

PRESTRESSED CONCRETE STRUCTURES

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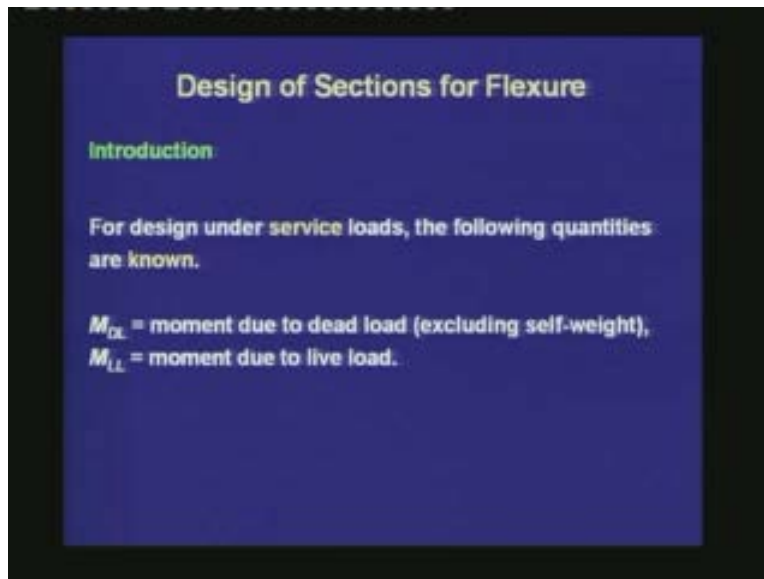
Indian Institute of Technology Madras

Module - 4: Design of Members

**Lecture -19: Design of Members for Flexure
(Type 2 and Type 3 Members)**

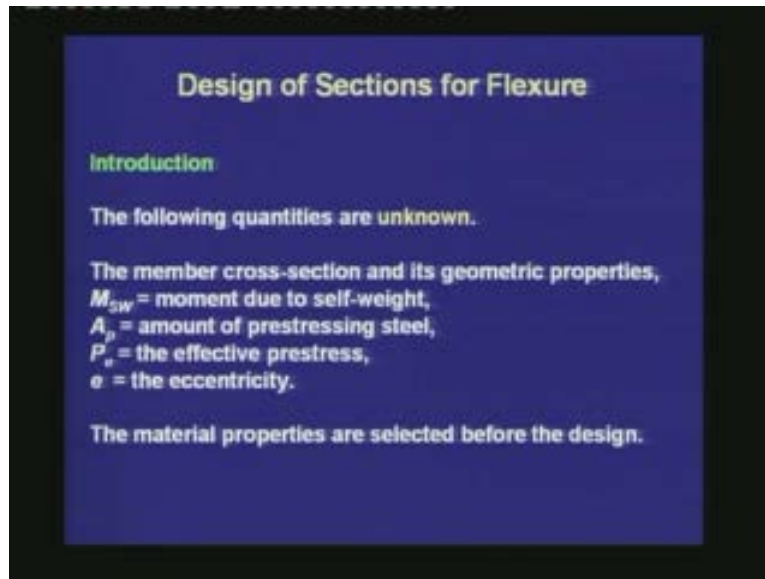
Welcome back to prestressed concrete structures. This is the third lecture in module four on design of members. In this lecture, we shall study the design of sections for flexure, specifically, the final design of Type 2 members.

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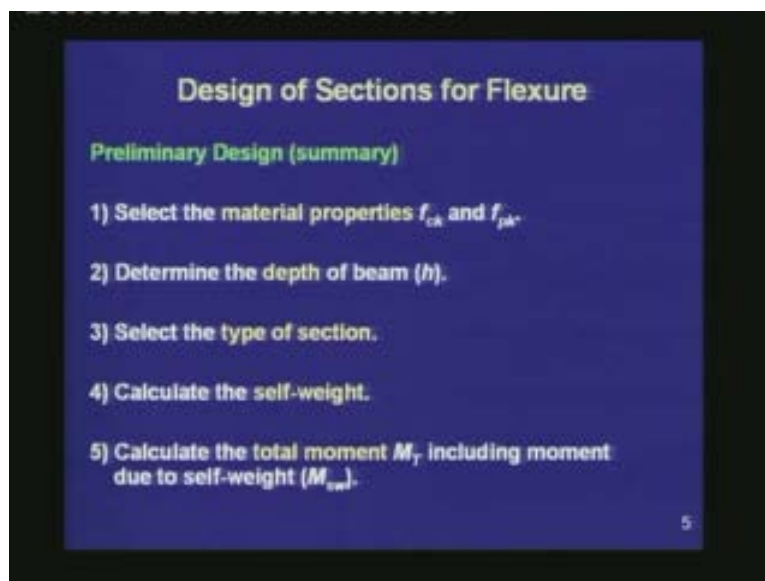
Before we start, let us recapitulate as to what we need in a design process. For design under service loads, the following quantities are known: M_{DL} , the moment due to dead load which is not including the self-weight yet, and M_{LL} which is the moment due to live load.

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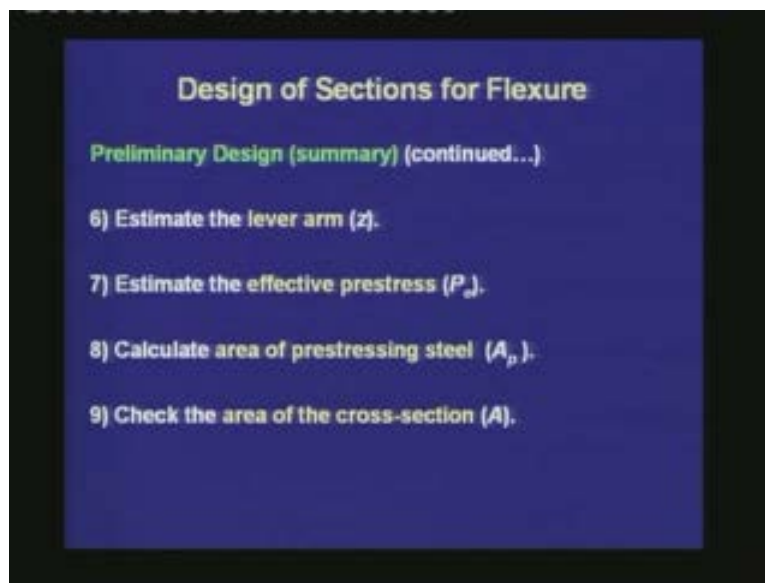
The unknown quantities are the member cross-section and geometric properties; M_{sw} : the moment due to self-weight, A_p : the amount of prestressing steel, P_e : the effective prestress, and e : the eccentricity. The material properties are selected at the onset of design.

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We had learnt earlier that the design process can be conveniently divided into two stages. The first is the preliminary design, and next is the final design. We are summarizing the preliminary design here. In the preliminary design, first select the material properties, f_{ck} and f_{pk} which are the characteristic strengths of concrete and the steel, respectively. Determine the depth of beam (h) which may be based on the architectural requirement. Select the Type of section. This depends on the application. Next, calculate the self-weight. Then, calculate the total moment (M_T) including the moment due to self-weight.

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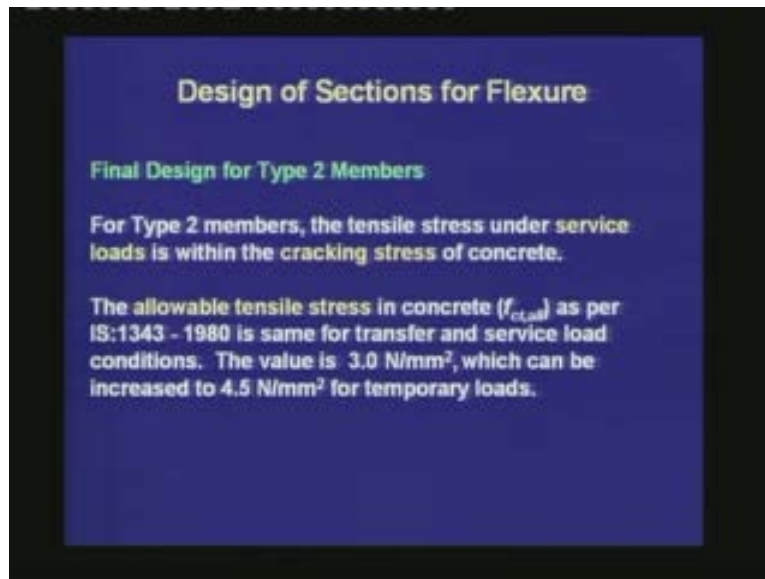
Next, estimate the lever arm (z). The lever arm is estimated as a fraction of the total height. Then estimate the effective prestress (P_e). The effective prestress is equal to the total moment divided by the lever arm. Calculate the area of prestressing steel (A_p) by assuming that the effective prestress is about 70% of the characteristic strength. Finally, check the area of the cross-section based on that the stress at the CGS is around 50% of the allowable compressive stress in the concrete.

Once we have done the preliminary design, we are moving on to the final design. Before we move on to the final design we need to know whether we are doing a Type 1 member, Type 2 member or Type 3 member. In a Type 1 member, no tensile stress is allowed under service loads or at transfer. For a Type 2 member, tensile stresses are allowed but

they should be within the cracking stress. For a Type 3 member, the tensile stress can exceed the cracking stress, but still it is limited to a certain value which limits the crack width.

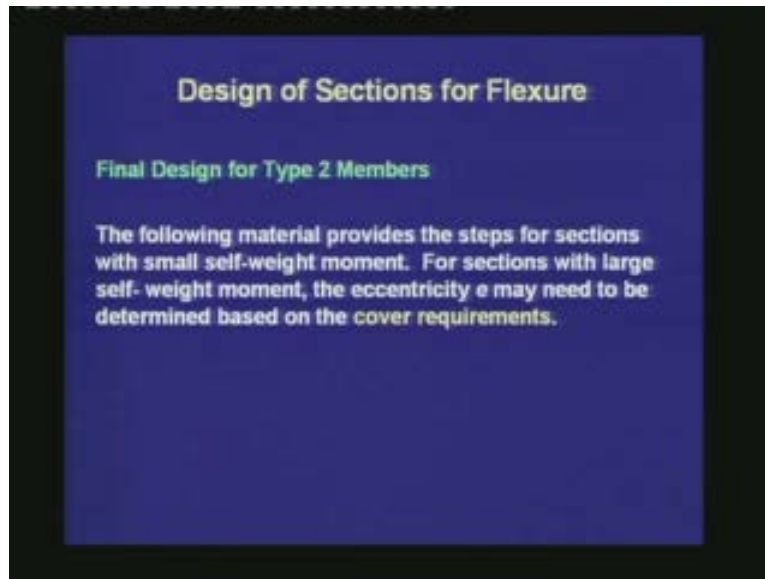
We had studied the final design of Type 1 members. Today, we are going to study the final design of Type 2 members. For Type 3 members, the design procedure is similar to Type 2 members except that the allowable tensile stress is different for Type 3 members as compared to Type 2 members. Now, we are moving on to the design of Type 2 members.

