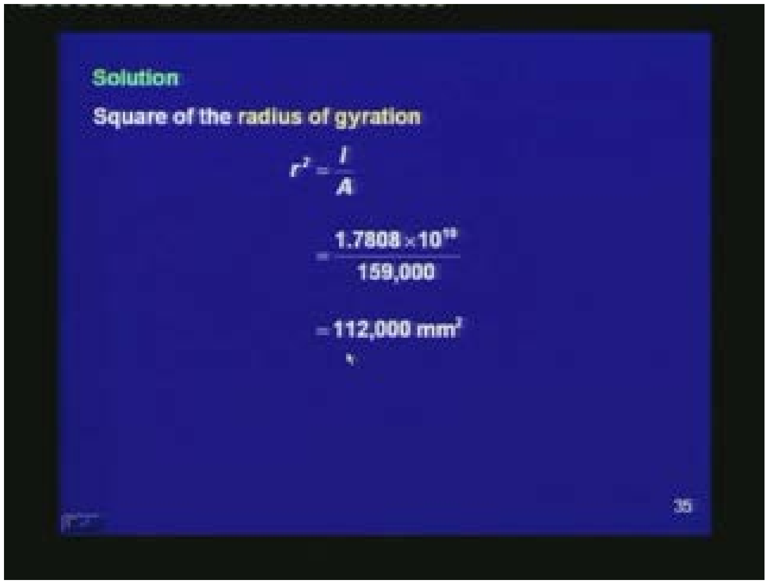

Square of the radius of gyration, $r^2 = I/A$. When we substitute the values of I and A, we find $r^2 = 112,000$ mm^2.

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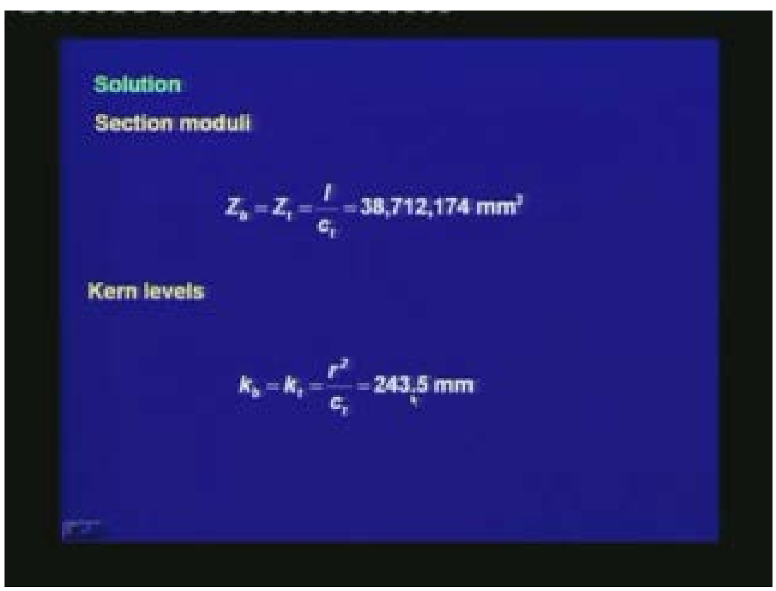

The section moduli $Z_b = Z_t = I/c_t$, and once we substitute the values of I and c_t, we get $Z_b = Z_t = 38,712,174$ mm^3. The kern levels $k_b = k_t = r^2/c_t$. Once, we substitute the values we get $k_b = k_t = 243.5$ mm.

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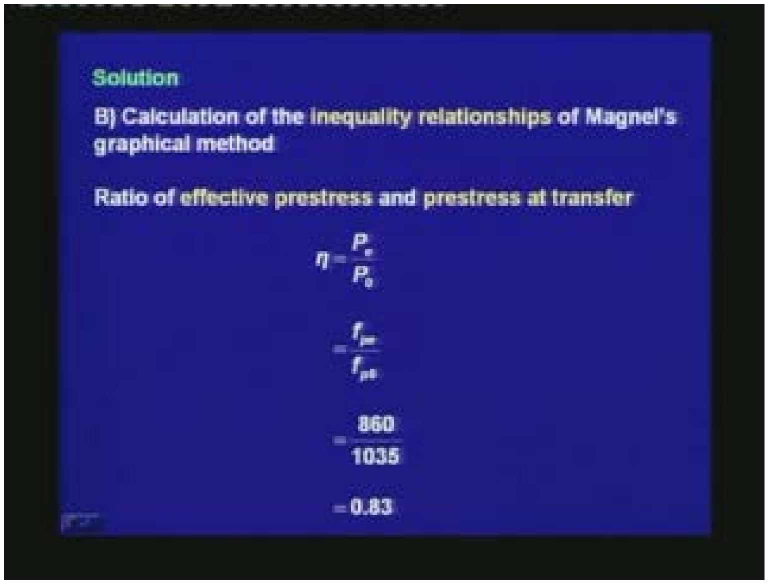

