PRESTRESSED CONCRETE STRUCTURES

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Module – 3: Analysis of Members

Lecture – 16: Analysis of Partially Prestressed Sections, Analysis of Behavior

Welcome back to prestressed concrete structures. This is the sixth lecture in module three on analysis of members. In today's lecture, we shall study analysis of members under flexure, specifically for the ultimate strength of partially prestressed sections and unbonded post-tensioned beams. We shall also look into the analysis of behavior of members.

First, we are studying the analysis of partially prestressed sections.

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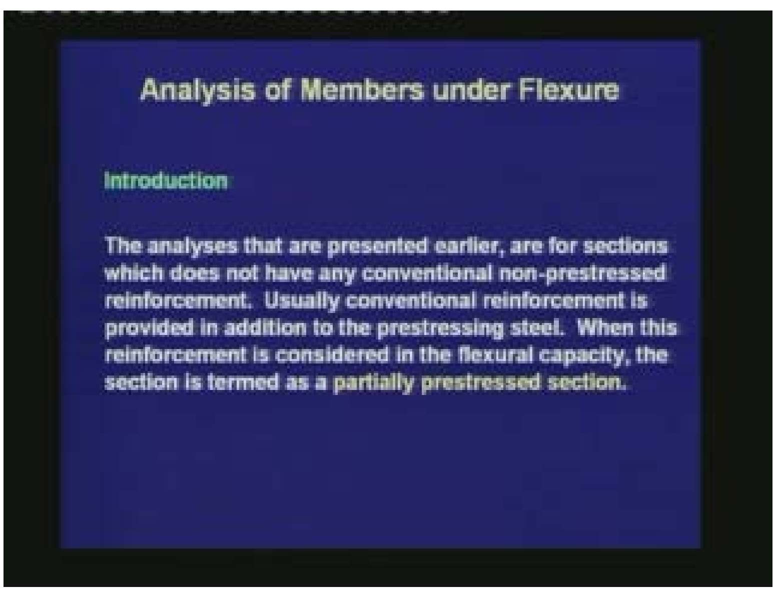

The analyses that are presented earlier are for sections which do not have any conventional non-prestressed reinforcement. Usually, conventional reinforcement is provided in addition to the prestressing steel. When this reinforcement is considered in the flexural capacity, the section is termed as a partially prestressed section.

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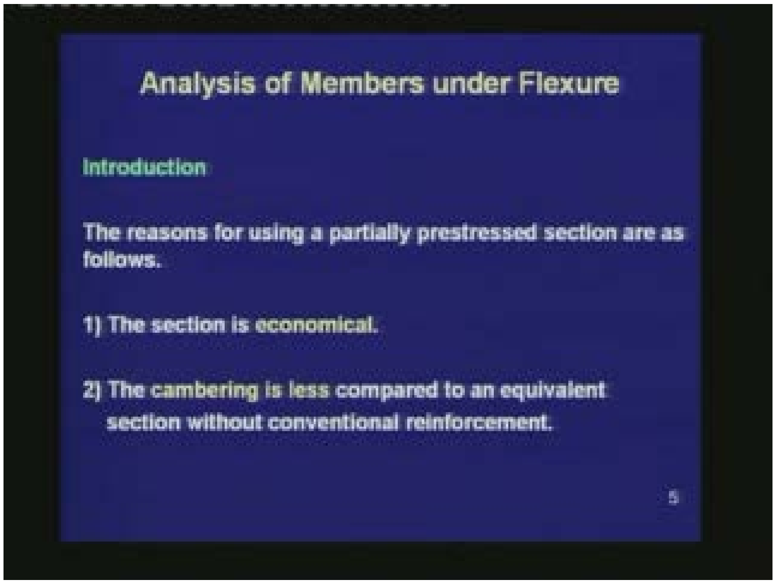

What are the reasons for using a partially prestressed section? The reasons are as follows. First, a partially prestressed section is economical compared to a fully prestressed section. We do not need as much amount of prestressing steel and anchorage units as compared to a fully prestressed section.

Second, the cambering is less compared to an equivalent section without conventional reinforcement. We have seen earlier that, when the prestress is transferred to a beam the beam tends to deflect upwards, which is termed as camber. The camber is large for beams with high prestress. For a partially prestressed section, the camber is lower and hence, possibility of cracking due to camber is reduced.

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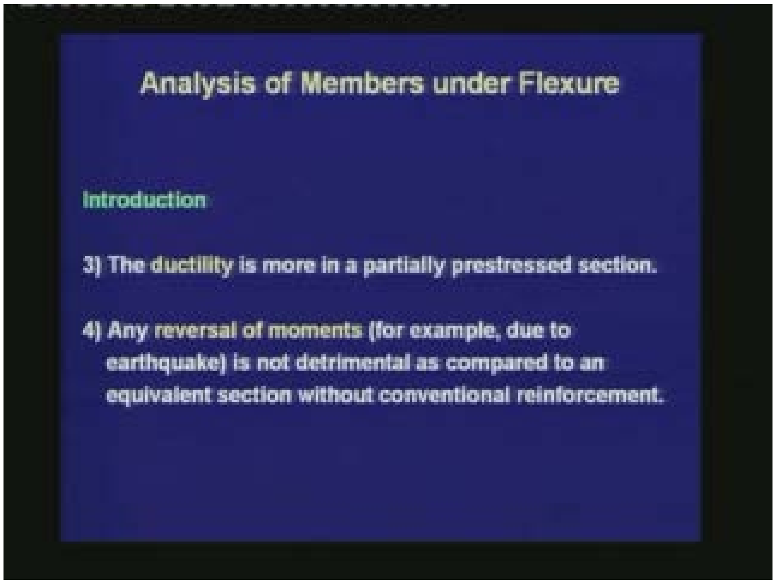

The third reason for using a partially prestressed section is that the ductility is more in such a section. Towards the end of this lecture we shall study about ductility.

The fourth reason is that any reversal of moments in the beam, for example due to an earthquake, is not detrimental in a partially prestressed beam as compared to an equivalent section without conventional reinforcement. In a fully prestressed beam, the prestress is designed for a certain value of the moment due to the load acting in a certain direction. But in case if there is any reversal of the moment, then the member is prone to premature failure compared to an equivalent reinforced concrete beam. A partially prestressed section is somewhere in between a fully prestressed section and a reinforced section. That means in a partially prestressed section, we try to have a balance between the benefits of a reinforced section and a prestressed concrete section. Depending on the situation, the partial prestressing is designed.

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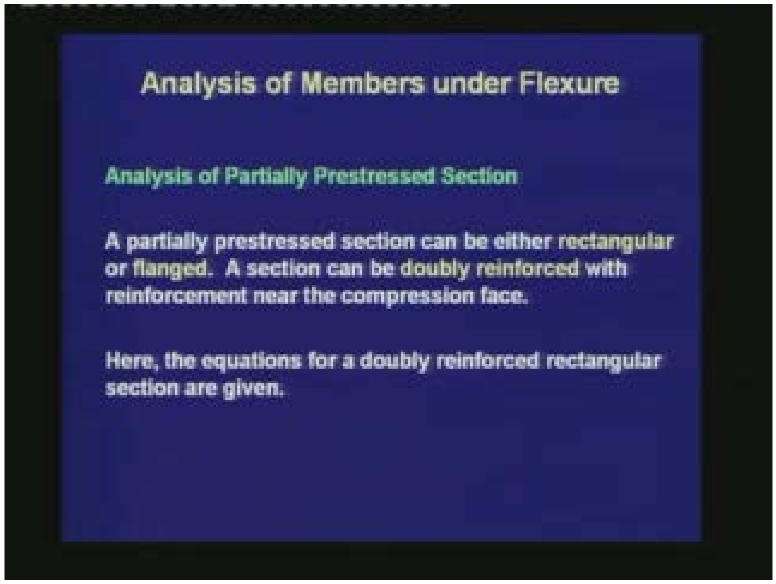

A partial prestressed section can be analyzed either as a rectangular or a flanged section. In our previous lectures, we have studied the analysis of fully prestressed sections for rectangular and flanged sections. We analyze a section as a flanged section, when the flanges are in compression and the depth of neutral axis is larger than the depth of the flange. In a partially prestressed section also, we can have either a rectangular section or a flanged section.

A section can be doubly reinforced with reinforcement near the compression face. We know about singly reinforced and doubly reinforced sections in conventional reinforced concrete. A similar classification is possible in partially prestressed members also. That means, in a doubly reinforced partially prestressed section there is reinforcement near the tension face, as well as near the compression face. In this lecture, we shall study the analysis of a rectangular doubly reinforced partially prestressed section.

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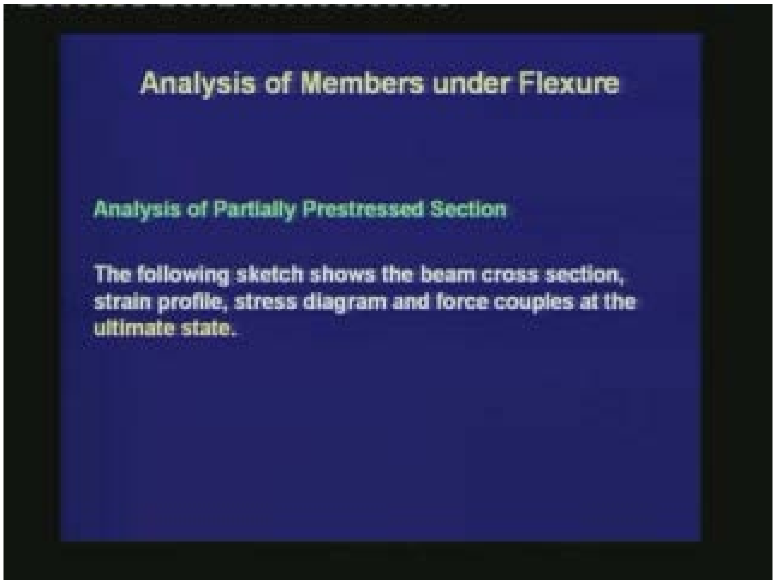

The following sketch shows the beam cross-section, strain profile, stress diagram and force couples at the ultimate state for a partially prestressed section.

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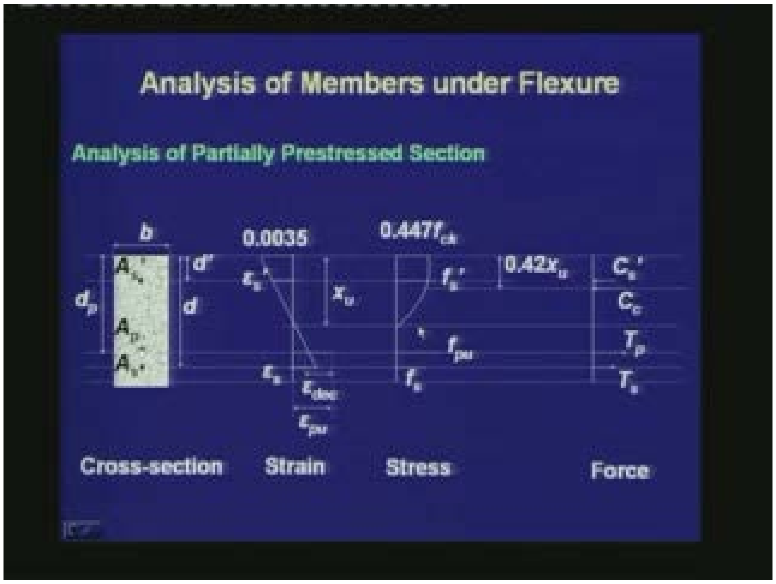

In the analysis for ultimate state, first we start with the strain diagram. The failure is defined as when the strain in the extreme compression fibre is equal to 0.0035. A linear strain diagram is drawn based on the principle, plane sections remain plane till ultimate.

The strains at the different levels of steel are identified. For a partially prestressed section, $A_s^{/}$ is the amount of steel near the compression face at a depth $d^{/}$. The area of prestressing tendon is represented as A_p and its CGS lies at a depth d_p. The steel near the tension face is represented as A_s and its centroid is at a depth d. Thus for this section, we have three layers of steel. For each layer of steel, we can identify the strain from the strain diagram. The strains are represented as $\varepsilon_s^{/}$ for the steel near the compression face, and ε_s for the steel near the tension face. For the prestressing steel, ε_{dec} is the difference between the strain in itself and in the surrounding concrete. The strain difference exists for any loading on the beam. When the strain in the concrete at the level of the CGS is added to ε_{dec}, the total strain in the prestressing tendon is obtained. For the ultimate state, the total strain is represented as ε_{pu}.

