PRESTRESSED CONCRETE STRUCTURES

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Module – 3: Analysis of Members

Lecture – 14: Analysis of Rectangular Sections

Welcome back to prestressed concrete structures. Today, we are covering the fourth lecture of module three on analysis of members.

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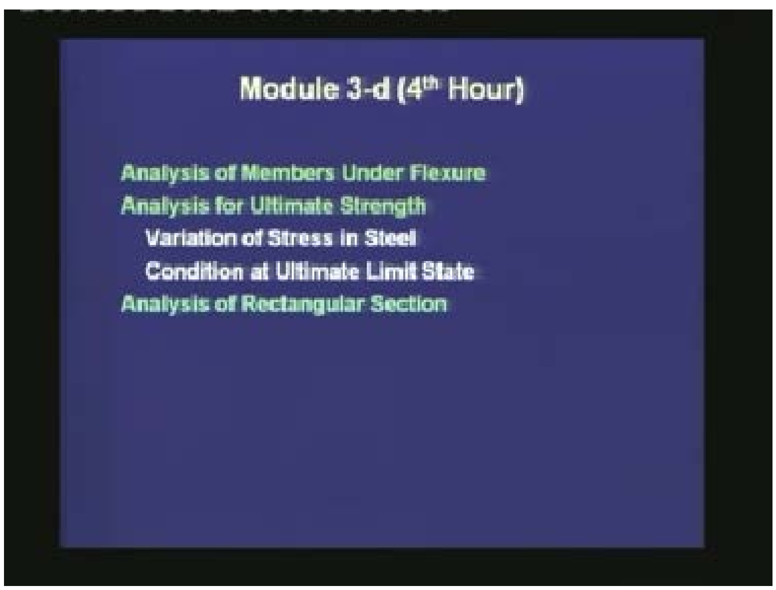

In today's lecture, we shall cover the analysis for ultimate strength for members under flexure. Under that, we shall understand the variation of stress in steel throughout the loading history. We shall also learn about the condition at the ultimate limit state and finally, we shall move on to the analysis of a rectangular section.

First is the analysis of ultimate strength.

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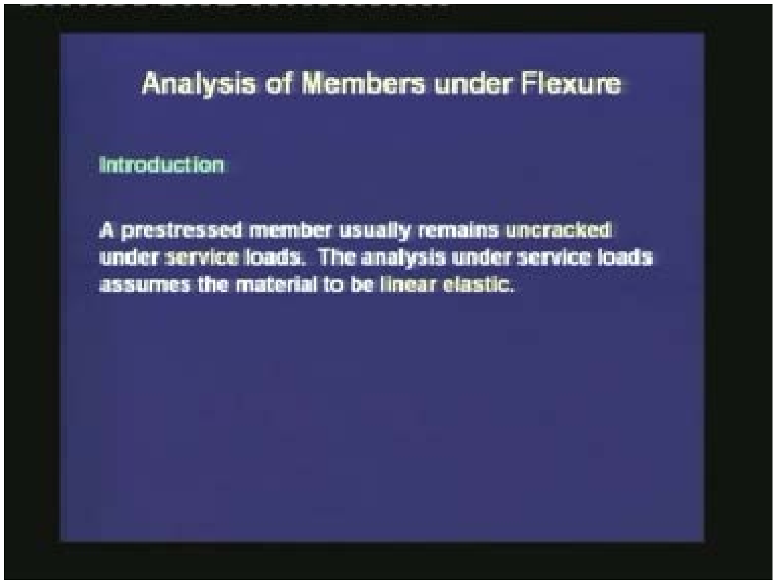

A prestressed member usually remains uncracked under service loads. The analysis under service loads assumes the material to be linear elastic. In whatever we have studied till now for the analysis under service loads, we had assumed linear stress–strain diagrams both for concrete and for steel.

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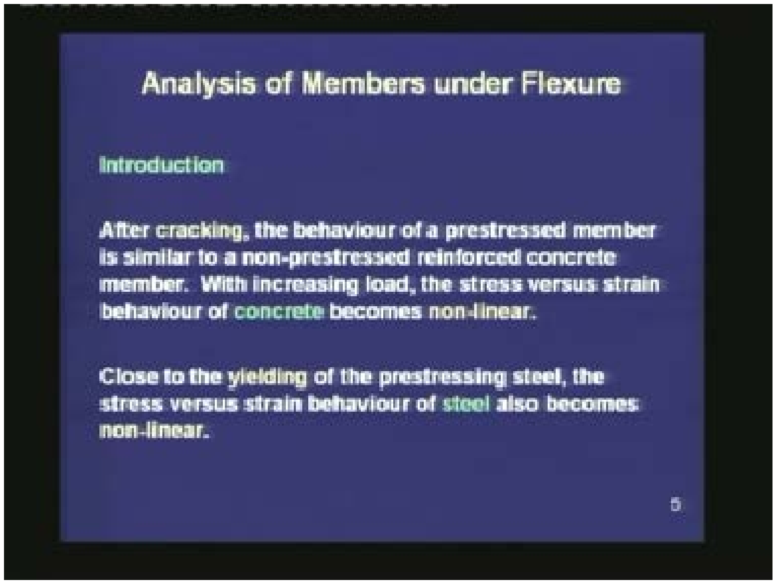

After cracking, the behaviour of a prestressed member is similar to a non-prestressed reinforced concrete member. With increasing load, the stress versus strain behavior of concrete becomes non-linear. Close to the yielding of the prestressing steel, the stress versus strain behavior of steel also becomes non-linear. This makes the analysis of ultimate strength different from the analysis under service loads.

Under analysis of service loads, we have assumed both the materials to be linear elastic. But when we are doing an analysis for ultimate strength, the stress–strain for concrete should be non-linear. Also, if the steel has started to yield, the stress–strain for steel should be non-linear.

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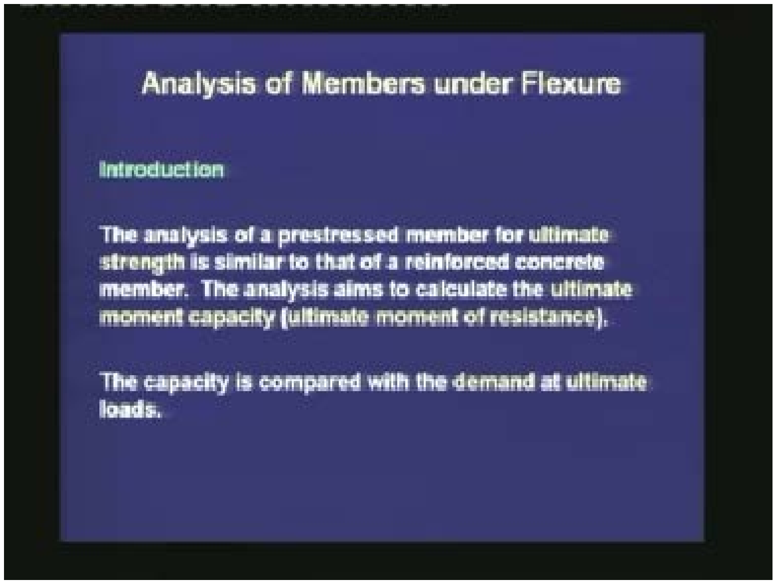

The analysis of a prestressed member for ultimate strength is similar to that of a reinforced concrete member. The analysis aims to calculate the ultimate moment capacity, which is also known as the ultimate moment of resistance. The capacity is compared with the demand at ultimate loads.

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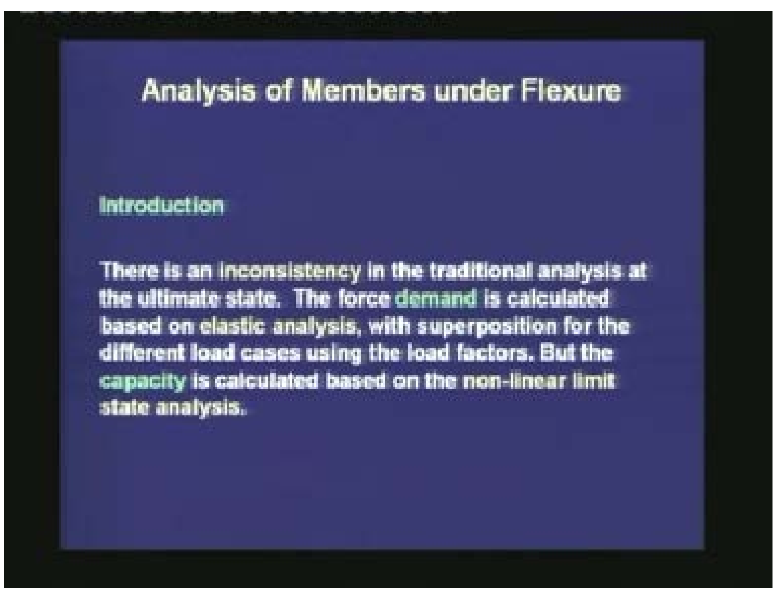

There is an inconsistency in the traditional analysis at the ultimate state. The force demand is calculated based on elastic analysis, with superposition for the different load cases using the load factors. But the capacity is calculated based on the non-linear limit state analysis. This inconsistency has stayed after the introduction of the limit states method. When we are calculating the demand, we are using a linear analysis with the principle of superposition being applied for the different load cases, whereas, when we are trying to find out the capacity, we are using a non-linear behaviour of the material.

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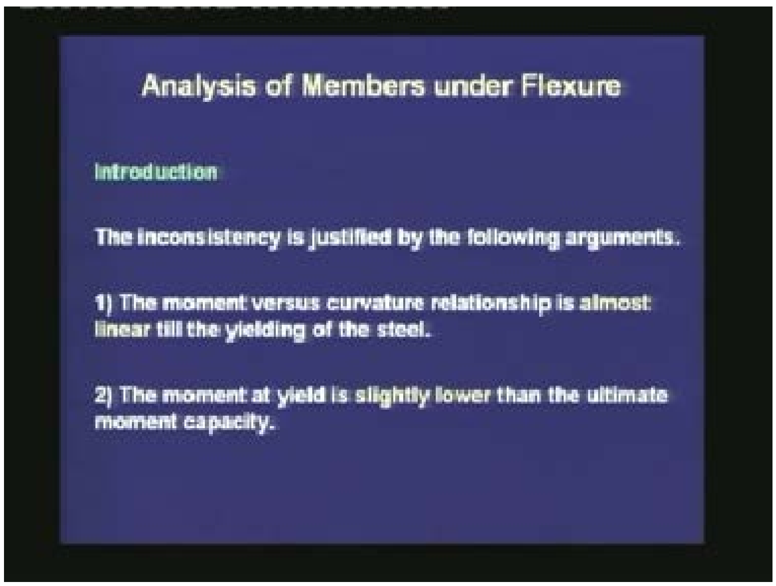

This inconsistency is justified by the following arguments: First, the moment versus curvature relationship is almost linear till the yielding of the steel. That is, although there is some non-linearity in the concrete, yet the non-linearity is not reflected to a great extent before the yielding of the steel.

Second, the moment at yield is slightly lower than the ultimate moment capacity. When we are talking of the behavior, it is almost linear up to the yield moment. That is, it is almost linear close to its ultimate capacity.

The third argument is that the calculated moment demand for a load case is based on elastic analysis that is well within the moment at yield. Say, if I pick up the load case either for the dead load or live load, then the moment demand for any of this particular load case is well within the yield moment. Hence, superposition for the different load cases is applicable to find out the force at ultimate.

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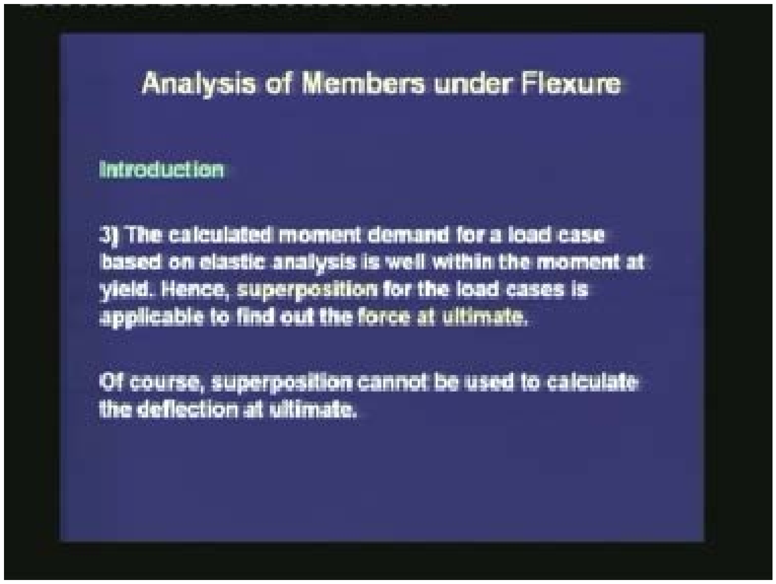

Of course, superposition cannot be used to calculate the deflection at ultimate. Here we have to be careful that, we may apply the principle of superposition to find out the demand, but we cannot use the principle of superposition to find out the deflection at ultimate. Because, after yielding there is substantial inelastic deformation which will not be accounted for if we use the principle of superposition for deflections of individual load cases.