Next, we are calculating the inequality relationships from the stress conditions. The ratio of effective prestress and prestress at transfer $\eta = P_e/P_0 = f_{pe}/f_{p0} = 860/1035 = 0.83$. Thus, 83% of the prestress at transfer stays as the effective prestress.

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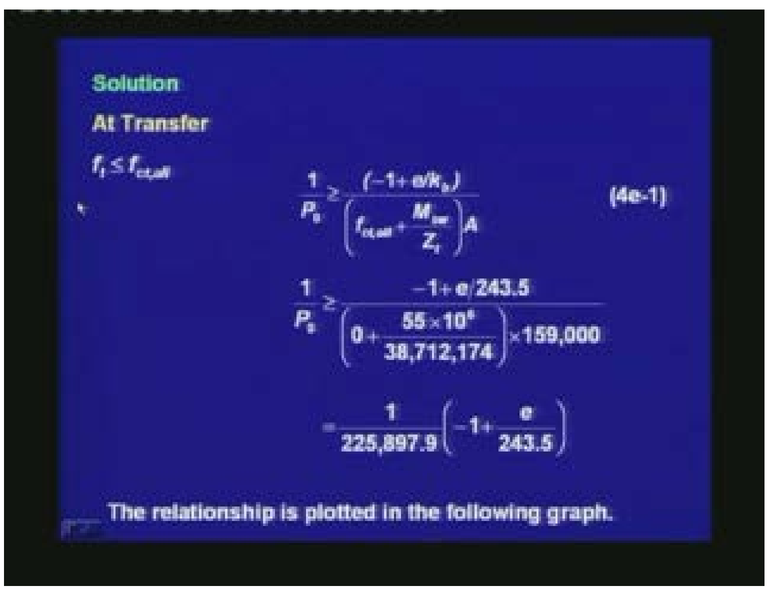

The first inequality relationship satisfies the condition that at transfer, the stress at the top should be less than the allowable tensile stress, which is $f_{ct,all}$. Since, it is a Type 1 section, $f_{ct,all} = 0$. Once, we have substituted the values of k_b, M_{sw}, Z_t and A on the right hand side, we find the relationship $1/P_0 \geq 1/225,897.9 \times (-1 + e/243.5)$.

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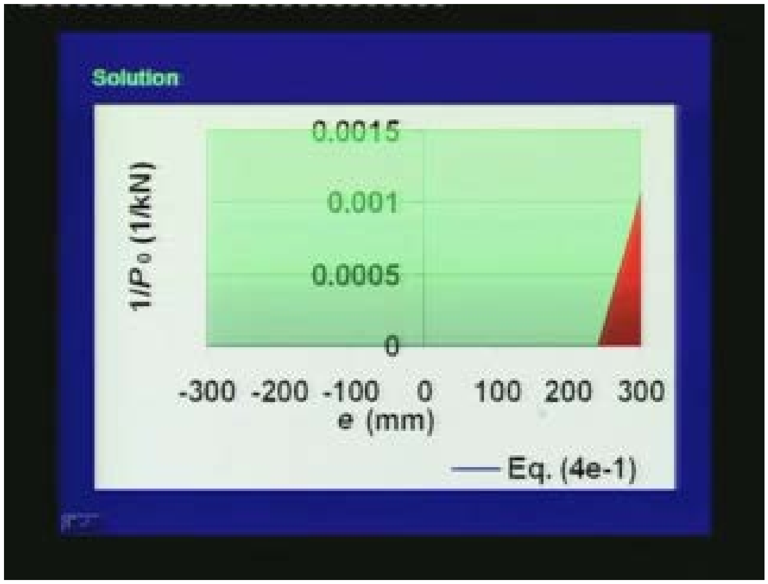

This equation is plotted in the $1/P_0$ versus e plane, and this passes through the point $e = k_b = 243.5$ mm. Any point lying above the line is acceptable, and hence that region is shaded as green. Any point lying below the straight line is not acceptable, and hence that region is represented as red. Hence, we see that the first stress condition gives a line which demarcates the combination of $1/P_0$ and e into two regions: one is a green region, which is acceptable and the other is red region, which is not acceptable.