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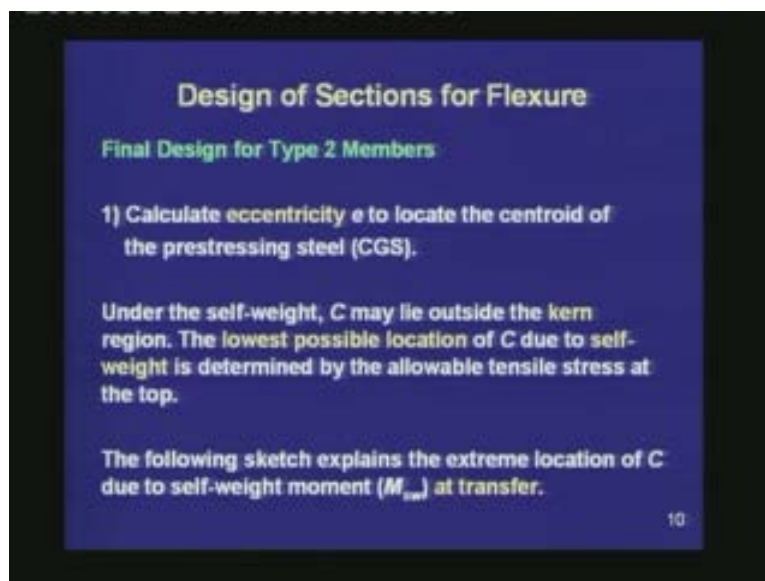
For Type 2 members, the tensile stresses under service loads and at transfer are within the cracking stress of concrete. The allowable tensile stress in concrete ($f_{ct,all}$) as per IS: 1343-1980 is same for transfer and service load conditions. The value is 3.0 N/mm², which can be increased to 4.5 N/mm² for temporary loads. However, the allowable tensile stress at transfer can be different from that at service, for international codes.

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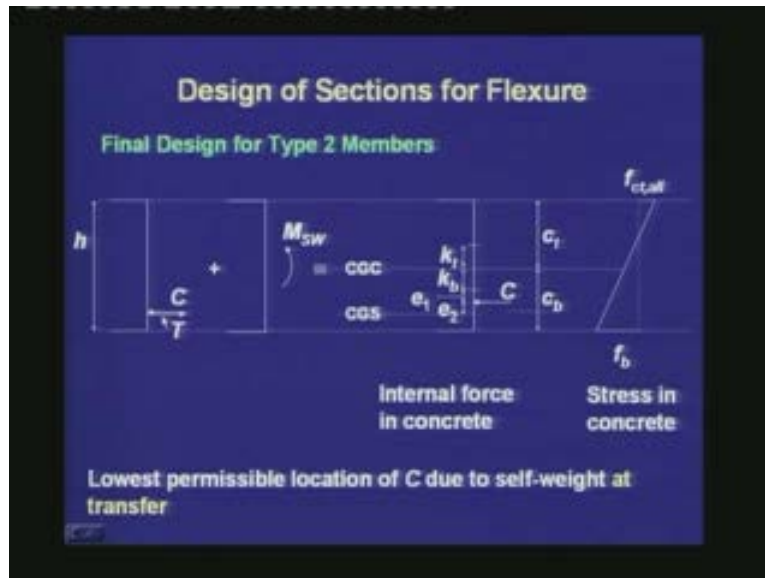
The following material provides the steps for sections with small self-weight moment. That means, the eccentricity that we are calculating does not violate the cover requirements. For sections with large self-weight moment, the eccentricity e may need to be determined based on the cover requirements.

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The first step in the final design is to calculate the eccentricity (e) to locate the centroid of prestressing steel, which is the CGS. Under the self-weight, C may lie outside the kern region for Type 2 members. The lowest possible location of C due to self-weight is determined by the allowable tensile stress at the top. The following sketch explains the extreme location of C due to self-weight moment (M_{sw}) at transfer.

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In this sketch, on the left hand side we find that C is located at the CGS in absence of any external moment. When the self-weight moment acts, C moves up from T through a distance equal to e_2 . For a Type 2 member, C may not enter the kern region so that tensile stress can generate during the acting of the self-weight. It is at a distance e_1 from the bottom kern point. Thus, C is outside the kern region unlike in a Type 1 member. The lowest permissible location of C is governed by how much tensile stress we are allowing at the top of the section ($f_{ct,all}$) at transfer.

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Design of Sections for Flexure

Final Design for Type 2 Members

From the previous figure, the shift of C due to self-weight gives an expression of e_2 . It is evident that if C is further shifted upwards by a distance e_1 to the bottom kern point, there will be no tensile stress at the top.

$$e_2 = \frac{M_{sw}}{P_s} \quad (4c-1)$$

From the figure, the shift of C due to self-weight gives an expression of e_2 . $e_2 = M_{sw}/P_0$. This is the lever arm which generates to counteract the self-weight moment. It is evident that if C is further shifted upwards by a distance e_1 to the bottom kern point, there will be no tensile stress at the top.

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Design of Sections for Flexure

Final Design for Type 2 Members

The value of e_1 is calculated from the expression of stress corresponding to the moment due to the shift in C by e_1 .

$$\frac{P_s e_1 c_1}{I} = f_{ccwt}$$
$$e_1 = \frac{f_{ccwt} I}{P_s c_1}$$

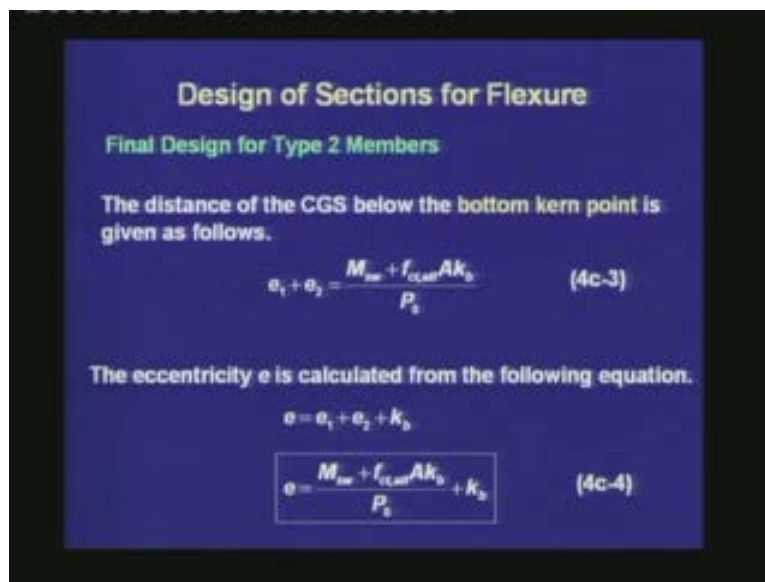
Substituting $I = Ar^2$
and $r^2/c_1 = k_b$

$$e_1 = \frac{f_{ccwt} A k_b}{P_s} \quad (4c-2)$$

The value of e_1 is calculated from the expression of stress corresponding to the moment required to shift C by e_1 . The additional moment generates a stress at the top which is equal and opposite to $f_{ct,all}$.

The moment is given by C times e_1 , which is equal to P_0 times e_1 . The stress at the top due to the additional moment is $P_0 e_1 c_t / I$, which is equal to $f_{ct,all}$. From this expression, we can calculate e_1 ; $e_1 = f_{ct,all} I / P_0 c_t$. If we substitute $I = A r^2$, where A is the cross-section and r is the radius of gyration, and if we substitute $r^2 / c_t = k_b$, we have an expression $e_1 = f_{ct,all} A k_b / P_0$.

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The distance of the CGS below the bottom kern point is given as follows: $e_1 + e_2 = (M_{sw} + f_{ct,all} A k_b) / P_0$. The eccentricity e is calculated from the following equation.

$$e = e_1 + e_2 + k_b = (M_{sw} + f_{ct,all} A k_b) / P_0 + k_b.$$

Thus we have got an expression of the eccentricity of the CGS with respect to CGC, which is a slightly more involved as compared to the expression for Type 1 member. We shall compare these expressions later.

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Design of Sections for Flexure

Final Design for Type 2 Members

2) Recompute the effective prestress P_e and the area of prestressing steel A_{pr} .

Under the total load, C may lie outside the kern region. The highest permissible location of C due to total load is determined by the allowable tensile stress at the bottom.

The following sketch explains the highest possible location of C due to the total moment (M_T).

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Once we have calculated the eccentricity, we are recomputing the effective prestress P_e and the area of prestressing steel A_p . Under the total load, C may lie outside the kern region which is unlike Type 1 members. The highest permissible location of C due to the total load is determined by the allowable tensile stress at the bottom. The following sketch explains the highest possible location of C due to the total moment M_T .

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Design of Sections for Flexure

Final Design for Type 2 Members

Diagram illustrating the highest permissible location of C due to total moment at service. The diagram shows a rectangular section of height h with center of gravity C at distance e from the top fiber. The kern points are k_t and k_b . The total moment M_T is shown. The internal force in concrete is C and the stress in concrete is f_t at the top and $f_{ct,all}$ at the bottom.

Internal force in concrete

Stress in concrete

Highest permissible location of C due to total moment at service

When the total moment is acting during the service condition, C has shifted up even beyond the kern region because we are allowing tensile stress at the bottom. The distance by which it crosses the top kern point is represented as e_3 , and it depends on the tensile stress that we allow at the bottom.