The second important aspect of the analysis at ultimate strength is to understand the variation of stress in steel.

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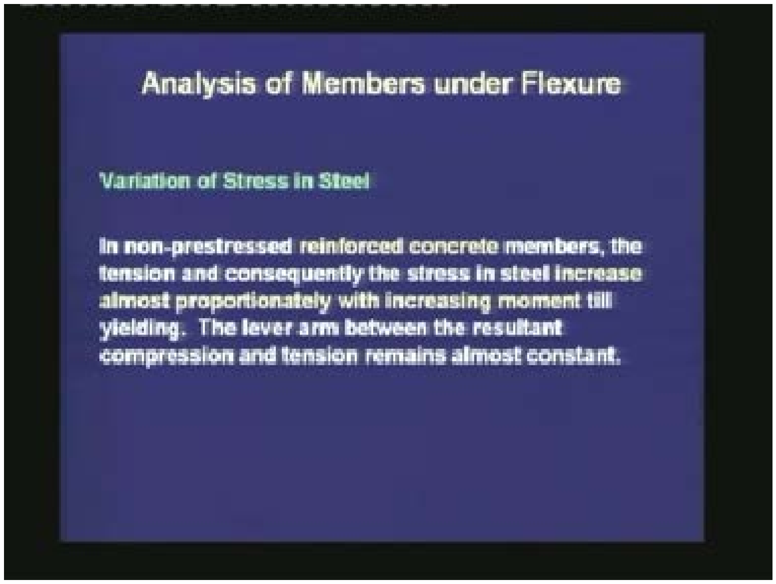

In non-prestressed reinforced concrete members, the tension and consequently the stress in steel increase almost proportionately with increasing moment till yielding. The lever arm between the resultant compression and tension remains almost constant. We had seen this in the analysis under service loads that for a reinforced concrete member, as the load is increased or as the external moment is increased, the stress in the steel increases almost proportionately till it starts to yield.

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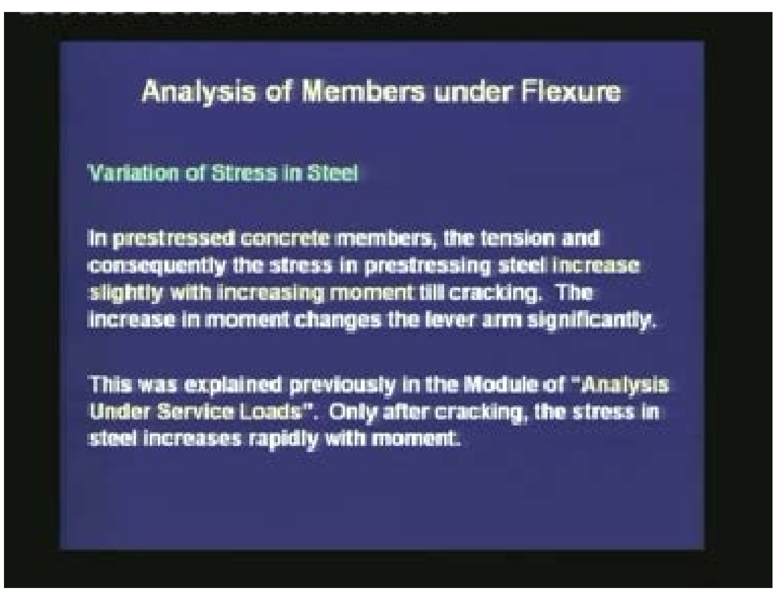

Whereas, in prestressed concrete members, the tension and consequently the stress in prestressing steel, increase slightly with increasing moment till the cracking. The increase in moment changes the lever arm significantly. This is in contrast to that in a reinforced concrete member. For a prestressed concrete member, as the load is increased, or as the external moment is increased, the tension in the steel does not increase significantly. It just increases slightly, whereas the lever arm between the compression and tension increases as the moment increases. The compression shifts from the level of the prestressing steel upwards, and that the increase in the lever arm is able to resist the increase in external moment. This was explained previously in the module of analysis under service loads.

Only after cracking, the stress in steel increases rapidly with moment. That means, before cracking the increase in the stress in steel is not significant, but after cracking there is a significant increase in the stress in the steel, and then it reaches a value close to its ultimate.

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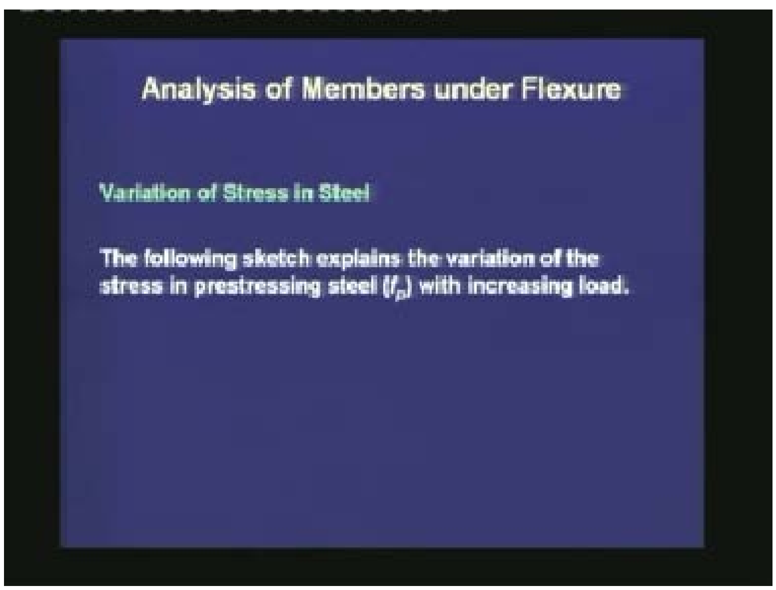

The following sketch explains the variation of stress in prestressing steel which is denoted as f_p, with increasing load.

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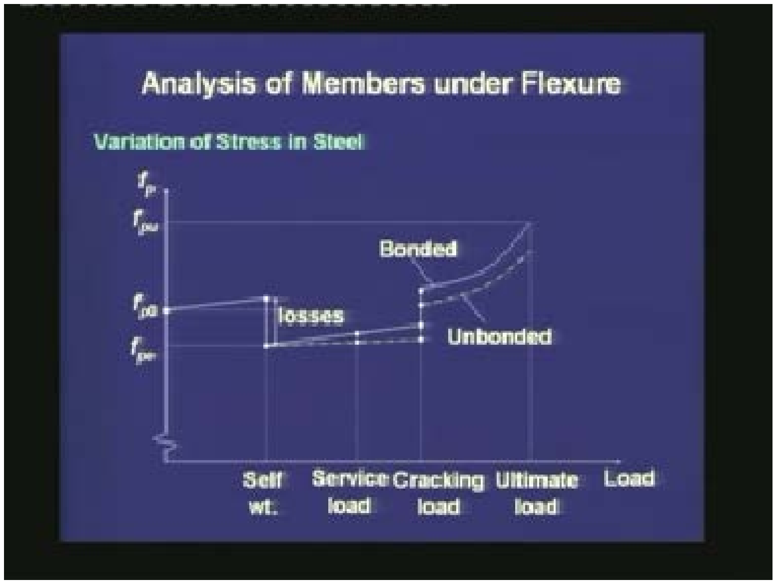

In this sketch, what we see is that first the prestressing steel in the tendons is stretched up to a stress of f_{p0}. The self-weight can act either while the member is hogging up, or it can act when the shuttering is removed. When the self-weight acts, there is a slight increase in the stress, and then with time there is a drop in the stress due to the long-term effects of shrinkage, creep and relaxation, and it stabilizes to a value of f_{pe}.

A drop in the prestress has been just denoted by a vertical line in this sketch, because we are not plotting this with respect to time. We are plotting the variation of the stress with respect to the increasing load. From the self-weight, once the service load starts acting, the stress again increases slightly until the member cracks. A Type 1 or Type 2 member does not crack under service load, and hence the variation of the stress in the steel is very small. Hence, it is neglected in the analysis under service loads. We assume that, the stress in the steel remains constant at a value of f_{pe}.

After cracking, there is a sudden increase in the stress in the steel. Here, we see that both for bonded and unbonded tendons, there is a jump in the stress in the steel. There is a redistribution of the stresses between the concrete and the steel, and then when the load is increased further, the stress increases till it reaches an ultimate value corresponding to the ultimate capacity of the member.

For a bonded tendon, the stress is higher at the locations of the cracks. For an unbonded tendon, the average stress over the length is not as high as the local stress for a bonded tendon.

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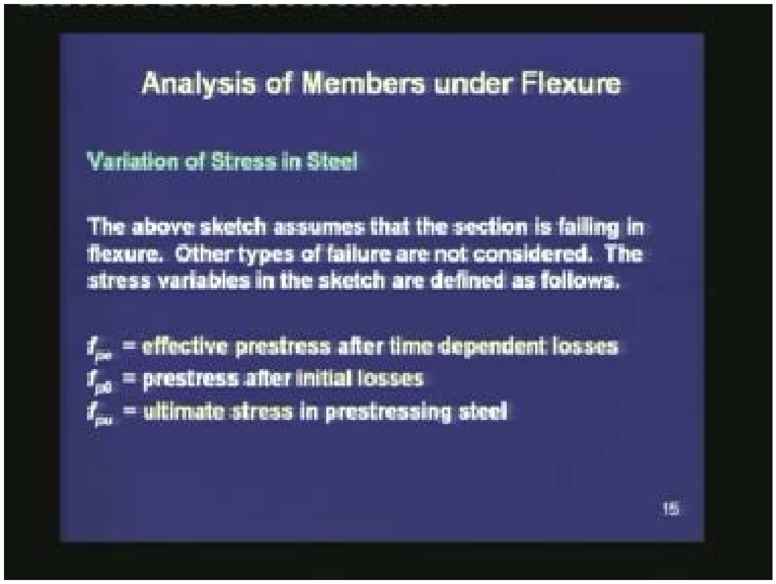

The above sketch assumes that the section is failing in flexure. Other types of failure are not considered here. The stress variables in the sketch are defined as follows: f_{pe} is the effective prestress after long-term time dependent losses, f_{p0} is the prestress after initial losses, f_{pu} is the ultimate stress corresponding to the capacity of the member.

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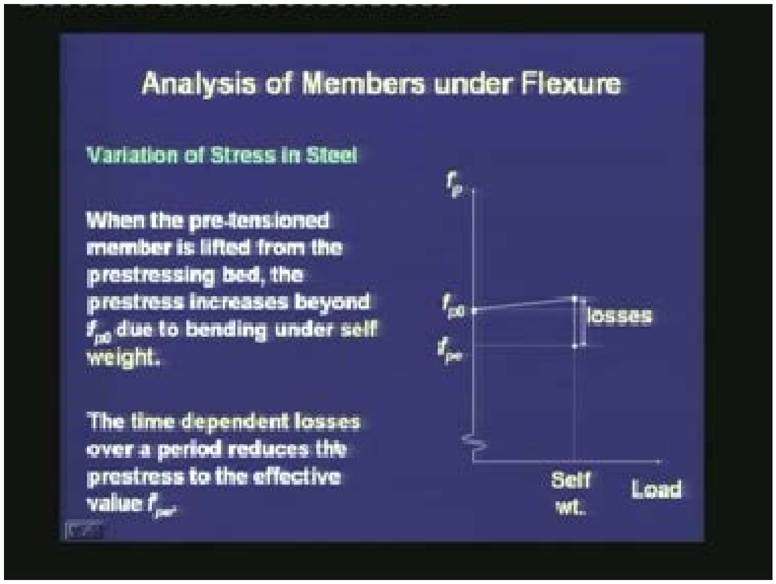

Let us try to understand this variation in a bit more details. When a pre-tensioned member is lifted from the prestressing bed, the prestress increases beyond f_{p0} due to bending under self-weight. If during the transfer of prestress the member hogs up, that means the prestress is high enough to counteract the self-weight, then the self-weight acts right after transfer.