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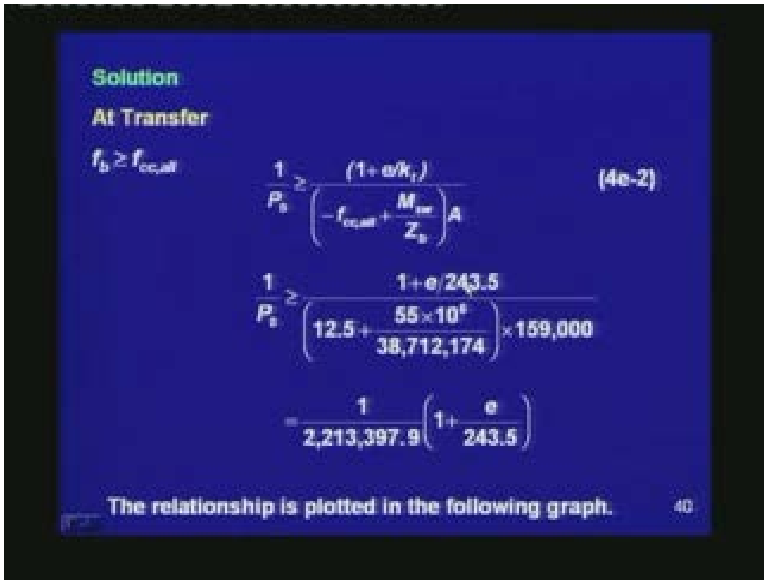

The second condition is at transfer, the stress at the bottom should be greater than the compressive stress at transfer. After substituting the values of k_t, M_{sw}, Z_b and A, the corresponding relationship is $1/P_0 \geq 1/2,213,397.9 \times (1 + e/243.5)$.

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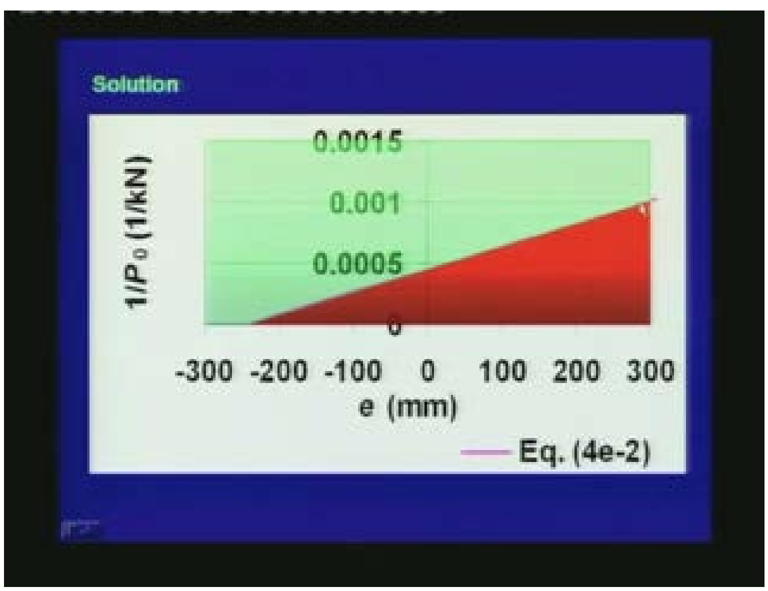

When we plot this relationship, the line passes through the point $e = -k_t = -243.5$ mm. As before, the straight line demarcates the $1/P_0$ versus e plane into two regions. Any combination which lies above the straight line is acceptable, and that is shown as green; any combination lying below the straight line is not acceptable, which is shown as red.

Next, we are moving onto the stress condition at service.

