PRESTRESSED CONCRETE STRUCTURES

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Module – 3: Analysis of Members

Lecture – 15: Analysis of Flanged Sections

Welcome back to prestressed concrete structures. This is the fifth lecture of module three on analysis of members.

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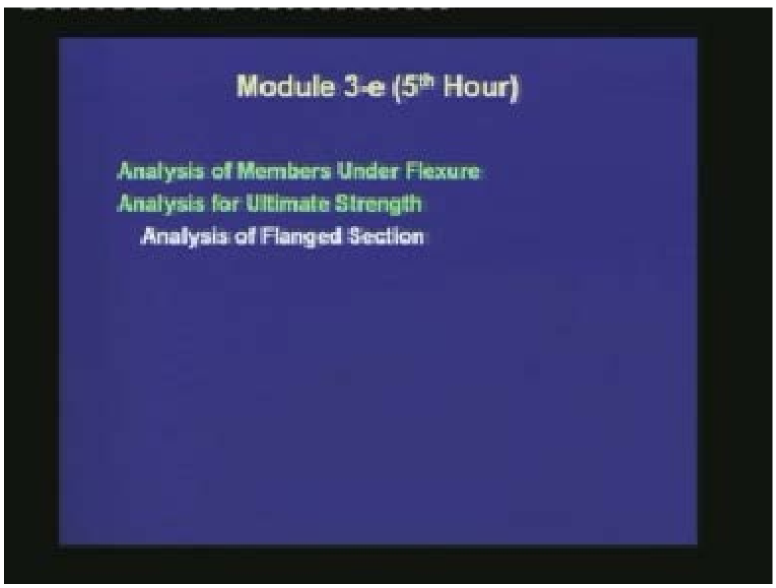

In this lecture, we shall study the analysis of members under flexure, specifically, the analysis for ultimate strength of flanged sections.

Analysis of flanged section:

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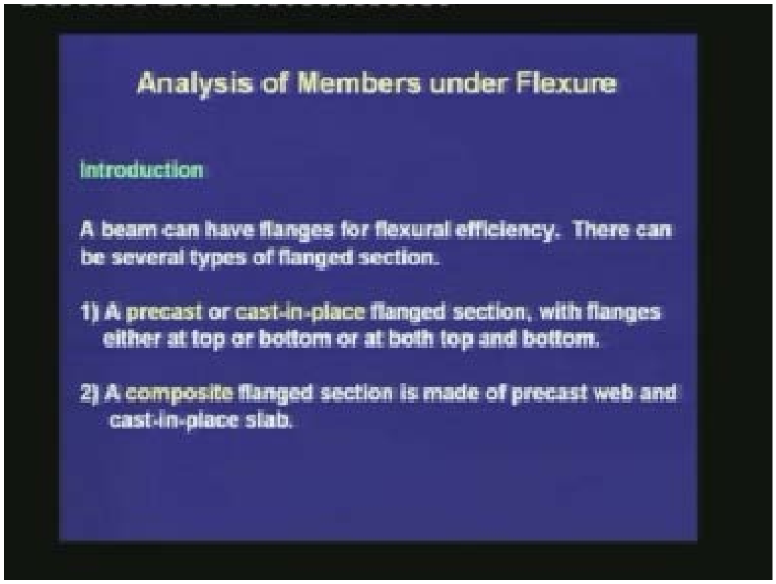

A beam can have flanges for flexural efficiency. There can be several types of flanged sections. First, a precast or cast-in-place flanged section, with flanges either at top or bottom, or at both top and bottom. Second, a composite flanged section is made up of precast web and cast-in-place slab. Let us now see a few examples of these types of flanged sections.

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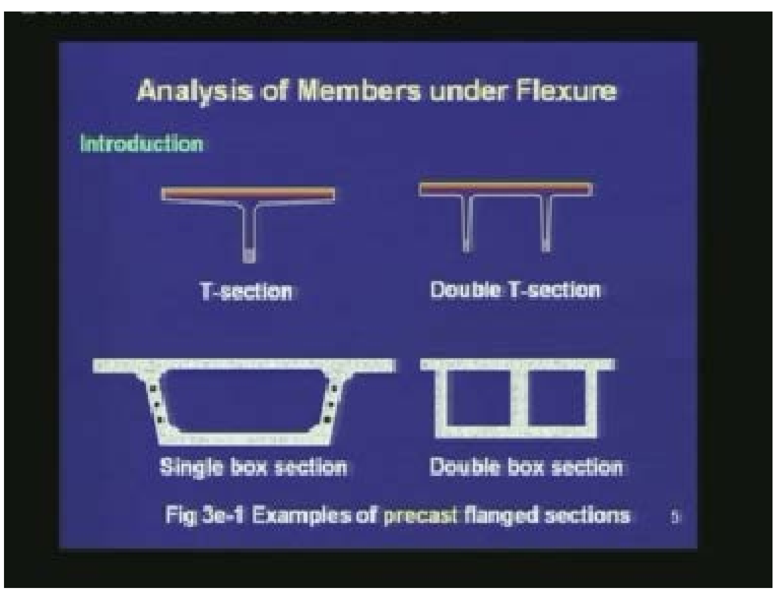

In the top, we see precast T- and double T-sections, which are frequently used as bridge girders. Sometimes, even they are used for making floors. In the lower part, we see a single box section and a double box section. These are also used as bridge girders. These types of sections can either be precast or it can be cast-in-place. But, more and more precast sections are being produced, and they are getting popular.

In the next figure, we see some other types of flanged sections where there can be flanges at the bottom to rest other members spanning on it. That means these members act as primary beams carrying some other secondary beams.

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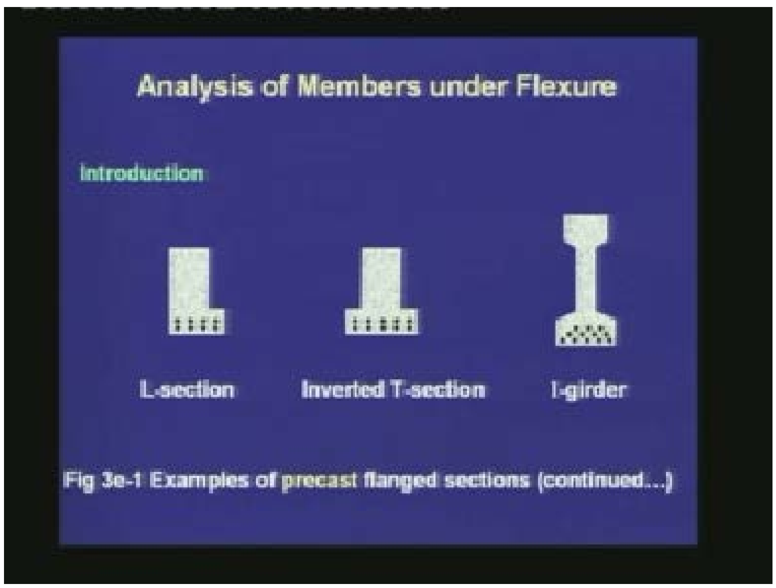

It can be an L-section with the flange only on one side. It can be an inverted T-section with flanges on both the sides. There can be I-girders which are very common in bridge construction.

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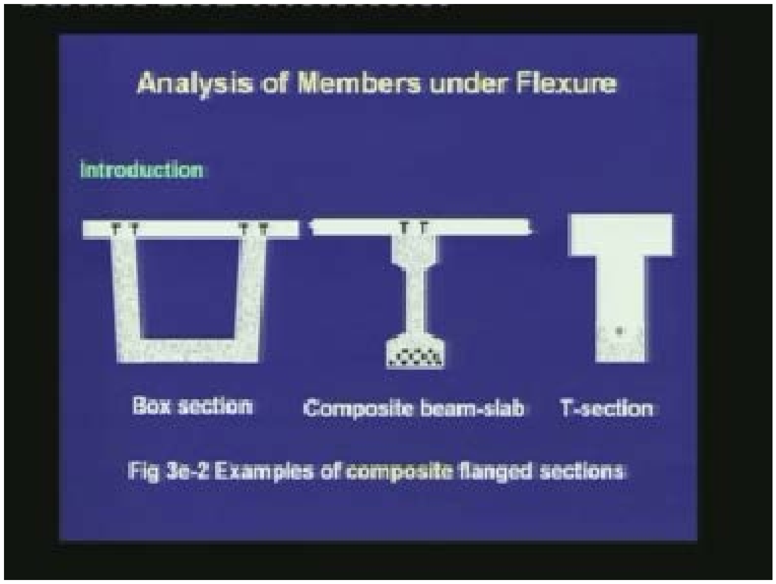

In this sketch, we see some composite sections. The web is precast and then moved to the site. The flange is cast-in-place made at the site. These types of composite sections are common in buildings or in bridges. There has to be proper shear connectors between the web and the flange to treat the section as a composite section. There can be various innovative types of composite sections. Some of them, we shall see in the module under composite sections.

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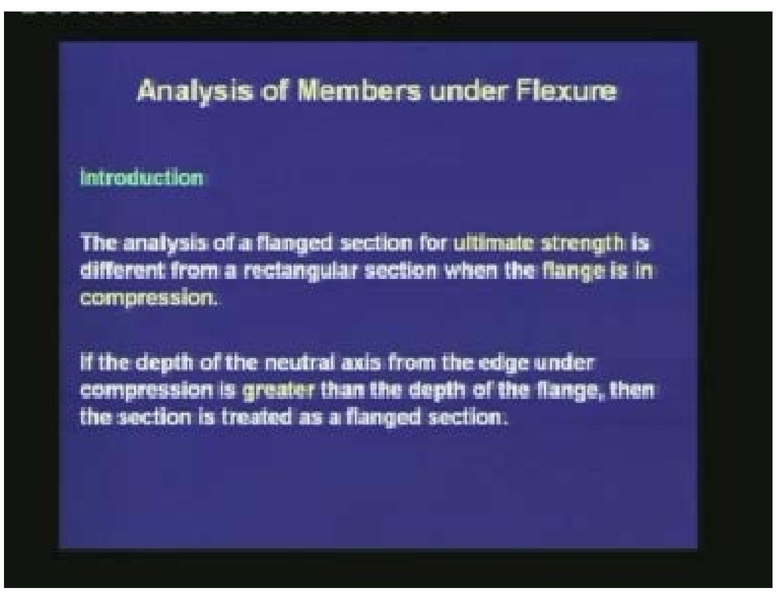

In this module, we shall study simple type of flanged sections with rectangular webs and rectangular flanges. The analysis of a flanged section for ultimate strength is different from a rectangular section when the flange is in compression. This is the first thing we have to be aware of, that we treat the section as a flanged section only when the flange is under compression. If the flange is under tension, then we need not have to consider the overhanging parts of the flange, and we can treat it as similar to a rectangular section.

The next consideration is that if the depth of the neutral axis from the edge under compression is greater than the depth of the flange, then the section is treated as a flanged section. Let us try to understand this from a sketch.