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Design of Sections for Flexure

Final Design for Type 2 Members
 From the previous figure, the expression of e_3 is obtained by the tensile stress generated due to the shift of C beyond the upper kern point.

$$\frac{P_e e_3 c_b}{I} = f_{ct,all}$$

$$e_3 = \frac{f_{ct,all} I}{P_e c_b}$$

Substituting $I = Ar^2$
 and $r^2/c_b = k_t$

$$e_3 = \frac{f_{ct,all} Ak_t}{P_e} \quad (4c-5)$$

From the previous figure, the expression of e_3 is obtained by the tensile stress generated due to the shift of C beyond the upper kern point. It is the same concept that we have used to calculate e_1 . Here, the moment is $P_e e_3$. The distance of the bottom fiber from the CGC is c_b . The generated stress is $P_e e_3 c_b / I = f_{ct,all}$. Thus, $e_3 = f_{ct,all} I / P_e c_b$. Substituting $I = Ar^2$ and $r^2/c_b = k_t$, we find $e_3 = f_{ct,all} Ak_t / P_e$.

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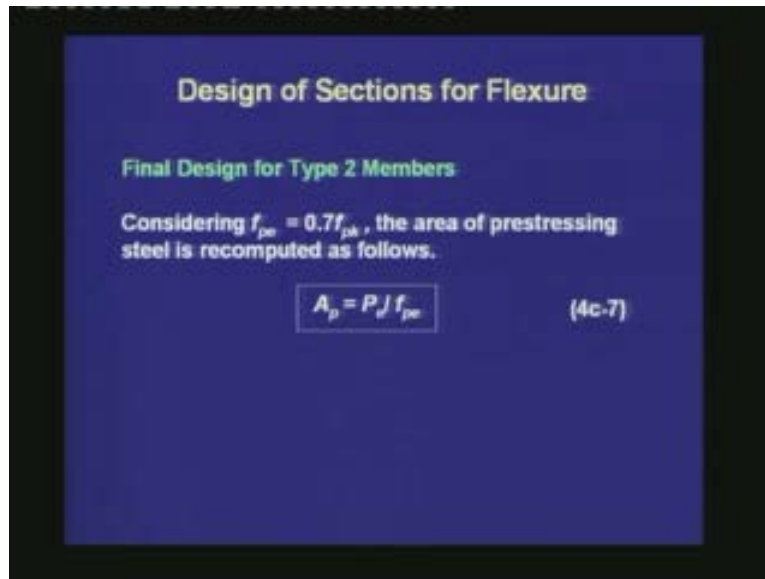
The slide is titled "Design of Sections for Flexure" and "Final Design for Type 2 Members". It states: "The shift of C due to the total moment gives an expression of P_e ." The equations shown are:

$$M_T = P_e(e + k_t + e_3)$$
$$= P_e(e + k_t) + f_{ct,all}Ak_t$$
$$\therefore P_e = \frac{M_T - f_{ct,all}Ak_t}{e + k_t} \quad (4c-6)$$

The shift of C from the CGS, due to the total moment, gives an expression of P_e . The total moment is equal to P_e times the total shift which is equal to $(e + k_t + e_3)$. Once we substitute the expression of e_3 and transpose the terms, $P_e = (M_T - f_{ct,all}Ak_t)/(e + k_t)$. Thus, we have got an expression of the effective prestress.

Thus, the procedure is based on the concept that first we are locating the CGS as down as possible, where under self-weight we are allowing some tension to be present at the top. Hence, C is below the bottom kern point. Next, we are considering maximum shift of C under service load conditions. C can shift even beyond the kern region depending on how much tensile stress we allow at the bottom. This procedure based on the lowest possible location of the CGS and the maximum distance travelled by C, gives an economical design in terms of the required amounts of prestressing steel and the prestressing force.

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Design of Sections for Flexure

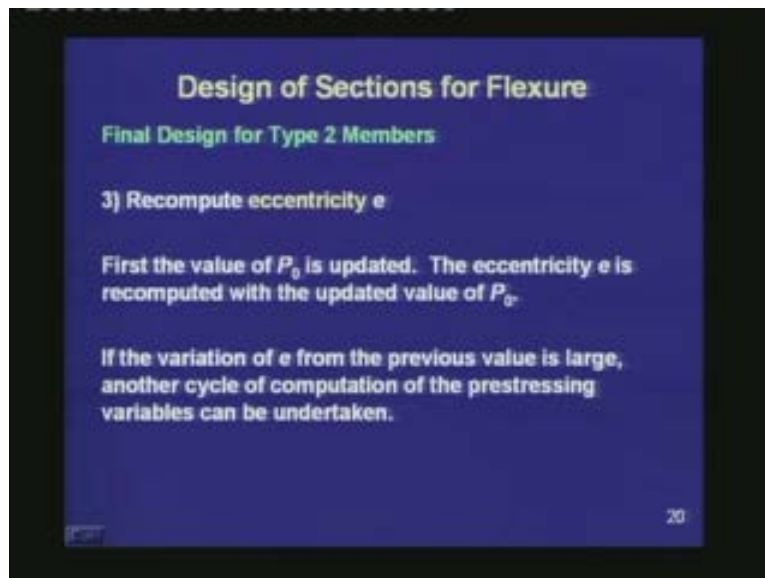
Final Design for Type 2 Members

Considering $f_{pe} = 0.7f_{pk}$, the area of prestressing steel is recomputed as follows.

$$A_p = P_e / f_{pe} \quad (4c-7)$$

Considering that the effective prestress is equal to about 70% of the characteristic tensile strength, the area of the prestressing steel is recomputed as follows: $A_p = P_e / f_{pe}$.

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Design of Sections for Flexure

Final Design for Type 2 Members

3) Recompute eccentricity e

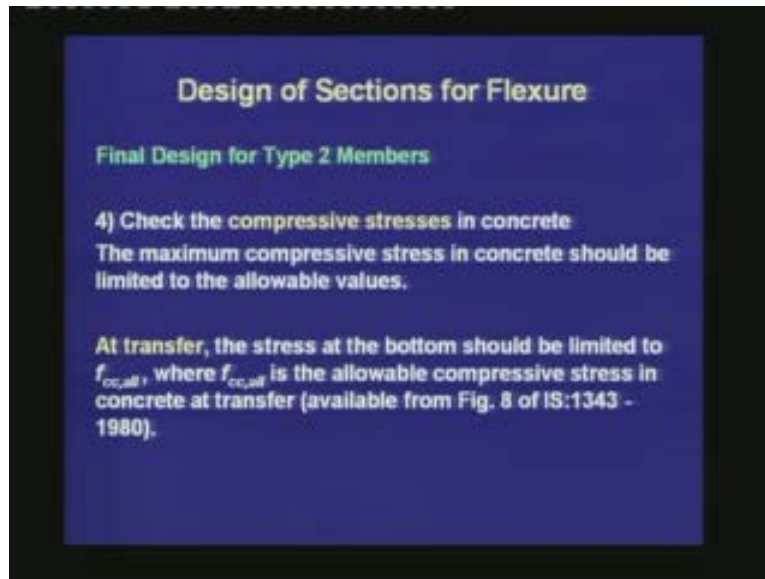
First the value of P_0 is updated. The eccentricity e is recomputed with the updated value of P_0 .

If the variation of e from the previous value is large, another cycle of computation of the prestressing variables can be undertaken.

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Next, the eccentricity is recomputed. First, the value of P_0 is updated. The eccentricity e is recomputed with the updated value of P_0 . If the variation of e from the previous value is large, another cycle of computation of the prestressing variables can be undertaken.

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Design of Sections for Flexure

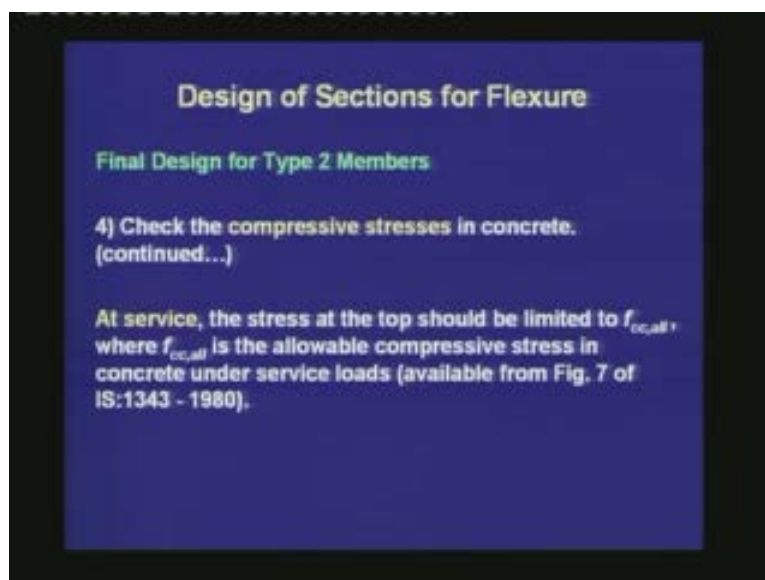
Final Design for Type 2 Members

4) Check the compressive stresses in concrete
The maximum compressive stress in concrete should be limited to the allowable values.

At transfer, the stress at the bottom should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete at transfer (available from Fig. 8 of IS:1343 - 1980).

The fourth step is to check the compressive stresses in concrete. The maximum compressive stress in concrete should be limited to the allowable values. At transfer, the stress at the bottom should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete at transfer. This is available from Fig. 8 of IS: 1343-1980.

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Design of Sections for Flexure

Final Design for Type 2 Members

4) Check the compressive stresses in concrete.
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At service, the stress at the top should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete under service loads (available from Fig. 7 of IS:1343 - 1980).

At service, the stress at the top should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete under service loads. This is available from Fig. 7 of IS: 1343-1980.

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Design of Sections for Flexure

Final Design for Type 2 Members

a) At Transfer
The stress at the bottom can be calculated from the stress diagram.