For a post-tensioned member, if the member is post-tensioned in the casting bed itself, then the self-weight may not act during prestressing. But, if the member is prestressed at the site where it is being supported only at the ends, then the self-weight is acting even before the prestress is applied.

The increase in stress from f_{p0} to a value slightly higher than f_{p0} is due to the self-weight of the member. The time dependent losses over a period reduce the prestress to the effective value f_{pe}. The drop in the prestress is represented by this vertical line, and this occurs over several years in the service life of the member.

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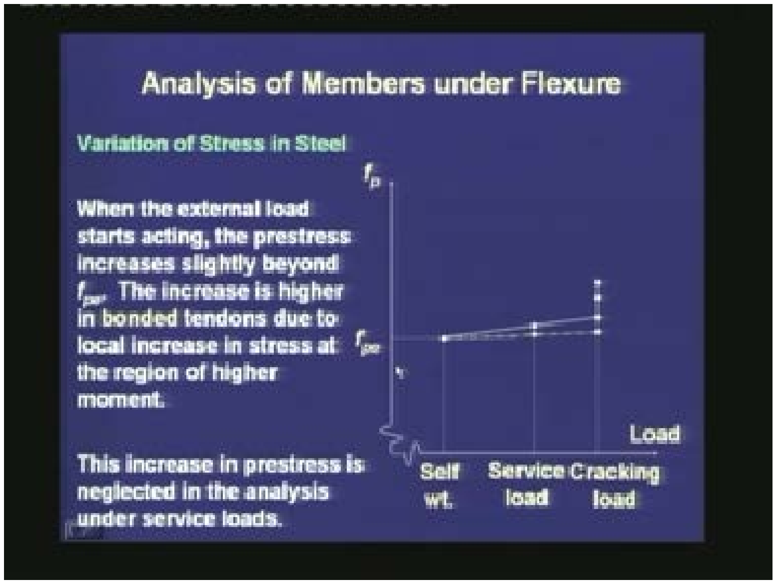

When the external load starts acting, the prestress increases slightly beyond f_{pe}. The increase is higher in bonded tendons due to the local increase in stress at the region of higher moments. For an unbonded tendon, the stress is averaged throughout the length of the member. The increase in prestress is neglected in the analysis under their service loads.

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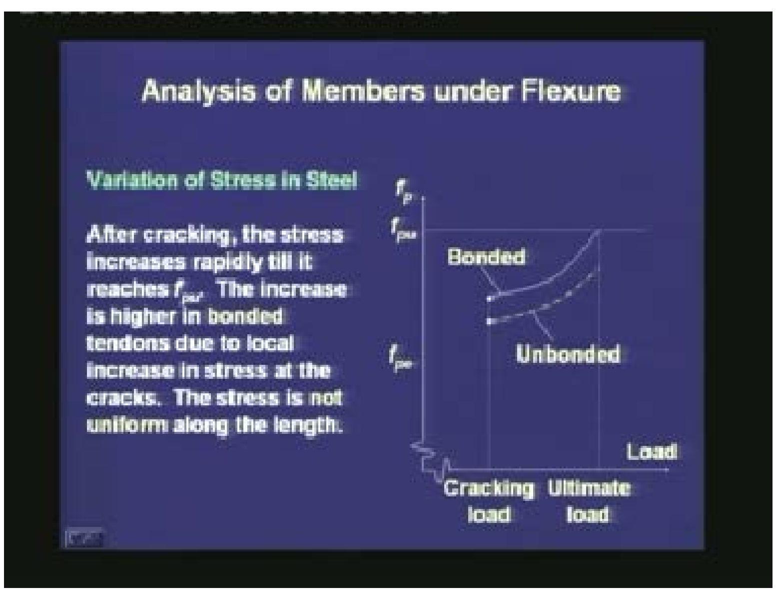

After cracking, the stress increases rapidly till it reaches f_{pu}. After cracking, the behaviour of a prestressed concrete member is very similar to a reinforced concrete member. The increase in stress is higher for bonded tendons, due to local increase at the cracks. The stress is not uniform along the length for a bonded tendon.

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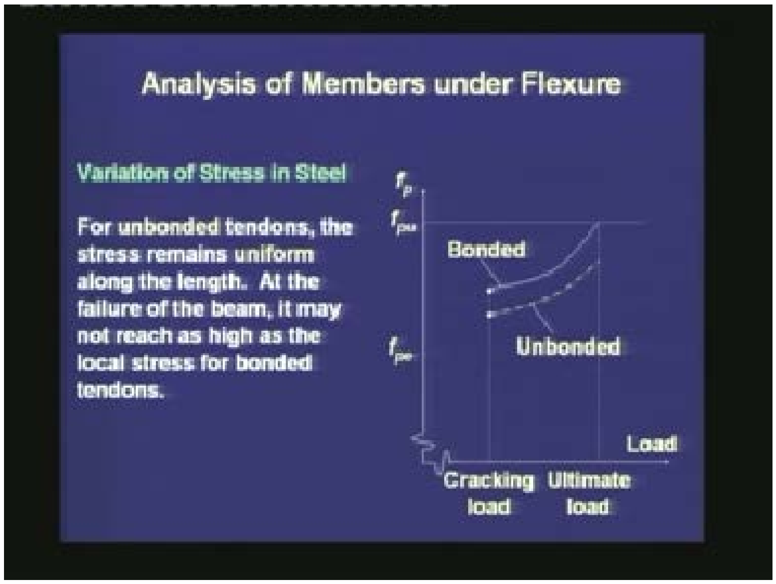

For an unbonded tendon, the stress remains uniform along the length. At the failure of the beam, it may not reach as high as the local stress for a bonded tendon. Thus, we have understood the variation of the stress in steel as the load is increased. To summarize, the stress in the tendon does not increase much under the service load. But after the member cracks, the increase in the stress is substantially high, and we cannot consider that the stress is remaining constant at f_{pe}.

Next we are trying to understand the conditions at ultimate limit state.

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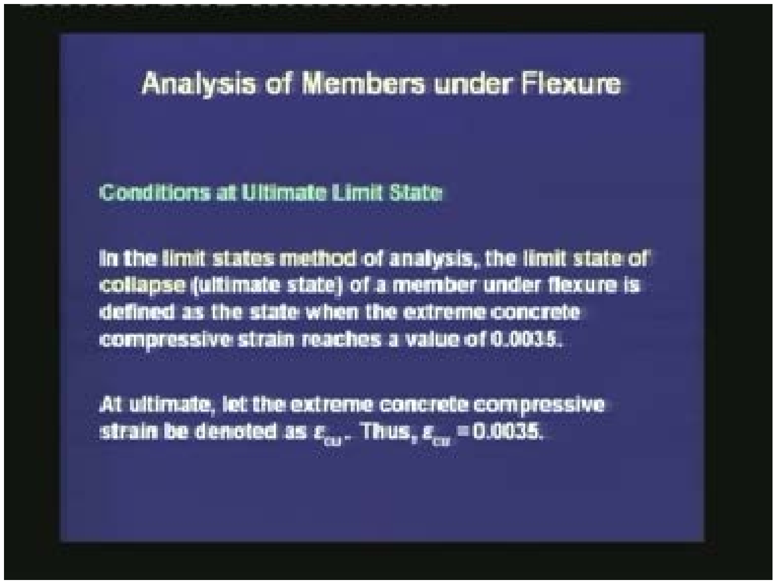

In the limit states method of analysis, the limit state of collapse (or the ultimate state) of a member under flexure is defined as the state when the extreme concrete compressive strain reaches a value of 0.0035. This is the definition of failure of both the reinforced concrete member and the prestressed concrete member, as per the Indian codes. At the ultimate state, the extreme concrete compressive strain is represented as ε_{cu}. Thus, ε_{cu} = 0.0035.

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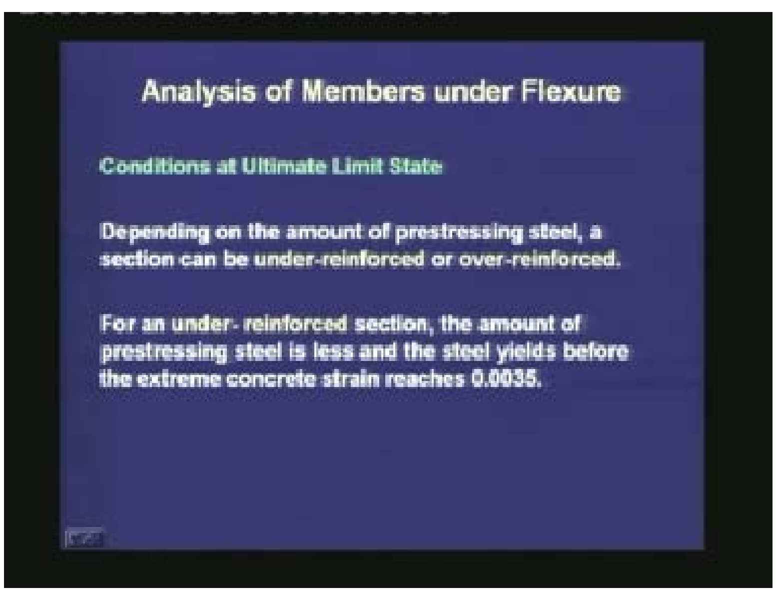

Depending on the amount of prestressing steel, a section can be under-reinforced or over-reinforced. For an under-reinforced section, the amount of prestressing steel is less and the steel yields before the extreme concrete strain reaches 0.0035. The concept is similar to that for reinforced concrete section, where an under-reinforced section means that the steel will yield before the concrete reaches its ultimate strain.

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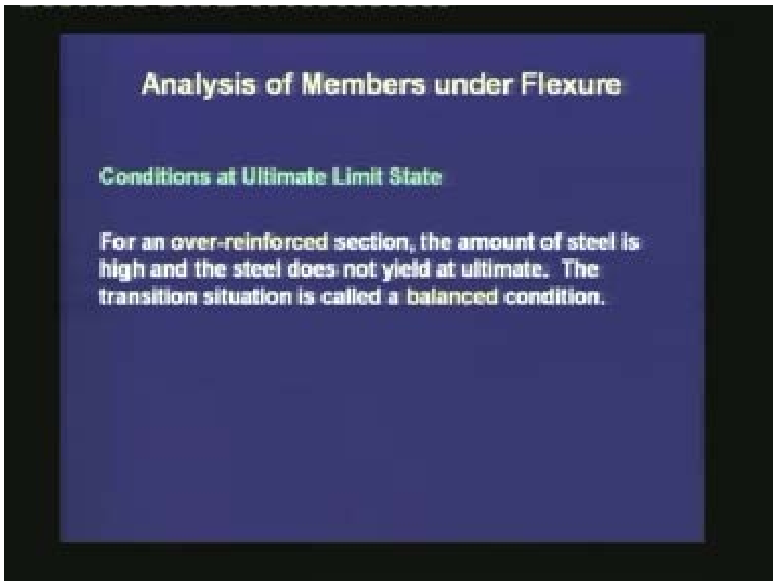

For an over-reinforced section, the amount of steel is high and the steel does not yield at ultimate. The transition situation is called a balanced condition, which we are familiar with in the analysis of reinforced concrete sections.

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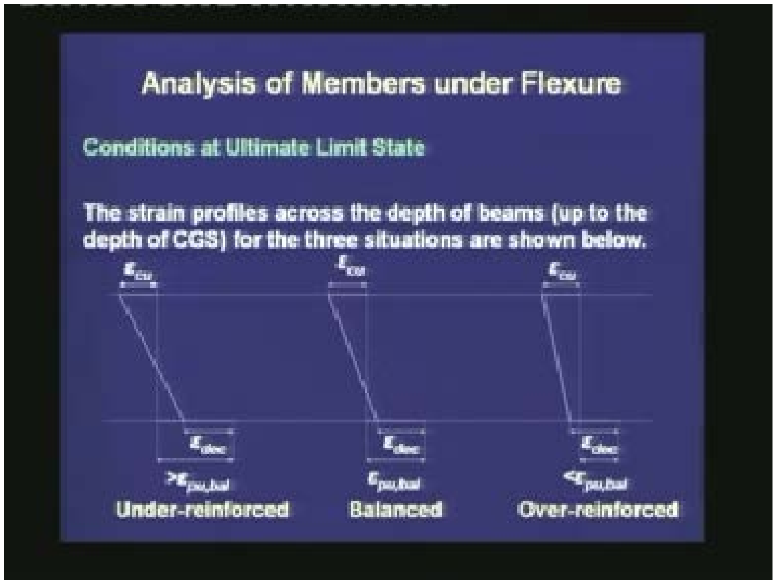

Let us try to understand the profiles of the strain across the depth of a section, at the ultimate, for the three situations. Here, we can see that the linear strain diagram comes from the assumption that the plane sections remain plane, till the failure. We have to also note the strain compatibility condition at the level of the prestressing steel. Here, there is a difference between the reinforced concrete and prestressed concrete sections. In reinforced concrete, the strain in steel is same as the strain in the concrete at the level of the steel; whereas, in prestressed concrete, there is a permanent difference between the strain in the prestressing steel and the strain in the concrete. Even at ultimate, the strain difference will be there and that is being denoted as ε_{dec} (strain at decompression). The total strain in the prestressing tendon is equal to the strain in the concrete at the level of the steel, plus ε_{dec}.

