PRESTRESSED CONCRETE STRUCTURES

Amlan K. Sengupta, PhD PE

Department of Civil Engineering

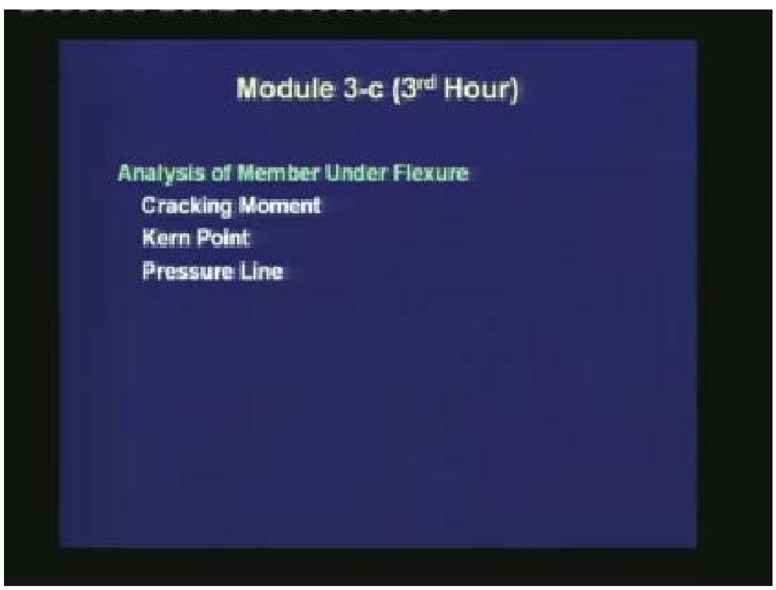
Indian Institute of Technology Madras

Module – 3: Analysis of Members

Lecture – 13: Cracking Moment, Kern Point and Pressure Line

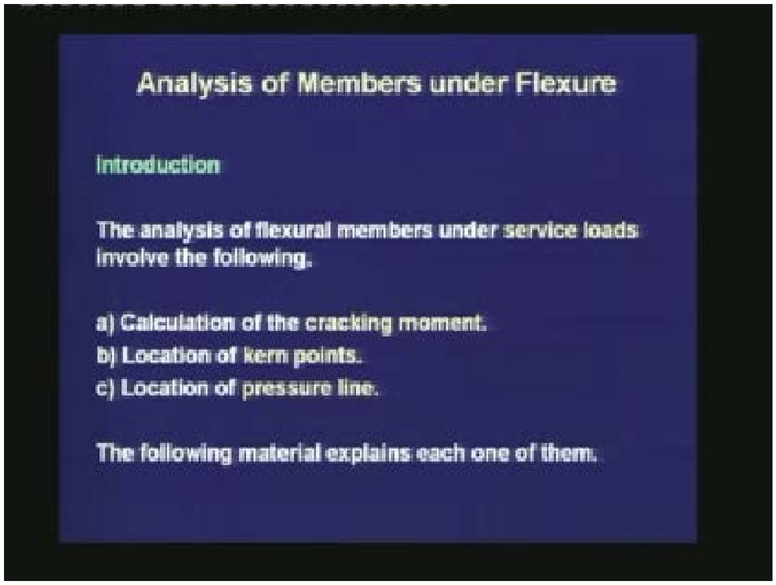
Welcome back to prestressed concrete structures. This is the third lecture of module three on analysis of members.

(Refer Slide Time: 01:18)



In today's lecture, we shall study the analysis of members under flexure for service condition, and we shall see some particular properties for the analysis. First, we shall learn about the cracking moment. Next, we shall learn about the kern point and kern zones. Then, we shall learn about the pressure line.