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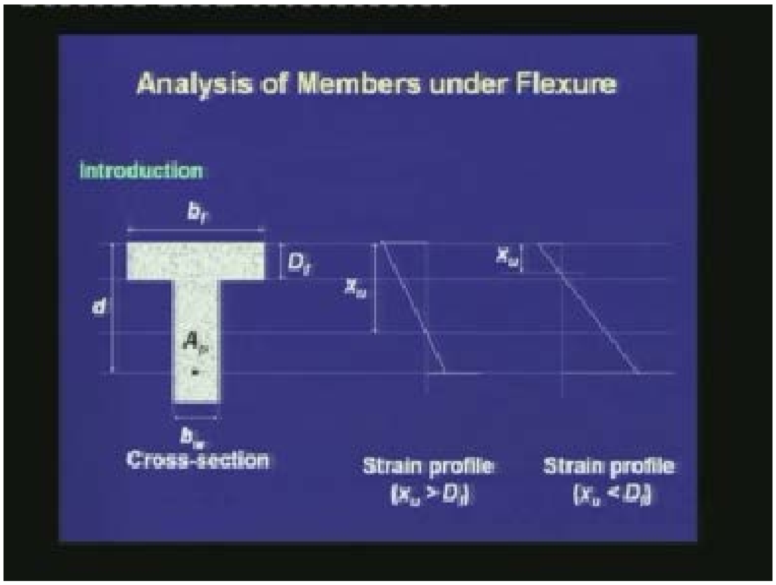

In this sketch, we see a flanged section where the flange in the top is under compression. Here, b_f denotes the breadth or the effective width of the flange, D_f denotes the depth of the flange, d denotes the depth of the CGS of the prestressing steel and, b_w is the breadth of the web. There can be two situations. First, the depth of the neutral axis at ultimate (which is represented as x_u) is greater than D_f, and in this case we treat the section as a flanged section. But, if the depth of the neutral axis is lower than D_f, then we can treat the section as a rectangular section with a width of b_f. In this lecture, we are studying the first case where x_u is greater than D_f.

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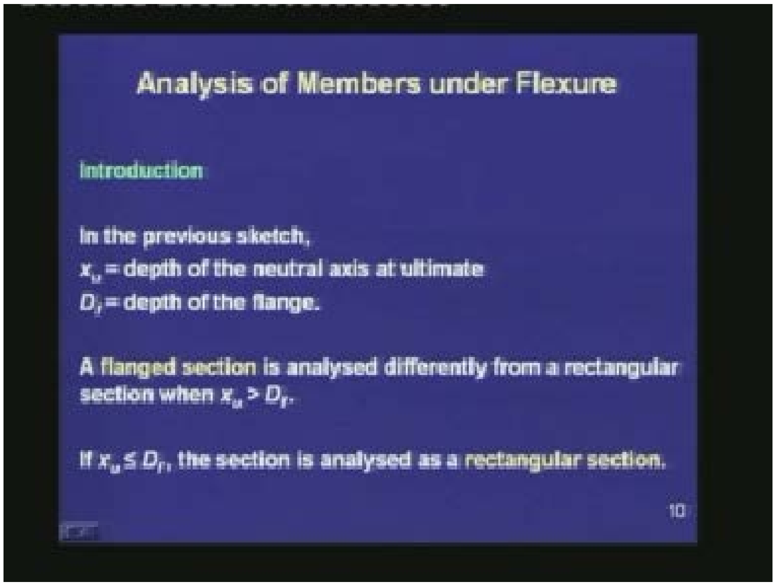

In the previous sketch, x_u is the depth of neutral axis at ultimate, D_f is the depth of the flange. Thus, a flanged section is analyzed differently from a rectangular section, when x_u is greater than D_f. If x_u is less than $D_{f,}$, the section is analyzed as a rectangular section.

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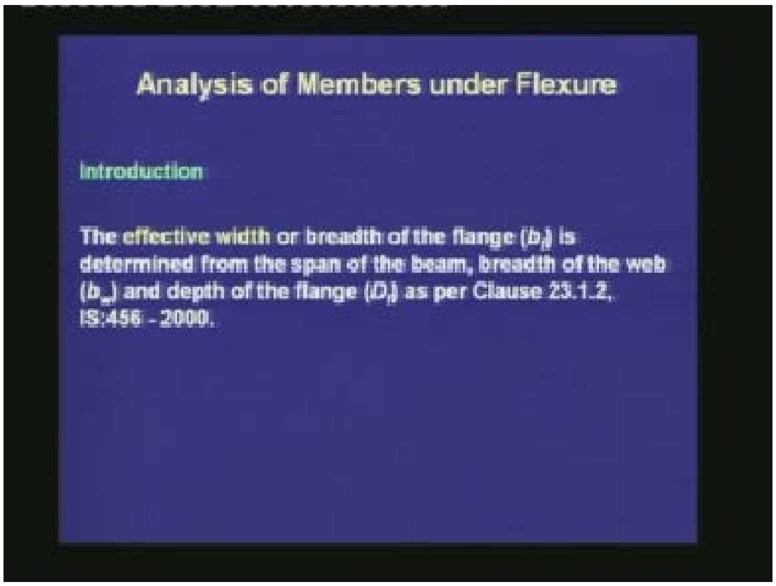

Another important consideration is to calculate the breadth of the flange or the effective width of the flange. If it is an isolated beam with a small width of the flange, then the full width of the flange can be considered as the breadth of the flange. But, if the flange is a part of a slab, then we have to calculate an effective width from the slab, where the assumption is that the compressive stress is uniform over the effective width.

The effective width of the flange is determined from the span of the beam, the breadth of the web, and the depth of the flange as per clause 23.1.2 of IS: 456-2000. Under this clause, there are several expressions for different types of sections to calculate the effective width of a flange. The expressions are simple in nature to be used in conventional design. These expressions are not repeated here because the students are expected to be familiar with these expressions in a course on reinforced concrete design.

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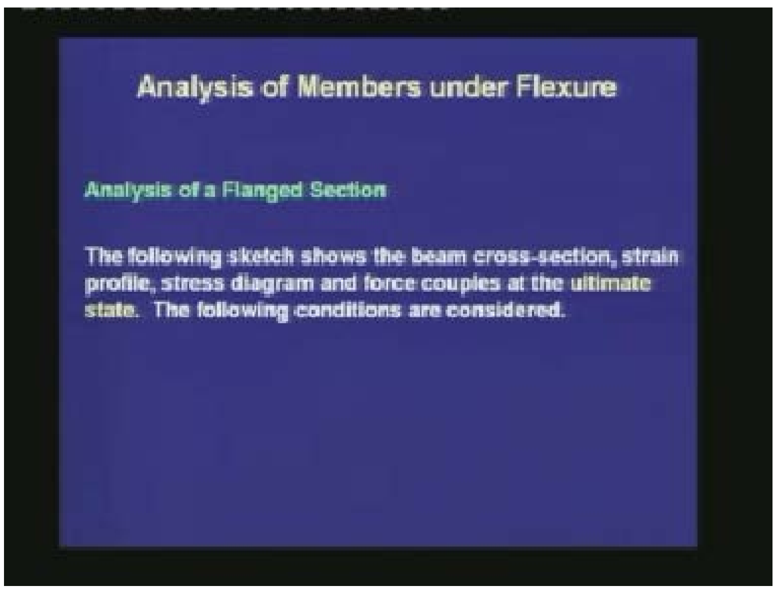

Next, we move on to the analysis of a flanged section. In the following sketch, the beam cross-section, the strain profile, the stress diagram and the force couples at the ultimate state are shown. The following conditions are considered in the sketch.

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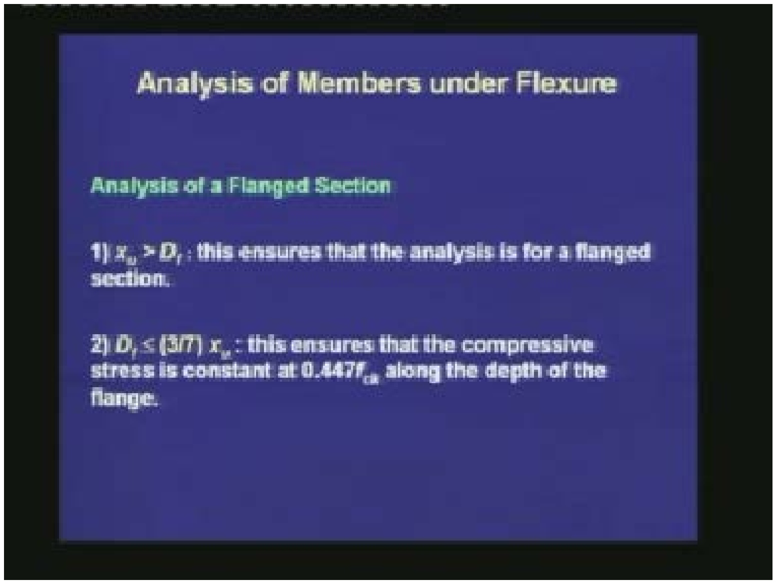

First, we consider that x_u is greater than D_f. This ensures that the analysis is for a flanged section, which is different from that of a rectangular section. Second, we consider that the depth of flange is smaller than 3/7 x_u. This ensures that the compressive stress is constant at $0.447f_{ck}$ along the depth of the flange.

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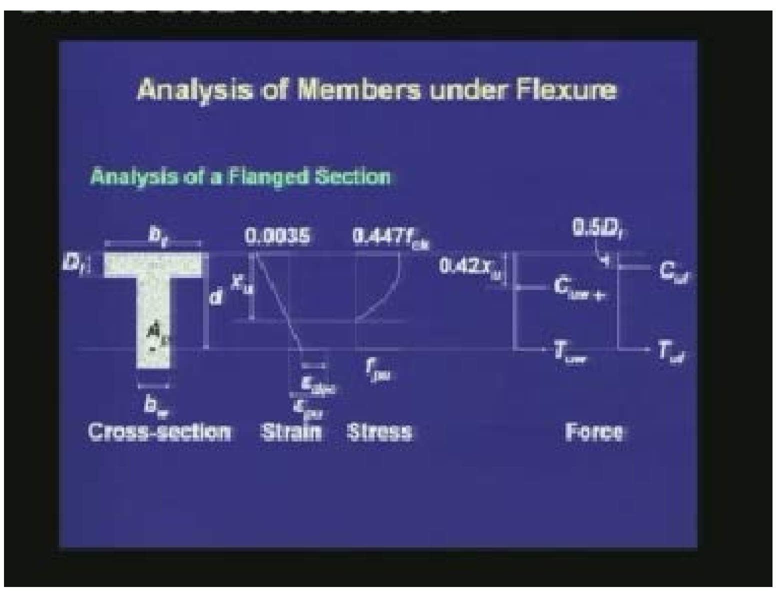

In the sketch, the left hand side shows the cross-section of a flanged section. Next, in the strain profile, the ultimate state is defined as the instance when the compressive strain in the concrete at the extreme edge becomes 0.0035. With the known strain in the extreme fibre, a linearly varying strain profile is drawn based on the hypothesis that plane sections remain plane, till the ultimate. For the prestressing tendon, the strain at ultimate is represented as ε_{pu}. It is equal to the strain in concrete at the level of the CGS plus the constant strain difference, which is denoted as ε_{dec}. In the last lecture, we had seen how to calculate ε_{dec} from the value of the prestressing force. The expressions are different for a pre-tensioned beam and a post-tensioned beam.

The stress profile is drawn based on the constitutive relationship of concrete. The stress f_{pu} is calculated from ε_{pu} using the design stress–strain curve for the prestressing tendon.

From the stress diagram, we move on to the force diagram. Here is the difference with respect to a rectangular section. We are decomposing the force diagram into two components: one, which exists in the outstanding parts of the flanges and part of the prestressing steel, and the other which exists in the web and the rest of the prestressing steel. The compression which acts in the web is denoted as C_{uw} and the corresponding component of the tension is denoted as T_{uw}. C_{uw} acts at the distance of $0.42x_u$ from the top fibre. The resultant compression in the outstanding parts of the flanges is denoted as C_{uf}. It acts at a distance of $0.5\ D_f$, that is at half of the flange depth. The corresponding component of the tension is denoted as T_{uf}.

The purpose of decomposing each force into two components is to identify the individual force couples and the respective lever arms.