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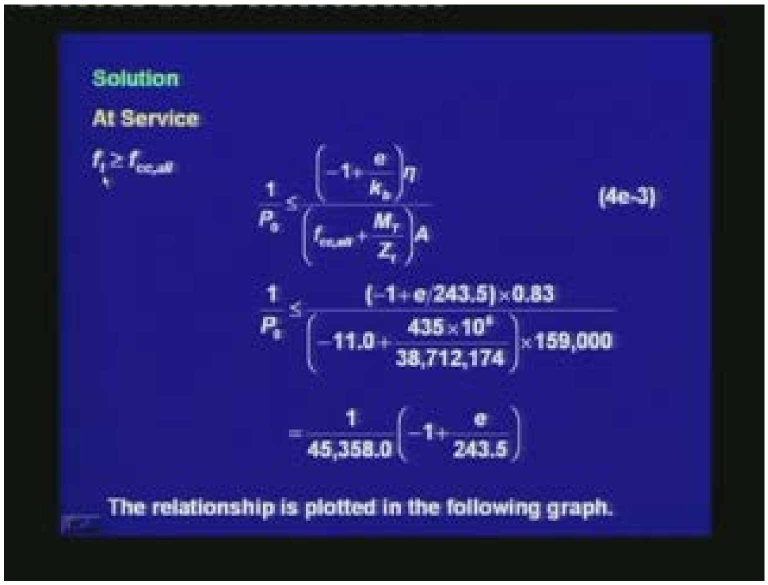
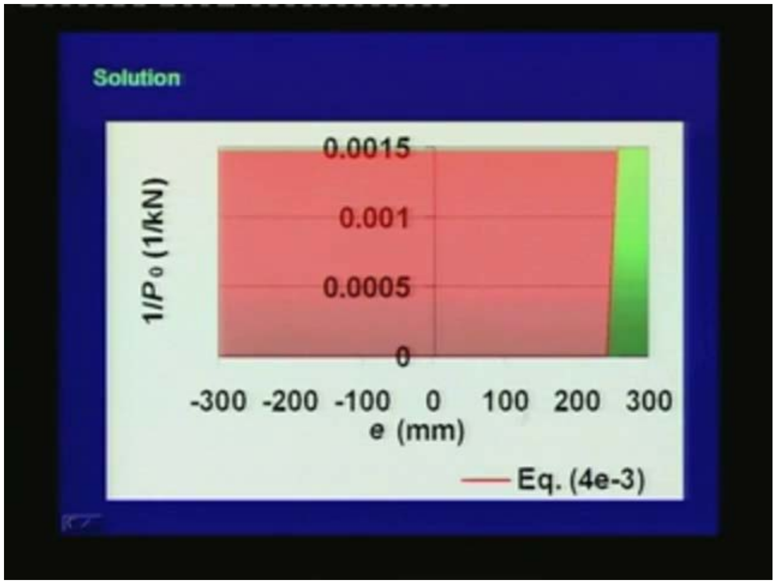


The first condition is that the stress at the top should be greater than $f_{cc,all}$. Once, we substitute the values of k_b, η, M_T, Z_t and A, we find the relationship as $1/P_0 \leq 1/45,358.0 \times (-1 + e/243.5)$.

(Ref Slide Time: 43:13)



After plotting in the $1/P_0$ versus e plane, the line passes through the point $e = k_b$. Any point which lies below the straight line is acceptable, here it is shown as green; and any point lying above the straight line is unacceptable, this is shown as red.

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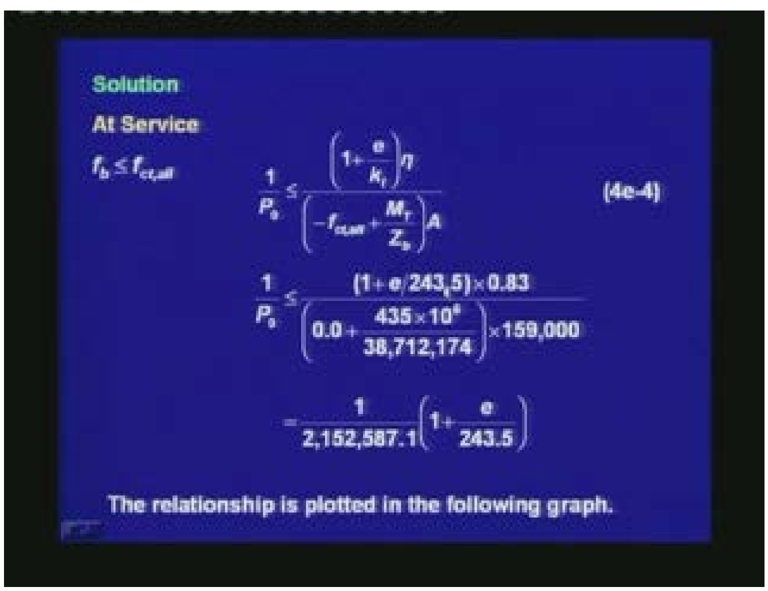

Next, we are moving on to the fourth condition which states that the stress at the bottom should be less than the allowable tensile stress at service. In the corresponding equation, after substituting the values of k_t, η, M_t, Z_b and A, we find that $1/P_0 \leq 1/2{,}152{,}587.1 \times (1 + e/243.5)$.