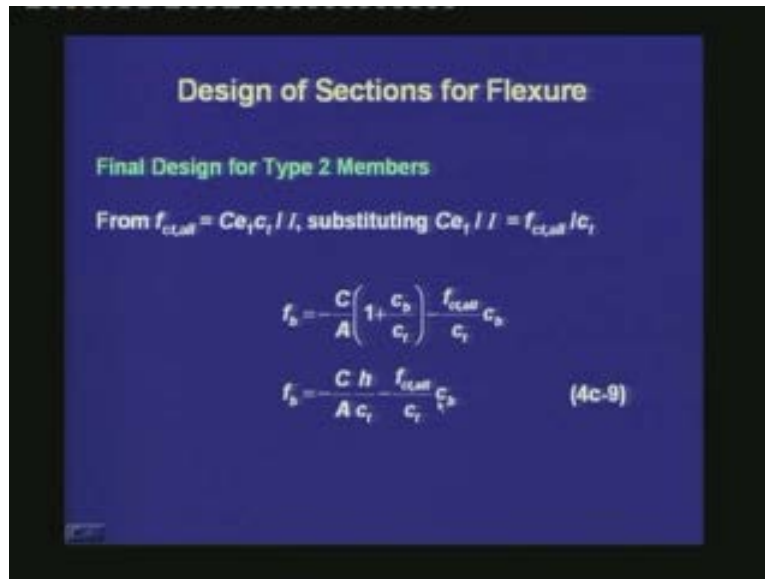
$$f_b = -\frac{C}{A} - \frac{C(k_b + e_1)c_b}{I}$$

$$f_b = -\frac{C}{A}\left(1 + \frac{k_b c_b}{r^2}\right) - \frac{C e_1 c_b}{I} \quad (4c-8)$$

Thus, in the fourth step we are checking the compressive stresses to be within the allowable values. How do we compute the compressive stresses? At transfer, the stress at the bottom can be calculated from the stress diagram as shown. C is located at a distance e_1 below the bottom kern point. Since, the stress block is not triangular we cannot calculate f_b the way we did for Type 1 members. We are calculating f_b based on first principles, from the individual components. The first component is $-C/A$. Next, the varying component due to the eccentricity of C with respect to the CGC, is given as $-C(k_b + e_1)c_b/I$. When we substitute the expression of e_1 , and regroup the terms, the following expression is obtained.

$$f_b = -C/A(1 + k_b c_b/r^2) - C e_1 c_b/I.$$

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Design of Sections for Flexure

Final Design for Type 2 Members

From $f_{ct,all} = Ce_1 c_1 / I$, substituting $Ce_1 / I = f_{ct,all} / c_1$

$$f_b = -\frac{C}{A} \left(1 + \frac{c_b}{c_t} \right) \frac{f_{ct,all}}{c_t} c_b$$
$$f_b = -\frac{C h}{A c_t} \frac{f_{ct,all}}{c_t} c_b \quad (4c-9)$$

Since e_1 is related with the allowable tensile stress at the top, we are substituting $Ce_1/I = f_{ct,all}/c_t$. The value of the stress in the bottom is given as

$$f_b = -C/A (1 + c_b/c_t) - f_{ct,all}C_b/c_t$$

In the numerator within the bracket, $c_t + c_b = h$. Hence,

$$f_b = - (C/A)h/c_t - (f_{ct,all}/c_t)c_b.$$

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Design of Sections for Flexure

Final Design for Type 2 Members

To satisfy $|f_t| \leq f_{cc,all}$

$$\frac{C}{A} \frac{h}{c_t} + \frac{f_{cc,all}}{c_t} c_b \leq f_{cc,all}$$
$$\frac{P_0 h}{A} \leq f_{cc,all} c_t - f_{cc,all} c_b$$
$$\therefore A \geq \frac{P_0 h}{f_{cc,all} c_t - f_{cc,all} c_b} \quad (4c-10)$$

If A is not adequate then the section has to be redesigned.

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We have to satisfy the bottom stress to be within the allowable value. In the inequality relationship, the terms are transposed. We find a condition for the area of the section.

$$A \geq P_0 h / (f_{cc,all} c_t - f_{cc,all} c_b)$$

If the area of the trial section after the preliminary design is not adequate, then the section has to be redesigned. Next, we are checking the compressive stress at the top under service conditions.


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Design of Sections for Flexure

Final Design for Type 2 Members

a) At Service

The stress at the top can be calculated from the stress diagram.



$$f_t = -\frac{C}{A} - \frac{C(k_t + e_s)c_t}{I}$$

$$f_t = -\frac{C}{A} \left(1 + \frac{k_t c_t}{r^2} \right) - \frac{C e_s c_t}{I} \quad (4c-11)$$

The stress at the top can be calculated from the stress diagram at service. It consists of two components.

$$f_t = -C/A - C(k_t + e_s)c_t/I$$

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Design of Sections for Flexure

Final Design for Type 2 Members

From $f_{ct,all} = C e_s c_b / I$, substituting $C e_s / I = f_{ct,all} / c_b$

$$f_t = -\frac{C}{A} \left(1 + \frac{c_t}{c_b} \right) - \frac{f_{ct,all} C_t}{c_b}$$

$$f_t = -\frac{C}{A} \frac{h}{c_b'} - \frac{f_{ct,all} C_t}{c_b} \quad (4c-12)$$

Earlier, we had found an expression of e_3 which is based on the allowable tension at the bottom. From that expression, $Ce_3/I = f_{ct,all}/c_b$. Substituting this in the expression of f_t and regrouping the terms

$$f_t = - (C/A)h/c_b - f_{ct,all}c_t/c_b$$

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The slide contains the following text and equations:

Design of Sections for Flexure

Final Design for Type 2 Members:

To satisfy $|f_t| \leq f_{cc,all}$

$$\frac{C h}{A c_b} + \frac{f_{ct,all} c_t}{c_b} \leq f_{cc,all}$$

$$\frac{P_e h}{A} \leq f_{cc,all} c_b - f_{ct,all} c_t$$

$$\therefore A \geq \frac{P_e h}{f_{cc,all} c_b - f_{ct,all} c_t} \quad (4c-13)$$

If A is not adequate then the section has to be redesigned.

We have to satisfy the stress at the top to be within the allowable value. In the inequality relationship, the terms are transposed. This gives another condition for the area of the section.

$$A \geq P_e h / (f_{cc,all} c_b - f_{ct,all} c_t)$$

The above equations are generalised form of the equations for Type 1 members. Let us compare the two sets of equations.

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Design of Sections for Flexure

Comparison of Equations for Type 1 and Type 2 Members

	Type 1	Type 2
Eccentricity	$e = \frac{M_{sw}}{P_s} + k_b$	$e = \frac{M_{sw} + \underbrace{f_{ct,all}Ak_b}_{\text{orange circle}}}{P_s} + k_b$
Effective prestress	$P_e = \frac{M_T}{e + k_t}$	$P_e = \frac{M_T - \underbrace{f_{ct,all}Ak_t}_{\text{orange circle}}}{e + k_t}$

First, we are writing the expressions of eccentricity e . For Type 1 member, e is equal to the shift of C due to the self-weight moment plus k_b . For Type 2 member, we find that the term which has been shown by an orange ellipse is an additional term. For Type 1 member, the allowable tensile stress is zero. That means, if we substitute $f_{ct,all}$ equal to zero in the additional term for Type 2 member then we find that the expression for e is same as that for Type 1 member. Next, for Type 1 member, the effective prestress is equal to the total moment divided by the shift of C , which is $e + k_t$. For Type 2 member, there is an additional term in the numerator which is related with the tensile stress that we are allowing at the bottom. Again, if we substitute $f_{ct,all}$ equal to zero, we find that the expression for Type 2 member is same as that for Type 1 member.

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Design of Sections for Flexure

Comparison of Equations for Type 1 and Type 2 Members
(continued...)

	Type 1	Type 2
Minimum area based on stress at bottom at transfer	$A \geq \frac{P_t h}{f_{ct,all} c_t}$	$A \geq \frac{P_t h}{f_{ct,all} c_t - f_{ct,all} c_b}$
Minimum area based on stress at top at service	$A \geq \frac{P_s h}{f_{ct,all} c_b}$	$A \geq \frac{P_s h}{f_{ct,all} c_b - f_{ct,all} c_t}$

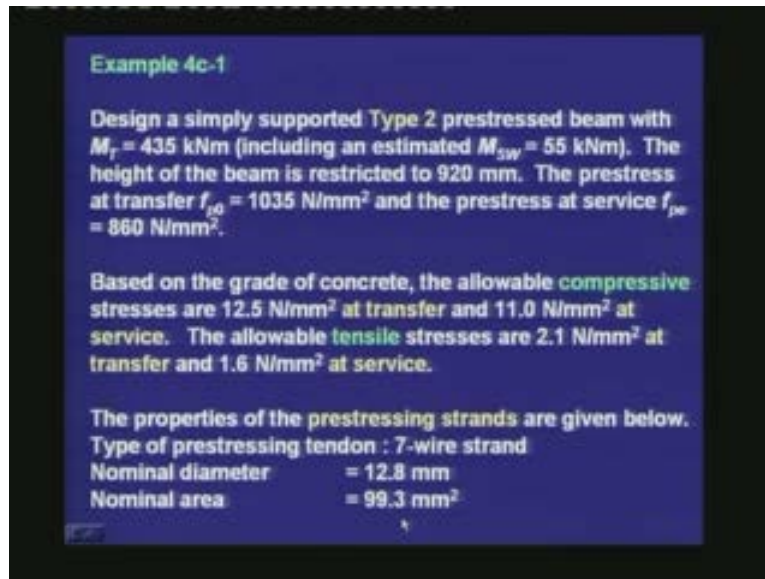
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Next, let us see the expressions for the areas. Regarding the minimum area based on stress at bottom at transfer, for Type 1 member, the expression is $P_0 h / f_{ct,all} c_t$ whereas, for Type 2 member we have an additional term in the denominator which contains $f_{ct,all}$. Similarly, for the minimum area based on stress at the top at service, we find that the expression for Type 2 member has an additional term in the denominator which is related to $f_{ct,all}$. Thus, for both the expressions for Type 2 member if we substitute $f_{ct,all}$ equal to zero, we get the expressions for Type 1 member. Thus, the expressions for Type 2 member are general form of the equations that we had seen for Type 1 member.

If we are designing a Type 3 member, then the only difference is that the allowable stress $f_{ct,all}$ for Type 3 member is different as compared to that for Type 2 member. Otherwise, the expressions for Type 2 member are applicable for a Type 3 member.

Let us understand the design procedure with the help of an example.

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Example 4c-1

Design a simply supported Type 2 prestressed beam with $M_T = 435$ kNm (including an estimated $M_{sw} = 55$ kNm). The height of the beam is restricted to 920 mm. The prestress at transfer $f_{p0} = 1035$ N/mm² and the prestress at service $f_{pe} = 860$ N/mm².

Based on the grade of concrete, the allowable compressive stresses are 12.5 N/mm² at transfer and 11.0 N/mm² at service. The allowable tensile stresses are 2.1 N/mm² at transfer and 1.6 N/mm² at service.

The properties of the prestressing strands are given below.