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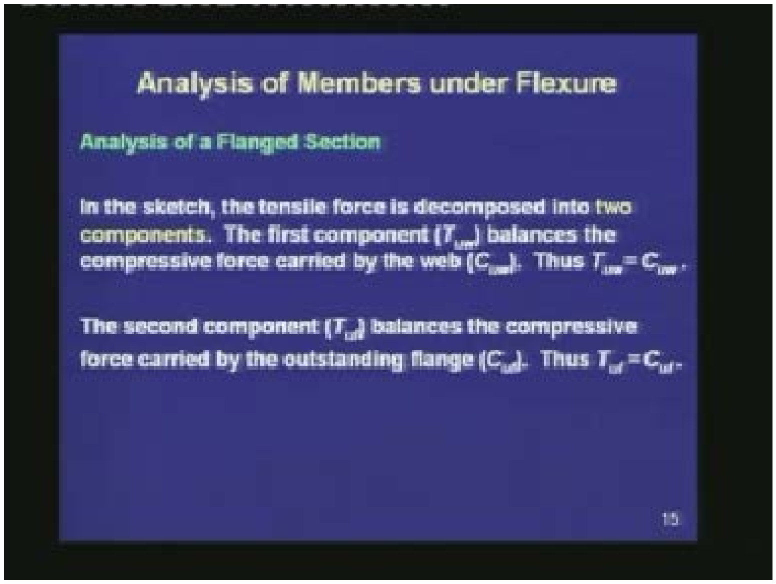

In this previous sketch, the tensile force is thus decomposed into two components. The first component T_{uw} balances the compressive force carried by the web, which is C_{uw}. Thus we have the equation $T_{uw} = C_{uw}$. The second component T_{uf} balances the compressive force carried by the outstanding flange which is C_{uf}. We have the second equation which is $T_{uf} = C_{uf}$.

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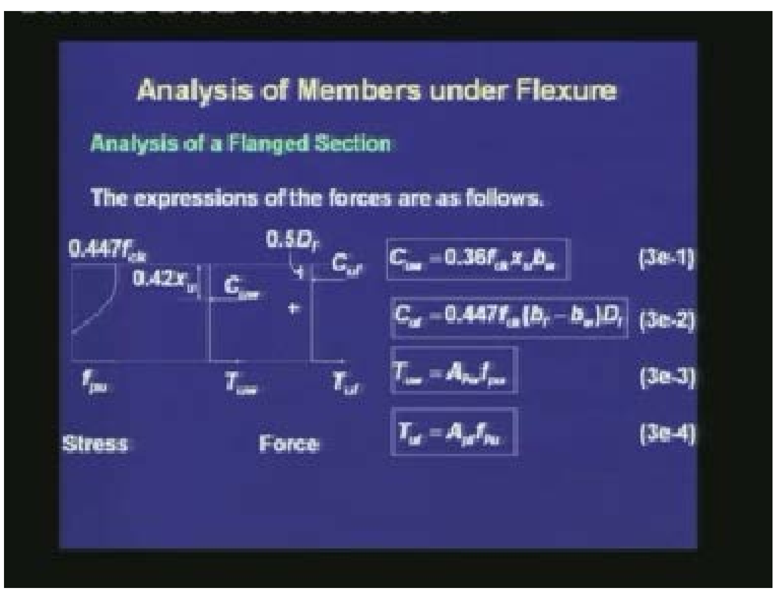

Next, we are writing the equations based on the principles of mechanics. Before that, we are writing the expressions of the resultant forces. C_{uw} is calculated from the stress block based on the design stress–strain curve of the concrete, and it is given as 0.36 f_{ck} x_u b_w. Next, C_{uf} is given as the constant stress over the flange which is 0.447 f_{ck}, times the width of the outstanding flanges which is the total width minus the width of the web, that is $b_f - b_w$, times the depth of the flange which is D_f. Next, T_{uw} is equal to a component of the total prestressing steel A_{pw} times f_{pu}. Finally, T_{uf} is equal to the rest of the prestressing steel, which is A_{pf} times $f_{pu.}$

Next, let us recollect the three principles of mechanics we use in our analysis of sections.

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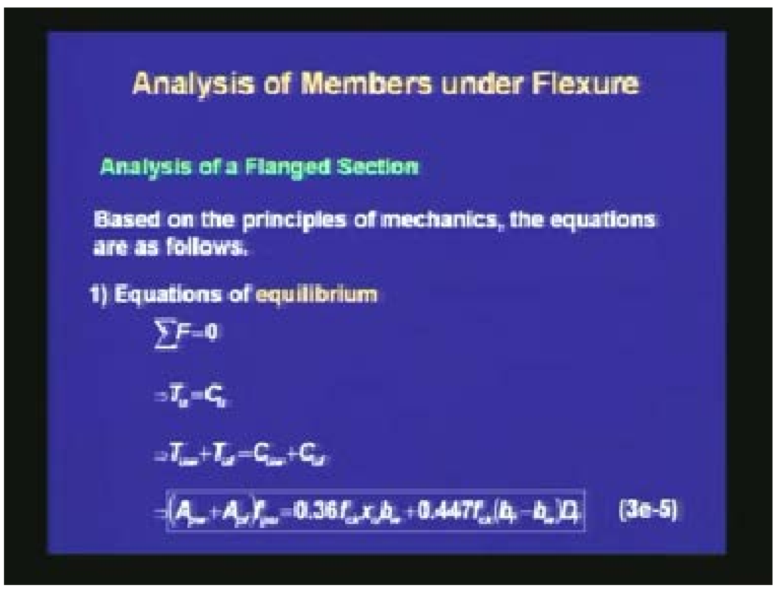

The first one is the equilibrium of forces. Here we have two equations: one is for the longitudinal force and the second is for the moment. The second principle we use is the strain compatibility. Here again there are two aspects. One is that the strain in the concrete in the vicinity of the tendon is equal to the increment of strain in the steel. The second consideration is that, plane sections remain plane and hence the strain profile is linear along the depth of the section. The third principle is the constitutive relationships which relate the stress and strain in each material, concrete and steel.

Let us see the equations based on these principles of mechanics. The first one is the equation for equilibrium of forces: $\Sigma F = 0$. The total tension $T_{uw} + T_{uf}$ is equal to the total compression $C_{uw} + C_{uf}$. With the expressions of the forces, $(A_{pw} + A_{pf})\, f_{pu} = 0.36 f_{ck} x_u b_w + 0.447 f_{ck}\, (b_f - b_w) D_f$.

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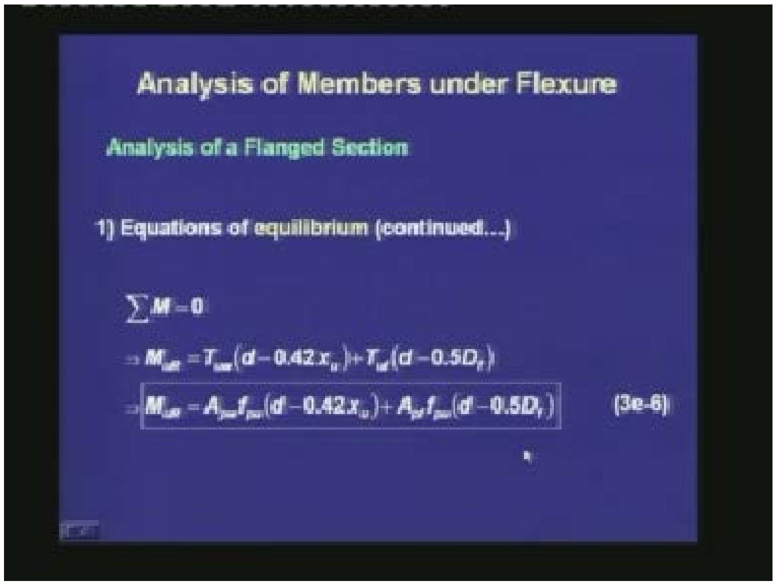

The second equilibrium equation is $\Sigma M = 0$, from which we can find out the ultimate moment capacity or the moment of resistance (M_{uR}), which is equal to the individual moments created by the two couples. The first component is equal to $T_{uw}\, (d - 0.42x_u)$, where $d - 0.42x_u$ represents the lever arm of the first couple. The second part of the moment is given as $T_{uf}\, (d - 0.5d_f)$. Once we add them up and substitute the expressions of T_{uw} and T_{uf}, we get an expression of the ultimate moment capacity of the section which is given as $M_{uR} = A_{pw}\, f_{pu}\, (d - 0.42x_u) + A_{pf}\, f_{pu}\, (d - 0.5D_f)$.

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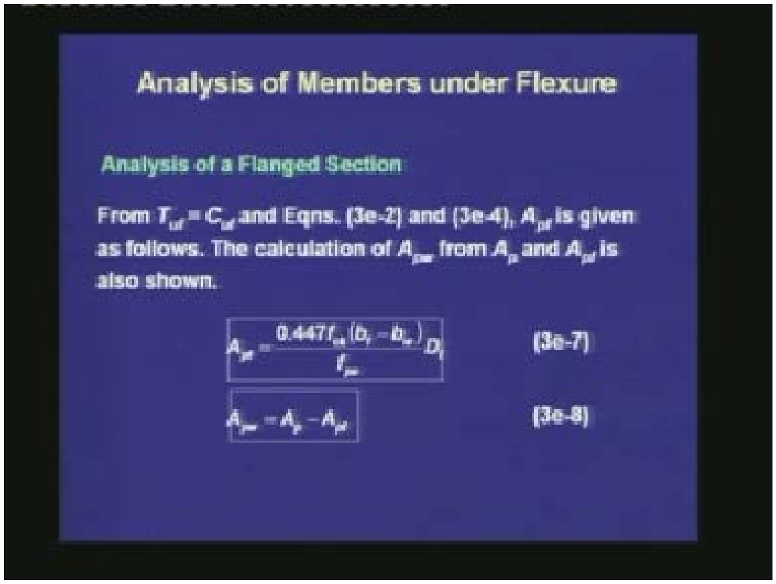

If we have to know the individual values of A_{pf} and A_{pw}, we can calculate them from the equation $T_{uf} = C_{uf}$. $A_{pf} = T_{uf} / f_{pu} = 0.447f_{ck} (b_f - b_w) D_f / f_{pu}$. Given the total amount of steel if we deduct the component $A_{pf,}$ we get the amount A_{pw}. That is, $A_{pw} = A_p - A_{pf.}$

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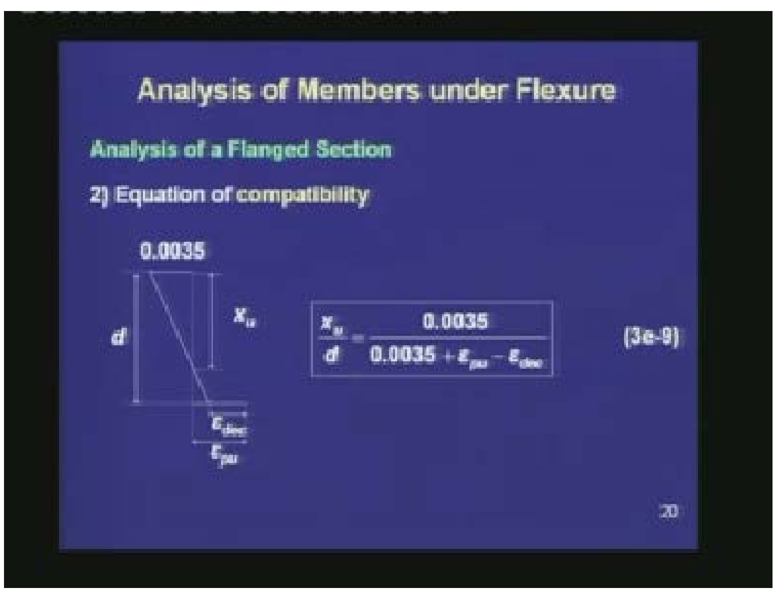

The second principle gives us the equation for compatibility. From the similarity of triangles, we can relate the compressive strain in the extreme fibre of concrete which is

0.0035, with the tensile strain in concrete at the level of CGS which is $\varepsilon_{pu} - \varepsilon_{dec}$. The expression is written as $x_u / d = 0.0035 / (0.0035 + \varepsilon_{pu} - \varepsilon_{dec})$.