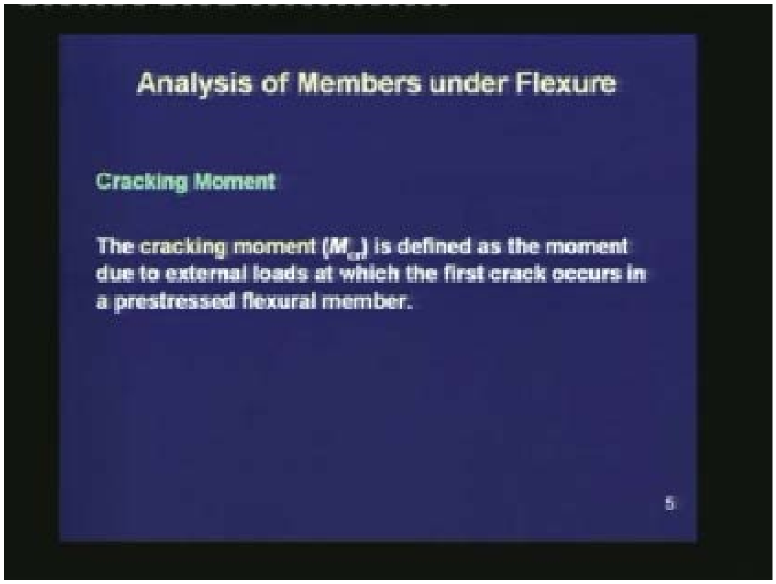
(Refer Slide Time: 01:47)



The analysis of flexural members under service loads, involve the following: First, is the calculation of the cracking moment; next, the location of the kern points and then, the location of pressure line. That means, these three particular quantities come within the analysis of the members under service loads. Now, we shall study each one of them individually.

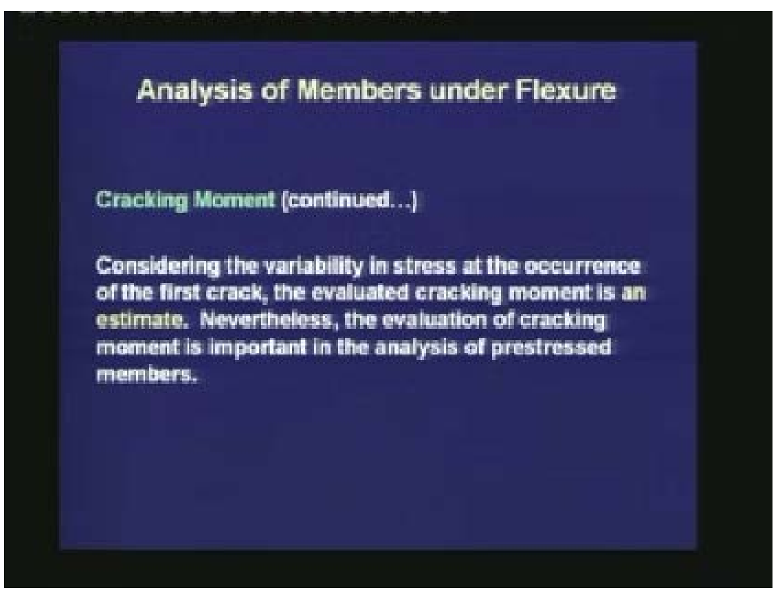
First, cracking moment.

(Refer Slide Time: 02:24)



The cracking moment is defined as the moment due to the external loads, at which the first crack occurs in a prestressed flexural member. Again to repeat, it is the moment corresponding to the first crack; after that there will be more cracking. Considering the variability in stress at the occurrence of the first crack, the evaluated cracking moment is just an estimate.

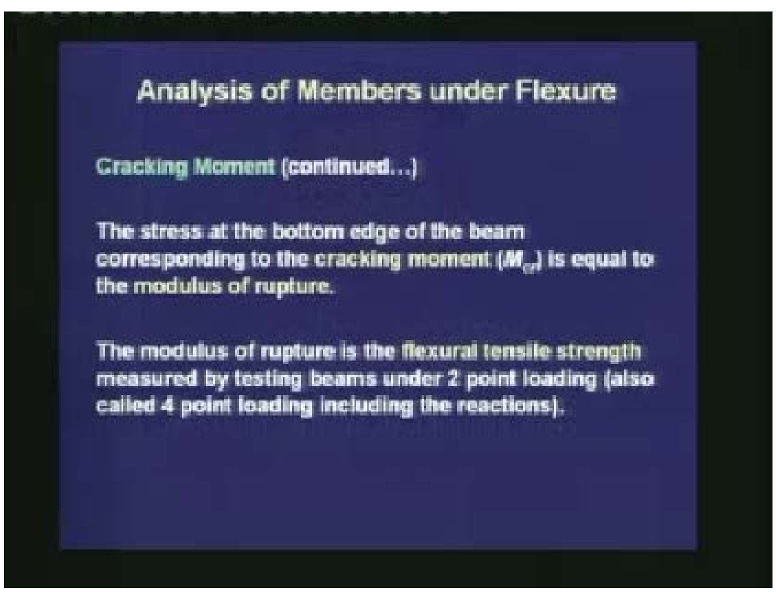
(Refer Slide Time: 03:03)