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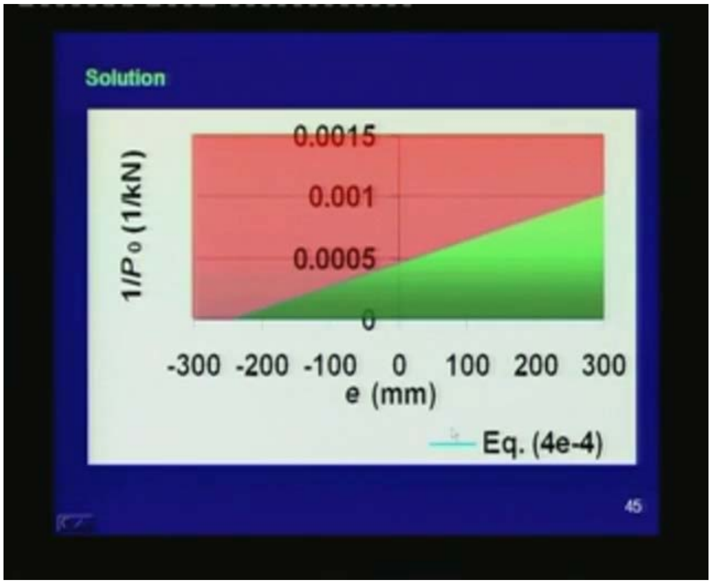

After plotting in the $1/P_0$ versus e plane, the straight line passes through the point $e = -k_t$. Any point lying below this straight line is acceptable, and hence this region is shown as green; any point lying above the straight line is not acceptable, and that is shown in red.

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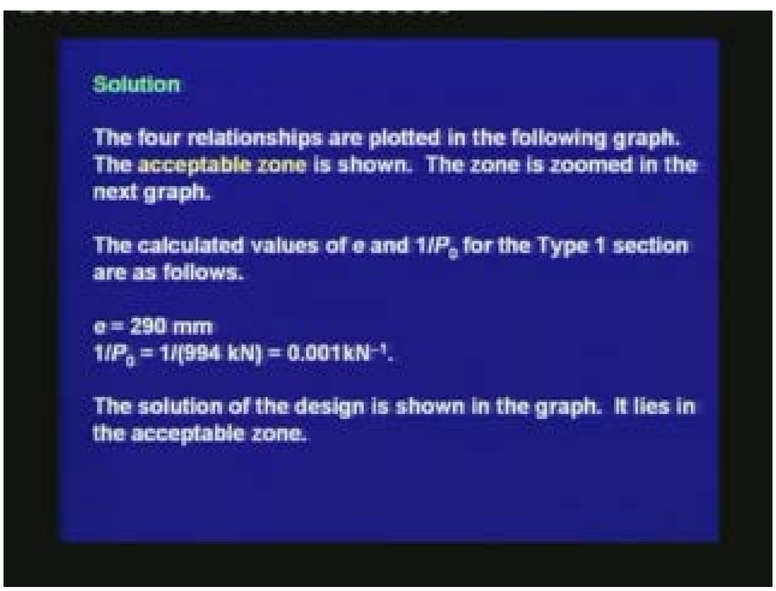

The four relationships are plotted in the following graph and the acceptable zone is shown. The zone is zoomed in the next graph. The calculated values of e and $1/P_0$ for the Type 1 section, from the earlier design are as follows: e = 290 mm, P_0 = 994 kN, $1/P_0$ = 1/994 = 0.001 kN^{-1}. The solution of the design is shown in the graph. It lies in the acceptable zone.