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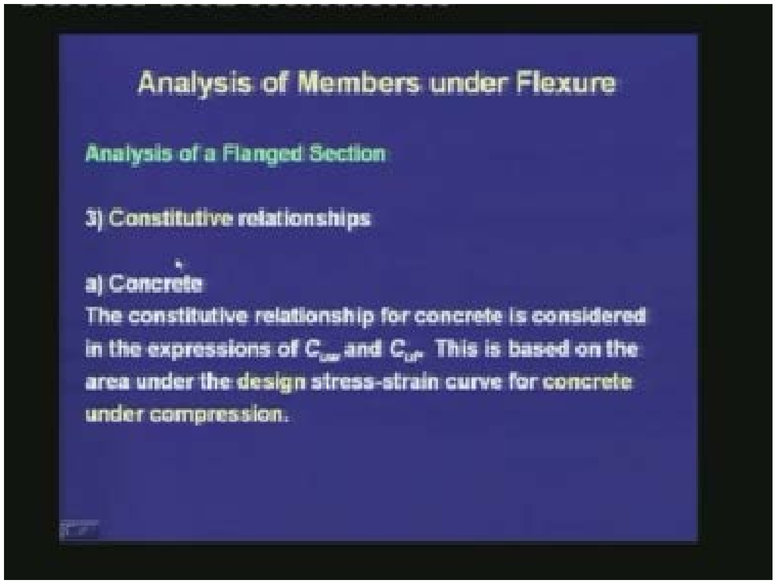

Finally, we move on to the constitutive relationships. For concrete the constitutive relationship is considered in the expressions of C_{uw} and C_{uf}. This is based on the area under the design stress–strain curve for concrete under compression.

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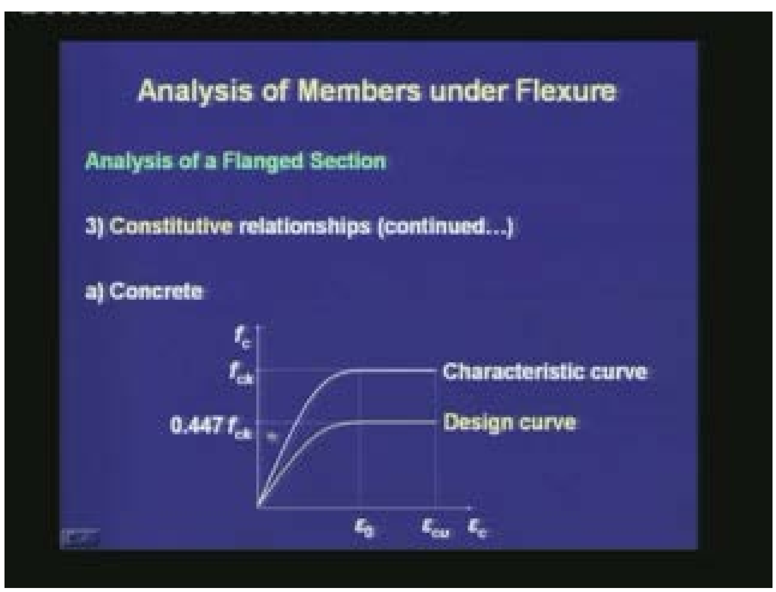

We had seen the design stress–strain curve in the module of material properties. From the characteristic curve, we define a design curve with a maximum stress of $0.447f_{ck}$. The area under this design curve gives the value of $C_{uw.}$ For C_{uf}, since the stress is constant, C_{uf} is equal to the constant stress times the area over which the stress acts on the outstanding flange.

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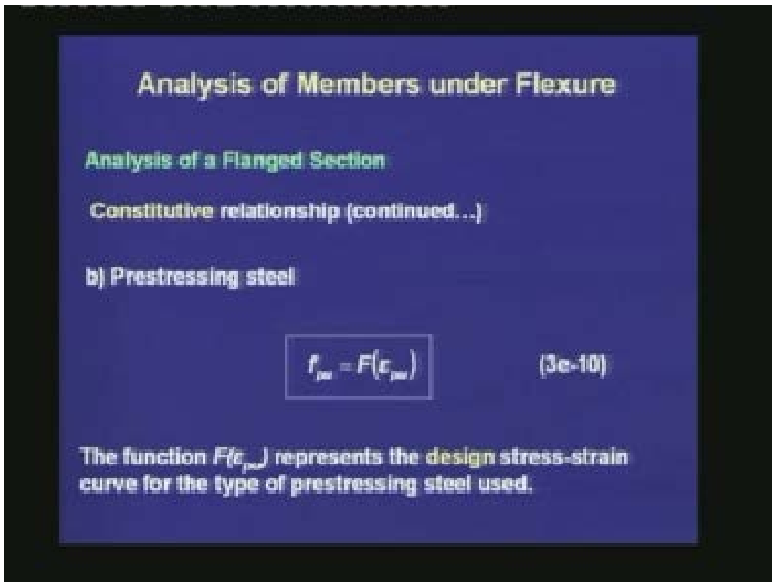

For the prestressing steel, the constitutive relationship can be expressed in the form $f_{pu} = F(\varepsilon_{pu})$, where the function $F(\varepsilon_{pu})$ represents the design stress–strain curve for the type of prestressing steel used.

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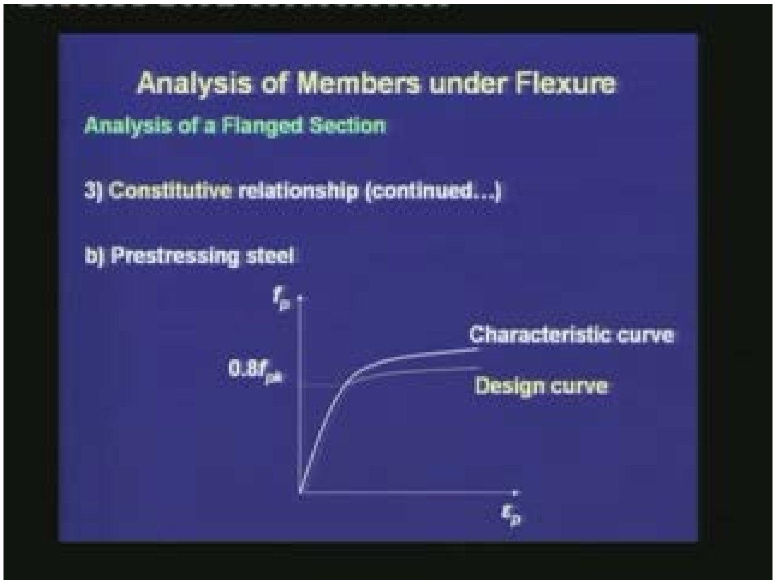

We had seen these curves in the module of material properties. We have the characteristic curve of the prestressing steel from which we define the design curve.

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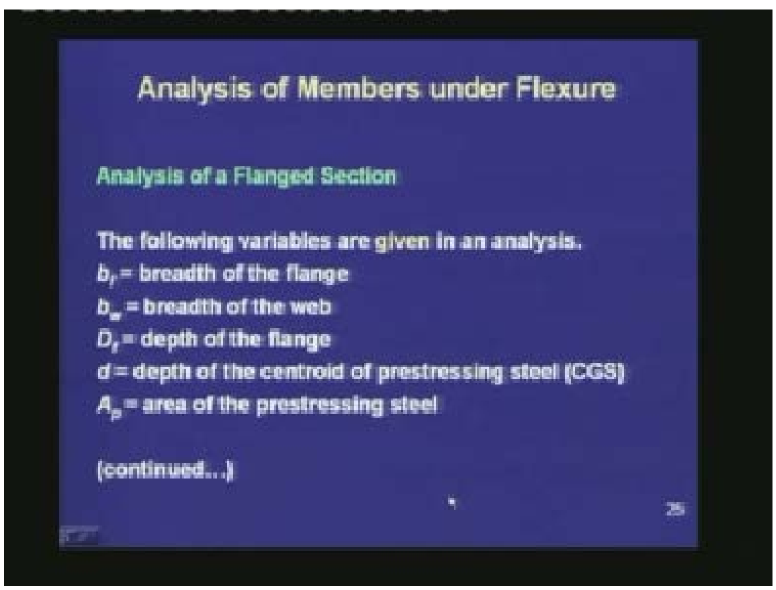

In the analysis of a flanged section, the following variables are given. We know the geometric properties of the flanged section: b_f the breadth of the flange, b_w the breadth of the web, D_f, the depth of the flange, d the depth of the centroid of the prestressing steel (CGS), and A_p the area of the prestressing steel.

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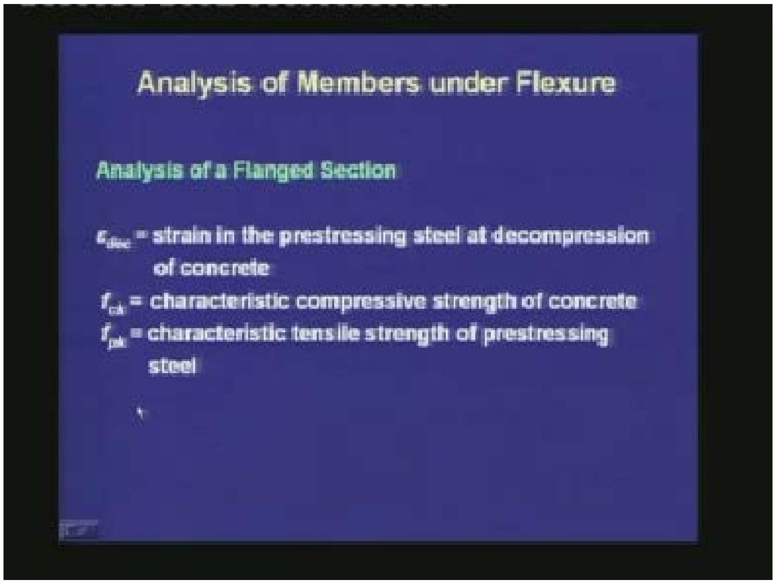

We know ε_{dec}, the strain in the prestressing steel at decompression of concrete, which is a function of the prestressing force applied. We also know the material properties: f_{ck} the characteristic compressive strength of concrete, and f_{pk} the characteristic tensile strength of the prestressing steel.

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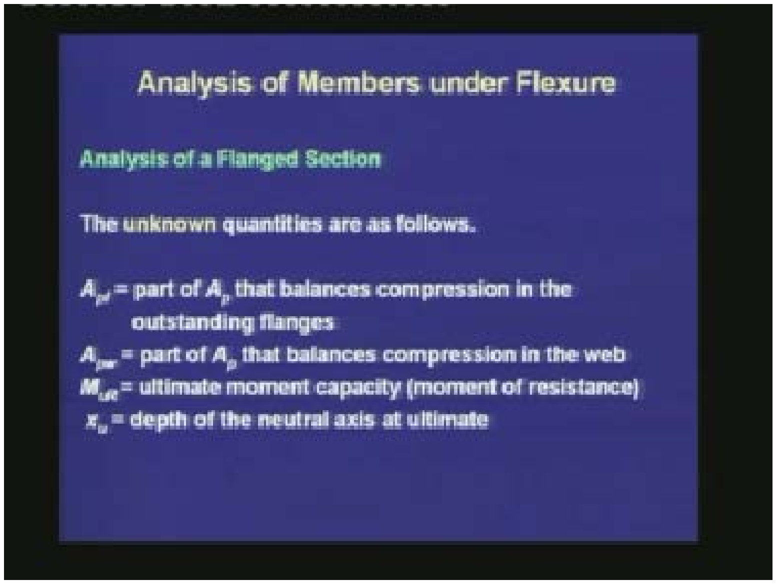

What are the unknown quantities in our analysis? First of all, we do not know the individual components of the prestressing steel, A_{pf} and A_{pw}. A_{pf} is the part of A_p that balances the compression in the outstanding flanges, and A_{pw} is the part of A_p that balances compression in the web. We also do not know the ultimate moment capacity, or the moment of resistance which is denoted as M_{uR}, and we do not know x_u, the depth of the neutral axis at ultimate.