For the profile at the left, at ultimate, the strain of the extreme concrete fiber is ε_{cu} and the strain in the prestressing steel is higher than the strain for a balanced condition ($\varepsilon_{pu,bal}$). This is an under-reinforced section. On the other hand, for the profile at the right, the strain in the steel is lower than $\varepsilon_{pu,bal}$ when the extreme compressive strain in the concrete is ε_{cu}. This is an over-reinforced section. At the transition, which is a balanced section, the strain in the prestressing steel is equal to $\varepsilon_{pu,bal}$.

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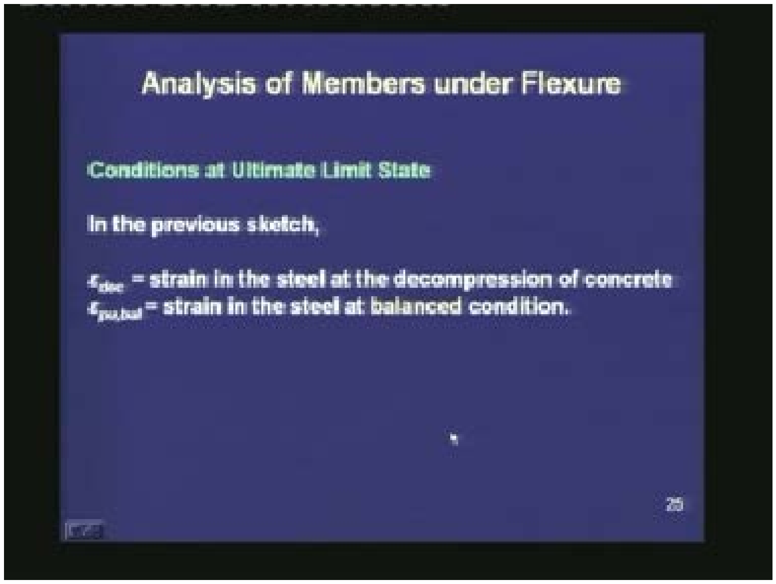

In the previous sketch, ε_{dec} is the strain in the steel at the decompression of concrete, and $\varepsilon_{pu,bal}$ is the strain in the steel at balanced condition.

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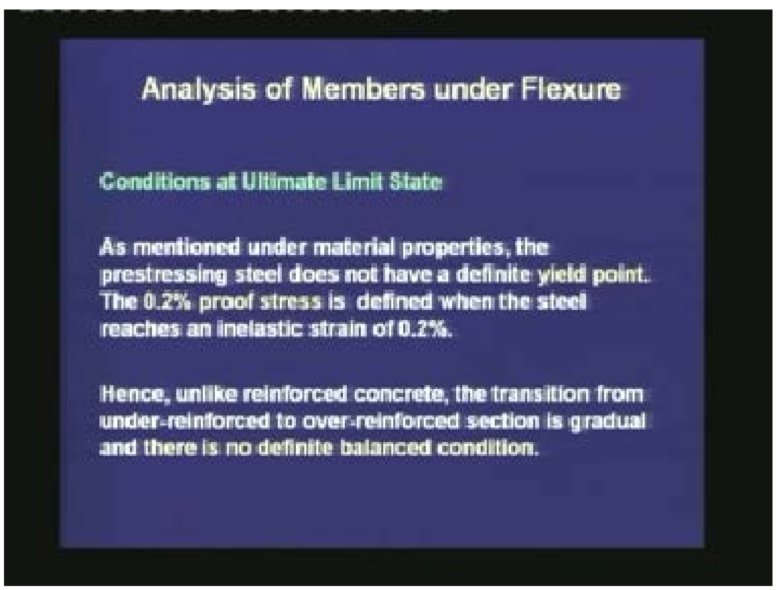

As mentioned under material properties, the prestressing steel does not have a definite yield point. Here, we are seeing a difference between the behavior of mild steel in reinforced concrete and the behaviour for prestressing steel. For prestressing steel, the

0.2% proof stress is defined when the steel reaches an inelastic strain of 0.2%. The 0.2% proof stress is like an equivalent yield point for prestressing steel.

Hence, unlike reinforced concrete with mild steel, the transition from under-reinforced to over-reinforced section in prestressed concrete is gradual, and there is no definite balanced condition.

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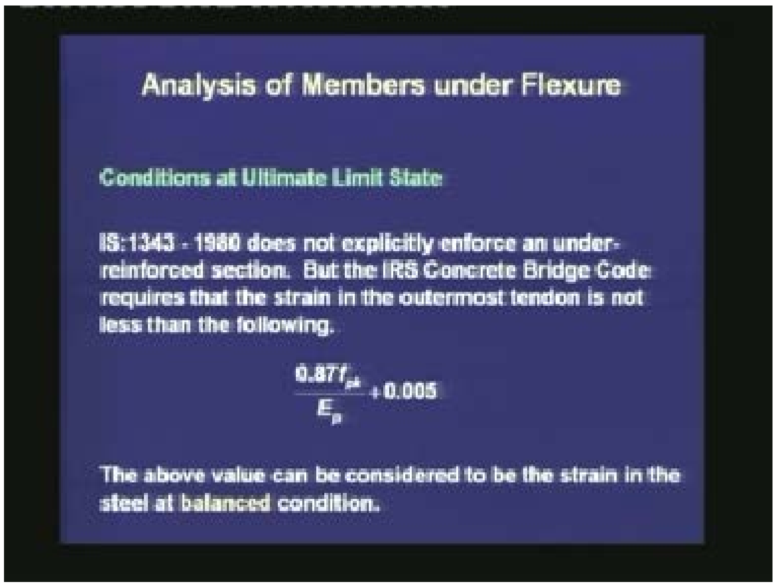

IS: 1343-1980 does not explicitly enforce an under-reinforced section. It just limits the maximum stress in the steel. But the Indian Railway Standard concrete bridge code requires that the strain in the outermost tendon is not less than $0.87f_{pk}/E_p + 0.005$. Here, f_{pk} and E_p are the characteristic strength and modulus of prestressing steel, respectively. $0.87f_{pk}/E_p$ is the elastic component and 0.005 is plastic component of the strain. The total value can be considered to be the strain in the steel at balanced condition.

Next, we are moving on to the assumptions of the analysis at ultimate strength.

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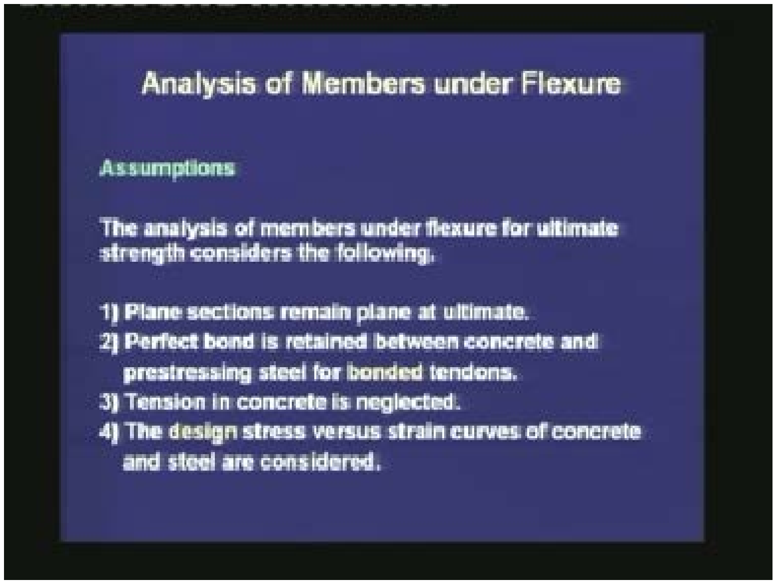

The analysis of members under flexure for ultimate strength considers the following:

First, a plane section perpendicular to the axis of the beam, remain plane at ultimate. This is the Bernoulli's hypothesis which is the basic assumption in flexural analysis.

Second, perfect bond is retained between concrete and prestressing steel for bonded tendons. That is, we assume strain compatibility between the concrete at the level of the steel and the prestressing tendon, till the failure of the member. This is not exactly true around the cracks, where there is disruption of the bond. But in an average sense, we assume that there is compatibility between the strains in the concrete and the steel.

Third, the tension in concrete is neglected. This is an assumption to simplify our calculations.

Fourth, the design stress versus strain curves of concrete and steel are considered. In service load analysis, we assume linear elastic behaviour for both the materials. We just need to know the moduli for the two materials. But in the analysis for ultimate strength, we should know the design stress-strain curves for both the concrete and the steel.

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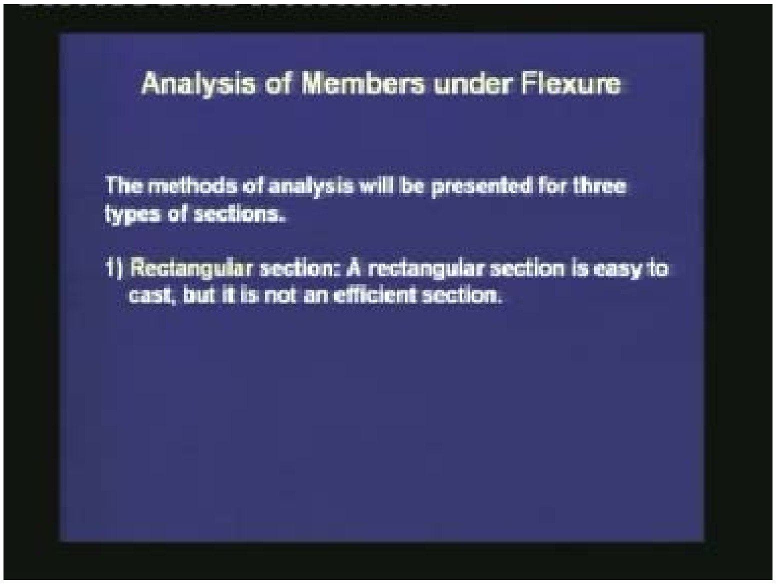

The method of analysis will be presented for three types of sections.

1) A rectangular section: A rectangular section is easy to cast, but it is not an efficient section.

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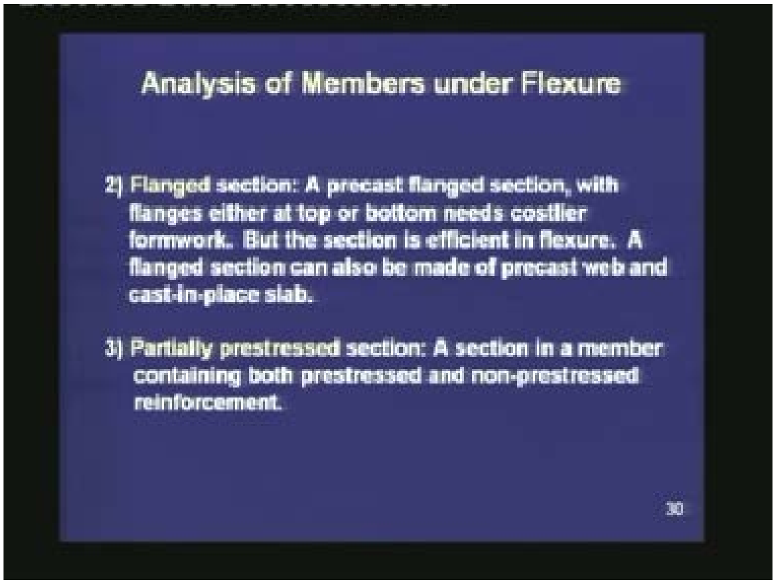

2) A flanged section: A precast flanged section, with flanges either at top or bottom needs costlier formwork. But, the section is efficient in flexure. A flanged section can also be made of precast web and cast-in-place slab

3) A partially prestressed section: A section in a member that contains both prestressed and non-prestressed reinforcement.

The comparison of different types of sections will be again elaborated when we study the design of prestressed members. Here, we shall learn about the analysis of the three types of sections.

We are moving on to the analysis of a rectangular section.