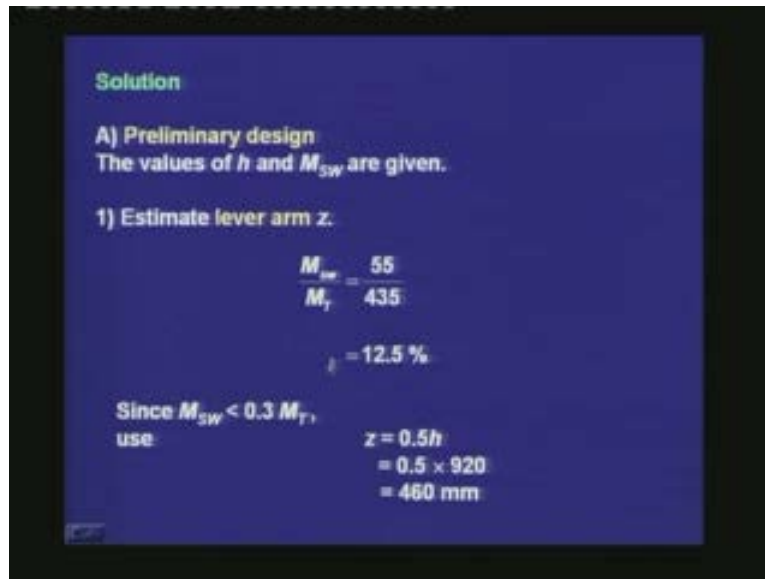
Type of prestressing tendon :	7-wire strand
Nominal diameter	= 12.8 mm
Nominal area	= 99.3 mm ²

Design a simply supported Type 2 prestressed beam with the total moment equal to 435 kNm, including an estimated self-weight moment M_{sw} equal to 55 kNm. The height of the beam is restricted to 920 mm. The prestress at transfer f_{p0} is equal to 1035 N/mm². The prestress at service f_{pe} is equal to 860 N/mm². Based on the grade of concrete, the allowable compressive stresses are 12.5 N/mm² at transfer and 11 N/mm² at service. The allowable tensile stresses are 2.1 N/mm² at transfer and 1.6 N/mm² at service.

The properties of the prestressing strands are given below. The type of prestressing strand is 7-wire strand, with nominal diameter = 12.8 mm, and the nominal area = 99.3 mm².

The problem is similar as that we have solved for Type 1 member, except that now we have to design the member as Type 2 member.

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Solution

A) Preliminary design
The values of h and M_{SW} are given.

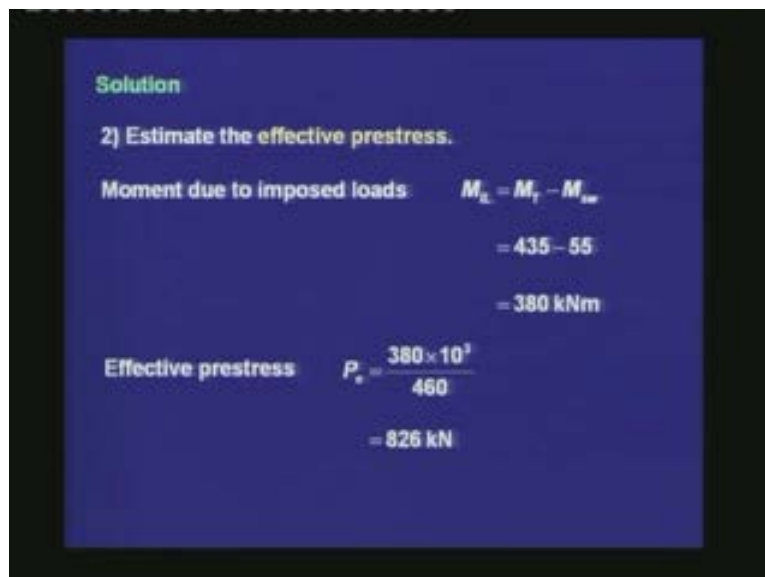
1) Estimate lever arm z .

$$\frac{M_{sw}}{M_T} = \frac{55}{435}$$
$$= 12.5\%$$

Since $M_{sw} < 0.3 M_T$,
use: $z = 0.5h$
 $= 0.5 \times 920$
 $= 460 \text{ mm}$

First, we are doing the preliminary design. The height and the self-weight moment are given. We have to estimate the lever arm based on the ratio of the self-weight moment to the total moment. The ratio is equal to $55/435 = 12.5\%$. Since, the self-weight moment is less than about 30% of the total moment, we are estimating the lever arm to be 50% of the total height, which is equal to 0.5 times 920 equal to 460 mm.

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Solution

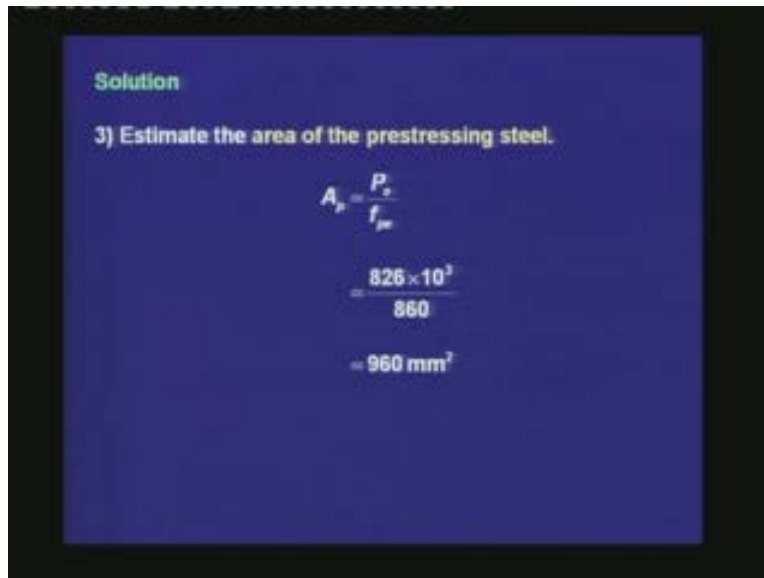
2) Estimate the effective prestress.

Moment due to imposed loads: $M_{im} = M_T - M_{sw}$
 $= 435 - 55$
 $= 380 \text{ kNm}$

Effective prestress $P_e = \frac{380 \times 10^3}{460}$
 $= 826 \text{ kN}$

Next, the effective prestress is estimated. The moment due to imposed loads M_{IL} is equal to $M_T - M_{sw} = 435 - 55 = 380$ kNm. The effective prestress P_e is equal to the imposed load moment divided by the lever arm. This is equal to $380 \times 10^3 / 460 = 826$ kN.

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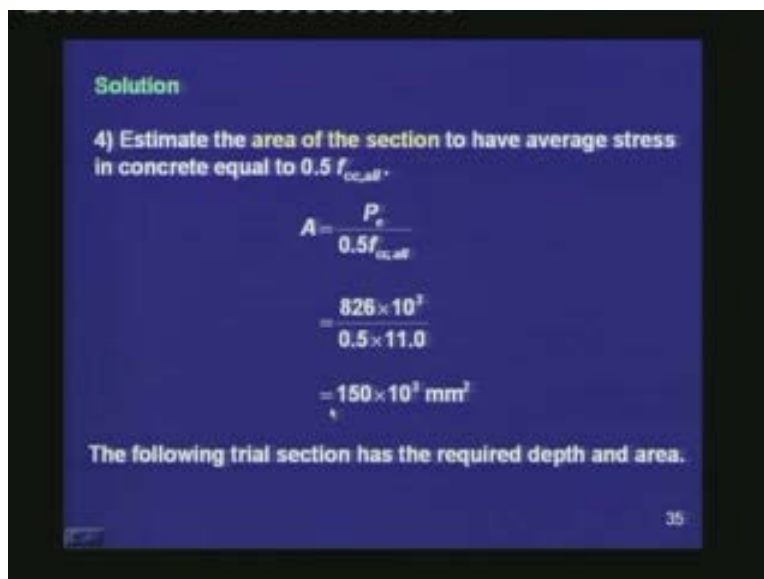
Solution

3) Estimate the area of the prestressing steel.

$$A_p = \frac{P_e}{f_{pe}}$$
$$= \frac{826 \times 10^3}{860}$$
$$= 960 \text{ mm}^2$$

The area of the prestressing steel A_p is equal to P_e divided by f_{pe} . Once we substitute the values of the two variables, we get $A_p = 960 \text{ mm}^2$.

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Solution

4) Estimate the area of the section to have average stress in concrete equal to $0.5 f_{cc,eff}$.

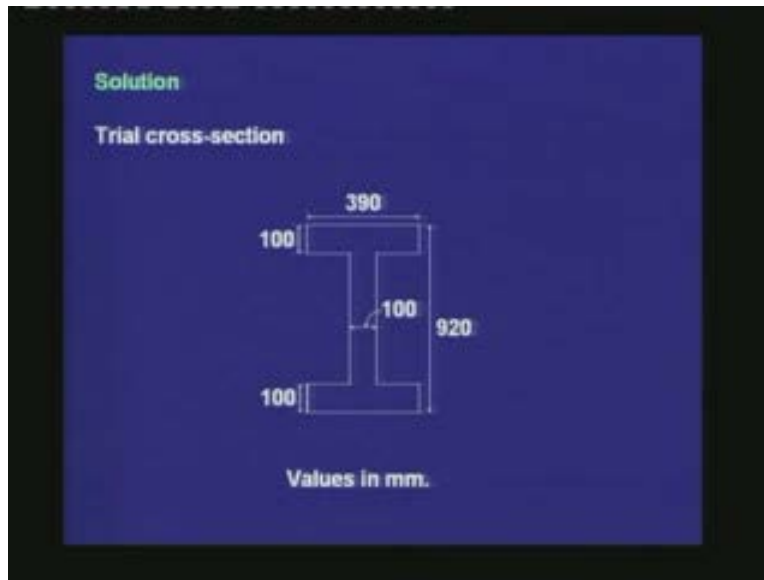
$$A = \frac{P_e}{0.5 f_{cc,eff}}$$
$$= \frac{826 \times 10^3}{0.5 \times 11.0}$$
$$= 150 \times 10^3 \text{ mm}^2$$

The following trial section has the required depth and area.

35

At the fourth step of the preliminary design, we are estimating the area of cross-section to have average stress in concrete equal to half of the allowable compressive stress. The area is equal to $P_e/0.5f_{cc,all} = 826 \times 10^3 / 0.5 \times 11 = 150 \times 10^3 \text{ mm}^2$. The following trial section has the required depth and the area.