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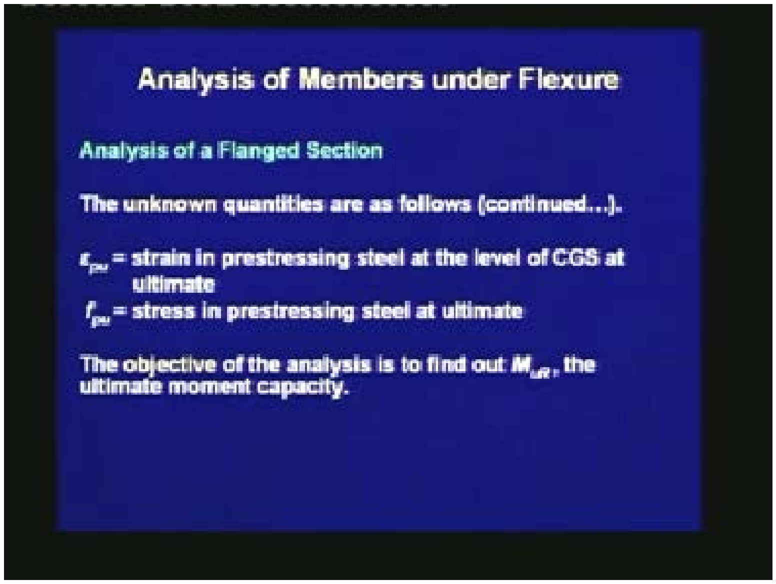

We do not know the strain in the prestressing steel at ultimate which is ε_{pu}, and we do not know the stress in the prestressing steel which is f_{pu}.

The objective of the analysis is to find out M_{uR}, the ultimate moment capacity. The set of simultaneous equations are solved in a certain procedure.

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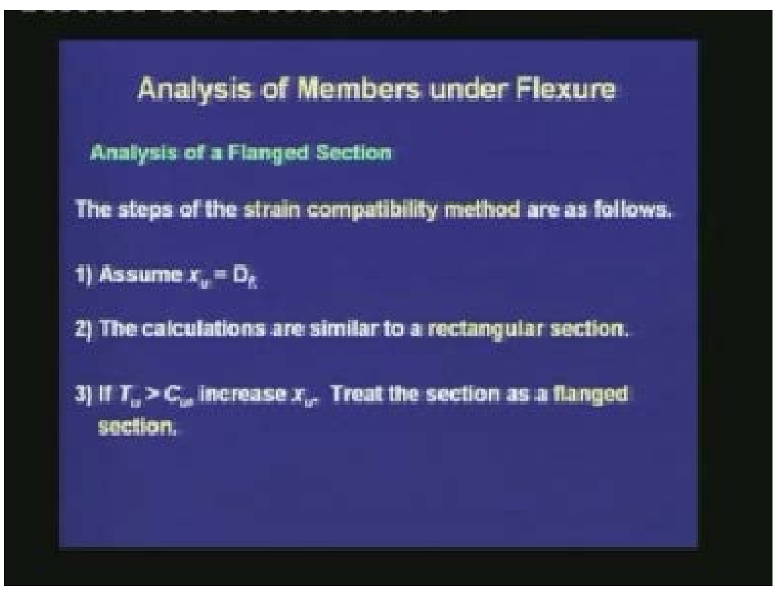

The method is called the strain compatibility method, and the steps are as follows. First check, whether the section behaves as a flanged section or not. We assume the depth of the neutral axis (x_u) equal to the depth of the flange (D_f). This is the border case of the transition from a rectangular section to a flanged section. With $x_u = D_f$, the calculations are similar to a rectangular section which we have seen in our previous lecture. We find the compressive force in the flange (C_u). We get the strain in the prestressing steel from the compatibility relationship, from which we get the stress using the design stress–strain curve. We calculate the tension in the prestressing tendon (T_u). If we find that $T_u > C_u$, then we need to increase x_u to have more compression. Once $x_u > D_f$, then the section is treated as a flanged section.

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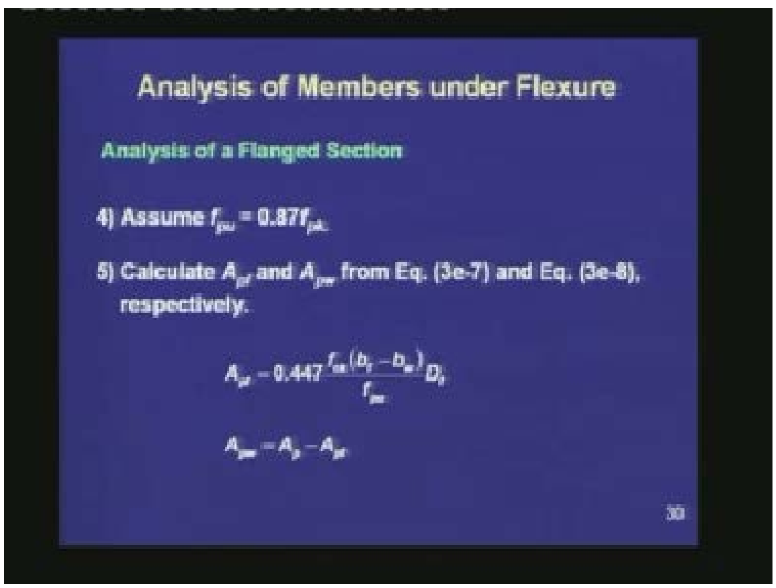

If $T_u < C_u$, then x_u has to be reduced. That means $x_u < D_f$, and we can proceed with the procedure of rectangular section, with the breadth of the flange equal to the breadth of the rectangular section.

Once we have determined whether the section behaves as a flanged section or not at ultimate, and if it behaves as a flanged section, then we move on to the next set of steps. Here we do an iteration to find out the strain and stress at the level of prestressing steel, and we have to satisfy the equilibrium equation of the longitudinal forces. In our next step in the analysis of a flanged section, we assume f_{pu} equal to the maximum allowed value as per the code which is equal to $0.87f_{pk}$. Once we have assumed f_{pu}, we can calculate the individual components A_{pf} and A_{pw} from the previous equations.

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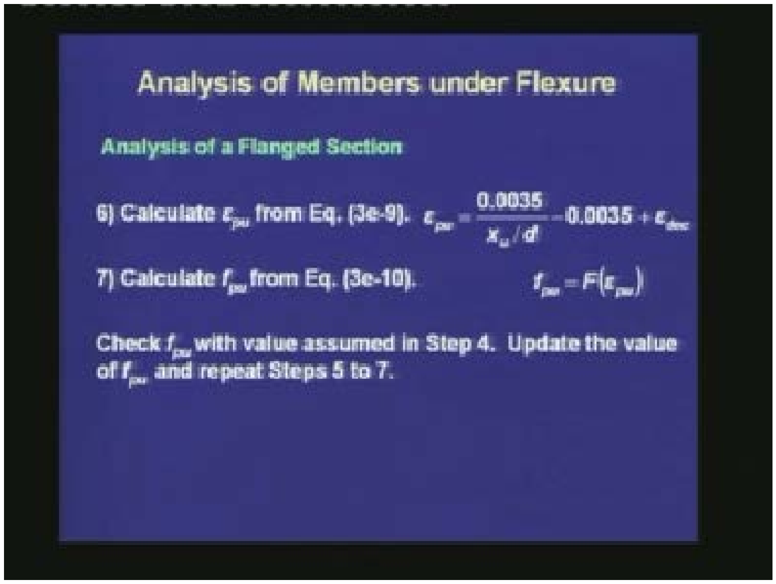

Next, we calculate ε_{pu} from the compatibility equation (Eq. 3e-9), which we have rewritten as $\varepsilon_{pu} = 0.0035/(x_u/d) - 0.0035 + \varepsilon_{dec}$. We calculate f_{pu} from the constitutive relationship $f_{pu} = F(\varepsilon_{pu})$.

Earlier, we had used a value of f_{pu} to calculate A_{pf} and A_{pw}, and now we have found out another f_{pu}. We have to check whether this f_{pu} is same as the value assumed in Step 4. If they are not same, then we update the value of f_{pu} and repeat Steps 5 to 7.

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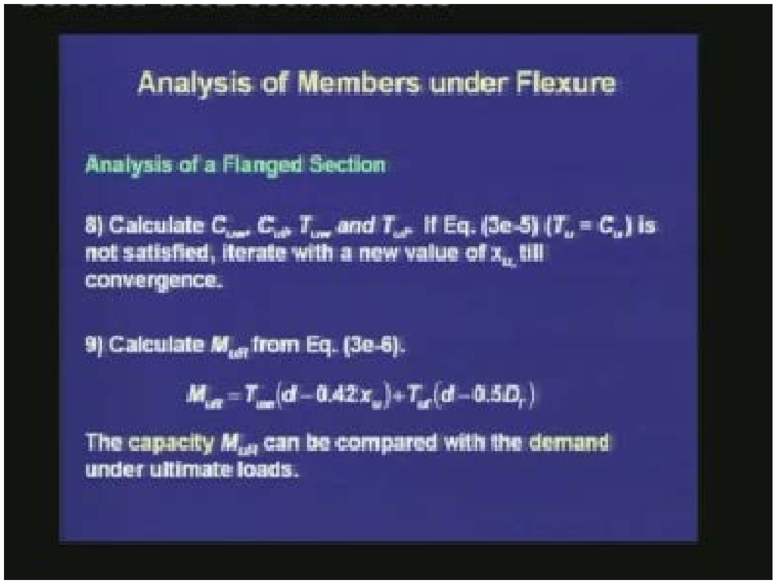

Next, we are able to calculate C_{uw}, C_{uf}, T_{uw} and T_{uf}. All the expressions for these individual forces were given before. Once we calculate the forces, we can check the first equilibrium equation which is $T_u = C_u$. If this is satisfied, then our assumed x_u is correct. If this is not satisfied, then we need to iterate with a new value of x_u till the convergence of T_u and C_u.

At this stage, we are able to calculate the ultimate moment of resistance from Eq. (3e-6), which is $M_{uR} = T_{uw} (d - 0.42x_u) + T_{uf} (d - 0.5D_f)$. The capacity M_{uR} can be compared with the demand under ultimate loads.

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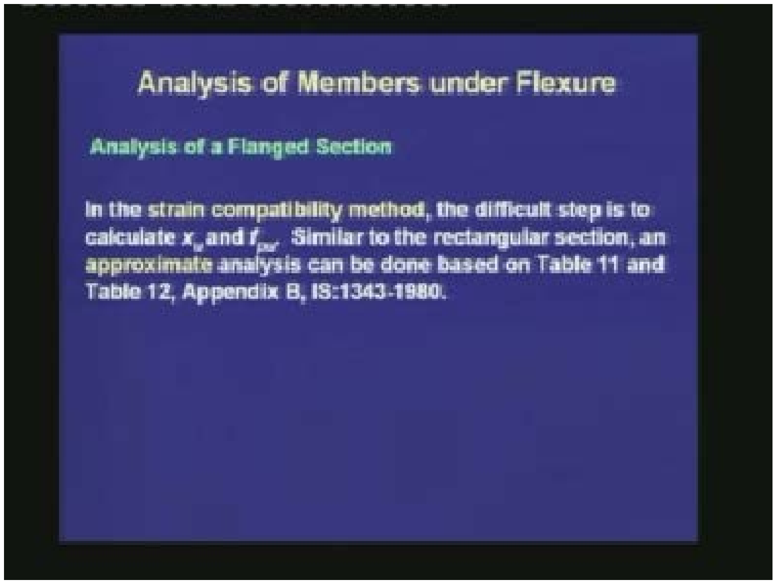

In the strain compatibility method, the difficult step is to calculate x_u and f_{pu} because we have to go through an iterative process, which satisfies the strain compatibility equation, the constitutive relationships and the equilibrium equations. If we want to bypass this method, the code IS: 1343-1980 allows us a simpler method which is similar to the approximate analysis for a rectangular section. This analysis is done based on Table 11 and Table 12 in Appendix B of the code.