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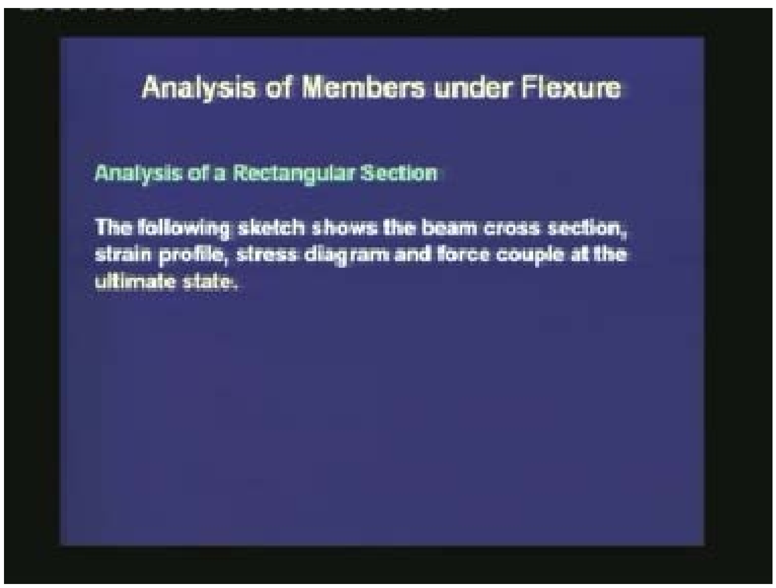

The following sketch shows the beam cross section, strain profile, stress diagram and force couple at the ultimate state.

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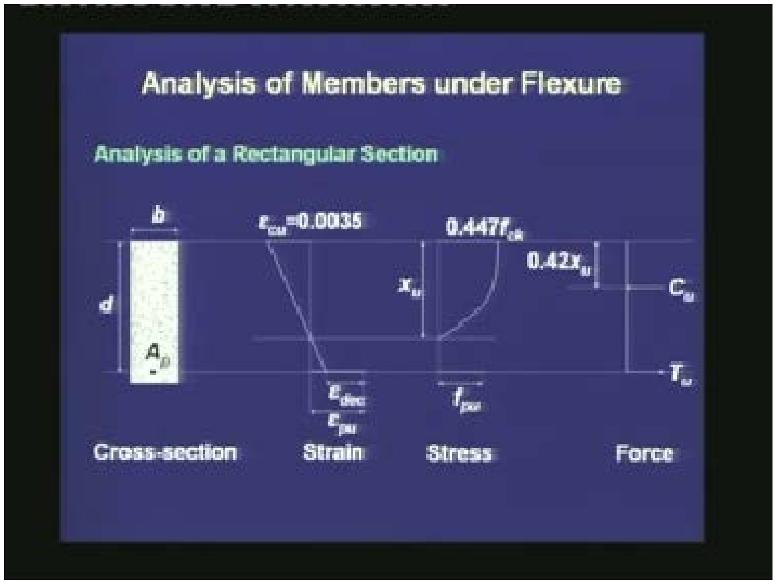

Before we start the analysis, we should understand these individual diagrams. The geometric variables in a rectangular section are the breadth of the section which is represented by b, and the depth of the CGS which is represented by d. At ultimate, we first draw the strain diagram. The strain is linear along the depth, which is from the assumption that a plane section remains plane. At failure, the ultimate compressive strain in concrete is 0.0035, and the tensile strain in the steel is equal to the tensile strain in the concrete at the level of the steel, plus the permanent strain difference which we are representing as ε_{dec}.

From the strain diagram, we move on to the stress diagram. The neutral axis is the level where the strain in the section is zero. The depth of the neutral axis at the ultimate is represented as x_u. In the stress profile, we are having a non-linear curve for concrete under compression, which we get from the design stress–strain curve. The maximum value of the stress is equal to 0.447 times the characteristic cube compressive strength (f_{ck}). The stress in the prestressing steel is denoted as f_{pu}, which corresponds to the strain of ε_{pu}. To determine f_{pu}, we need the design stress strain–curve for the prestressing steel.

From the stress diagram, we move on to the force couple. The resultant compression is represented as C_u, which acts at a depth of 0.42 times x_u from the face under compression. The resultant tension T_u is equal to the area of the prestressing steel which is denoted as A_p, times the stress f_{pu}.

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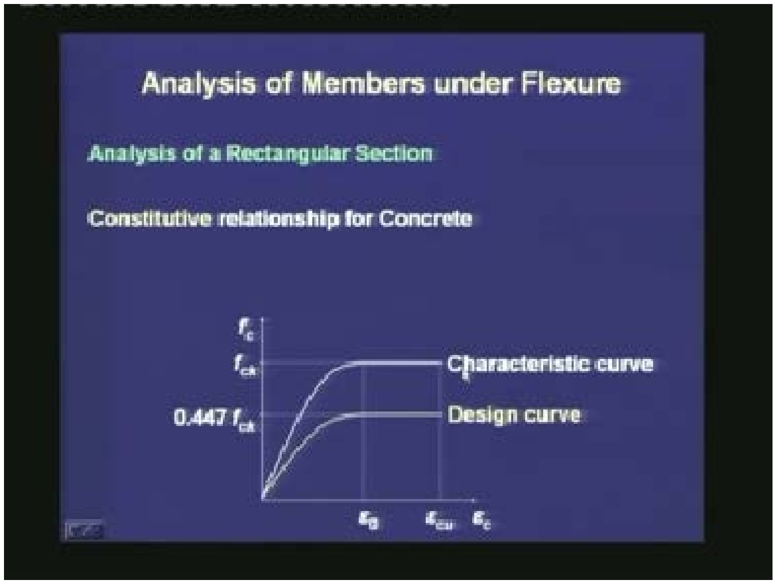

The constitutive relationship for concrete is parabolic up to a strain of ε_0, and then the stress is constant up to the ultimate strain of ε_{cu} = 0.0035. From the stress block based on the design curve, we can find out the expression of the resultant compression.

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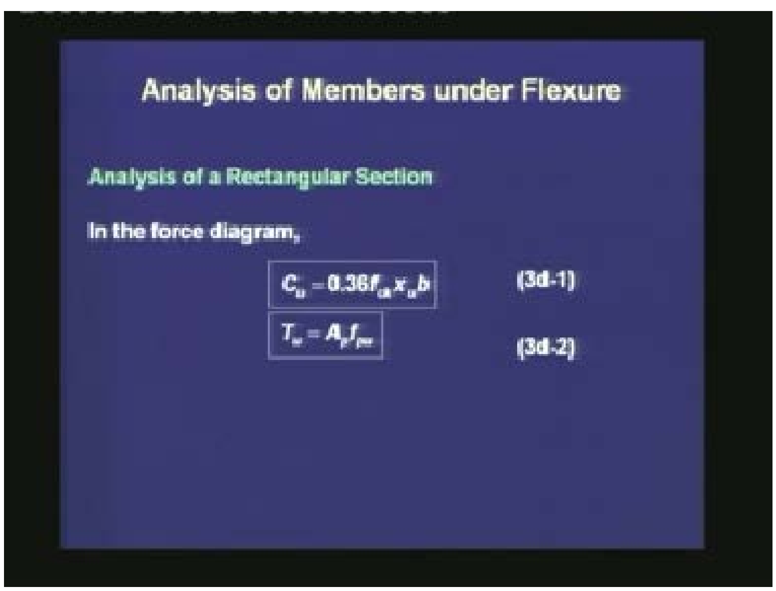

This expression is $C_u = 0.36\, f_{ck}x_ub$. Also, $T_u = A_pf_{pu}$.

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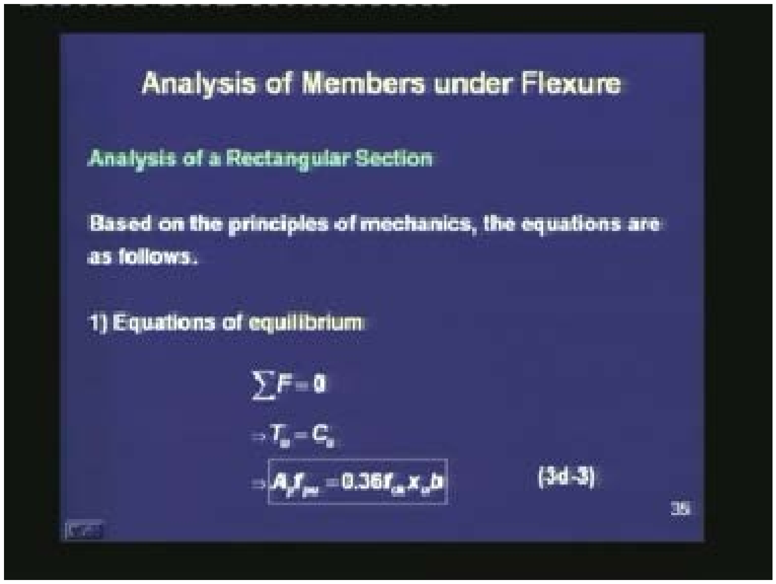

The analysis at ultimate strength is based on three principles of mechanics.

1. The first principle is based on the equilibrium of forces. The axial forces are under equilibrium, that means $T_u = C_u$ which gives, $A_pf_{pu} = 0.36f_{ck}x_ub$.

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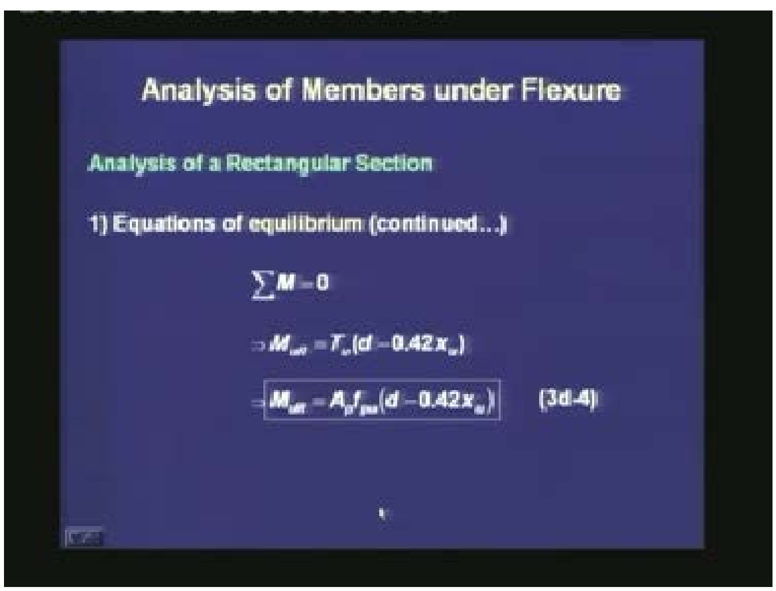

The second equilibrium equation is that the ultimate moment of resistance is equal to the tension times the lever arm. The lever arm is given as the depth of the CGS (d) minus the depth of the resultant compression (which is $0.42x_u$). Hence, the lever arm is $d - 0.42x_u$. Thus, the ultimate moment capacity is $M_{uR} = T_u (d - 0.42x_u) = A_p f_{pu} (d - 0.42x_u)$.

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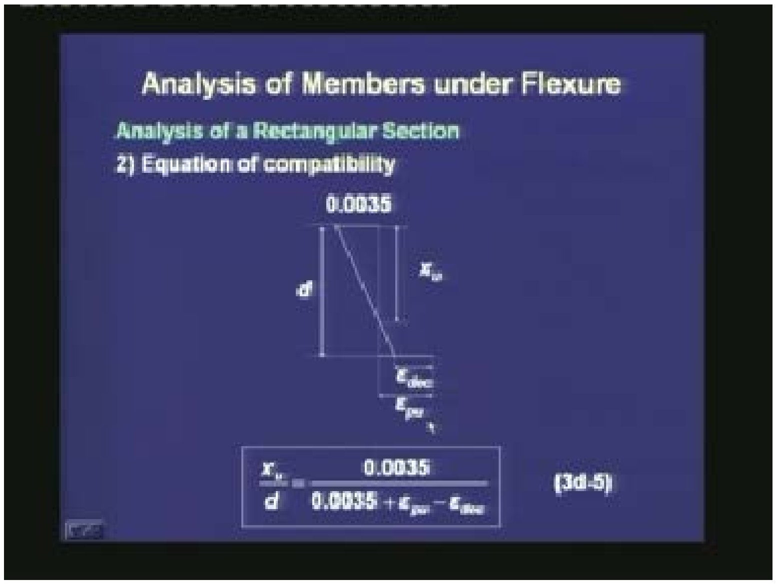

2. The second principle is based on the compatibility of strains. The equation also involves the assumption of plane sections remaining plane. We are able to relate the strain in the steel with the ultimate strain in the concrete. By the similarity of triangles, $\mathbf{x}_u/d = 0.0035/(0.0035 + \varepsilon_{pu} - \varepsilon_{dec})$. Here, $\varepsilon_{pu} - \varepsilon_{dec}$ is the strain in the concrete at the level of the prestressing steel, at the ultimate state.