From the strain diagram, we are moving on to the stress diagram where we calculate the stress block for concrete and the stresses for the three layers of steel, from the constitutive relationships. For the stress in concrete, the diagram is parabolic and rectangular with a maximum stress of 0.447 f_{ck}. The stress in the compression steel is represented as $f_s^{/}$ and is calculated from the strain $\varepsilon_s^{/}$. The stress in the tension steel is represented as f_s and is calculated from the strain ε_s. The stress in the prestressing tendon f_{pu}, is calculated from the total strain ε_{pu}.

From the stress diagram, we move to the force diagram. $C_s^{/}$ is the force acting in the compression steel. C_c is the resultant of the stress block in concrete. T_p is the tension carried by the prestressing steel, and T_s is the force carried by the reinforcement near the tension face.

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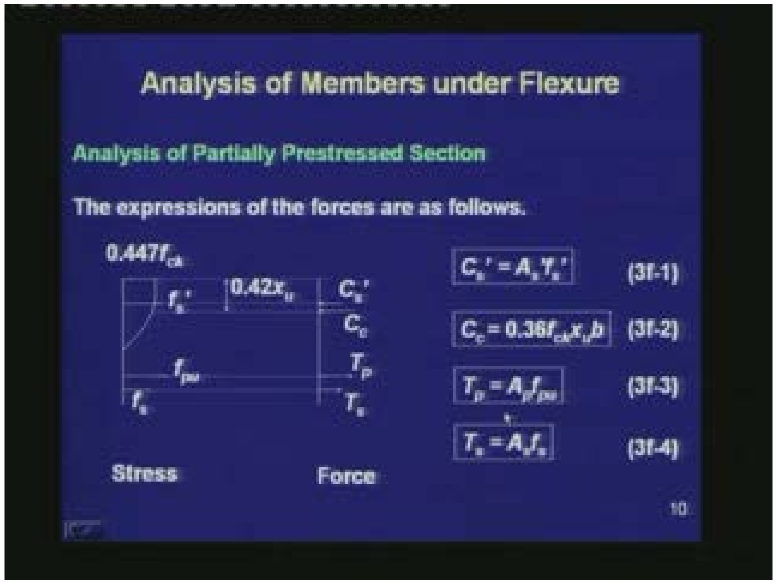

The expressions of the forces are as follows:

$C_s^{/} = A_s^{/}f_s^{/}$, $C_c = 0.36f_{ck}x_ub$, $T_p = A_pf_{pu}$ and $T_s = A_sf_s$.

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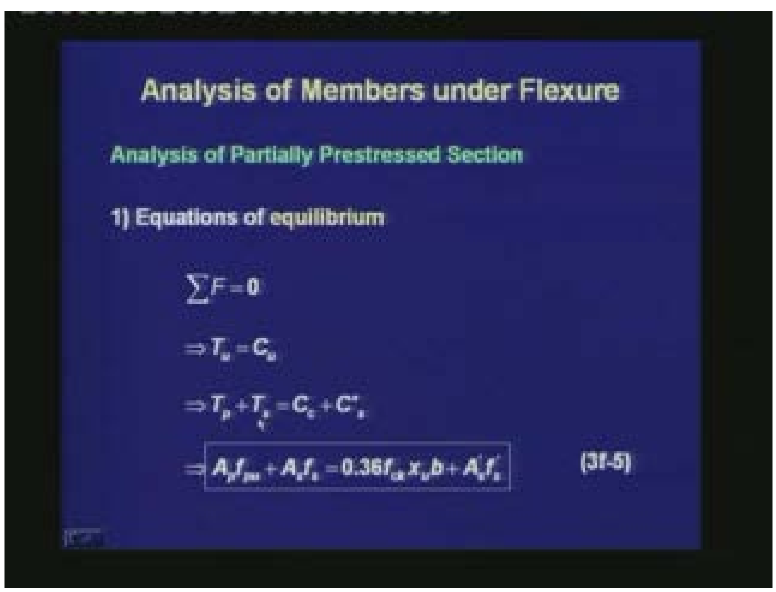

Next, we write the equations based on the principles of mechanics. First, let us start with the equations of equilibrium. For the longitudinal forces, $\Sigma F = 0$, which gives the total tension is equal to the total compression. The tension constitutes of T_p, the tension in the prestressing tendon, and T_s which is the tension in the reinforcement steel near the tension face. The compression also consists of two components, C_c which is the component taken by the concrete and $C_s^{/}$ which is the compression taken by the compression steel. We are writing the expressions of the individual forces, and this gives us the first equilibrium equation.

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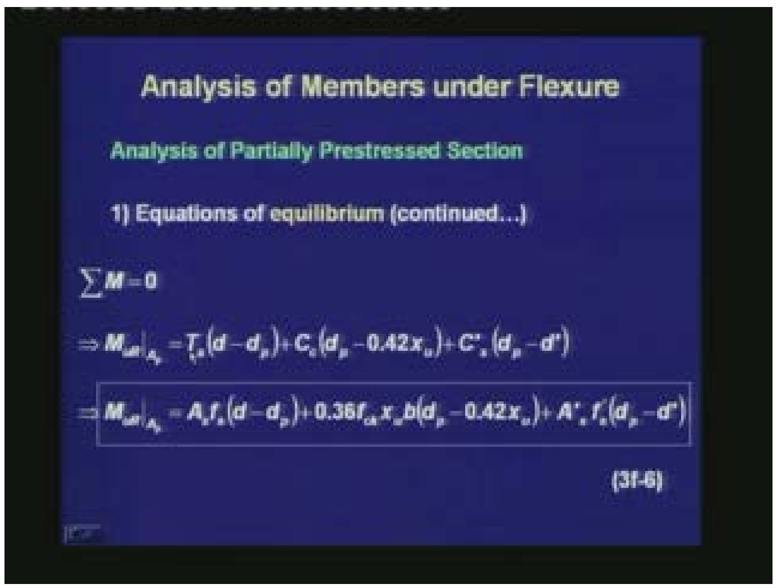

The second equilibrium equation is for the moment. Here we are writing the moment with respect to the axis passing through the CGS. The moment at ultimate is represented as M_{uR} or the ultimate moment of resistance. That is equal to the summation of the individual forces times the respective lever arms. The first term is $T_s(d - d_p)$, where $d - d_p$ is the lever arm between the tension steel and the CGS. The second term is $C_c\,(d_p - 0.42x_u)$, where $d_p - 0.42x_u$ is the lever arm between the CGS and the resultant of the compression in concrete. The third term is $C_s^{/}\,(d_p - d^{/})$, where $d_p - d^{/}$ is the lever arm between the CGS and the compression steel. Once we substitute the expressions of T_s, C_c and $C_s^{/}$ from the previous equations, we get an expression of the ultimate moment of resistance.

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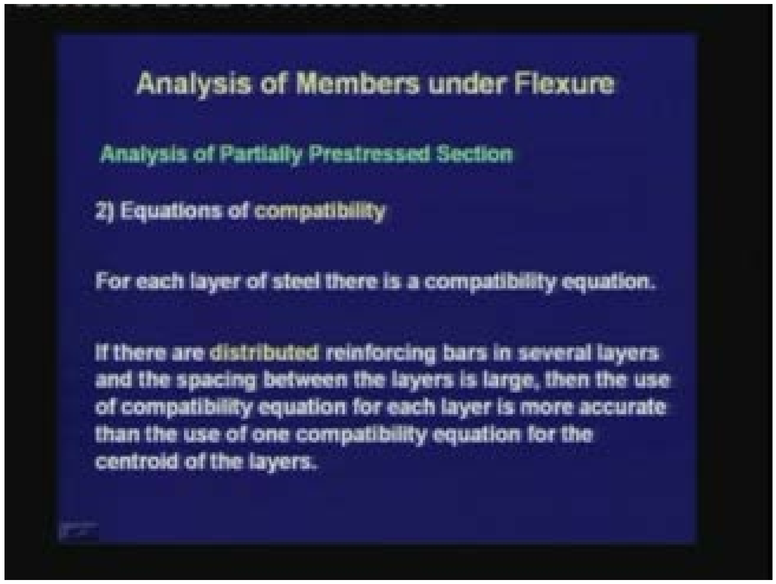

The second principle provides the equations of compatibility of strains. For each layer of steel there is a compatibility equation. The compatibility of strains means that the steel and the surrounding concrete are deforming together. If there are distributed reinforcing bars in several layers and the spacing between the layers is large, then the use of compatibility equation for each layer is more accurate than the use of one compatibility equation for the centroid of the layers. The equations that we shall learn now, will help us to analyze any beam where we have distributed steel in several layers, with wide spacing between the layers.

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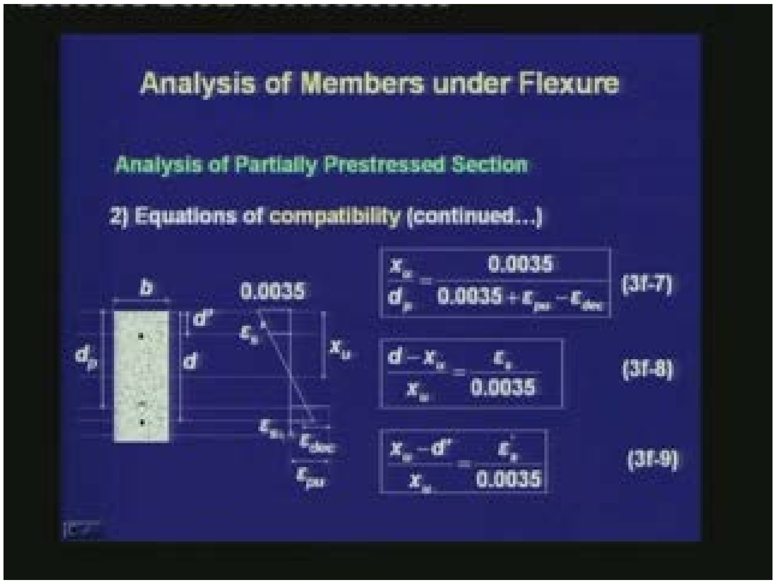

In this figure, we can see that there are three layers of steel. The top layer is the steel near the compression face. Then we have the prestressing steel which is at a depth d_p, and finally we have the steel near the tension face. The first equation is related with the prestressing steel. The depth of the neutral axis which is represented as x_u, is related to d_p by the similarity of the triangles. The second equation is related with the steel near the tension face. The depth of the steel from the neutral axis $(d – x_u)$ divided by x_u is equal to ε_s divided by 0.0035. The third equation is for the steel near the compression face, which relates the distance of the compression steel from the neutral axis $(x_u – d')$ to x_u as ε_s' divided by 0.0035.

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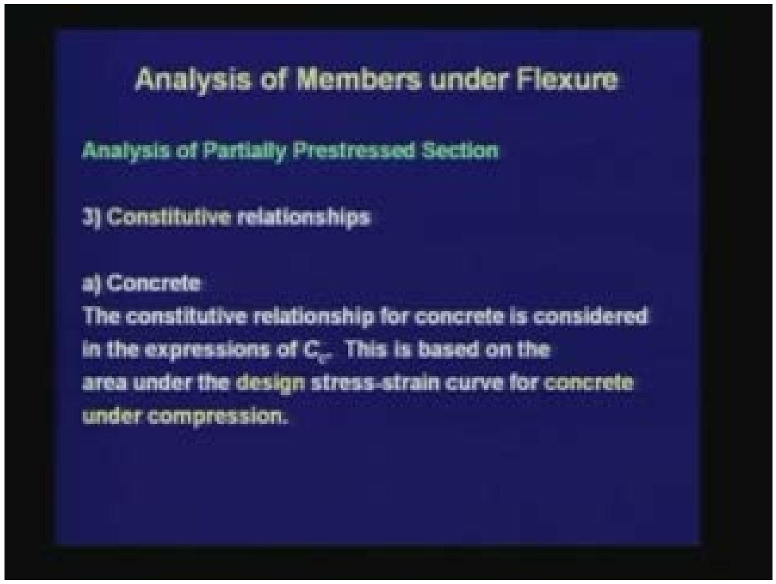

Next we move on to the constitutive relationships. A constitutive relationship relates the stress and strain in a material. For concrete, the constitutive relationship is considered in the expression of C_c which we had seen earlier. This is based on the design stress–strain curve for concrete under compression.