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We are selecting an I-section with a depth of 920 mm to satisfy the depth requirement. We are selecting the width of the web and the depth of the flange to be 100 mm. Then, we are finding out the width of the flange to satisfy the requirement of the area. The width of the flange has come out to be 390 mm.

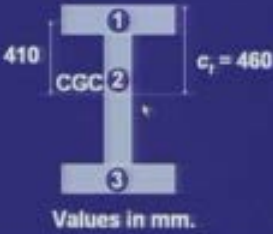
At the end of the preliminary design, we have come up with a trial section.

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Solution

B) Calculation of geometric properties

The section is symmetric about the horizontal axis. Hence, the CGC lies at mid depth. The section is divided into three rectangles for the computation of the geometric properties.



Values in mm.

Next, we have to calculate the geometric properties of the trial section. The section is symmetric about the horizontal axis. Hence, the CGC lies at mid depth. The section is divided into 3 rectangles for the computation of the geometric properties. We see that the centroid of the top rectangle is at a distance of 410 mm from the CGC, and the distance of the top fiber c_t is equal to 460.

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Solution

Check area of the section

$$A = 2 \times (390 \times 100) + (720 \times 100)$$
$$= 150,000 \text{ mm}^2$$

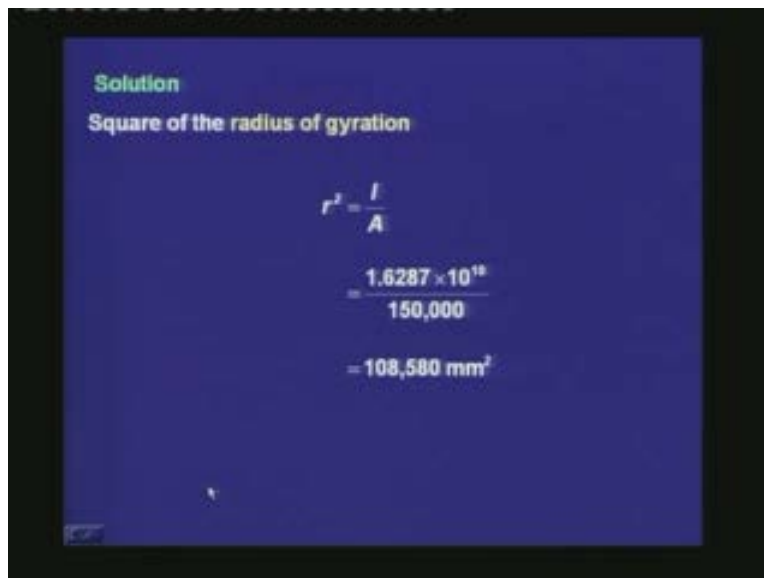
Moment of inertia of the section about axis through CGC

$$I = 2I_1 + I_2$$
$$= 2 \left[\frac{1}{12} \times 390 \times 100^3 + (390 \times 100) \times 410^2 \right] + \frac{1}{12} \times 100 \times 720^3$$
$$= 1.6287 \times 10^{10} \text{ mm}^4$$

First, we are checking the area of the section. A is equal to 2 times the area of the first rectangle, plus the area of the second rectangle. $A = 150,000 \text{ mm}^2$. Thus, the section satisfies the requirement of the area from the preliminary design. Next, we are calculating the moment of inertia of the section. The moment of inertia about an axis through the CGC is equal to two times the moment of inertia of the first rectangle, plus the moment of inertia of the second rectangle.

Using the parallel axis theorem, the moment of inertia of the first rectangle is equal to the moment of inertia about its centroidal axis, plus its area times the distance square, where the distance is that between the centroid of the rectangle and CGC. After substituting the terms, $I = 1.6287 \times 10^{10} \text{ mm}^4$.

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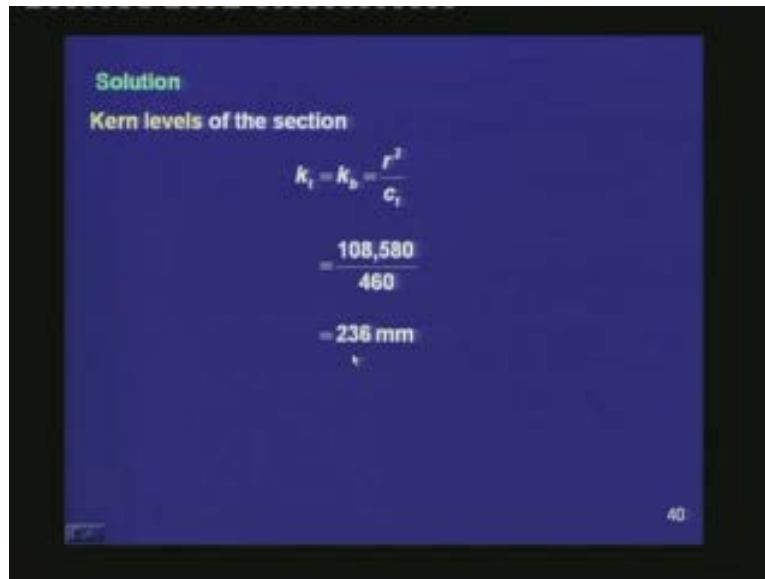


Solution
Square of the radius of gyration

$$r^2 = \frac{I}{A}$$
$$= \frac{1.6287 \times 10^{10}}{150,000}$$
$$= 108,580 \text{ mm}^2$$

Square of the radius of gyration, $r^2 = I/A = 1.6287 \times 10^{10} / 150,000 = 108,580 \text{ mm}^2$.

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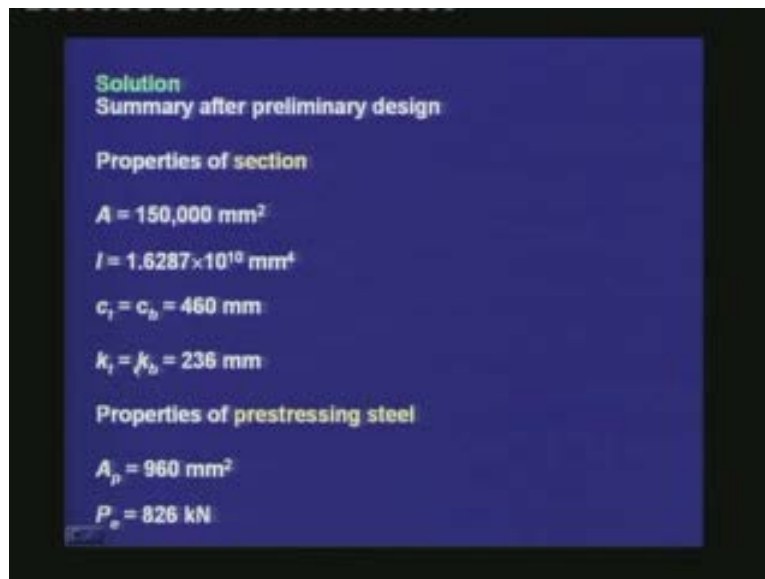
Solution
Kern levels of the section

$$k_t = k_b = \frac{r^2}{c_t}$$
$$= \frac{108,580}{460}$$
$$= 236 \text{ mm}$$

40

Kern levels of the section: $k_t = k_b = r^2/c_t = 108,580/460 = 236 \text{ mm}$.

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Solution
Summary after preliminary design

Properties of section

$A = 150,000 \text{ mm}^2$
 $I = 1.6287 \times 10^{10} \text{ mm}^4$
 $c_t = c_b = 460 \text{ mm}$
 $k_t = k_b = 236 \text{ mm}$

Properties of prestressing steel

$A_p = 960 \text{ mm}^2$
 $P_e = 826 \text{ kN}$

Summary after the preliminary design:

The following properties of the section are available: A , I , the distances of the extreme fibers from CGC, the kern levels from CGC. The estimates for prestressing are $A_p = 960 \text{ mm}^2$ and $P_e = 826 \text{ kN}$.

(Refer Slide Time: 38:34)

Solution
C) Final design
1) Calculate eccentricity e

$$P_s = A_p f_{ps}$$
$$= 960 \times 1035$$
$$= 993.6 \text{ kN}$$
$$e_1 + e_2 = \frac{M_{sw} + f_{ct,all} A k_b}{P_s}$$
$$= \frac{55 \times 10^3 + \frac{2.1}{10^3} \times 150,000 \times 236}{993.6}$$
$$= 130 \text{ mm}$$
$$\therefore e = e_1 + e_2 + k_b$$
$$= 130 + 236$$
$$= 366 \text{ mm}$$

Next, we are moving on to the final design. Here, we are designing as a Type 2 member. First, we are calculating the prestress at transfer, which is equal to the estimated area of steel times the prestress at transfer. $P_0 = 993.6$ kN. From this we are calculating $e_1 + e_2$ which is equal to $(M_{sw} + f_{ct,all} A k_b) / P_0$. After substituting the values of variables, $e_1 + e_2 = 130$ mm. The eccentricity of the CGS from the CGC is equal to $e = e_1 + e_2 + k_b = 130 + 236 = 366$ mm.

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Solution

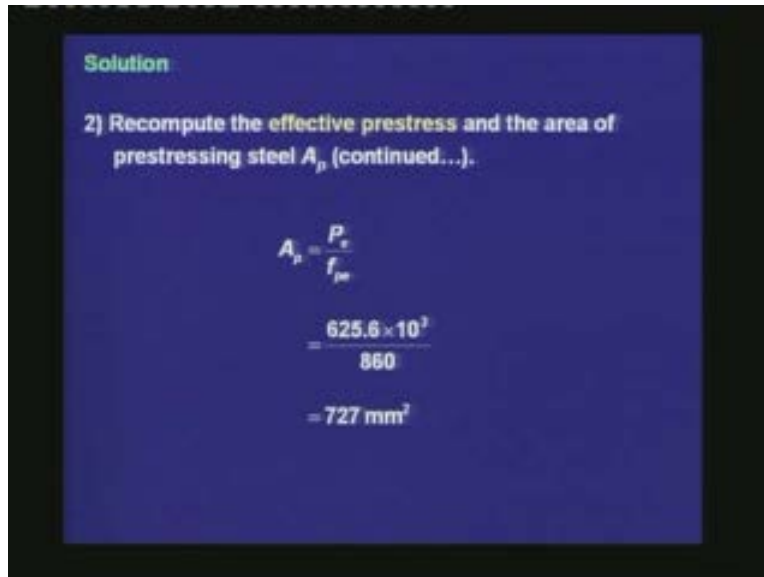
2) Recompute the effective prestress and the area of prestressing steel A_{pr}

$$P_e = \frac{M_T - f_{ct,all} A k_t}{e + k_t}$$
$$= \frac{435 \times 10^3 - \frac{1.65}{10^3} \times 150,000 \times 236}{336 + 236}$$
$$= 625.6 \text{ kN}$$

Since P_e is substantially lower than the previous estimate of 826 kN, A_{pr} , P_0 and e need to be recalculated.