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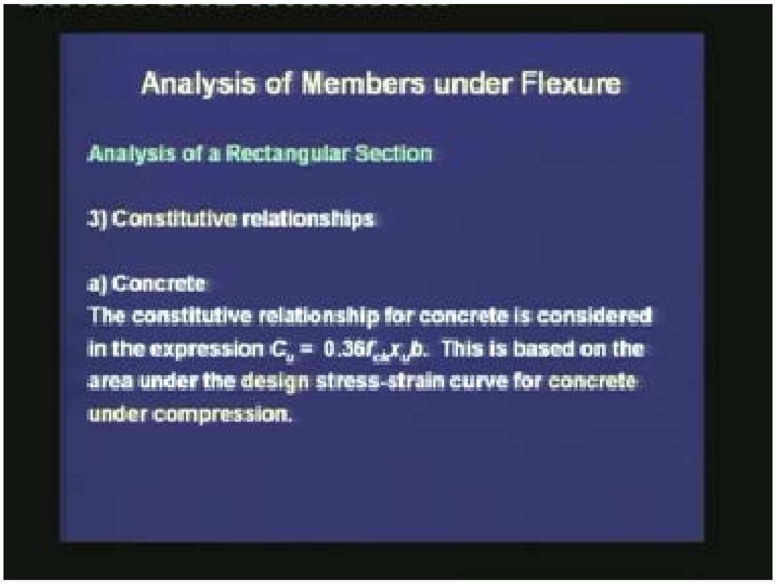

3. The third principle is based on the constitutive relationships. The constitutive relationship for concrete has already been considered in the expression of C_u. This is based on the area under the design stress–strain curve for concrete under compression.

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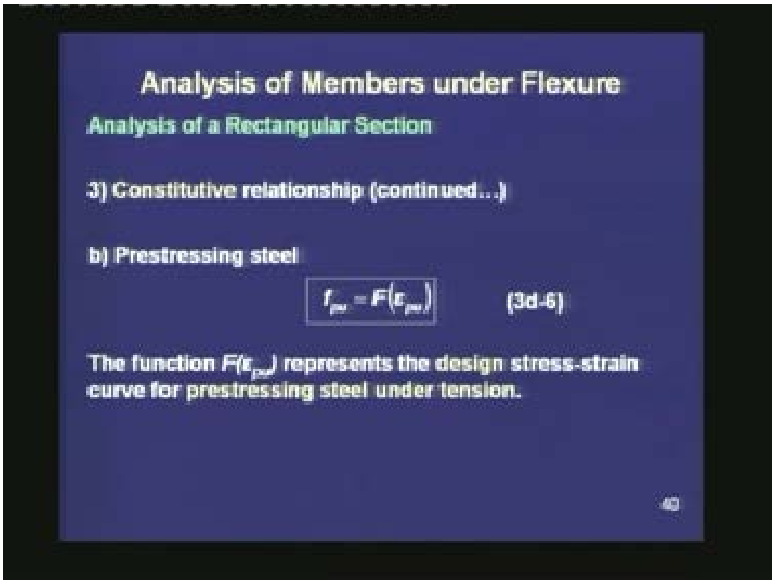

For the steel, the constitutive relationship can be expressed as $f_{pu} = F(\varepsilon_{pu})$, where $F(\varepsilon_{pu})$ represents the design stress–strain curve for prestressing steel under tension.

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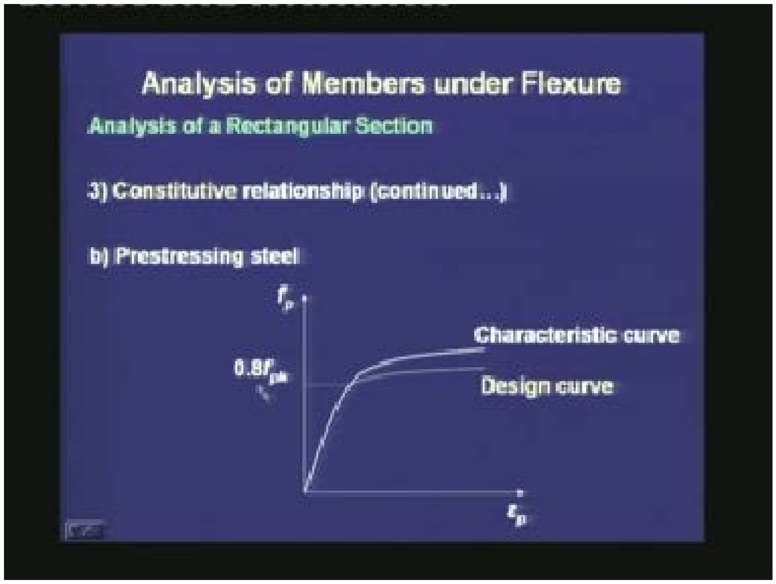

We have seen this curve earlier under material properties. Thus, given the design stress–strain curve, for any value of ε_{pu}, we can determine f_{pu}.

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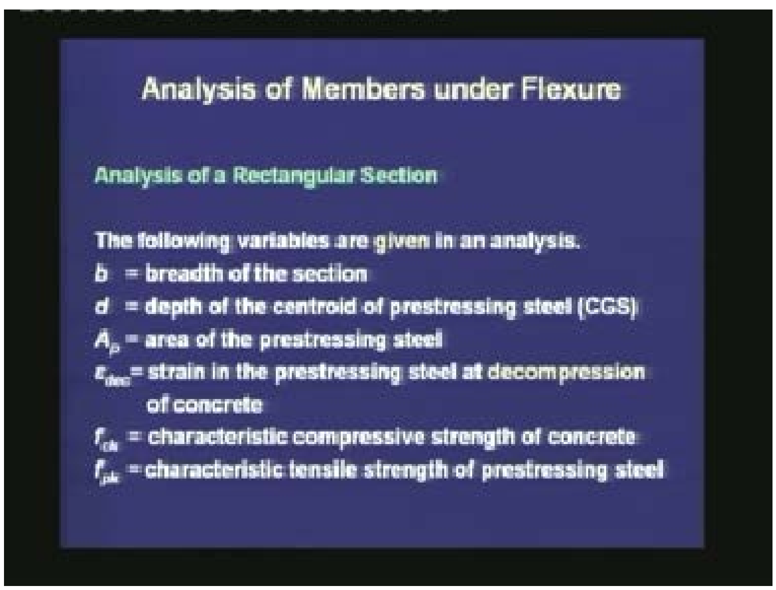

In the analysis of a rectangular section, the following variables are given. We know the section of the member, that means, we know b and d. We know the area of the prestressing steel (A_p). We also know ε_{dec}, which is the strain in the prestressing steel at decompression of concrete, or which is the strain difference between the concrete and the steel at the level of the steel. This is further explained later. We also know the strengths of the two materials, for concrete it is represented as f_{ck} and for the steel it is represented as $f_{pk.}$ What are the unknown quantities that we need to find out?

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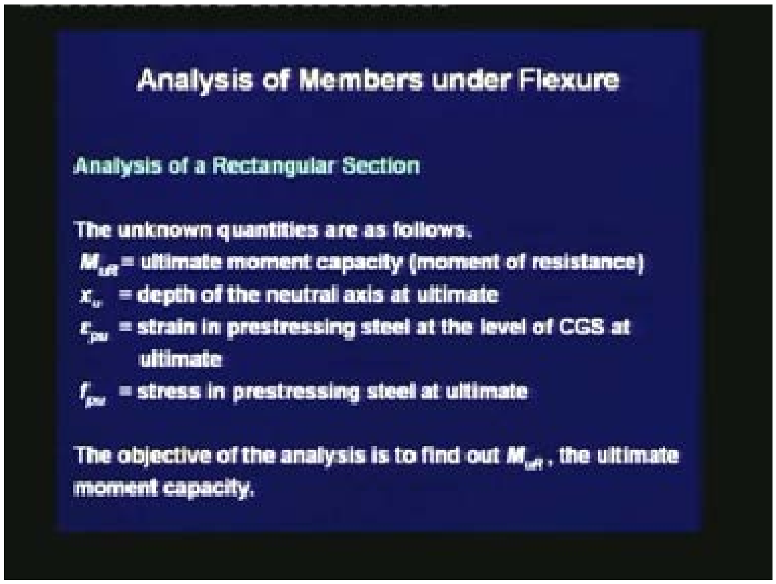

The first one, what we are looking for is M_{uR} which is the ultimate moment capacity or the ultimate moment of resistance. We have to know x_u, which is the depth of neutral axis at ultimate. We also need to know ε_{pu} and f_{pu} which are the strain and stress, respectively, in the prestressing steel at the ultimate. The objective of the analysis is to find out M_{uR} from the equations that we have in our hand.

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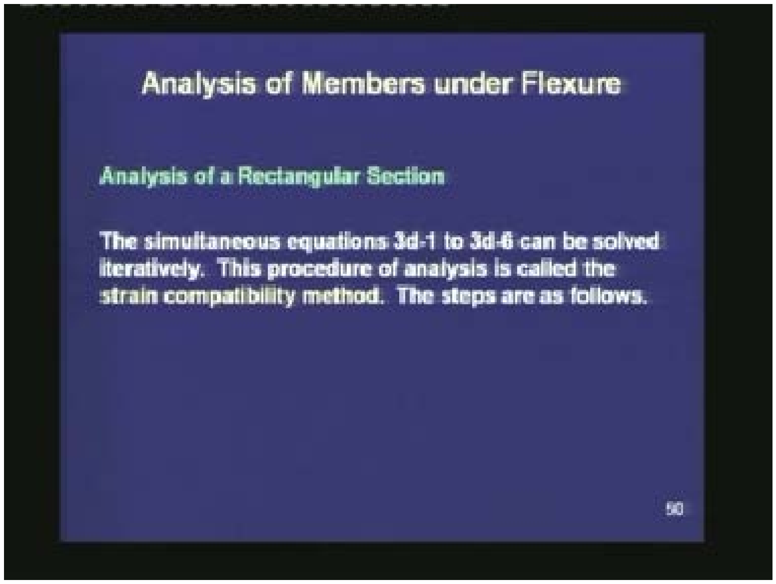

The simultaneous equations 3d-1 to 3d-6 can be solved iteratively. This procedure of analysis is called the strain compatibility analysis. The steps are as follows:

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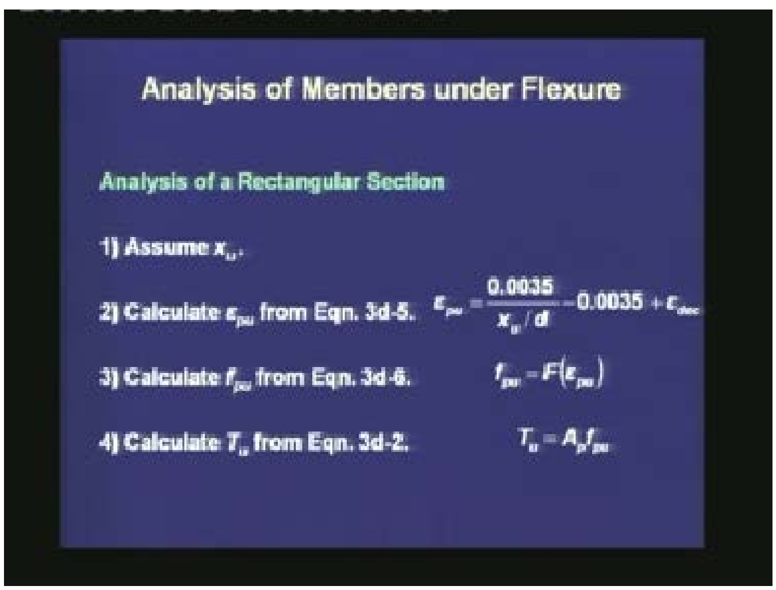

First we assume a depth of the neutral axis, which is x_u. Second we calculate ε_{pu} from Eqn. 3d-5, which is the strain compatibility equation. By rewriting the equation in a slightly different form, we can find out that $\varepsilon_{pu} = 0.0035/(x_u/d) - 0.0035 + \varepsilon_{dec}$. The third step is to calculate f_{pu} from Eqn. 3d-6, which is the design stress–strain curve for the steel. Calculate T_u from Eqn. 3d-2, where the tension is equal to the area of the prestressing tendon times the stress.