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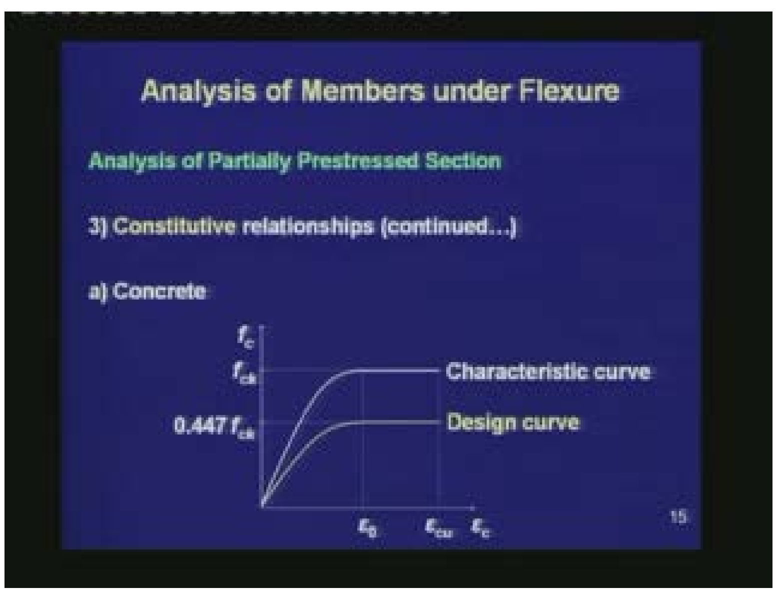

The design curve has a maximum stress of $0.447f_{ck}$, and the maximum strain ε_{cu} is equal to 0.0035. When we calculate the area of the stress block, we get the expression of C_c equal to $0.36f_{ck}x_ub$.

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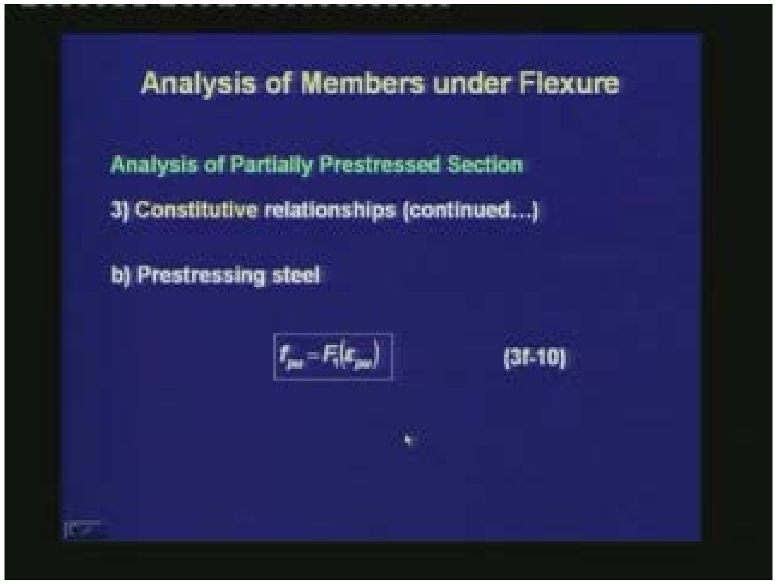

The second constitutive relationship is for the prestressing steel, where the relationship is denoted as $f_{pu} = F_1(\varepsilon_{pu})$.

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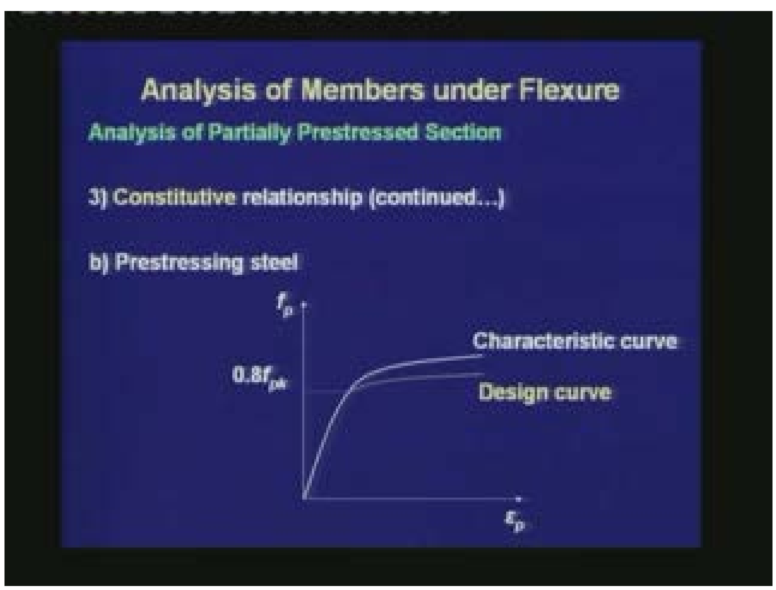

This function is available from the design curve for the prestressing steel.

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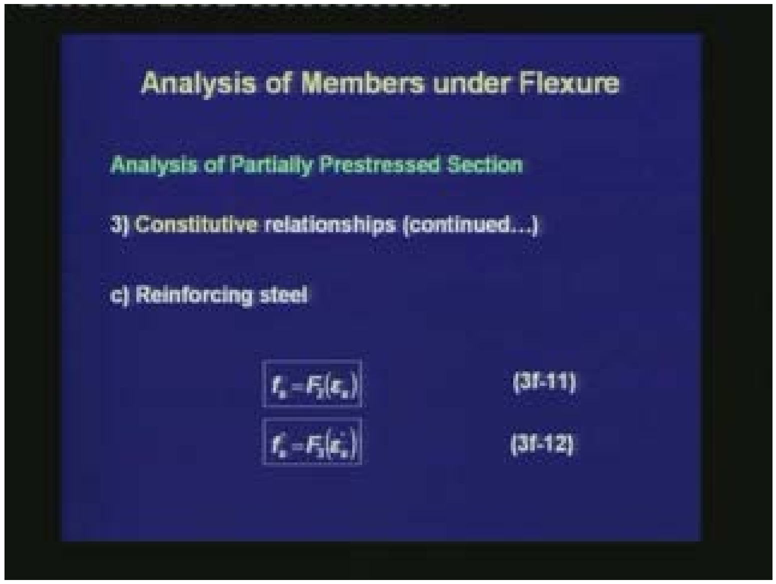

The third constitutive relationship is for the reinforcing steel of two layers. The equation for the bottom layer near the tension face is represented as $f_s = F_2(\varepsilon_s)$. The equation for the steel near the compression face is equal to $f_s^{/} = F_3(\varepsilon_s^{/})$.

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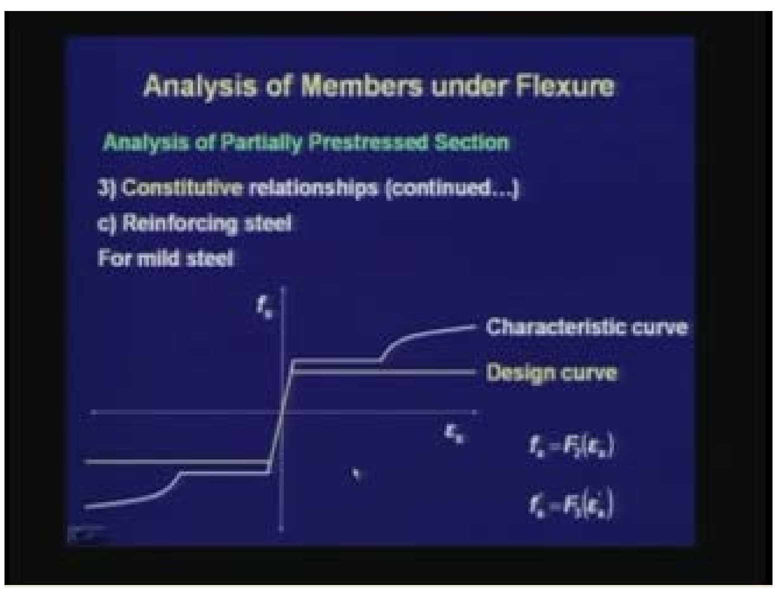

Both the functions F_2 and F_3 are same, if we are using the same grade of steel for the bottom steel and the top steel. For mild steel bars, the design curve is an elasto-plastic curve without any strain hardening.

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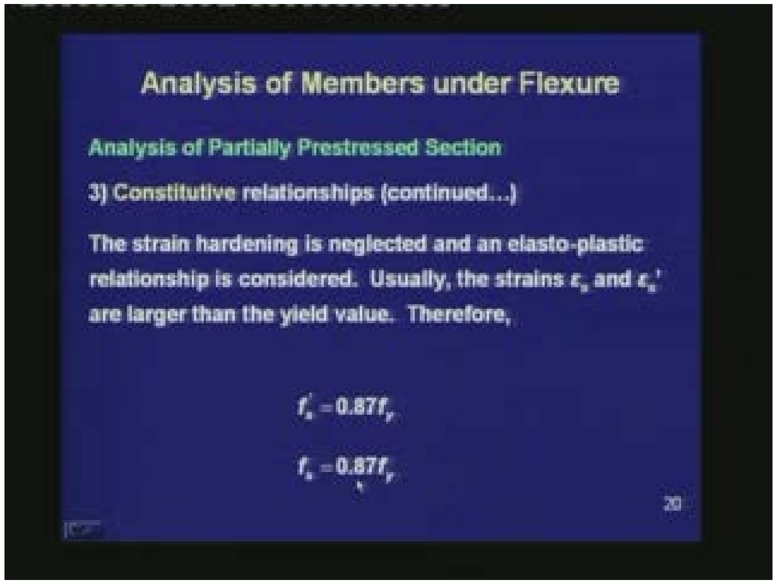

Usually, the strains ε_s and $\varepsilon_s^{/}$ are larger than the yield strain. Therefore, $f_s = 0.87\ f_y$ and $f_s^{/} = 0.87\ f_y$.

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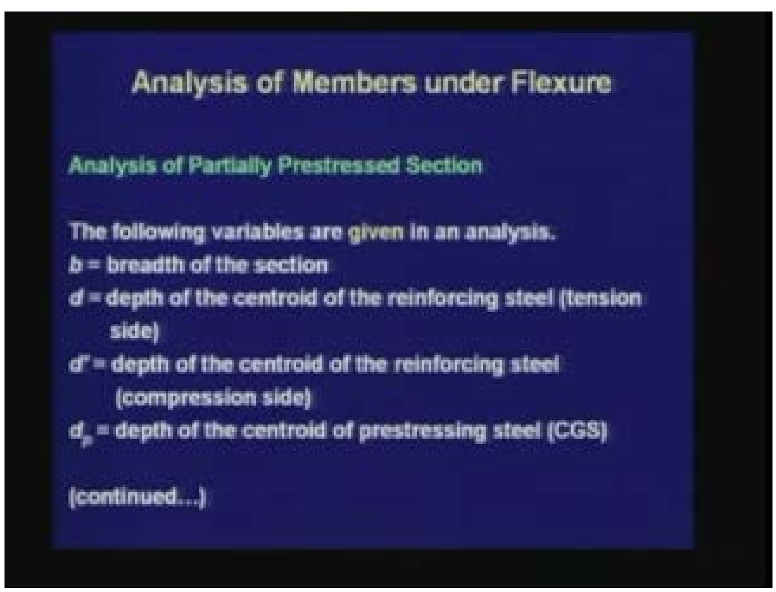

In an analysis, the following variables are given:

b = breadth of the section, d = depth of the centroid of the reinforcing steel near the tension side, $d^{/}$ = depth of the centroid of the reinforcing steel near the compression side, d_p = depth of the centroid of prestressing steel (CGS).

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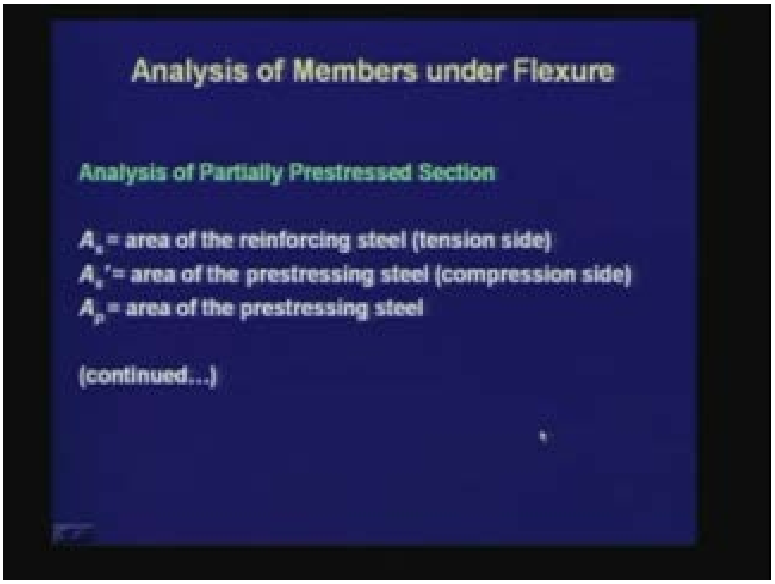

We know the sectional areas of steel. A_s = area of the reinforcing steel in the tension side, $A_s^{/}$ = area of the reinforcing steel in the compression side, A_p = area of the prestressing steel.