The above tables are given with respect to a variable which is a measure of the total prestressing steel times its strength divided by the area of the concrete times the characteristic strength of concrete.

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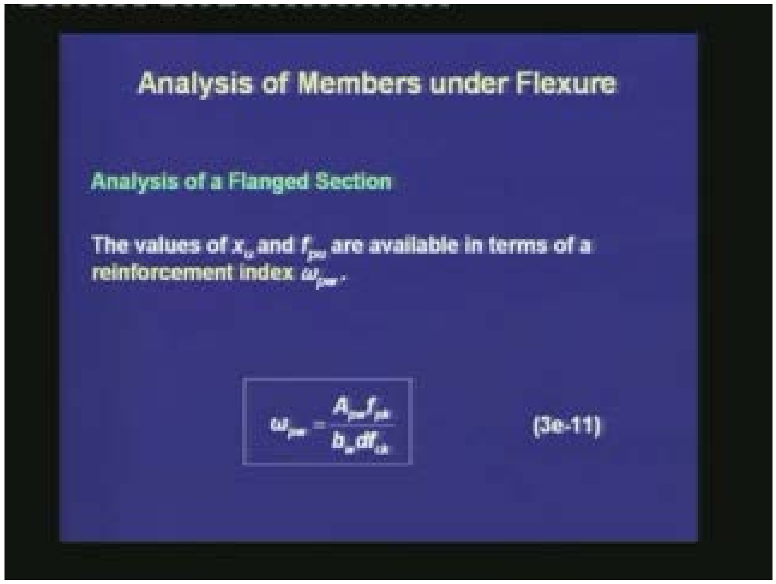

This quantity is denoted as the reinforcement index (ω). When we use this table for a flanged section, then the area of concrete that we have to take is equal to the breadth of the web times the effective depth of the CGS. Since this is different from a rectangular section, we are denoting the reinforcement index with a different subscript, and it is represented as ω_{pw}. Here, $\omega_{pw} = A_{pw}f_{pk} / (b_w d f_{ck})$. Thus, ω_{pw} uses a fraction of the prestressing steel which is A_{pw}, and it also uses only the breadth of the web in the denominator.

As the prestressing steel increases, ω_{pw} will increase and the value of x_u will go up. That is, with increasing amount of steel we need larger depth of the compression to balance the tension in the steel. Also, with increasing ω_{pw} beyond a certain value, the stress in the steel decreases. That means, a lower amount of stress in the steel will be able to equilibrate the compression in the concrete.

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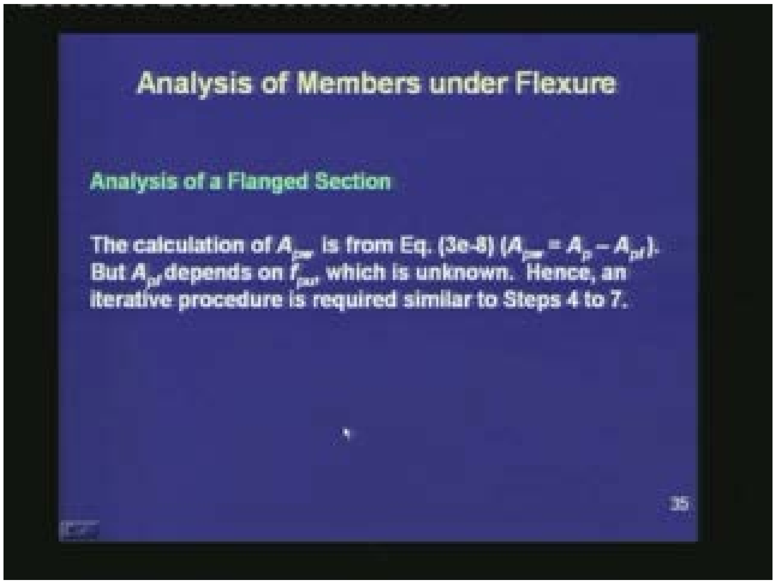

When we are using the approximate analysis for a flanged section, we need to calculate A_{pw} from the total prestressing steel. The calculation of A_{pw} is from Eq. (3e-8) which is $A_{pw} = A_p - A_{pf}$. But A_{pf} depends on f_{pu} which is unknown. Hence, an iterative procedure which is similar to that we have seen in the strain compatibility method, is required. That means, we have to assume a value of f_{pu}. Next, calculate A_{pf}, A_{pw} and ω_{pw}. From ω_{pw} we can calculate f_{pu} and check this f_{pu} with our assumption. If they are not close, then we have to update our assumption and then make another cycle of calculation.

In all these analyses, we had assumed that the depth of the flange (D_f) is less than $(3/7)x_u$. This assumption makes the stress in the flange constant, and the calculation of C_{uf} is simple using the constant stress $0.447f_{ck}$.

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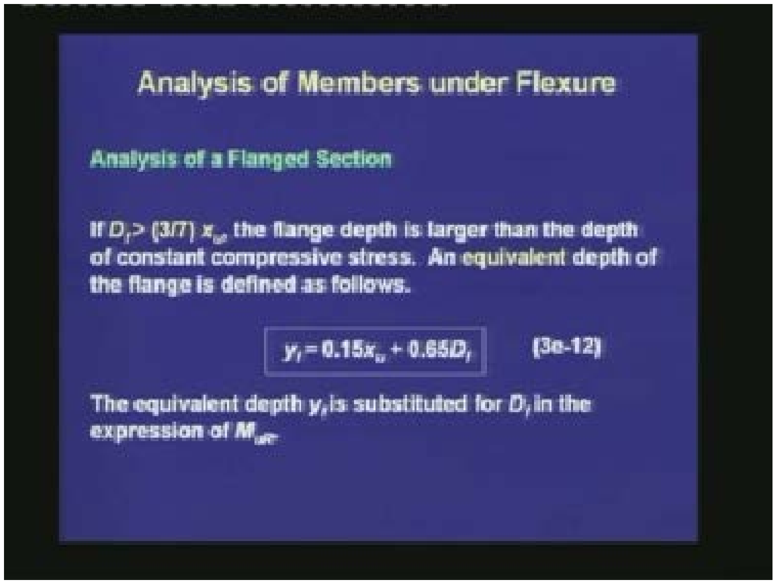

But if $D_f > (3/7)x_u$, then D_f is larger than the depth of constant compressive stress in the stress block. In that case, the code allows us to define an equivalent depth of the flange. The equivalent depth means that, the compressive force in the original flange will be equal to the compressive force in the new flange. The expression of this equivalent depth is $y_f = 0.15x_u + 0.65D_f$. Thus, the equivalent depth depends both on the depth of neutral axis x_u and the depth of the flange, which is D_f. The equivalent depth y_f is substituted for D_f in the expression of M_{uR}. Thus, the only difference we have when the depth of flange is large is that we are using an equivalent depth instead of the actual depth of the flange in the expression of the ultimate moment of resistance. Otherwise all other steps are similar.

Let us now understand the procedure of analysis for a flanged section with the following example.

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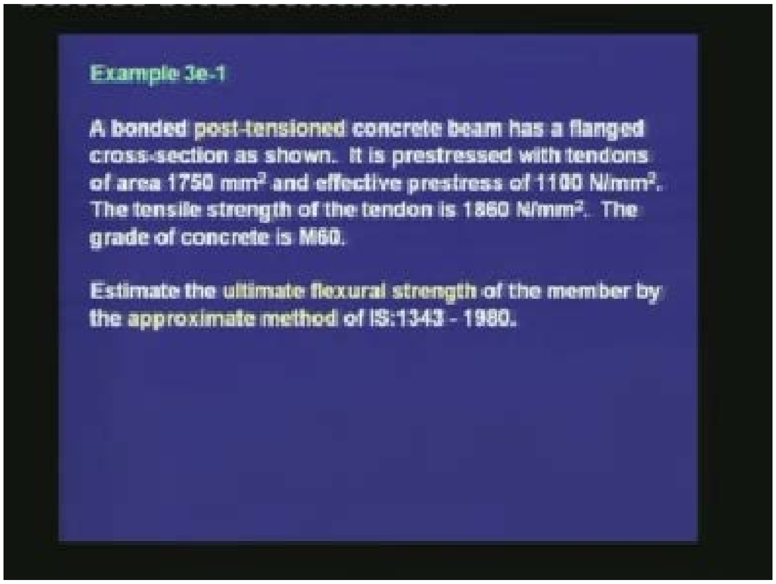

A bonded post-tensioned concrete beam has a flanged cross-section as shown in the following sketch. It is prestressed with tendons of area 1750 mm^2, and effective prestress of 1100 N/mm^2. The tensile strength of tendon is 1860 N/mm^2 and the grade of concrete is M60.

Estimate the ultimate flexural strength of the member by the approximate method of IS: 1343-1980.

In our last example for a rectangular section, we had solved by the strain compatibility method. Here, we are solving by the approximate method as given in IS: 1343. The cross-section of the section is as follows.

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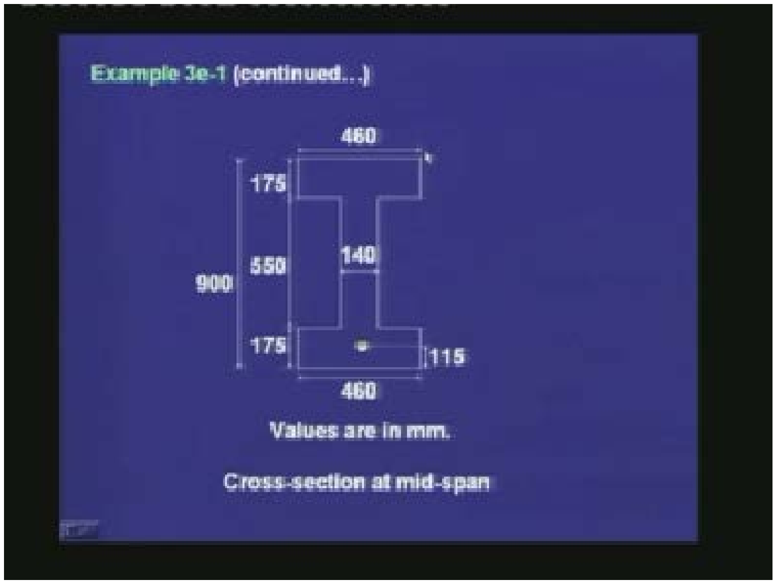

The effective width of the flange is 460 mm. The depth of the flange is 175 mm. There is also a flange at the bottom, but we will not consider this flange in our analysis because this flange is under tension. The CGS is located at a distance of 115 mm from the soffit of the beam. The width of the web is 140 mm. The total depth of the member is 900 mm.