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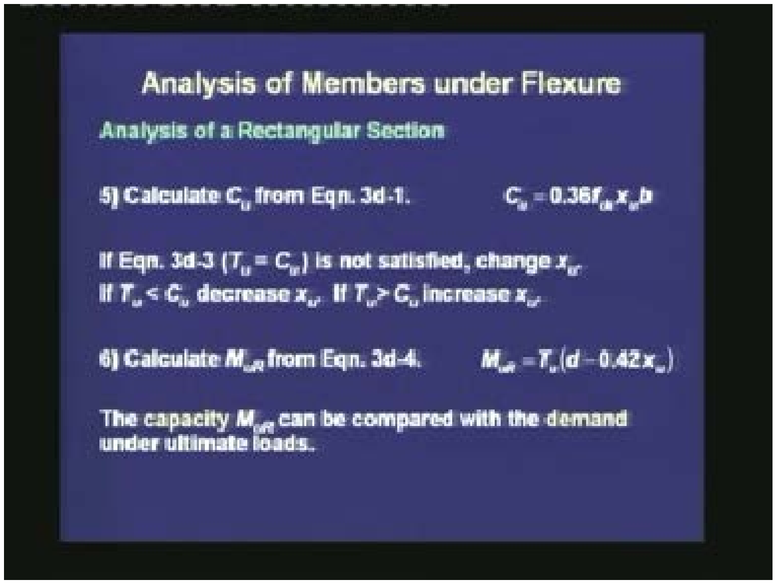

Fifth, we calculate C_u from Eqn. 3d-1, which is $C_u = 0.36f_{ck}x_ub$. Next, we check the first equilibrium equation (Eqn. 3d-3), that is whether $T_u = C_u$ as per our calculations.

If it is not satisfied, then we have to change x_u. If the calculated T_u is less than C_u, then decrease x_u to reduce compression. If the calculated T_u comes out to be greater than C_u, then increase x_u to increase compression. We do this iteration process till the values of T_u and C_u are reasonably close. Once we have converged to this condition, then we calculate M_{uR} from Eqn. 3d-4, which is $M_{uR} = T_u\ (d - 0.42x_u)$. The capacity M_{uR} is then compared with the demand under ultimate loads.

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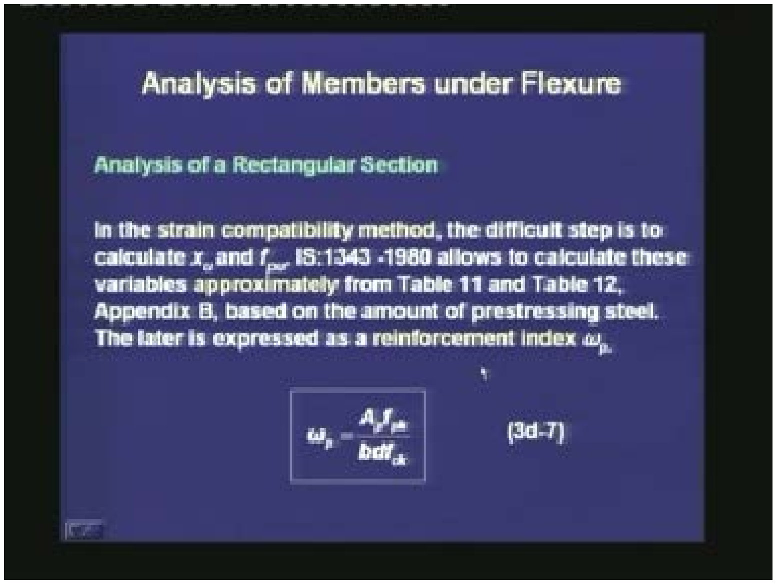

In the strain compatibility method, the difficult step is to calculate x_u and f_{pu} which we have to do in an iteration process. The code IS: 1343-1980 allows to calculate these values approximately from Table 11 and Table 12, Appendix B, based on the amount of prestressing steel. The later is expressed as a reinforcement index $\omega_p = A_p f_{pk}/bdf_{ck}$. That means, the code gives us a simple way to calculate approximate values of the depth of the neutral axis and the stress in the prestressing steel at ultimate.

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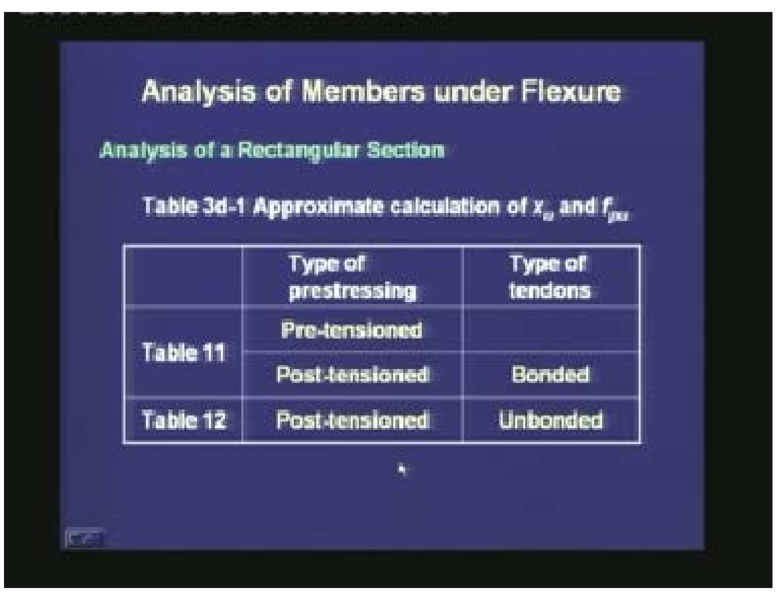

Table 11 gives the values for pre-tensioned members, and post-tensioned members with bonded tendons. Table 12 gives the values for post-tensioned members with unbonded tendons. As I had said earlier, that the analysis of pre-tensioned members and post-tensioned members with bonded tendons are similar because, we assume strain compatibility for both of them. Whereas, the analysis for post-tensioned members with unbonded tendons is different, because there we cannot assume strain compatibility at a section. We assume a compatibility of the overall deformation of the member.

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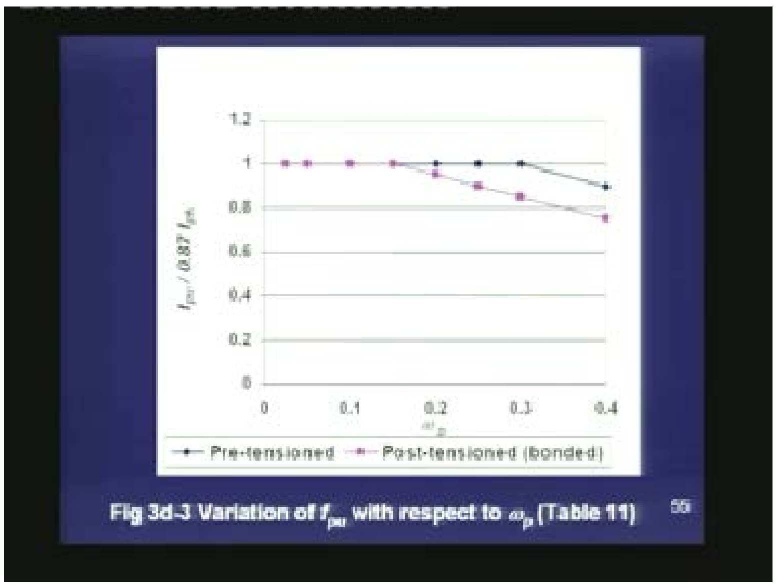

To understand the tabulated values in Table 11, this plot shows the values of f_{pu} (normalized with respect to the maximum allowed stress $0.87f_{pk}$) with respect to ω_p. As ω_p increases, we observe that beyond a certain value f_{pu} reduces. If we put more steel, beyond a certain point the stress in the steel will go down, which is evident. On the other hand, if we reduce the amount of steel, then the stress in steel goes up till it reaches the maximum value which is allowed by the code. The code does not allow the design stress in the steel to go beyond $0.87f_{pk.}$

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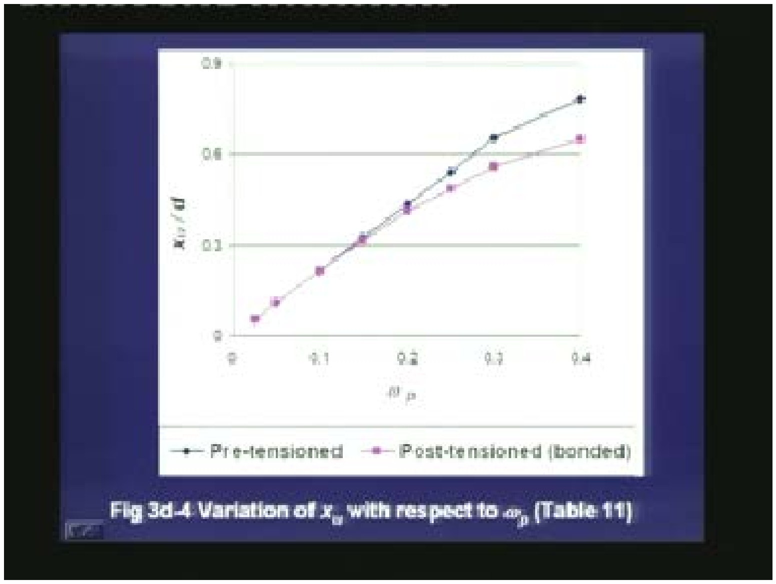

Fig 3d-4 Variation of x_u with respect to ω_p (Table 11)

On the other hand, if we try to understand the variation of the depth of the neutral axis (x_u), then as the amount of steel (or ω_p) is increased, x_u also increases. If we have more steel, we need more concrete to balance the force in the steel. Hence, the depth of the neutral axis increases with the increasing amount of the reinforcement. Thus, the values of f_{pu} and x_u given in the tables follow our intuition.

Next, the analysis of a rectangular section is demonstrated with the help of an example.

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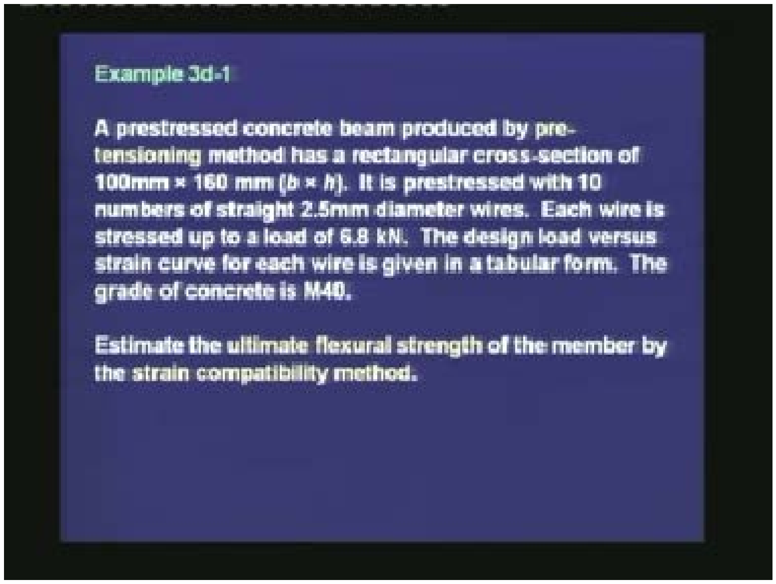

A prestressed concrete beam produced by pre-tensioning method has a rectangular cross section of 100 mm × 160 mm, which is breadth times the total depth. It is prestressed with 10 numbers of straight 2.5 mm diameter wires. Each wire is stressed up to a load of 6.8 kN. The design load versus strain curve for each wire is given in a tabular form. The grade of concrete is M40. Estimate the ultimate flexural strength of the member by the strain compatibility method.