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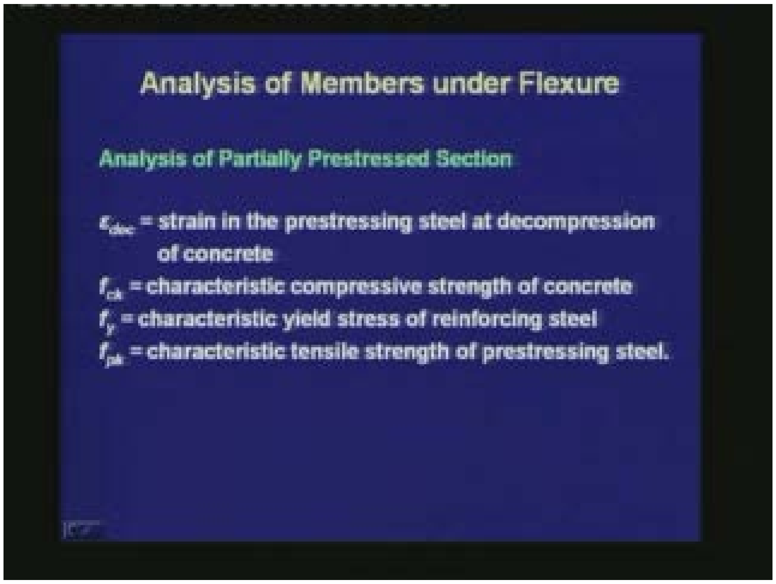

ε_{dec} is known from the amount of prestressing. This is the strain in the prestressing steel at decompression of concrete. The expression of ε_{dec} for a pre-tensioned beam is different from that for a post-tensioned beam. For a pre-tensioned beam, ε_{dec} is the same as the strain in the steel just before the transfer of the prestress. But for a post-tensioned beam, ε_{dec} is the strain in the prestressing steel plus the strain in the concrete after the prestressing operation has been done. We also know the material properties: f_{ck} = characteristic compressive strength of concrete, f_y = characteristic yield stress of the reinforcing steel, f_{pk} = characteristic tensile strength of the prestressing steel.

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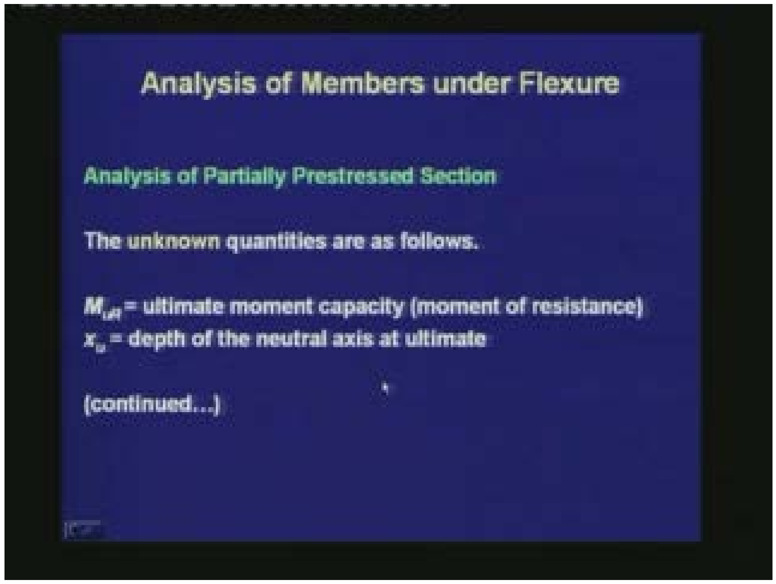

What are the unknown quantities? The first unknown quantity is M_{uR} which is the ultimate moment capacity. The second unknown quantity is x_u, the depth of the neutral axis at ultimate.

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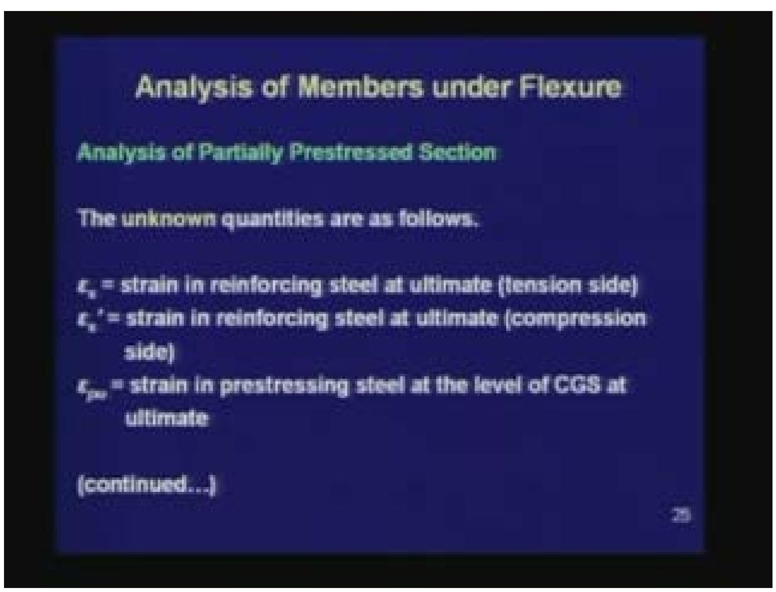

We do not know the strains in the steel at ultimate: ε_s = strain in reinforcing steel for the tension side, $\varepsilon_s^{/}$ = strain in the reinforcing steel for the compression side, ε_{pu} = strain in the prestressing steel at the level of CGS.

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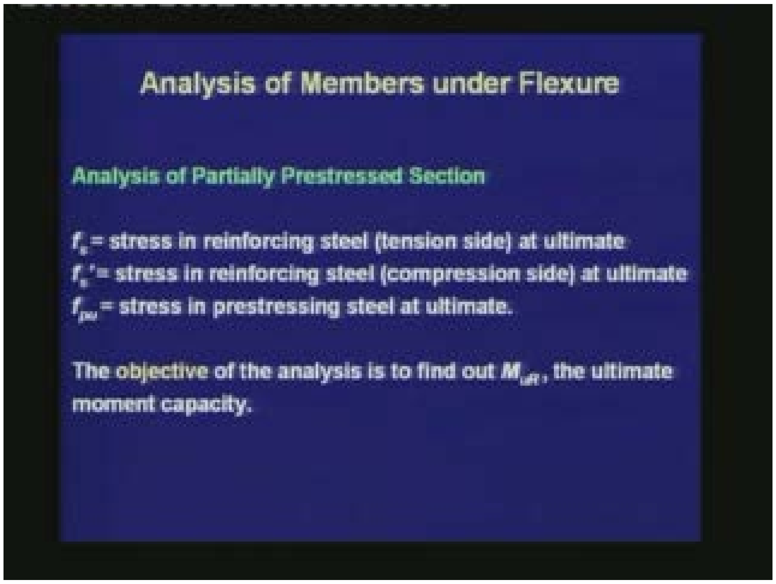

We also do not know the stresses in the steel at ultimate: f_s = stress in reinforcing steel for the tension side, $f_s^{/}$ = stress in reinforcing steel for the compression side, f_{pu} = stress in prestressing steel. With this set of known and unknown quantities, the objective of an analysis is to find out M_{uR}, the ultimate moment capacity.

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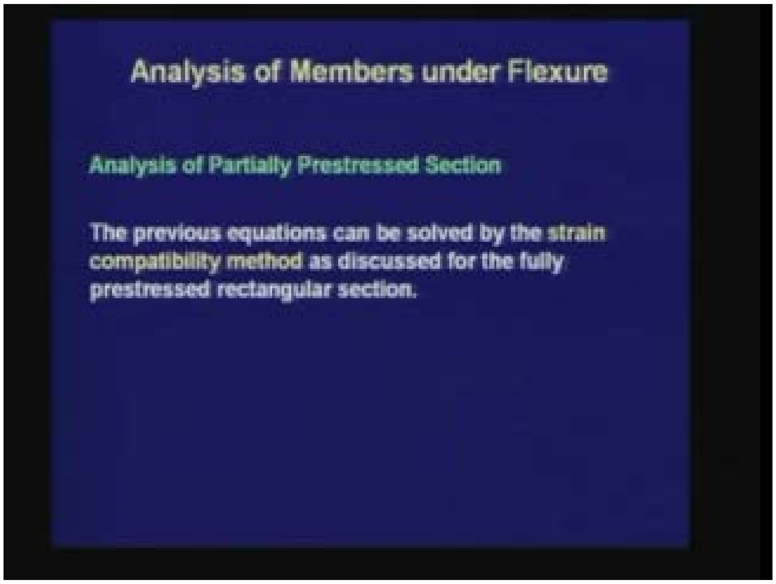

The set of equations can be solved iteratively by the method of strain compatibility, as we have learnt for a fully prestressed section.

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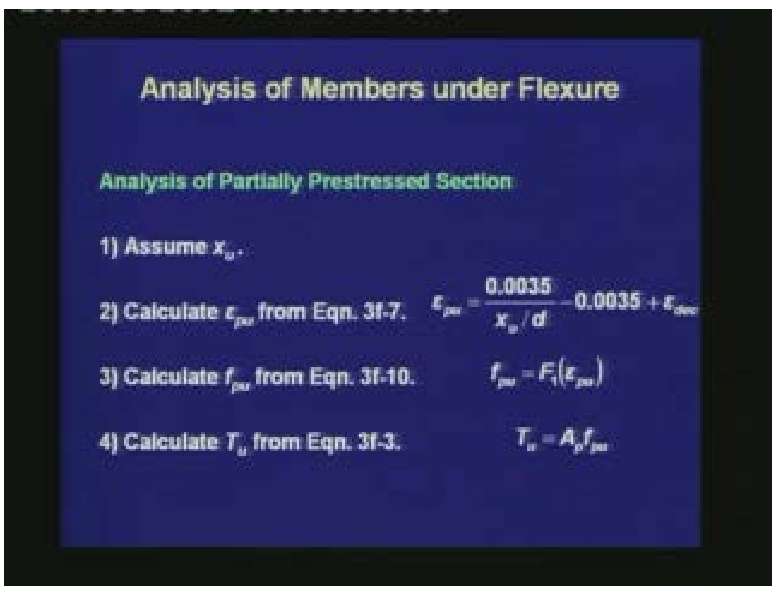

First, we are assuming the depth of the neutral axis which is x_u. Next, we are calculating ε_{pu} from Eqn. 3f-7, where the equation is rewritten as $\varepsilon_{pu} = 0.0035/(x_u/d) - 0.0035 + \varepsilon_{dec}$. Once we know ε_{pu}, we can calculate f_{pu} from Eqn. 3f-10 which is the constitutive relationship for the prestressing tendon, $f_{pu} = F_1(\varepsilon_{pu})$. Next, we calculate the tension in the prestressing steel which is given as $T_u = A_p f_{pu}$.

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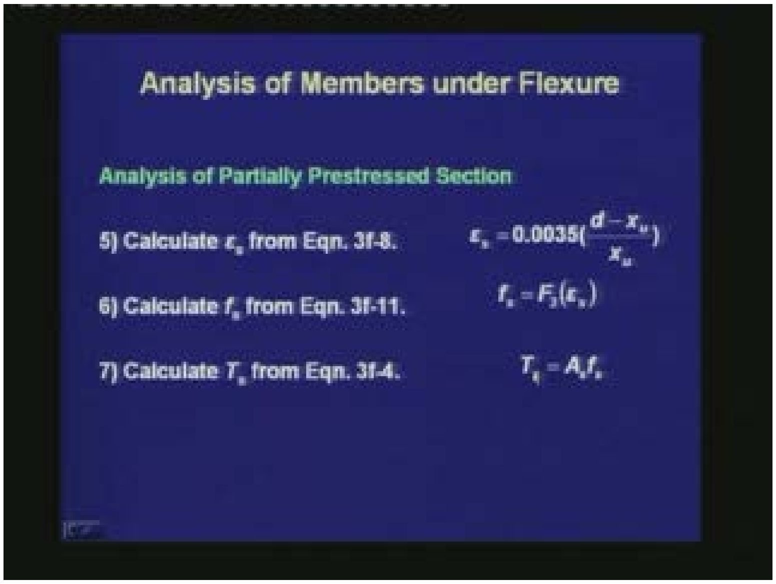

Next, from Eqn. 3f-8 we calculate $\varepsilon_s = 0.0035(d–x_u)/x_u$. From Eqn. 3f-11, $f_s = F_2(\varepsilon_s)$. The tension in the reinforcement steel at the tension face is equal to $T_s = A_s f_s$.