We are recomputing the effective prestress and the area of the prestressing steel. The effective prestress is equal to $(M_T - f_{ct,all}Ak_t)/(e + k_t)$. Once we substitute the values of the variables, we get $P_e = 625.6$ kN. Since, P_e is substantially lower than the previous estimate of 826 kN, A_p , P_0 and e need to be recalculated. Here, we find a situation where the effective prestress is quite different from the value after the preliminary design, and hence we are recomputing the eccentricity and the other prestressing variables.

(Refer Slide Time 40:31)



Solution

2) Recompute the effective prestress and the area of prestressing steel A_p (continued...).

$$A_p = \frac{P_e}{f_{pe}}$$
$$= \frac{625.6 \times 10^3}{860}$$
$$= 727 \text{ mm}^2$$

$A_p = P_e/f_{pe} = 625.6 \times 10^3/860 = 727 \text{ mm}^2$. Again, this value is substantially lower than the value after the preliminary design which was 960 mm^2 .

(Refer Slide Time: 40:52)

Solution

3) Recompute eccentricity e

$$P_0 = A_p f_{p0}$$
$$= 727 \times 1035$$
$$= 752.4 \text{ kN}$$

45

To recompute e, we are first calculating $P_0 = A_p f_{p0} = 727 \times 1035 = 752.4 \text{ kN}$.

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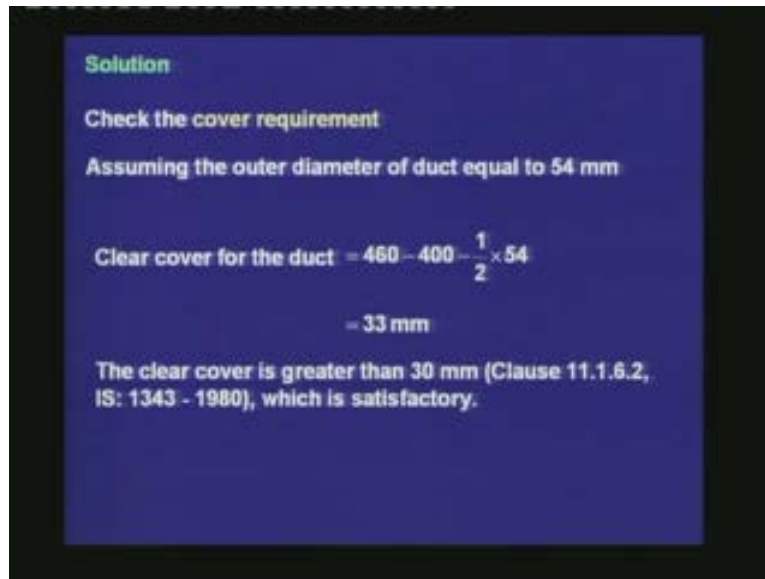
Solution

3) Recompute eccentricity e (continued...)

$$e = \frac{M_{sw} + f_{ct,all} A k_b}{P_0} + k_b$$
$$= \frac{55 \times 10^3 + \frac{2.1}{10^3} \times 150,000 \times 236}{752.4} + 236$$
$$= 172 + 236$$
$$= 408 \rightarrow 400 \text{ mm}$$

Then, the eccentricity is given as $e = (M_{sw} + f_{ct,all} A k_b) / P_0 + k_b$. Once we substitute the variables, $e = 408 \text{ mm}$. Here, we have to check whether e is satisfying the cover requirement or not. Based on the cover requirement, we have to reduce e to 400 mm.

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Solution

Check the cover requirement

Assuming the outer diameter of duct equal to 54 mm

Clear cover for the duct = $460 - 400 - \frac{1}{2} \times 54$

= 33 mm

The clear cover is greater than 30 mm (Clause 11.1.6.2, IS: 1343 - 1980), which is satisfactory.

To check the cover requirement, let us assume the outer diameter of duct to be equal to 54 mm. This information is available from the supplier of the ducts depending on how many strands we need to place within the duct. Then the clear cover for the duct is equal to 460 minus the eccentricity, minus half of the outer diameter of the duct, which gives a value of 33 mm. The clear cover is greater than 30 mm, which is satisfactory as per Clause 11.1.6.2 of IS: 1343-1980. If we need to have larger cover for extreme environment, then we have to reduce e . Thus, in this case the maximum value of the eccentricity is governed by the cover requirement and not by the expressions based on the stress.

(Refer Slide Time 42:56)

Solution

Since the value of e has changed from 366 mm to 400 mm, prestressing variables are recomputed.

$$P_e = \frac{M_T - f_{ct,all} A k_t}{e + k_t}$$
$$= \frac{435 \times 10^3 - \frac{1.65}{10^3} \times 150,000 \times 236}{400 + 236}$$
$$= 592.0 \text{ kN}$$

P_e has further reduced from 625.6 kN. A_p and P_0 are recalculated.

Since the value of e has changed from 366 mm to 400 mm, the prestressing variables are recomputed. We have been able to increase the eccentricity, and hence we shall be able to reduce the prestressing force. $P_e = (M_T - f_{ct,all} A k_t) / (e + k_t)$. After substituting the values of the variables, $P_e = 592$ kN. Thus, P_e has further reduced from 625.6 kN. A_p and P_0 are recalculated.

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Solution

$$A_p = \frac{592 \times 10^3}{860}$$
$$= 688.5 \text{ mm}^2$$

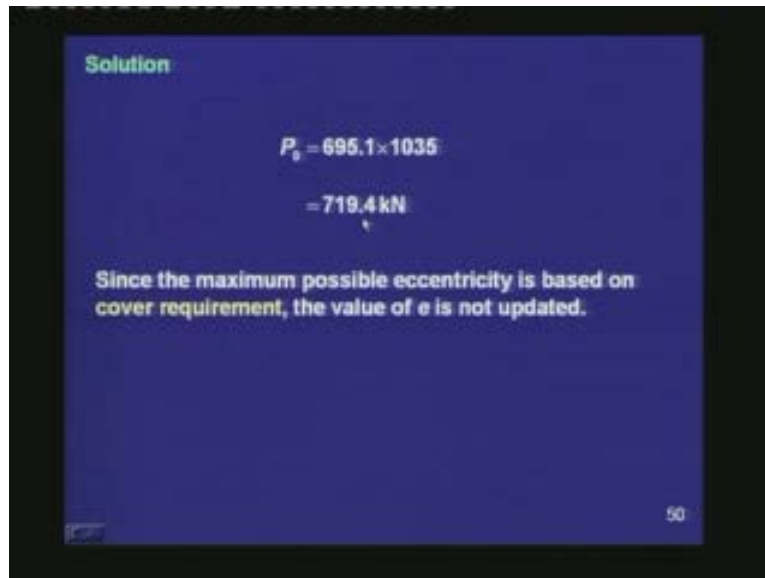
Select (7) 7-wire strands with $A_p = 7 \times 99.3$

$$= 695.1 \text{ mm}^2$$

The tendons can be placed in one duct. The outer diameter of the duct is 54 mm.

$A_p = 592 \times 10^3 / 860 = 688.5 \text{ mm}^2$. Select seven 7-wire strands, with $A_p = 7 \times 99.3 = 695.1 \text{ mm}^2$. The tendons can be placed in one duct and the outer diameter of the duct is 54 mm.

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P_0 the prestress at transfer, is equal to the area of the prestressing steel times f_{p0} , which is equal to 719.4 kN. Since the maximum possible eccentricity is based on the cover requirement, the value of e is not updated. Thus, we have come to the end of the final design for the prestressing variables, and next we have to check the compressive stresses in the concrete.

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Solution

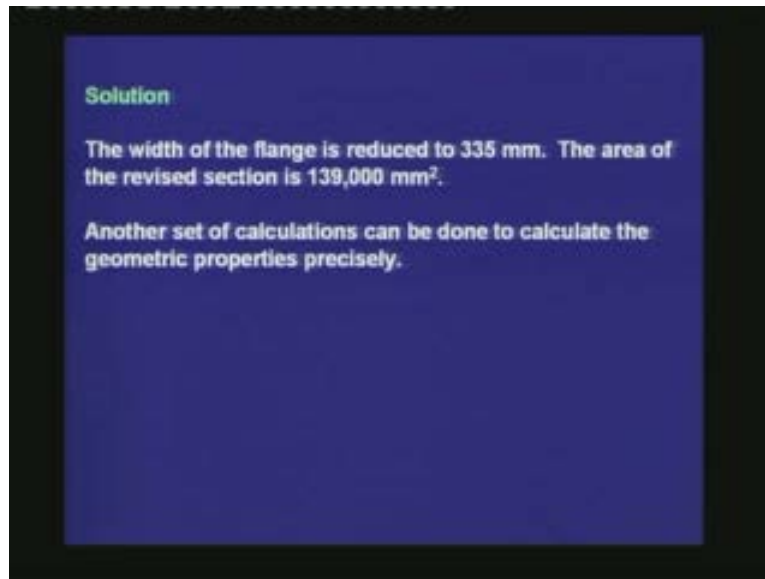
4) Check the compressive stresses in concrete.

At transfer	At service
$A \geq \frac{P_e h}{f_{ct,at} c_T - f_{ct,at} c_b}$	$A \geq \frac{P_e h}{f_{ct,at} c_b - f_{ct,at} c_T}$
$= \frac{719,4 \times 10^3 \times 920}{12,5 \times 460 - 2,1 \times 460}$	$= \frac{592 \times 10^3 \times 920}{11 \times 460 - 1,65 \times 460}$
$= 138,352 \text{ mm}^2$	$= 126,631 \text{ mm}^2$

The governing value of A is $138,352 \text{ mm}^2$. The section can be revised.