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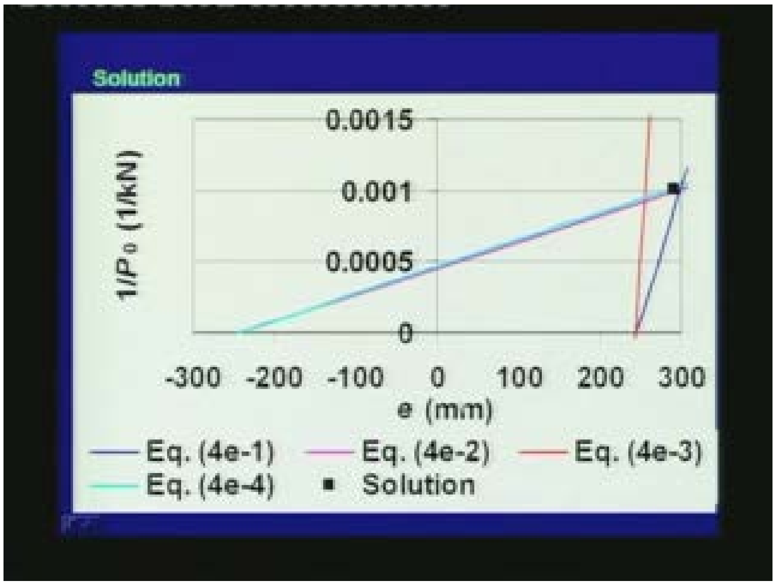

Once we plot all these four lines, we see that for this particular section, the allowable zone is a small strip on the right hand side; the solution is shown as a dark rectangle. Let us zoom into this small region which will show that the solution indeed lies in the acceptable zone.

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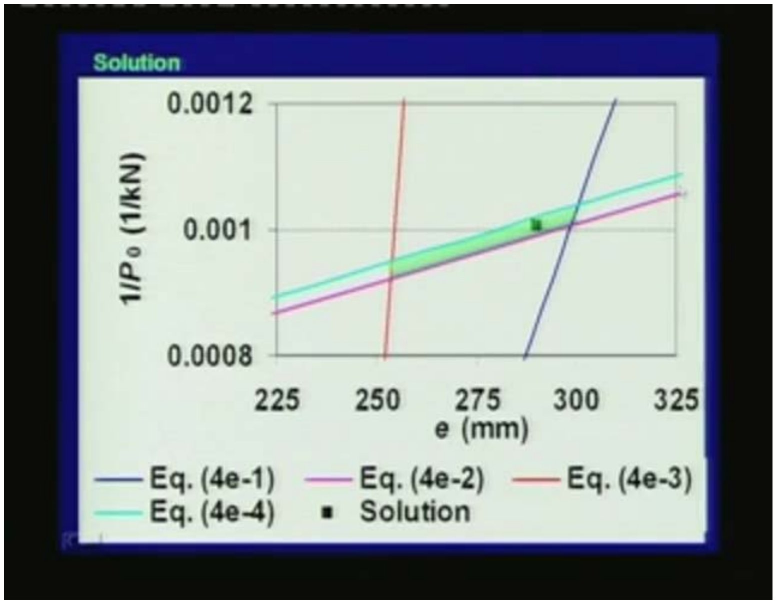

When we zoom, we find that the four lines give a region which is shaded as green. This is the acceptable zone for this particular section. The selected values of e and P_0 indeed lie within the acceptable zone. Hence, the design values are appropriate. If we shift the design value more towards the right, then we are increasing the eccentricity and reducing P_0; but the eccentricity will not be a rounded off value. Thus, this graph explains the application of Magnel's method for the type of section which has been selected for this particular problem.

Summary

In today's lecture, we studied the design of sections for flexure by the Magnel's graphical method. In a real situation, calculations based on the first principles of the stress conditions can be quite involved, especially, if you are having different types of section to select from. In that situation, Magnel's method can be quite handy. It can be implemented in a program and it can give a visual interpretation of the equations that we are using.

The stress conditions are: at transfer, the stress at the bottom is numerically lower than the allowable compressive stress, and the stress at the top is lower than the allowable tensile stress. At service, the stress at the bottom is less than the allowable tensile stress, and the stress at the top is less than the allowable compressive stress. This concept of satisfying the stress conditions at transfer and at service has been implemented in a graphical form by the Magnel's method.

When we plot the four inequality relationships in the $1/P_0$ versus e plane, we may get an acceptable zone for that particular section, under the given loading conditions. Then that section is adequate. If there is no acceptable zone, we cannot use that section for the given loading conditions.

We saw the application of the method through an example, which we had solved earlier. First, we calculated the geometric properties of the section. Next, we found out the inequality relationships. Third, we plotted the four relationships in the $1/P_0$ versus e plane. Once we plotted, we found that there is a small strip which is an acceptable zone, and our previous solution indeed lies within the acceptable zone.

Thus, today we are ending the design of members with the Magnel's graphical method, and in our next class we shall move on to the detailing of members for flexure.

Thank you.