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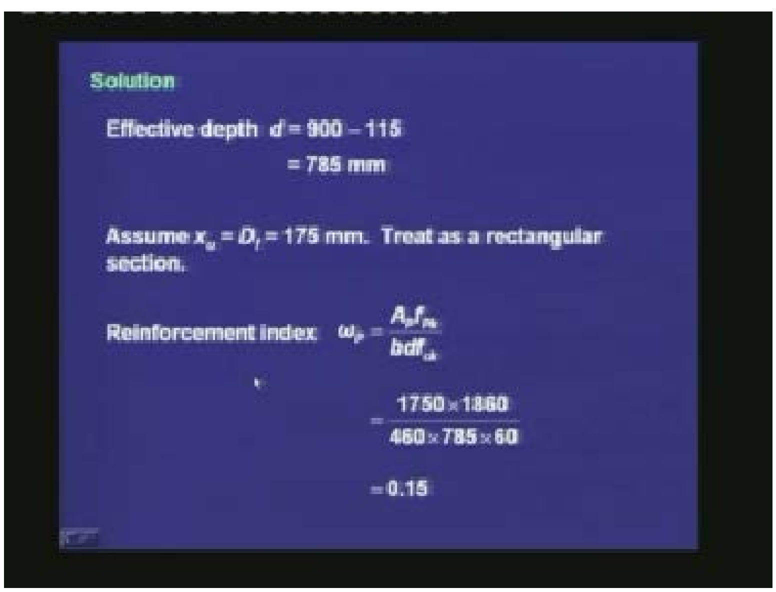

First, we are calculating the effective depth which is the total depth minus the distance of the CGS from the soffit, d = 900 – 115 = 785 mm. Next, we are assuming the depth of the neutral axis equal to the depth of the flange to check whether the section will behave as a rectangular section or as a flange section. Once we assume $x_u = D_f = 175$ mm, we can treat the section as a rectangular section. For a rectangular section, $\omega_p = A_p f_{pk}/bdf_{ck}$. Substituting the values of the variables, $\omega_p = 0.15$. Note that, here b = 460 mm which is the breadth of the flange, and $A_p = 1750$ mm^2 which is the total amount of prestressing steel.

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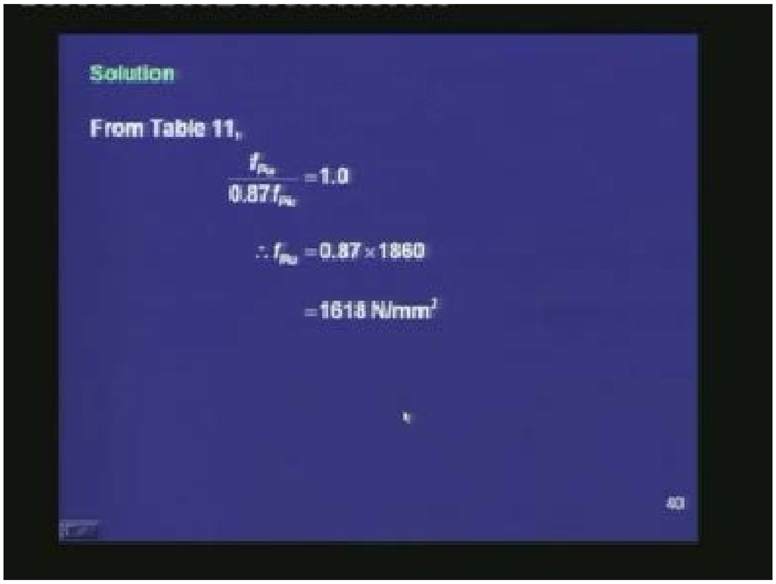

From Table 11 corresponding to $\omega_p = 0.15$, $f_{pu}/0.87f_{pk} = 1.0$. The prestressing steel has a stress which is the maximum allowable value by the code, and is equal to 0.87 f_{pk} = 1618 N/mm^2.

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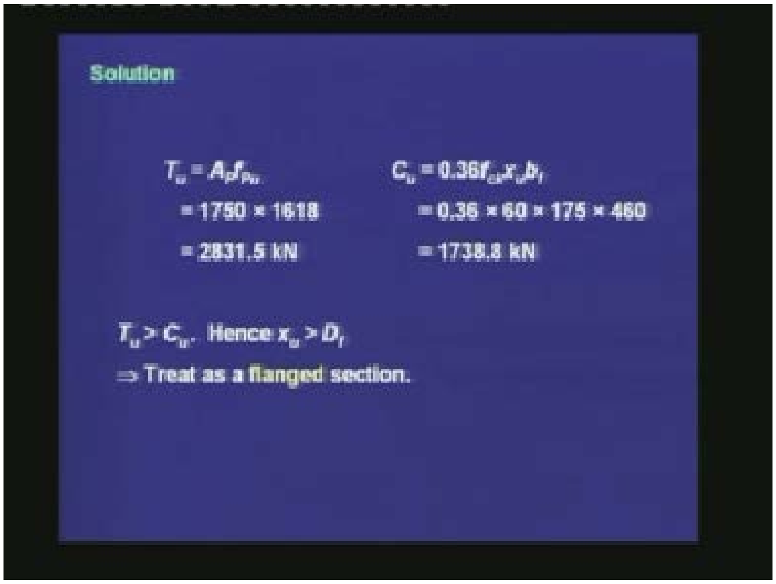

The tension $T_u = A_p f_{pu} = 2831.5$ kN. The compression $C_u = 0.36 f_{ck} x_u b_f = 1738.8$ kN. Comparing these two values, we find that T_u is much larger than C_u and hence we need to increase x_u beyond D_f. Thus, we have to treat the section as a flanged section at ultimate.

From now onwards we are treating the section as a flanged section, and hence the expression of the reinforcement index will also be different.

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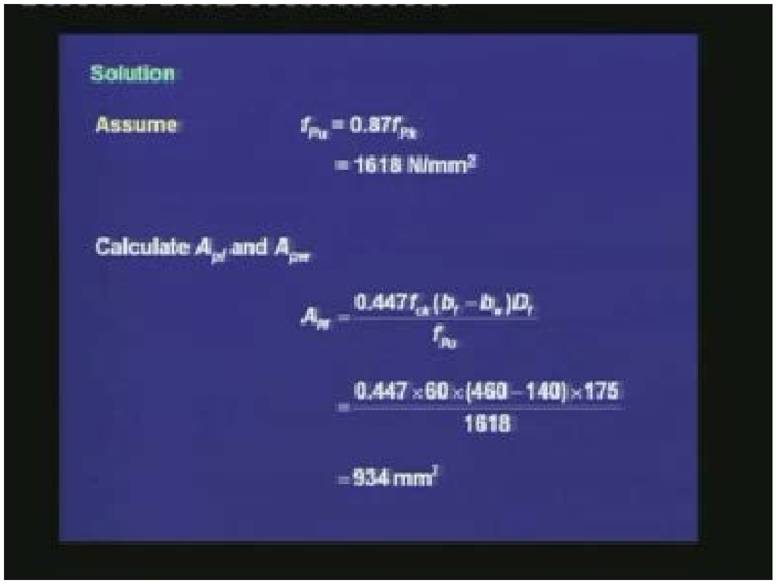

First, we are assuming $f_{pu} = 0.87 f_{pk} = 1618$ N/mm^2. Next, we are calculating A_{pf} which is the amount of steel balancing the compression in the outstanding part of the flanges. $A_{pf} = 0.447 f_{ck}(b_f - b_w)D_f / f_{pu} = 934$ mm^2.

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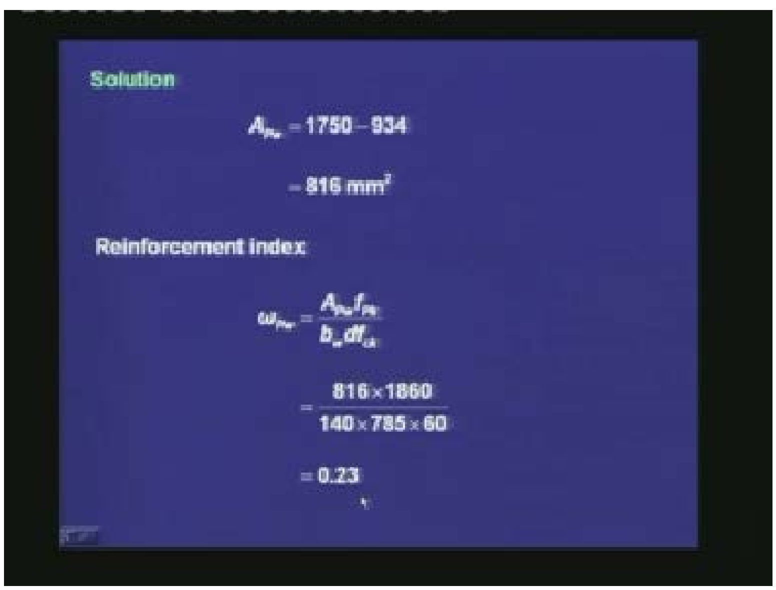

From this we calculate $A_{pw} = 1750 - 934 = 816$ mm^2. The reinforcement index for the flanged section is $\omega_{pw} = A_{pw} f_{pk} / b_w d f_{ck} = 0.23$.

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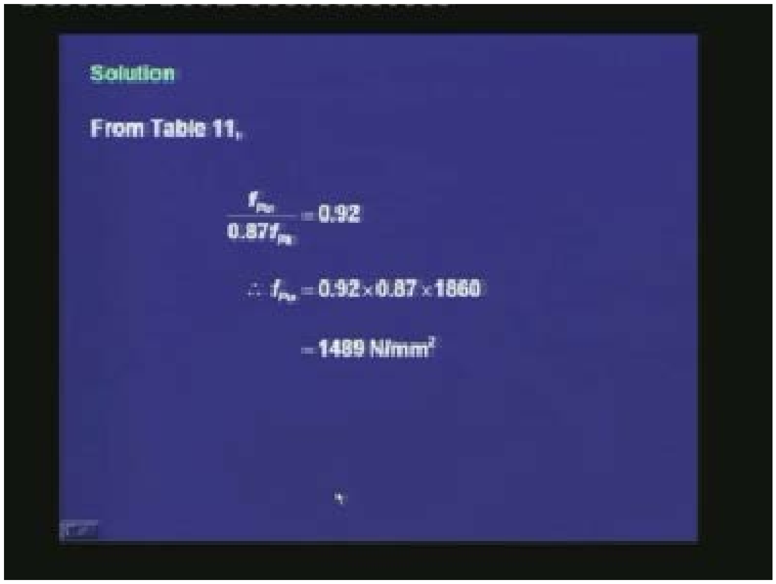

Again going back to Table 11, $f_{pu}/0.87f_{pk}$ = 0.92 from which we get f_{pu} = 1489 N/mm^2. Thus, with increased x_u, there is a drop in the prestressing stress in the tendons. In our second iteration, the new value of f_{pu} is used to calculate A_{pf}.

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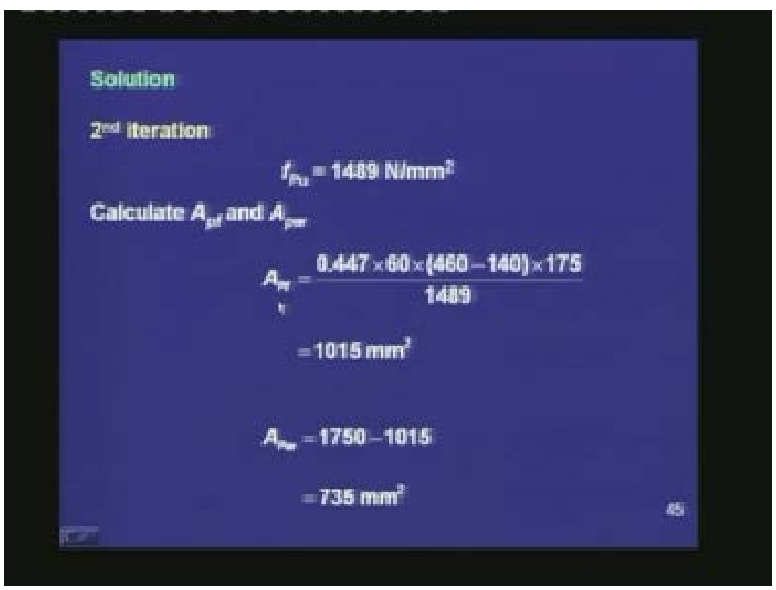

We are recalculating A_{pf} by the previous expression where we have substituted 1489 in the denominator, and we find A_{pf} = 1015 mm^2. We are recalculating A_{pw} as 1750 – 1015 = 735 mm^2.