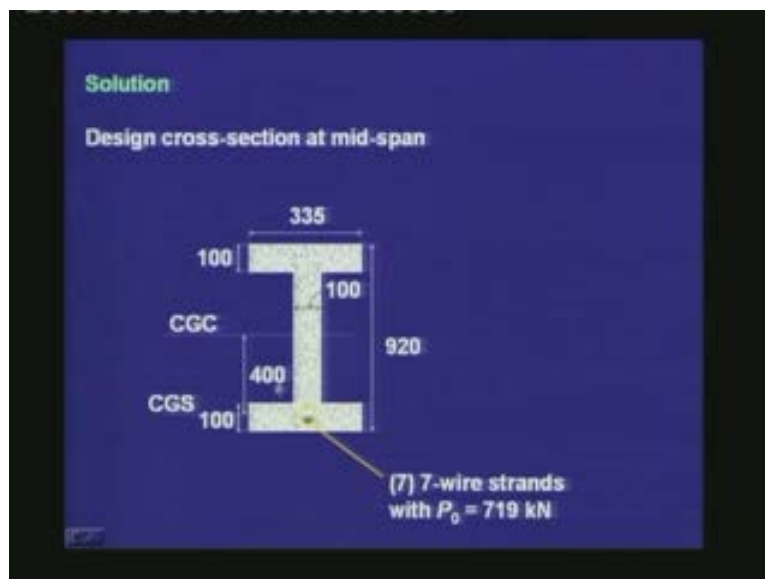
At transfer, we have the expression of A based on the prestressing force at transfer, and once we substitute the variables A should be greater than $138,352 \text{ mm}^2$. At service, the expression is based on the effective prestress at service, and the value is $126,631 \text{ mm}^2$. Thus, we have two requirements of area and we are selecting the higher value. The governing value of A is $138,352 \text{ mm}^2$. Initially, we had assumed a section which has an area of $150,000 \text{ mm}^2$, and now we find that the requirement of the area is much lower and hence we can revise the section.

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The width of the flange is reduced to 335 mm. The area of the revised section is 139,000 mm² which satisfies the minimum requirement based on the allowable compressive stresses. Another set of calculations can be done to calculate the geometric properties precisely. But, we shall find that the prestressing variables will not change much based on this new set of geometric variables.

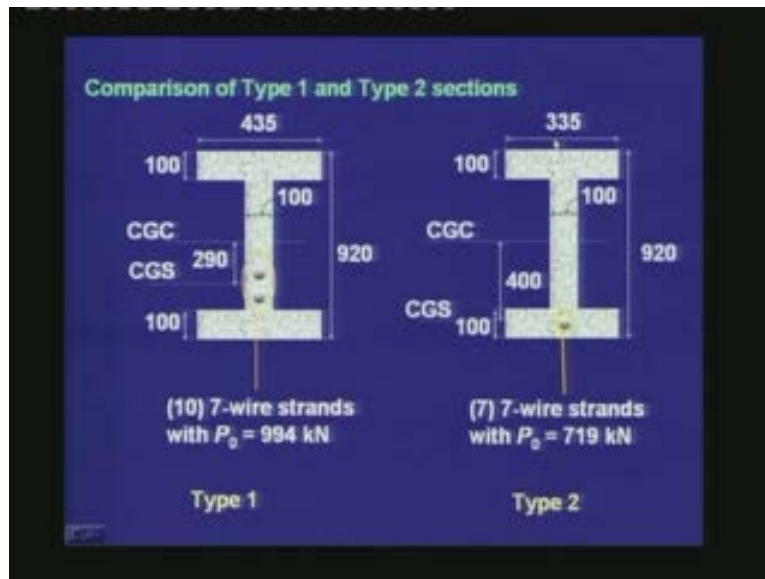
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Thus, at the end of the final design, we have come to a cross-section which first satisfies the depth requirement of 920 mm. The flange width is 335 mm, the width of the web is 100 mm, and the depth of the two flanges is also 100 mm. Based on the cover requirement, the CGS is located at a depth of 400 mm from the CGC. The amount of prestressing steel is seven of 7-wire strands with $P_0 = 719$ kN. This is the end result of the design for a Type 2 member.

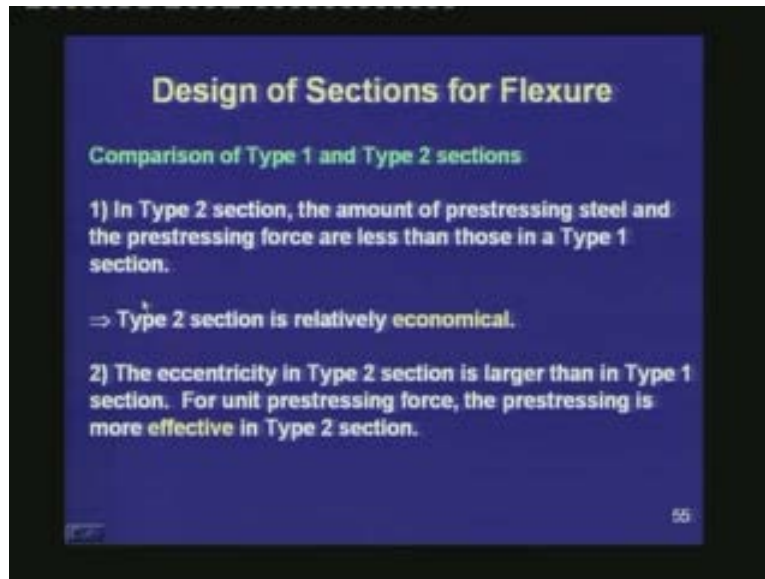
Let us compare what we had achieved for the design of a Type 1 member compared to the one that we have achieved for the Type 2 member.

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Once we compare the two sections, we find that for the Type 1 section the flange width is larger because the required area for the Type 1 section was larger as compared to that for the Type 2 section. The eccentricity is smaller for the Type 1 section. The amount of steel required for the Type 1 section is larger, which is ten of 7-wire strands. For the Type 2 section, the amount of steel is seven of 7-wire strands. The prestress at transfer for the Type 1 section is higher, which is 994 kN as compared to 719 kN for Type 2 section.

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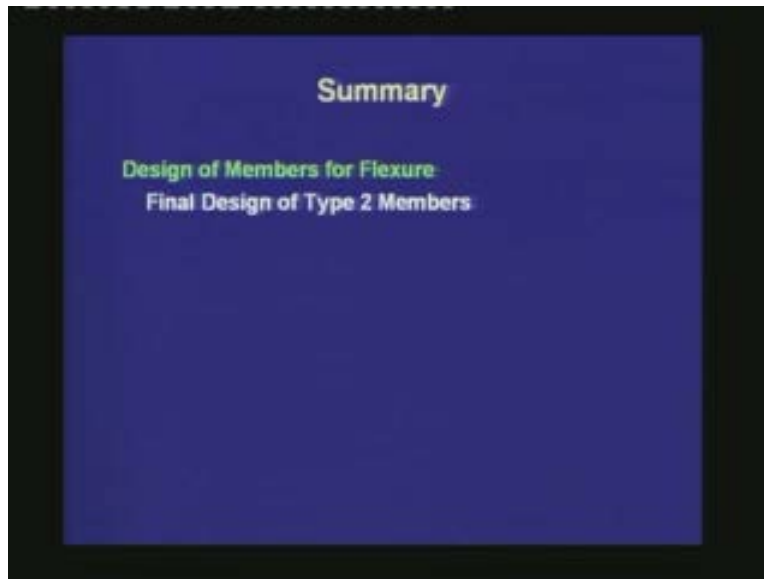
Comparing the two sections, we can draw the following conclusions.

- 1) In Type 2 section, the amount of prestressing steel and the prestressing force are less than those in Type 1 section. Also, the sectional area is small for Type 2 section. Hence, the Type 2 section is relatively economical as compared to Type 1 section.

This is the reason why we allow some tension in the concrete. In the early developments of prestressed concrete, the designers were hesitant to have any tensile stress in the concrete. It was found later that we can allow tensile stress in the concrete because most of the time the structure does not have the characteristic load. Hence, allowing some tensile stress is not detrimental to the section.

- 2) The eccentricity in Type 2 section is larger than in Type 1 section. Thus for unit prestressing force, the prestressing is more effective in Type 2 section. If we compare, say the camber (the upward deflection) due to unit prestressing force at that particular eccentricity, we find that the camber in the Type 2 member will be larger than in Type 1 member.

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Today we studied the final design of a Type 2 member. First, we recapitulated the variables that are available for design, and the variables that need to be calculated.

The design process is conveniently grouped into two stages. The first is the preliminary design, where we select a section and estimate the prestressing variables. We select the material properties, then we select the depth. We estimate the lever arm as a fraction of the depth. We calculate the effective prestressing force, which is the moment divided by the lever arm. From the effective prestressing force, we estimate the amount of prestressing steel. We also make a check for the area of the concrete section.

After the preliminary design, we move on to the final design. In the final design, we compute the eccentricity accurately. The eccentricity is given as the total moment divided by the estimated prestressing force. From the eccentricity, we may need to calculate the effective prestress once again, and we may recompute the eccentricity. If the second value of the eccentricity is quite different from the first value, then we do another cycle of calculations. Also, we need to check that the eccentricity satisfies the cover requirement.

Once we have designed the eccentricity, the prestressing force and the amount of prestressing steel, we need to check the section for compressive stresses in the concrete.

There are two conditions; one is based on the compressive stress in the bottom during transfer, and the other is based on the compressive stress at the top during service. These two conditions give two minimum values of the area of the section. If the trial section is not satisfactory, then we have to again redesign the section. Once the section is redesigned we can go through another round of computations for the prestressing variables.

We have found that the procedure is same for Type 1 and Type 2 members. The only difference is that in a Type 1 member we are not allowing any tension during transfer or service, but for a Type 2 member we are allowing tension. We compared the two sets of equations, and found that the equations for Type 2 members are generalised in nature. If we equate the allowable tensile stress to be zero then we get back the equations for Type 1 members. If we design a Type 3 member, then we can use the equations for Type 2 members where we allow higher tensile stress.

Finally, we solved a problem. Earlier, we had designed the section as Type 1 and now we designed it as Type 2. When we compared the two results, we found that the area of the Type 2 section is less than that of the Type 1 section. The eccentricity of the Type 2 section is larger. The amounts of prestressing steel and force were smaller for the Type 2 section. These are the reasons why a Type 2 section is considered to be economical as compared to a Type 1 section.

With this we are ending the design of prestressed concrete sections for flexure. In our next class, we shall study about some more design variables which are helpful in placing the strands along the member.

Thank you.