Nevertheless, the evaluation of cracking moment is important in the analysis of prestressed concrete members. We have to be aware that concrete inherently shows variations in its properties, especially so, for the cracking stress. Hence, the cracking moment that we evaluate is an estimate. It may not be the exact value when we test a beam under the testing machine. But still, the evaluation of the cracking moment helps us to check the properties of the member under study.

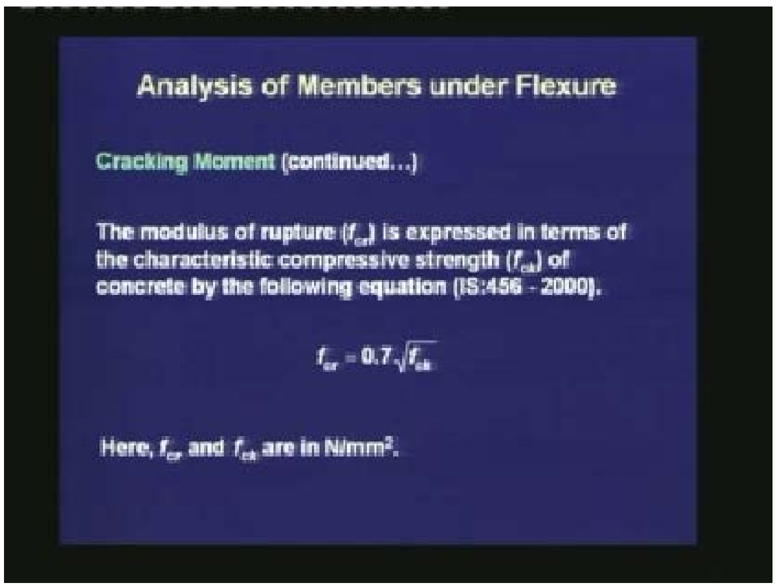
For Type 1 and Type 2 prestressing members, cracking is not allowed under service loads. Type 1 prestressed members are considered to be fully prestressed, where no tensile stress is allowed under service loads. Type 2 members are called limited prestressed members where tensile stress is allowed, but cracking is not allowed under service loads. Hence, it is imperative to check that the cracking moment is greater than the moment due to service loads for these members. One purpose of calculating the cracking moment is that once we have an estimate of the cracking moment, we can compare it with the moment due to the service loads. If we find that the moment due to the service loads is less than the cracking moment, then we can expect that the members will not crack under the service loads.

(Refer Slide Time: 04:59)



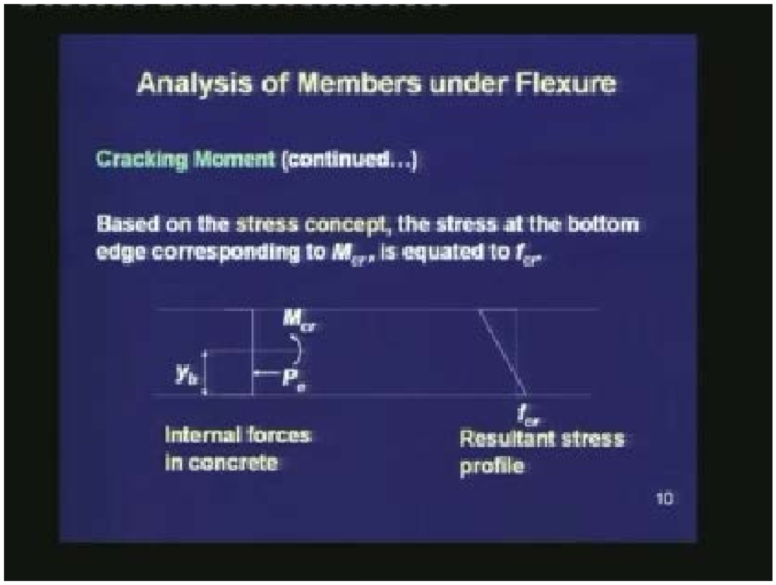
The stress at the bottom edge of the beam corresponding to the cracking moment (which is denoted as M_{cr}) is equal to the modulus of rupture. The modulus of rupture is the flexural tensile strength measured by testing plain concrete beams under 2-point loading, which is also called 4-point loading including the reactions. Earlier, in the module of material properties, we had studied about the modulus of rupture, which is a measure of the tensile strength of concrete. This value corresponds to the stress, when the cracking moment occurs in a particular member.