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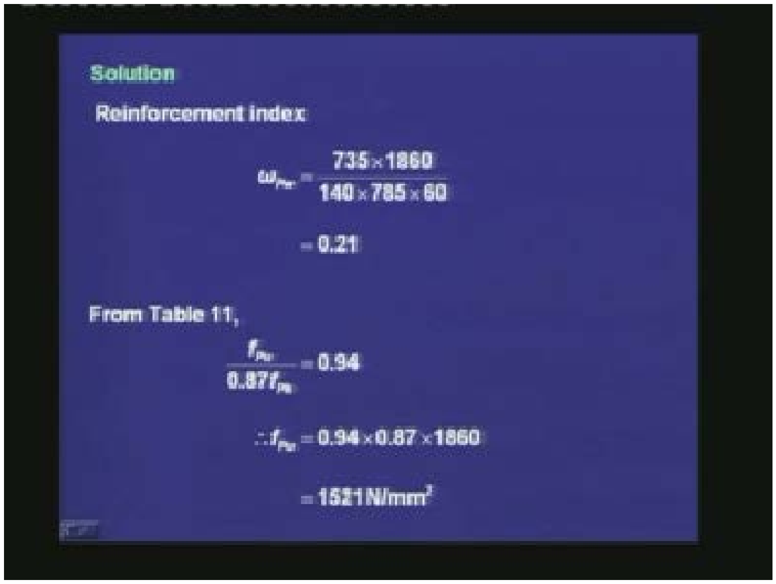

From that, we are recalculating ω_{pw} = 0.21. Last time, we had got a value of ω_{pw} = 0.23 and this time we are getting ω_{pw} = 0.21. Thus, we can see that we are converging gradually. With this new value of ω_{pw} we are going back to Table 11, and we are finding f_{pu} / $0.87f_{pk}$ = 0.94 from which f_{pu} = 1521 N/mm^2.

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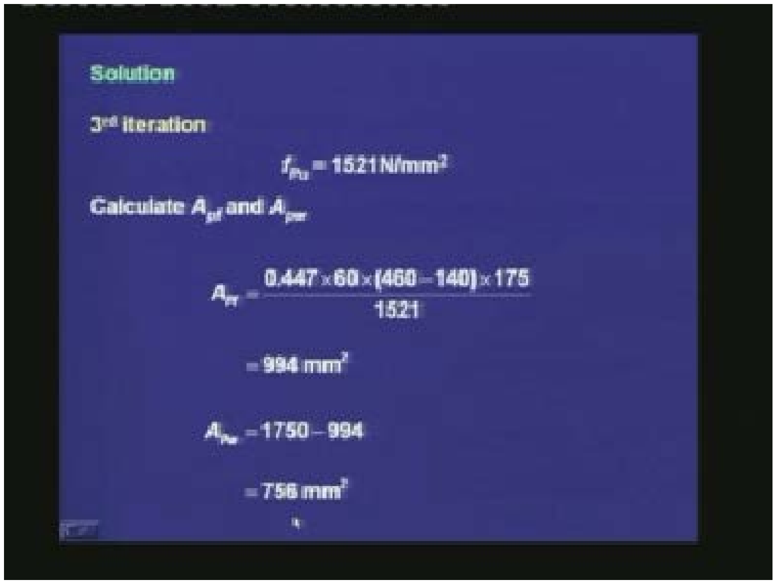

In the third iteration, if we substitute $f_{pu} = 1521 \text{ N/mm}^2$, we calculate $A_{pf} = 994 \text{ mm}^2$ and $A_{pw} = 756 \text{ mm}^2$.

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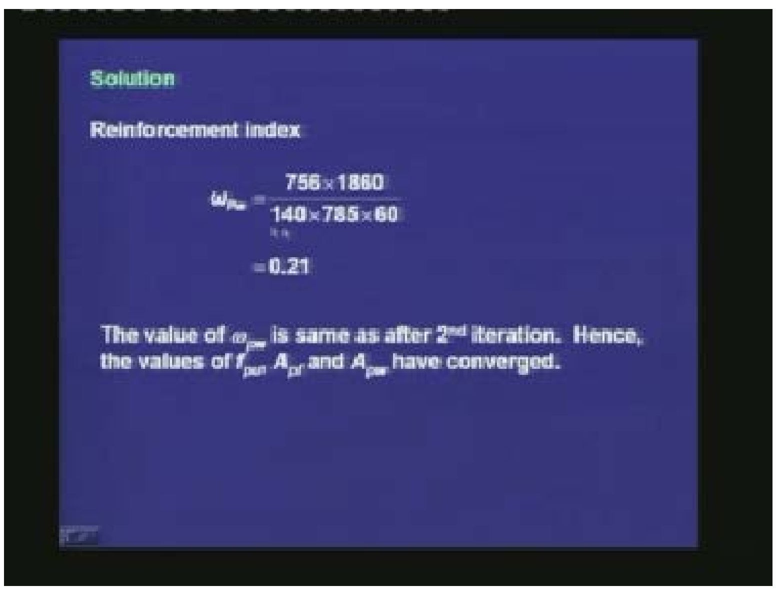

We recalculate ω_{pw} and it turns out to be same as that of the second iteration, which is 0.21. That means, we have converged to a value of ω_{pw} which is not changing beyond the third iteration. Hence, the number of iterations is sufficient and the values of f_{pu}, A_{pf} and A_{pw} have converged.

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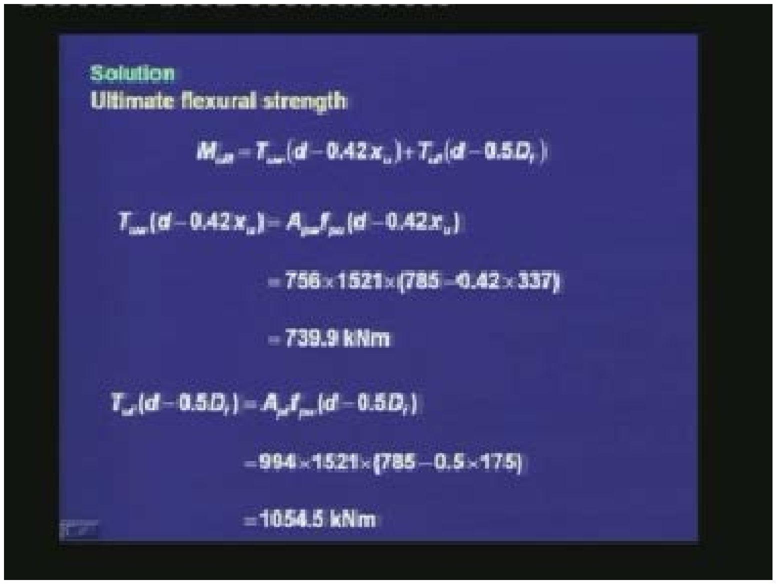

From that, we are now calculating the ultimate moment of resistance which is given as T_{uw} (d – 0.42x_u) + T_{uf} (d – 0.5D_f). The first component of the moment is equal to 739.9 kNm. The second component of the moment is 1054.5 kNm.

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Thus adding these two values, the total ultimate moment of resistance is equal to 1794.4 kNm.

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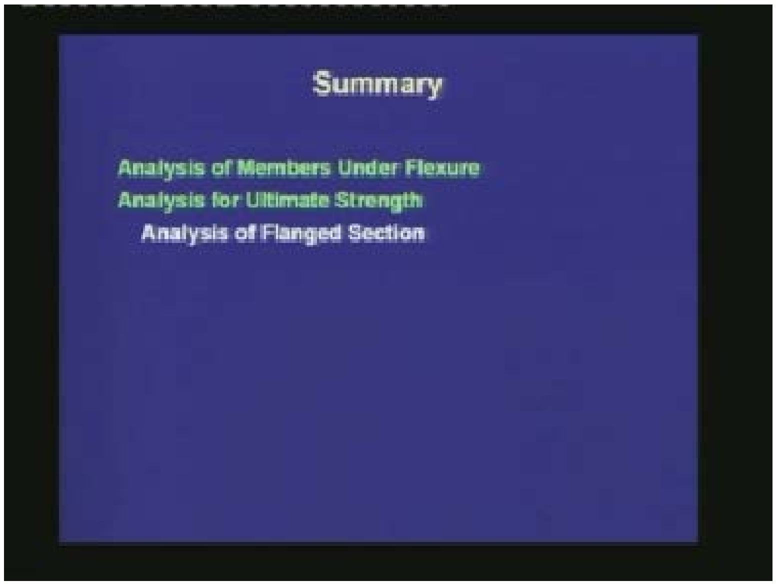

Today, we studied the analysis of members under flexure. Specifically, we studied the analysis of flanged sections for the ultimate strength. We first saw the different types of flanged sections that are possible in the construction. We treat a section as a flanged section under two considerations. The first is that the flanges have to be in compression so that we get benefit out of the concrete under compression. If the flange is under tension, then we neglect the part of the concrete in the flange and treat it similar to a rectangular section. The second consideration is that, the depth of the neutral axis should be larger than the depth of the flange. If the depth of the neutral axis is smaller than the depth of the flange, then we can use the expressions of the rectangular section to find out the ultimate moment of resistance, where the breadth of the rectangular section is same as the breadth of the flange.

In this lecture, we had considered a flange section which is constituted of a rectangular web and a rectangular flange. We found that when $x_u > D_f$, in the analysis we decompose the force couple into two components. In one couple, the compression in the web is balanced by a part of the tension in the tendons, and in the second couple the compression in the outstanding flanges is balanced by the rest of the tension. This type of decomposition helps us to identify the individual lever arms of the two couples. From the expressions of the individual lever arms, we got an analytical expression of the ultimate moment of resistance of a flanged section.

In an analysis, we are using three principles of mechanics; first is the equilibrium of forces, which has two equations. Next is the equation of compatibility which relates the maximum strain in concrete with the strain in the prestressing steel, and third, we have the constitutive relationships of the concrete and the steel. In the analysis, there are a set of variables which are given, and there are few variables which are unknown.

There is a rational procedure which is called the strain compatibility method by which we solve the simultaneous equations in a logical manner. First, we assume the depth of the neutral axis and then we find out the values of the forces in the compression and the tension. If they are equal, then our assumption is correct. If not, then we change the depth of the neutral axis and do another cycle of calculations.

The steps of the strain compatibility method can be involved if the section is more complicated than what we had studied. This can be implemented in a computer program where once the variables are input, the program itself will do the iteration process. Else, if we do not want to use the strain compatibility method, then we can use an approximate method proposed by the code. Here, we calculate the depth of the neutral axis and the stress in the prestressing tendon based on a quantity called the reinforcement index. For a flanged section, the calculation of reinforcement index is different from that of a rectangular section. Here we take the part of the tendon A_{pw} in the numerator and we take b_w in the denominator. Once we know the reinforcement index, from the tables we can calculate the depth of the neutral axis and the stress in the prestressing tendon. From this we can calculate the ultimate moment of resistance of a flanged section. We solved a problem based on the approximate method. In the next class, we shall move on to the third type of section, the partially prestressed section.

Thank you.