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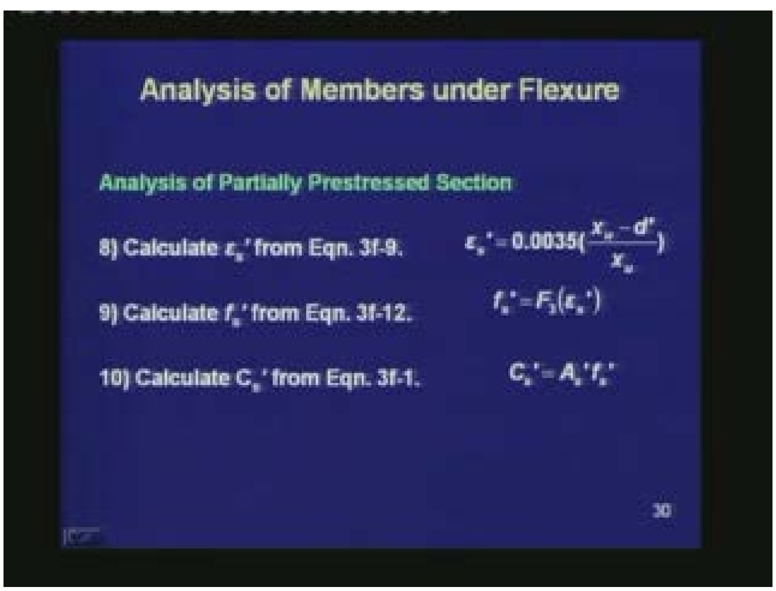

Finally, we calculate the strain in the steel close to the compression face. From Eqn. 3f-9, $\varepsilon_s^{/} = 0.0035(x_u – d^{/})/x_u$. From Eqn. 3f-12, $f_s^{/} = F_3(\varepsilon_s^{/})$. Next, $C_s^{/} = A_s^{/} f_s^{/}$.

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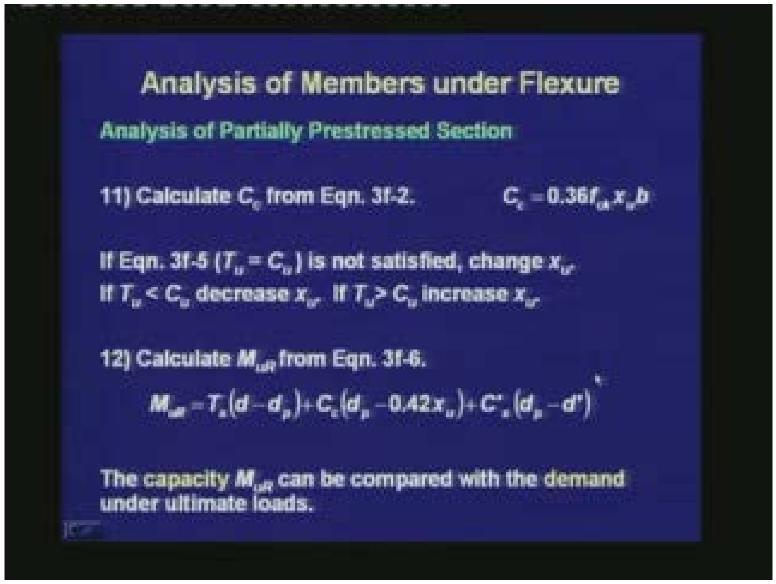

Thus, what we did till now is from the compatibility equations, we calculated the strain in each layer of the steel. From the strain, we calculated the stress from the constitutive relationship, and from the stress we calculated the force carried by each layer of steel.

Next, we are calculating the compression in the concrete C_c from Eqn. 3f-2. $C_c = 0.36f_{ck}x_ub$. If the equilibrium equation $T_u = C_u$ is not satisfied, then we need to change x_u. If $T_u < C_u$, then decrease x_u; if $T_u > C_u$, then increase x_u. Thus, if the tension is less compared to the compression, then we need to reduce x_u such that the amount of compression in concrete is less, and the tension in the steel is more. On the other hand, if the tension comes out to be larger than C_u, then we need to increase x_u thus having more compression in the concrete, and lesser tension in the steel. Like that, we iterate till the first equilibrium equation is satisfied within tolerable limits.

Next, we are calculating M_{uR} from the second equilibrium equation. $M_{uR} = T_s (d – d_p) + C_c(d_p – 0.42x_u) + C_s^{/}(d_p – d^{/})$. Thus, M_{uR} consists of three components which come from the three forces multiplied by the respective lever arms with respect to the CGS. Once we have calculated the capacity M_{uR}, we compare it with the demand under the ultimate loads.

Next we move on to the analysis of an unbonded post-tensioned beam.

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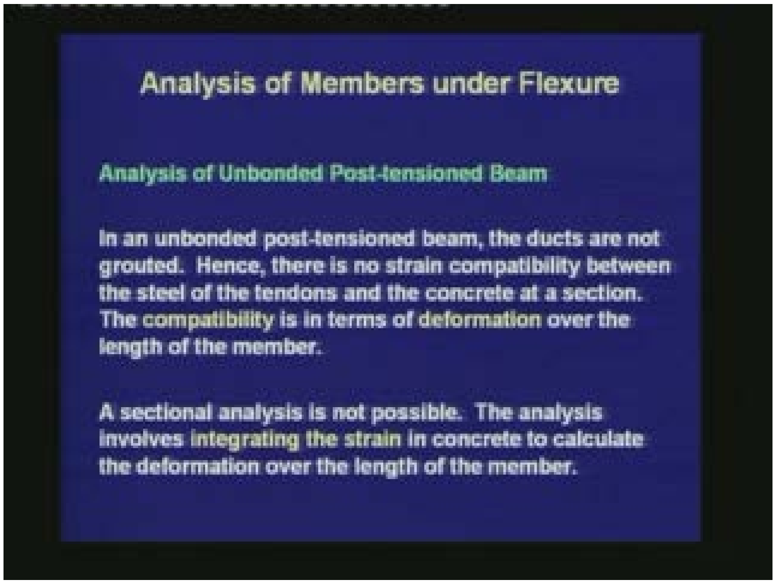

In an unbonded post-tensioned beam, the ducts are not grouted. Hence, there is no strain compatibility between the steel of the tendons and the concrete at a section. The compatibility is in terms of deformation over the length of the member. A sectional analysis is not possible. The analysis involves integrating the strain in concrete to calculate the deformation over the length of the member.

Till now, whatever we have studied is for a bonded tendon and we have assumed a perfect bond between the steel and the concrete. From this, we wrote the strain compatibility equations. A perfect bond means that, there is a relationship between the strain in the steel with the strain in the concrete at the level of the steel. If the tendons are not grouted, then we cannot apply the strain compatibility equation and we cannot do a sectional analysis. A sectional analysis means that the analysis is based on the strains, stresses and forces at a section. Since the strain in the concrete at the level of the prestressing steel and the strain in the prestressing steel are not related by an equation throughout the loading, we cannot use the conventional analysis for an unbonded tendon.

For an unbonded tendon, we write the compatibility equation in a different way. We write the compatibility in terms of the total deformation of the concrete at the level of the prestressing steel, and the deformation of the prestressing steel. Since we need to calculate the deformation of the concrete throughout the length of the member, we need to integrate the strain in the concrete at the level of prestressing steel.

The equation of compatibility is given as follows.

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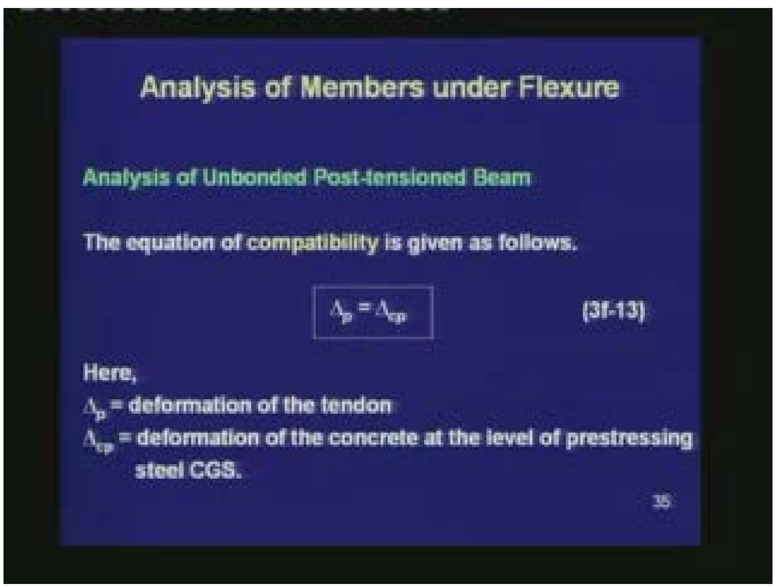

$\Delta_p = \Delta_{cp}$, where Δ_p is the deformation of the tendon and Δ_{cp} is the deformation of the concrete at the level of prestressing steel, which is the CGS.

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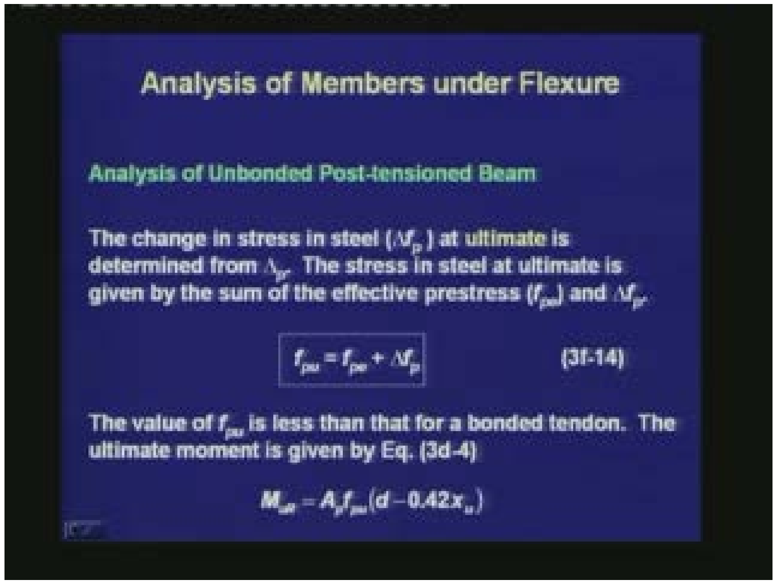

The change in stress in steel at ultimate which is denoted as Δf_p, is determined from Δ_p. The stress in steel at ultimate is given by the sum of effective prestress f_{pe} and Δf_p. Unlike bonded tendons, where we calculated the stress from the strain at a particular location, here we are calculating the stress from the total deformation. Once we know the total deformation, we know the change in the stress in the tendon at ultimate, and then we calculate the total stress in the tendon at ultimate.

The value of f_{pu} for an unbonded tendon is less than that for a bonded tendon. We have seen earlier that for a bonded tendon, the stress at the cracks can get substantially high. Whereas for an unbonded tendon, the stress is averaged out for the length of the member, and hence when the member fails, the stress does not reach a value which a bonded tendon achieves. Once we know f_{pu}, the ultimate moment is given by the conventional equation, $M_{uR} = A_p f_{pu}(d – 0.42x_u)$.

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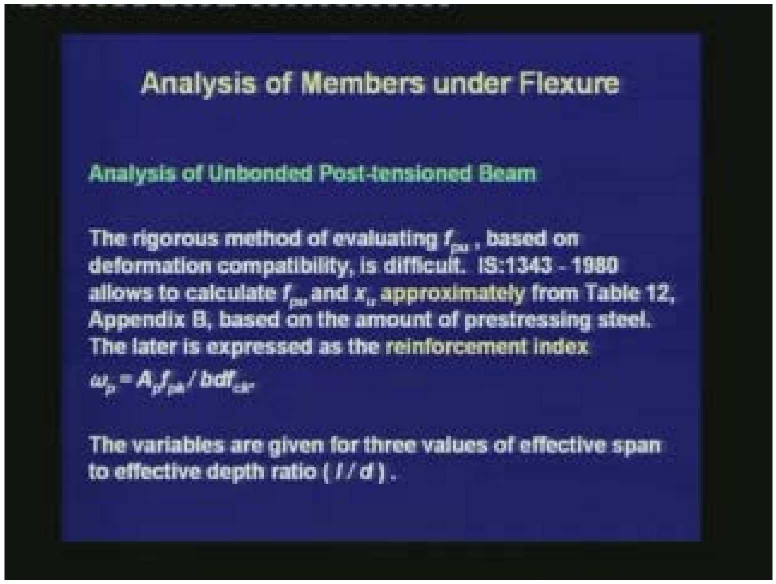

The rigorous method of evaluating f_{pu}, based on deformation compatibility is difficult. IS: 1343-1980 allows the calculation of f_{pu} and x_u approximately from Table 12 Appendix B, based on the amount of prestressing steel. The later is expressed as the reinforcement index $\omega_p = A_p f_{pk}/bdf_{ck}$. The variables are given for three values of effective span to effective depth ratio, which is denoted as *l*/d.