(Refer Slide Time: 05:59)



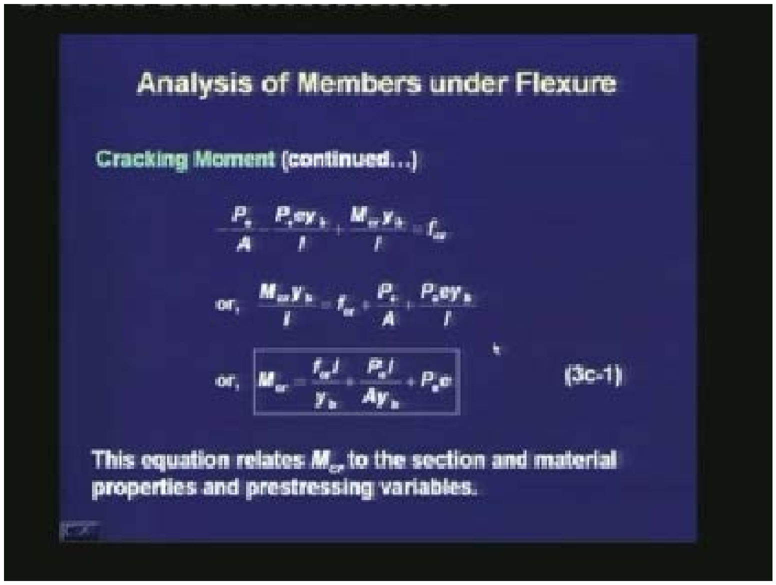
Hence, before calculating the cracking moment, we need to estimate the modulus of rupture. The modulus of rupture (which is denoted as f_{cr}) is expressed in terms of the characteristic compressive strength of concrete (which is denoted as f_{ck}) by the following equation as per IS: 456 – 2000. The equation is $f_{cr} = 0.7 \sqrt{f_{ck}}$. Here, both f_{cr} and f_{ck} are in N/mm^2. If we know the characteristic strength of the concrete, we can estimate the modulus of rupture by this simple expression. Next, when we know the modulus of rupture, we are able to estimate the cracking moment for a particular beam.

(Refer Slide Time: 07:02)



Based on the stress concept, the stress at the bottom edge corresponding to M_{cr} is equal to the modulus of rupture f_{cr}. That means, to estimate M_{cr}, we are going back to the stress concept of analysis, and we are equating the stress at the bottom to be tensile, with the value of f_{cr}. On the left hand side, we see the effective prestress occurring at a certain eccentricity. M_{cr} is the moment due to the external load, which also includes its self-weight. Corresponding to the occurrence of M_{cr} in the resultant stress profile, the stress at the bottom is tensile with a value of f_{cr}. This is the state of stress along the depth of the section, when the moment due to the external load is equal to M_{cr}.

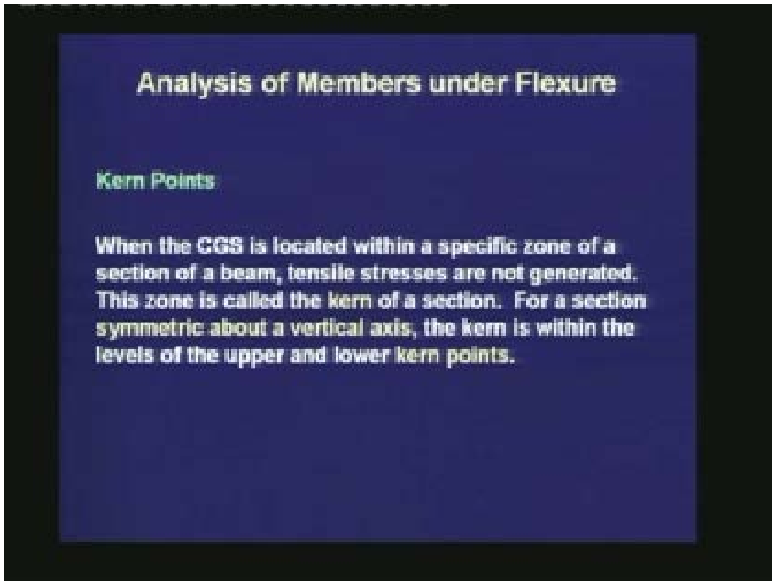
(Refer Slide Time: 08:13)