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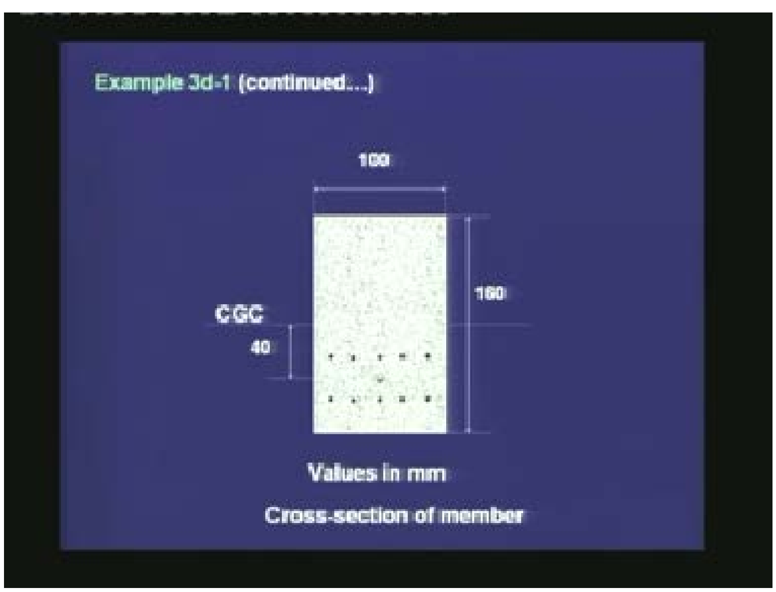

The depth of the CGS is 40 mm below the CGC. The design load versus strain curve for each wire is given for the range under consideration. This is different from the stress–strain curve because, the load versus strain curve is readily available from the tests, and it can be used to calculate the ultimate strength of a wire.

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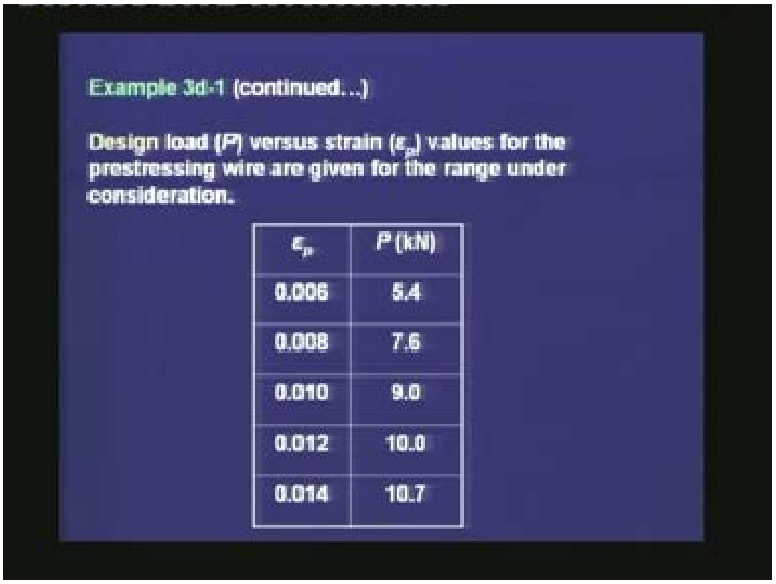

ε_p	P (kN)
0.006	5.4
0.008	7.6
0.010	9.0
0.012	10.0
0.014	10.7

For a strain of 0.006, the load in a wire is 5.4 kN. Similarly, for a strain of 0.014, the load is 10.7 kN. The values outside this range are not required.

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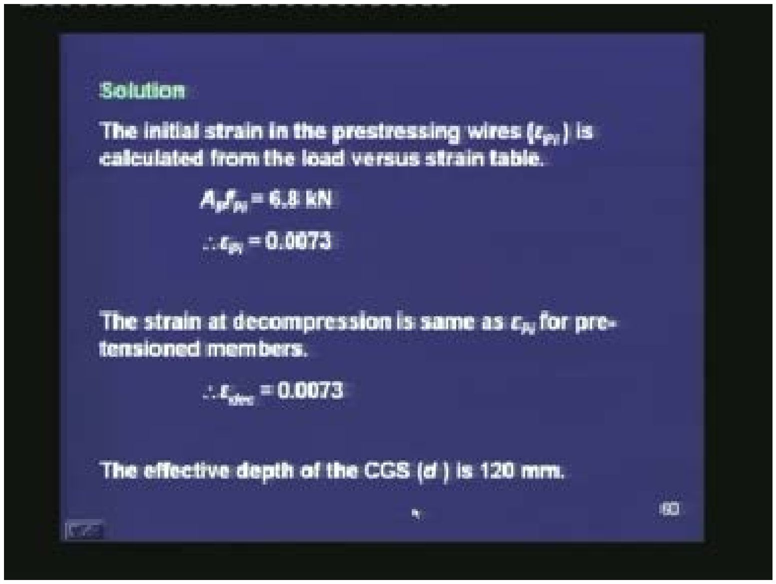

First, we are calculating the initial strain in the prestressing wires, ε_{pi} to calculate the strain at decompression of the concrete. Since, we know the initial load in each wire, we can calculate the strain from the given load versus strain table, and that strain is 0.0073. Since, for a pre-tensioned member the strain at decompression is same as ε_{pi}, we can write ε_{dec} = 0.0073. From the section, we also know the depth of the CGS which is equal to 120 mm.

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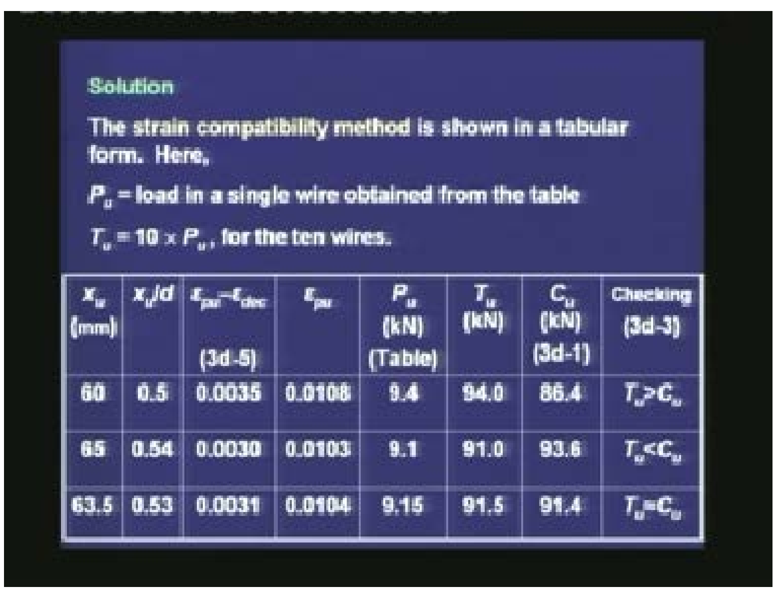

We are using the strain compatibility method to solve this problem. The strain compatibility method is an iterative procedure where, we start by assuming a depth of the neutral axis (x_u). This iterative procedure can be nicely written down in a tabular form to have the results in a compact way. In this table, we are first writing the assumed value of x_u. We are writing the normalized value x_u/d. From the compatibility equation, we can find out the strain in the concrete at the level of the steel, which is $\varepsilon_{pu} - \varepsilon_{dec}$. Once we know this, we can calculate the strain in the steel because ε_{dec} has been calculated. Then, from the strain we are calculating the load based on the given table.

Once we know the load in single wire, we can find the total tensile force in the tendons. It is 10 times the load in a single wire, since there are 10 wires. We can also calculate the compression based on the assumed value of x_u. Then we are checking the first equilibrium equation, whether $T_u = C_u$?

If the starting value of x_u is 60 mm, which is half of the effective depth ($x_u/d = 0.5$), T_u = 94 kN and C_u = 86.4 kN. For this step, $T_u > C_u$. Hence, we need to have a larger x_u to give more compression to the concrete. We are increasing x_u to 65 mm, for which x_u/d = 0.54. After going through the steps, T_u has dropped from 94 to 91 kN, whereas, C_u has increased from 86.4 to 93.6 kN. For this step, $T_u < C_u$. At this stage we should reduce x_u to give more tension in the tendons. After a few iterations, for x_u = 63.5 mm, T_u = 91.5 kN and C_u = 91.4 kN, which are reasonably close. Thus, x_u and the forces have converged to the final values.

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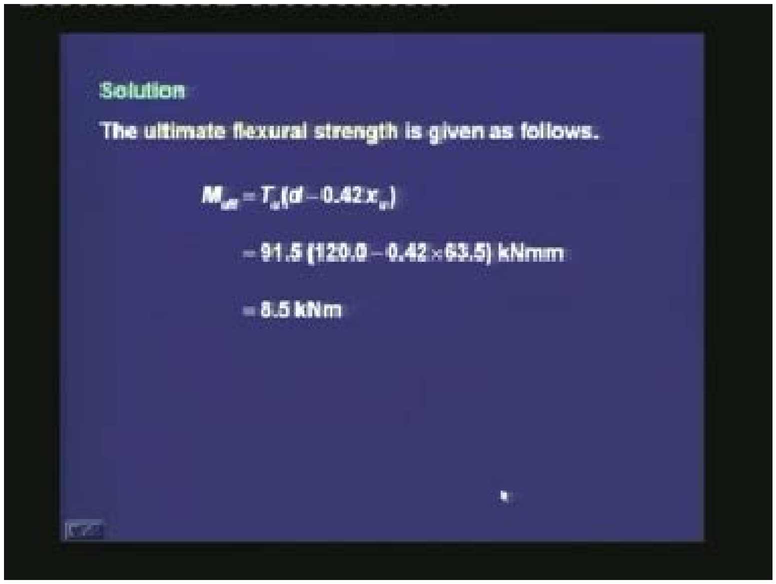

The ultimate flexural strength is given as the tension or the compression times the lever arm. Once we substitute the variables, M_{uR} = 8.5 kNm.

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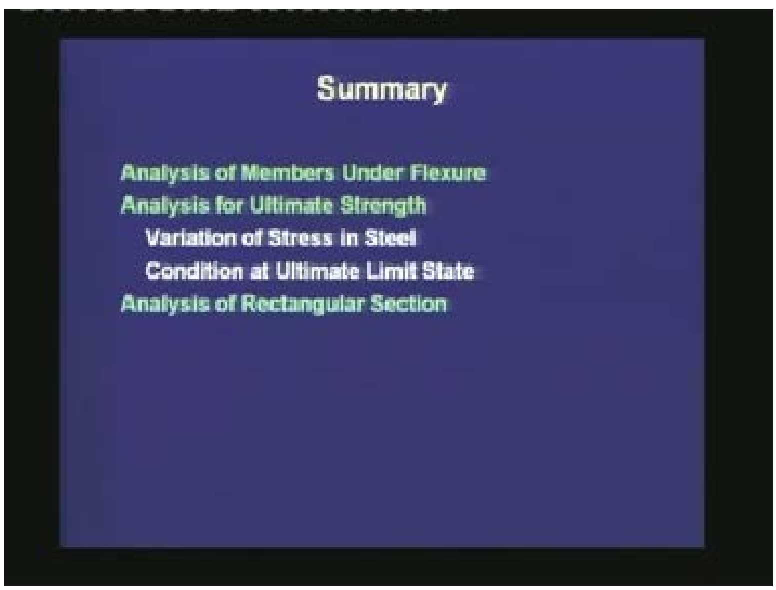

Thus in today’s lecture, we studied the analysis of members for ultimate strength. First, we understood the variation of stress in the steel. In our analysis for service loads, we did not consider any variation in the stress in the steel. We said that the stress is constant at the effective prestress $f_{pe.}$ Today, what we observed is that before cracking the increase in stress with increasing load is very small and hence, we can neglect the increase in stress. It is only after cracking that the stress in steel increases suddenly. As the ultimate strength is approached, the stress will increase quite rapidly.

We learned the condition to define the ultimate limit state of a member under flexure. As per the code, it is defined based on the compressive strain in the extreme concrete fiber to be 0.0035. During that, if the steel has substantially yielded then it is called an under-reinforced section. But if the steel has not yielded when the concrete has reached its ultimate strain, then it is called an over-reinforced section. For a prestressed member, there is no definite balanced point, because the steel itself does not have a true yield point. We may define a balanced condition by considering a particular value of strain which the steel should have, when the concrete attains the strain of 0.0035.

We learned the analysis of rectangular sections. We first draw the strain diagram, where we assign a strain of 0.0035 for the extreme concrete fiber under compression. From the strain diagram, we draw the stress diagram based on the constitutive relationships. For concrete, the stress–strain behaviour is non-linear. For the steel also, the behaviour is non-linear at ultimate. From the stress diagram, we get the force couple.

There are three principles of mechanics. The first is the equilibrium of forces. The first equation is the equilibrium of the tension and the compression. The second is the equilibrium of the moment, from which we have an expression of the ultimate moment capacity. The second principle gives the compatibility equation. The third principle is related to the constitutive relationships. The set of equations can be solved iteratively by a strain compatibility method, where we start with an assumed depth of the neutral axis. If we want to bypass the strain compatibility method, we can use the code tabulated values of the depth of neutral axis, and the stress in the steel. From these we can evaluate the ultimate moment of resistance for a rectangular section.

In the next lecture, we shall study flanged sections.

Thank you.