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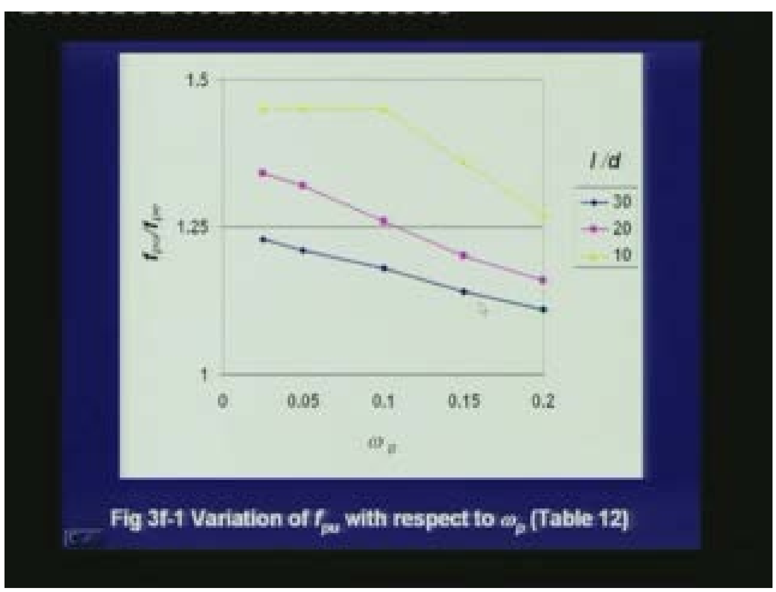

As we had seen for bonded tendons, the code allows us to use some approximate values for x_u and f_{pu}, based on the reinforcement index. In the tables for a bonded tendon, f_{pu} is normalized with respect to $0.87f_{pk}$, the maximum allowed stress in the tendon. In the tables for an unbonded tendon, f_{pu} is normalized with respect to f_{pe}, the effective prestress.

The code has given the values of f_{pu}/f_{pe} for three values of l/d equal to 30, 20, and 10. The variations by f_{pu}/f_{pe} with respect to ω_p are shown. With increasing amount of the prestressing steel f_{pu} decreases, which follows the logic that with more steel, the less stress the tendon will carry at ultimate.

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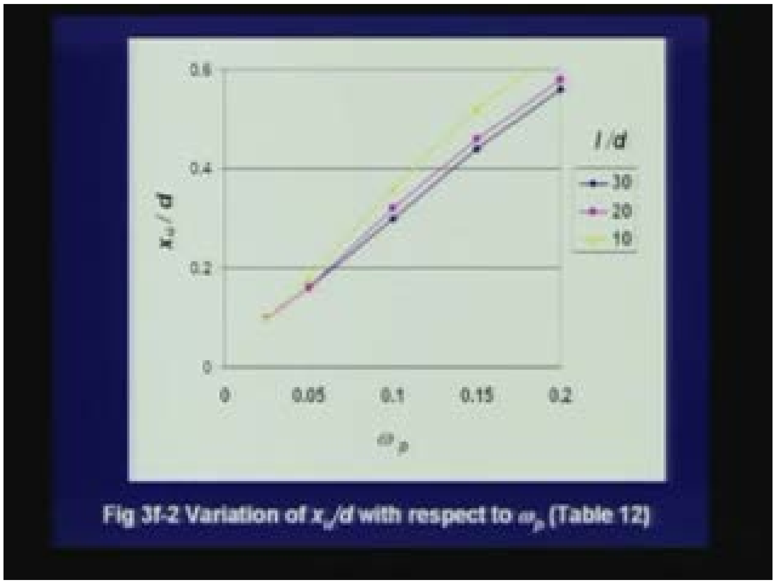

The other table gives us the values of the depth of the neutral axis, which is normalized with respect to the depth of the prestressing steel. x_u/d increases with ω_p. That means the more steel we have, we need to have a larger depth of the neutral axis to have higher compression in the concrete, to balance the stress in the steel at ultimate. Here also, the values of x_u/d are given for the three values of l/d.

Next, we move on to the analysis of behavior of a prestressed member.

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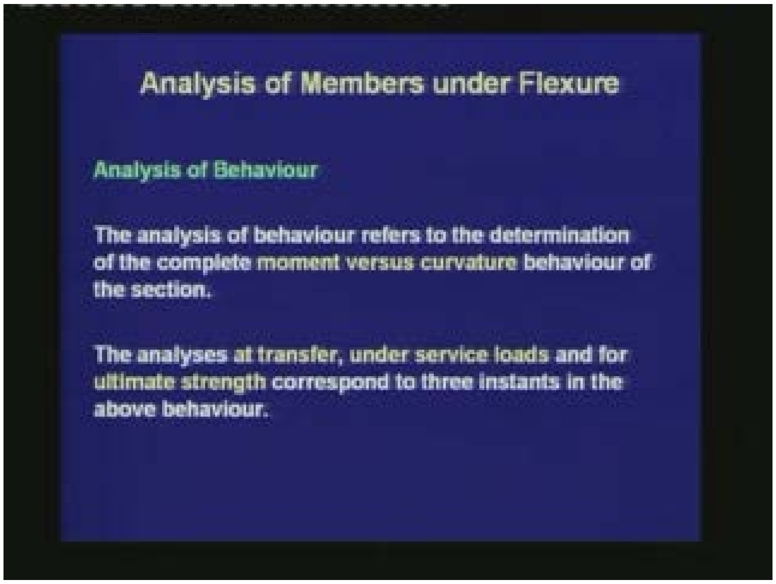

The analysis of behavior refers to the determination of the complete moment versus curvature behavior of the section. The analyses at transfer, under service loads and for ultimate strength, correspond to three instances in the above behavior. Till now, for the analysis of members under flexure, first we had studied analysis at transfer and service loads which are similar, based on the elastic analysis. We had checked the stresses to be within the allowable values. For the analysis at ultimate, we found out the ultimate moment of resistance and we compared this capacity with the demand under factored loads. In both these type of analysis, we were interested only in either the stress or the strength.

But when we talk of analysis of behavior, we are interested not only on the stress or strength, we are also interested in the deformation of the section which is given in terms of curvature. That means the analysis of behavior refers to the generation of the moment versus curvature curve for a section.

How do we define curvature for a beam?

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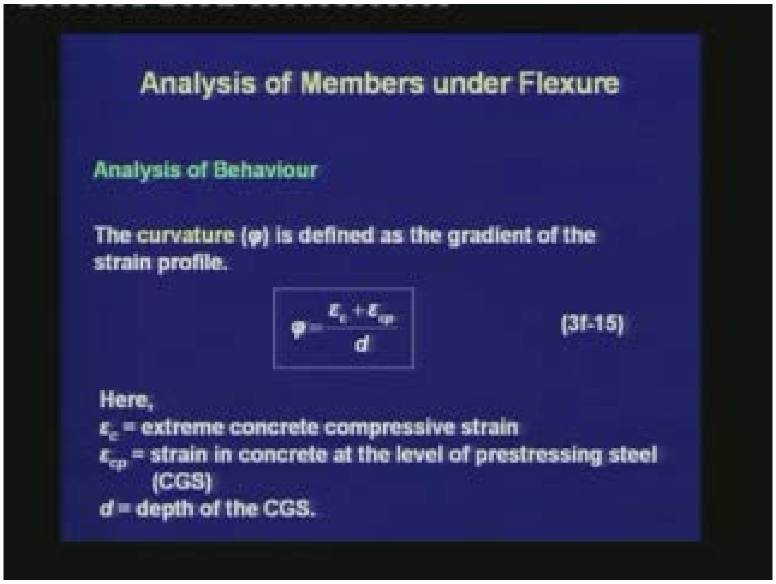

The curvature which is denoted as φ, is defined as a gradient of the strain profile. $\varphi = (\varepsilon_c + \varepsilon_{cp})/d$, where ε_c is the extreme concrete compressive strain and ε_{cp} is the strain in concrete at the level of prestressing steel (the CGS), and d is the depth of CGS. Remember that, we are talking of curvature not just at ultimate, but throughout the loading history, and hence ε_c is variable. It varies from zero till a value of 0.0035 at failure, as the load is increased.

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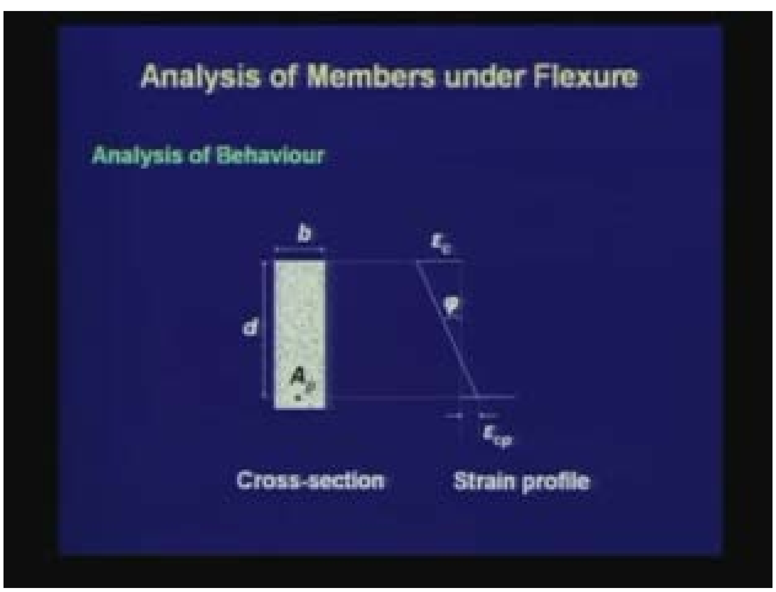

This sketch shows the definition of curvature which is equal to the gradient of the strain profile.

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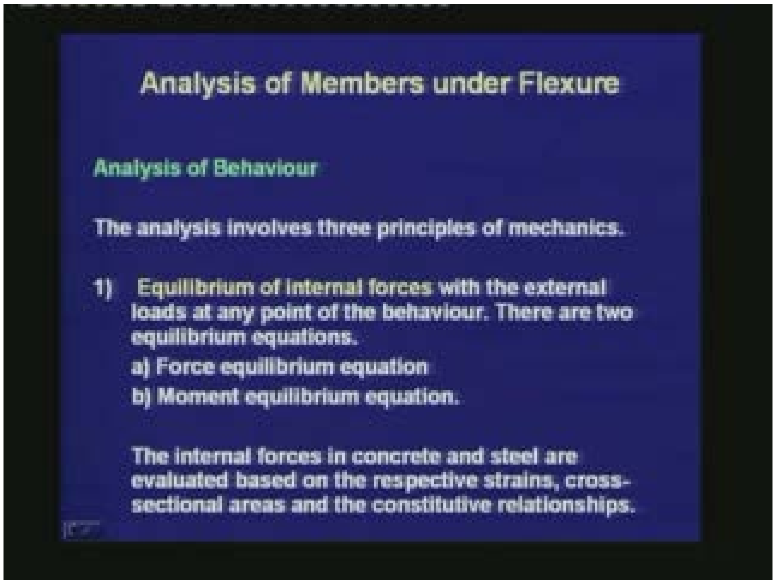

The analysis involves three principles of mechanics: first, the equilibrium of internal forces with the external loads at any point of the behavior. There are two equilibrium equations: a) force equilibrium equation, b) moment equilibrium equation.

Earlier in our analysis for ultimate strength, we had come across the equilibrium equations at ultimate. But, now we are saying that the equilibrium is maintained throughout the loading history. That means, at any point of the loading history, the longitudinal forces are under equilibrium, and the internal moment is equal to the externally applied moment. The internal forces in concrete and steel are evaluated based on the respective strains, cross-sectional areas and the constitutive relationships.

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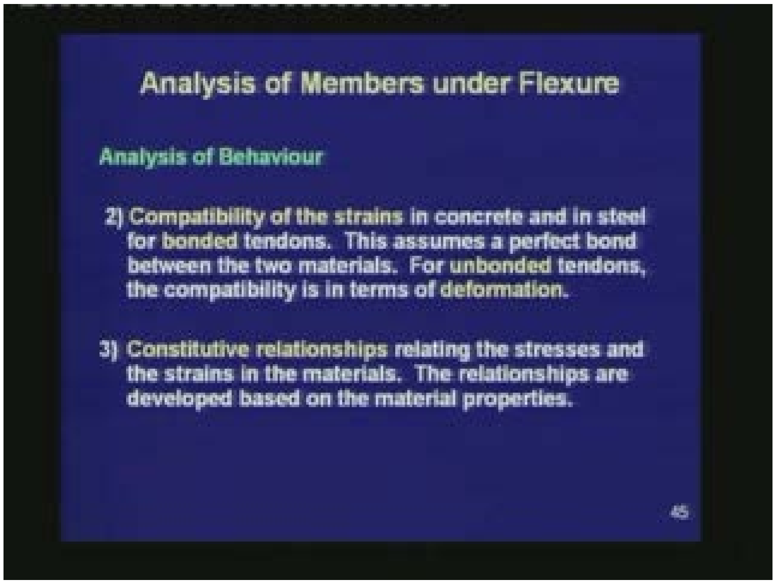

The second principle that we use is the compatibility of the strains in concrete and in steel for bonded tendons. This assumes a perfect bond between the two materials. Thus, to relate the strains in the several layers of steel with the strain in the concrete, we use the strain compatibility equations. If we have unbonded tendon, then the strain compatibility equation cannot be used. For an unbonded tendon, we use the compatibility in terms of deformation.

The third principle that we use, is the constitutive relationships relating the stresses and strains in the materials. These relationships are developed based on the material properties.

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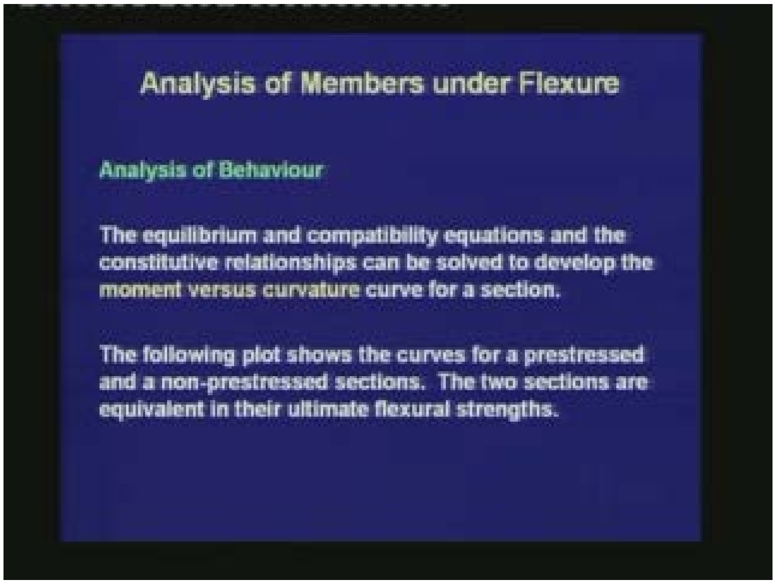

The equilibrium and compatibility equations, and the constitutive relationships can be solved to develop the moment versus curvature curve for a section. In this lecture, we are not going into the details of evaluating the moment versus curvature curve, but we are just learning the basic idea of how to develop the moment versus curvature curve. That means, we have set of equations which are written based on the principles of mechanics. Then, we solve those equations iteratively with an increasing amount of a certain variable say the maximum compressive strain in the concrete. Once we solve the set of equations for each value of ε_c, we get a complete moment versus curvature curve.

The following plot shows the curves for a prestressed and a non-prestressed section. The two sections are equivalent in their ultimate flexural strengths.

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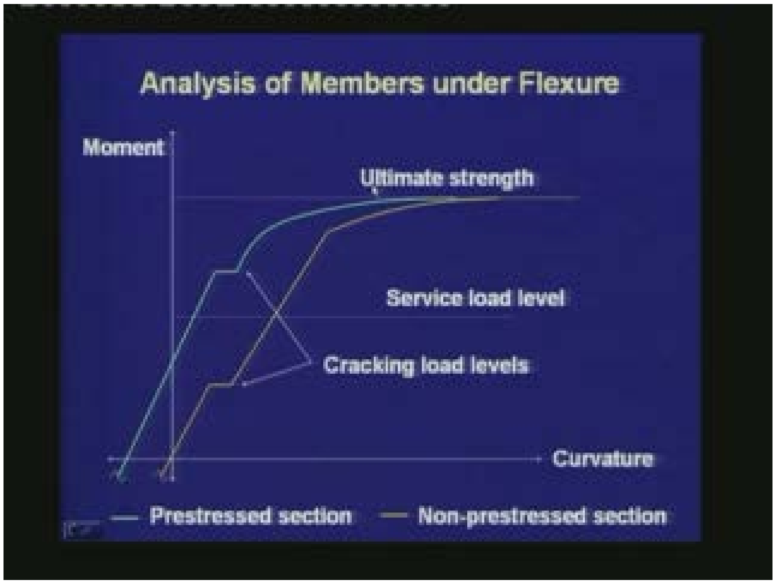

The idea of studying the behavior is to understand the effect of prestressing in the moment versus curvature curve for a section. For a conventional reinforced concrete section which is non-prestressed, in the initial part we have an elastic behavior where the moment is proportional to the curvature. Then after cracking, there is an increase in the curvature, and then there is non-linearity in the curve. At the yielding of the steel, we can get a short kink in the curve if we are using mild steel. After the yielding, we see a substantial reduction of the slope of the curve. The curve goes up to a point when the extreme strain in the concrete reaches a value 0.0035.

On the other hand, for a prestressed section we see that the curve has shifted from the origin. We have a higher cracking load for a prestressed section, and then after cracking we have a yielding of the steel. But here we do not see any sharp kink in the curve because the prestressing steel does not have a specific yield point. Finally, the strength in the prestressing steel also tapers off to a maximum value. We are comparing the curves of two sections whose ultimate strengths are same.

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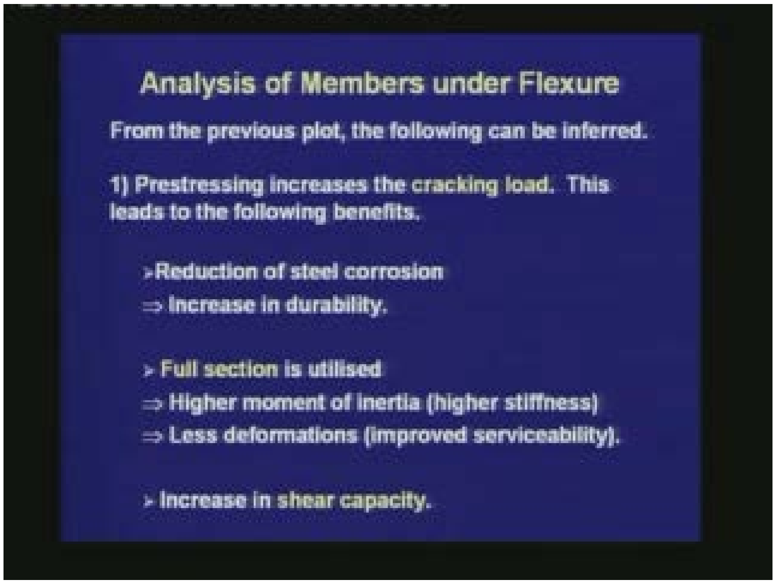

From the previous plot the following can be inferred:

1) The prestressing increases the cracking load. This is the primary reason why we use prestressing in a concrete member. The increase in the cracking load leads to the following benefits: first, it leads to the reduction of steel corrosion, which in effect increases the durability of a section. Second, the full section is utilized; that means we have a higher moment of inertia for a prestressed section corresponding to a reinforced section, which gives higher stiffness. Thus, we have less deformation for a prestressed concrete section of equivalent strength, and thus it gives improved serviceability. Also, since the section is uncracked, we have increase in the shear capacity for a prestressed section.

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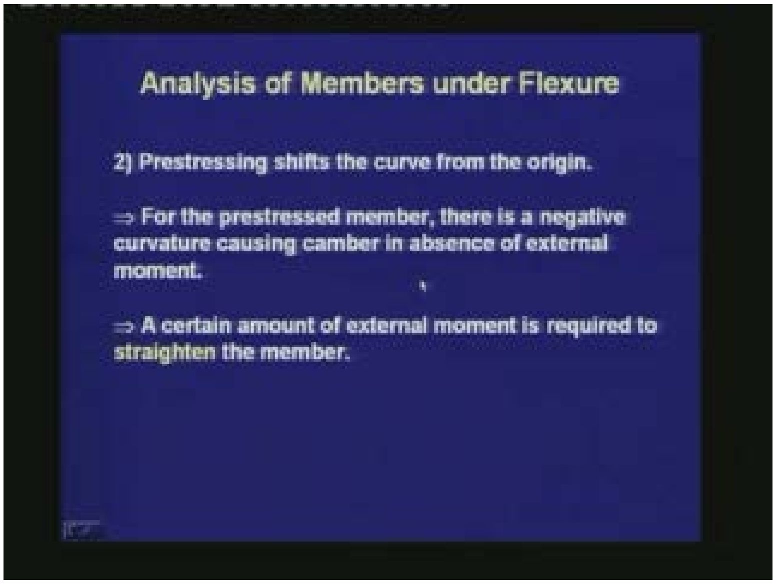

2) The second observation is that the prestressing shifts the curve from the origin which means, for the prestressed member there is a negative curvature causing camber in absence of external moment. We have seen earlier that when the prestress is transferred to the concrete member, the member bends up. That means, now it has a negative curvature in absence of any external load. Even the self-weight may not be adequate to bring it down. Although the self-weight is acting downwards, still due to prestressing the member may hog up causing camber. A certain amount of external moment is required to straighten the member. That means, we need some external load to have zero curvature in a prestressed member; whereas, in a reinforced concrete member the curvature is zero in absence of any external load.

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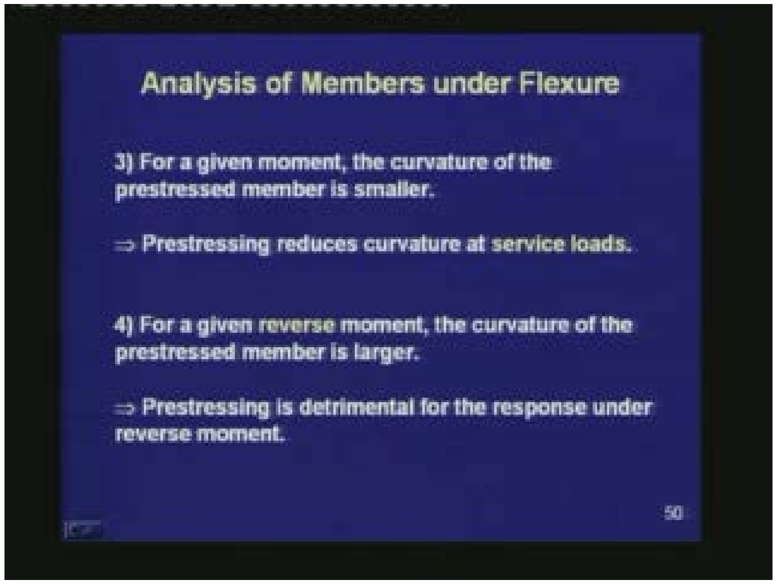

The third observation is for a given moment, the curvature of the prestressed member is smaller.

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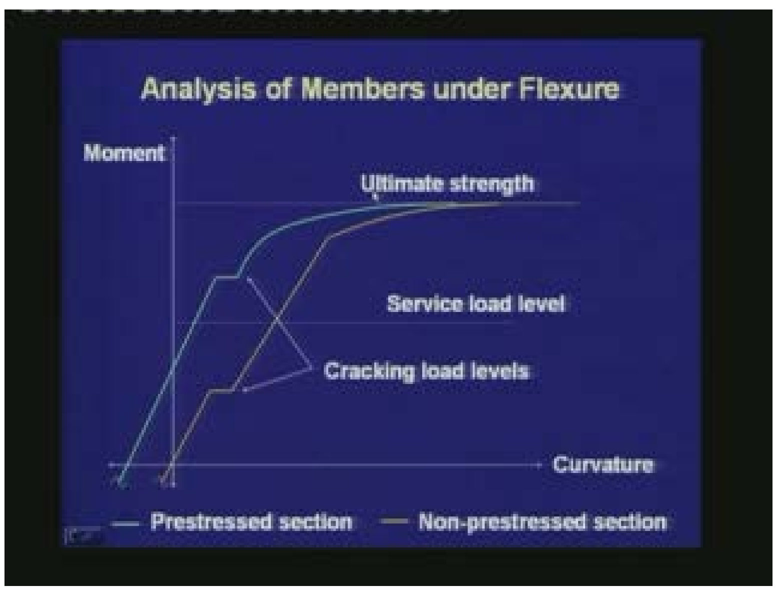

If we look at the two values of curvature from the two curves at a particular value of the moment, we see that the curvature in the prestressed concrete member is less than the curvature of the reinforced concrete member. Hence, prestressing reduces curvature at service loads which gives less deflection at service loads. The fourth inference is, for a given reverse moment the curvature of the prestressed member is larger. In case if the moment reverses, then the curvature of the prestressed concrete member will be higher than that of a reinforced concrete section. We can conclude that prestressing is detrimental for the response under reverse moment.

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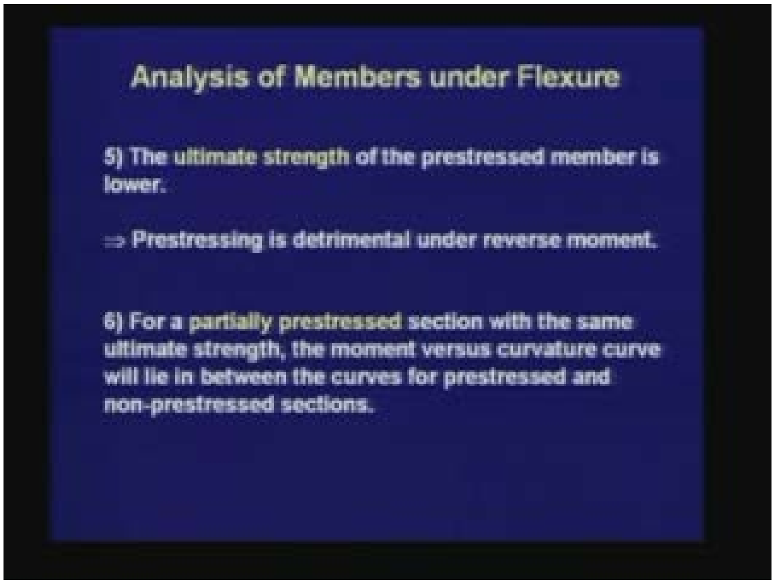

The fifth inference is, the ultimate strength of the prestressed member is lower for reverse moments and hence, prestressing is detrimental under reverse moments. The sixth inference is for a partially prestressed section, with the same ultimate strength, the moment versus curvature curve will lie in between the curves of prestressed and non-prestressed sections. Thus, for a partially prestressed section, we may get the benefits of both reinforced concrete section and a prestressed concrete section depending on the amount of prestressing. For a partially prestressed section, the cracking moment will be higher compared to a reinforced concrete section, but in case if there is a reversal in the moment, a partially prestressed section will not be as detrimental as a fully prestressed section. Hence, a partially prestressed section is preferable when there is a chance of reversal of moments.

Another important concept, we come to know by studying the behavior is the concept of ductility. The ductility is the measure of energy absorption.

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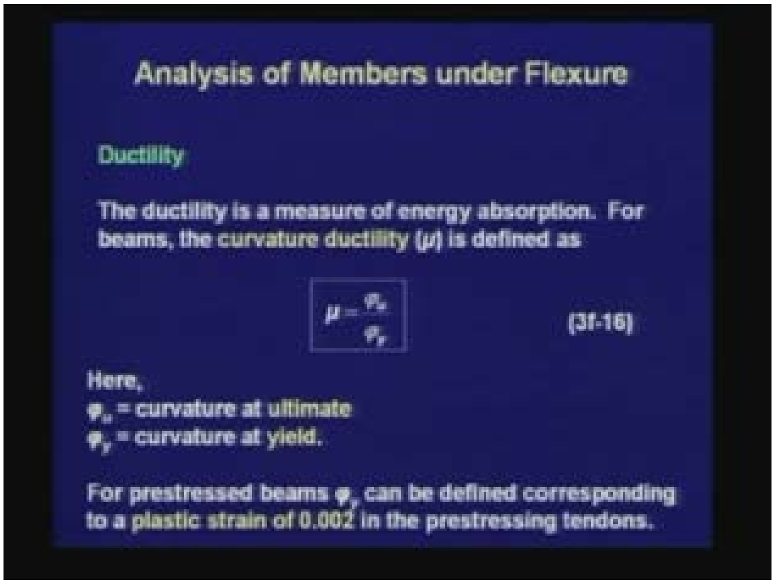

For beams, we define the ductility in terms of curvature and we are denoting it as μ. The curvature ductility is the ratio of the curvature at ultimate which is denoted as φ_u, divided by the curvature at yield which is denoted as φ_y. When we are designing for seismic forces, the ductility becomes an important consideration.

We know that for a prestressing tendon there is no definite yield point. Then how do we calculate φ_y in the moment versus curvature curve of a prestressed section?

We have to define a point in the stress–strain curve of the steel for which we define a corresponding yield curvature. This point we can select as the 0.2% proof stress point; that is the point with a plastic strain of 0.002. The definition can be changed if we want to have more plastic strain corresponding to the defined yield point. We can have a value equal to $0.87f_{pk}/e_p + 0.005$, which gives a plastic strain of 0.005 for the defined yield point. Thus, the definition of yield point for a prestressed section is left to the analyst based on how much plastic strain the analyst wants at the onset of yielding.

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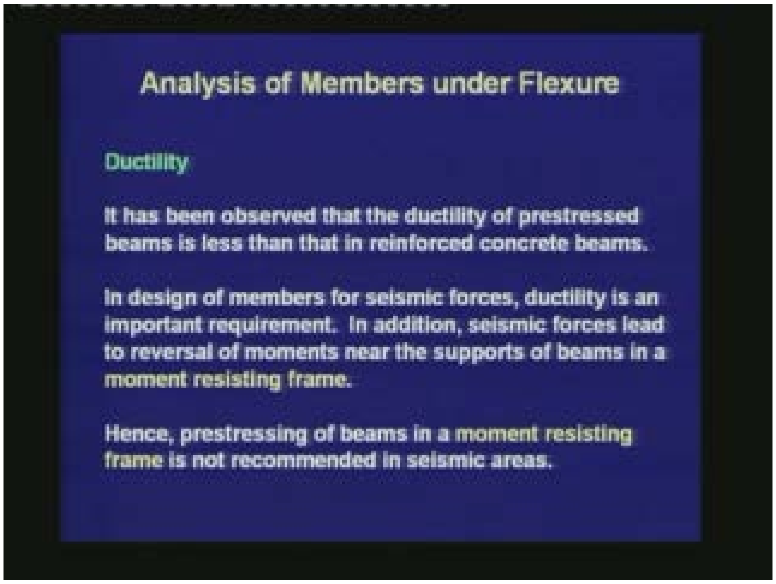

It has been observed that, the ductility of prestressed beams is less than in reinforced concrete beams. That is, the ratio of φ_u/φ_y is less for a prestressed concrete beam as compared to a reinforced concrete beam. In design of members for seismic forces, ductility is an important requirement. In addition, seismic forces lead to reversal of moments near the supports of beams in a moment resisting frame. Hence, prestressing of beams in a moment resisting frame is not recommended in seismic areas.

There are two considerations why prestressing may not be helpful in a seismic region. First, the ductility of a prestressed member is lower than a reinforced concrete member. Second, there is a chance of reversal of moments under seismic forces near the supports of the beams. Hence, prestressing is not recommended for beams in a moment resisting frame. We may have prestressed members which are simply supported, but those members are not part of the primary lateral load resisting frame. The primary lateral load resisting frame should have conventional reinforced concrete beams with appropriate detailing, to have adequate ductility and to have adequate capacity under reversal of moments.

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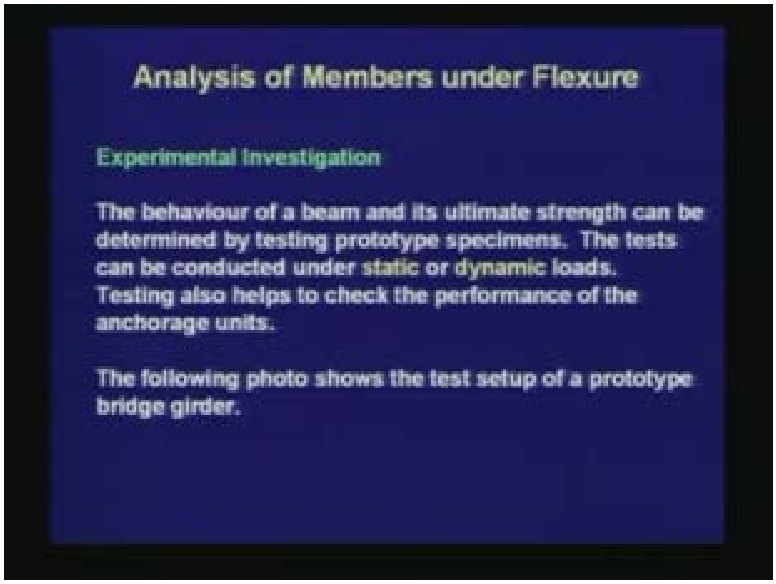

The behavior of a beam and its ultimate strength can be determined by testing prototype specimens. The tests can be conducted under static or dynamic loads. Testing also helps to check the performance of the anchorage units. Whatever we have studied till now was based on analysis. But, it is also possible to understand the behavior of a prestressed member and to calculate its ultimate capacity based on experiments because, there can be beams which are prestressed in a non-conventional way for which the analysis can be difficult. There can be cross-sections which are not very easy for computation. In that case, an experimental program is helpful to understand the behavior.

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This photograph shows a prestressed concrete beam being tested in a laboratory. The distributed load has been simulated by eight point loads along the span of the beam. We see the reaction frame which supports the jacks. The jacks are applying loads on the brown steel beam, which is transferring the loads to the blue beams, and finally the blue beams are transferring the loads to the prestressed beam, through the yellow loading stubs. In this test, the distributed load is simulated by a series of point loads, which is more applicable for a bridge structure. The section is a flanged section, where the flange width is constant throughout the length. But the width of the web has been increased near the ends, so as to have increased shear capacity and reduced stress concentration for anchoring the tendons. The beam was tested to failure to observe the behavior and to calculate its ultimate moment capacity.

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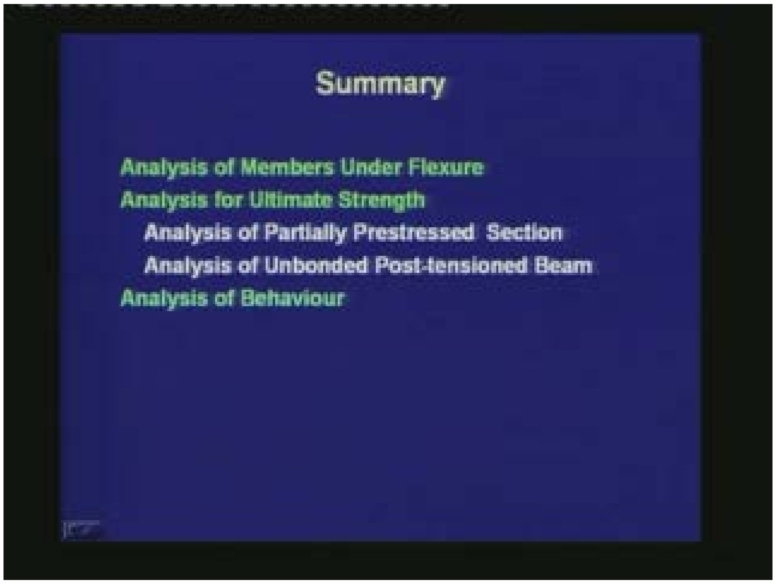

Today we studied the analysis for ultimate strength of a partially prestressed section. A beam may not be fully prestressed. There can be conventional reinforcement along with the prestressing tendon. There are some advantages of partially prestressed sections. First of all, the sections are economical. Next, the sections have more ductility. It is better for reversal of moments. In a beam with unbonded tendons, in presence of bonded reinforcement, the cracks are more distributed.

A partially prestressed section has several layers of steel. There is a compatibility equation for each layer of steel. We have the set of equilibrium equations and the set of constitutive relationships. When we solve these equations for the ultimate state, we get the ultimate moment of resistance for a partially prestressed section.

We also studied the analysis of unbonded post-tensioned beams. Here, we cannot use the strain compatibility equations as we have used for bonded beams. For an unbonded post-tensioned beam, the compatibility is based on overall deformation. Thus it needs an integration of the strain in the concrete at the level of the CGS. The code allows us to use approximate values of the depth of the neutral axis and the force in the tendon, from tabulated results.

Finally, we studied the analysis of behavior, where we understood the moment versus curvature curve for a prestressed section as compared to that for a reinforced concrete section. We learned the concept of ductility. We understood the benefits and certain drawbacks of prestressing. With this, we are ending the analysis of members. Next, we shall move on to the design of prestressed concrete members.

Thank you.