From the expression that we have seen under the analysis based on stress concept, we are writing that the total stress is composed of the uniform compressive stress, then the compressive stress due to the prestressing force with an eccentricity e, and next the tensile stress, which is from the cracking moment. From all these terms, we are finding out the stress at the bottom of the beam and we are equating that to the modulus of rupture f_{cr}.

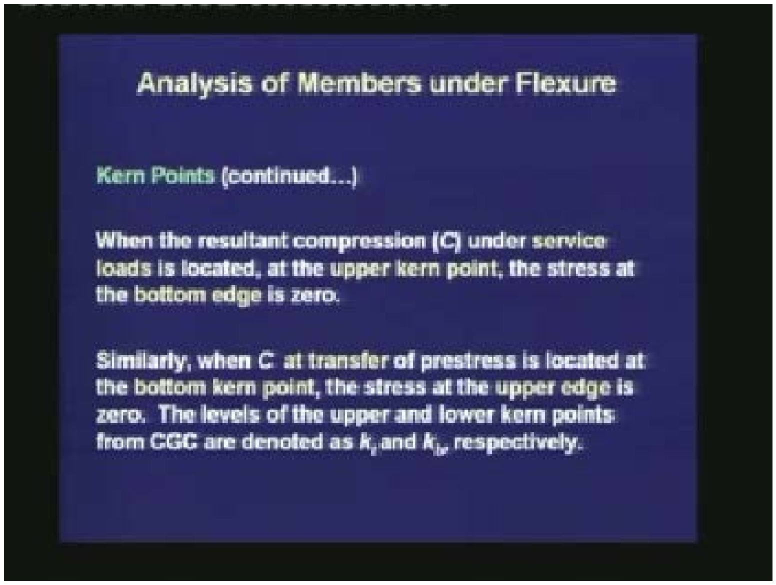
After that we are transposing the term with M_{cr} on the left side and rest of the terms on the right side. Then we get an expression of M_{cr},, which is equal to $f_{cr}I/y_b + P_eI/Ay_b + P_ee$. Thus, we have an analytical expression which relates M_{cr} to the section and material properties, and the prestressing variables. We understand that M_{cr} depends not only on the section properties and the material properties, but it also depends on the amount of prestressing force.

(Refer Slide Time: 10:16)



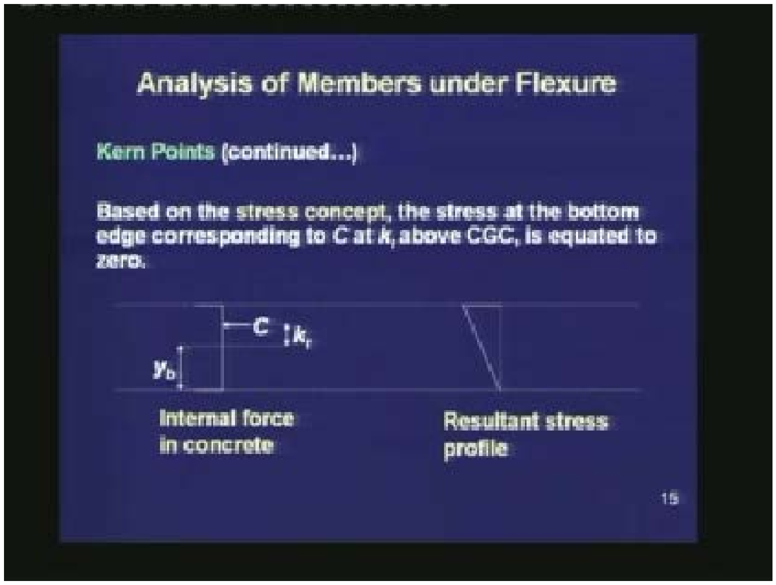
We move on to the study of kern points. When the resultant of the compression is located within a specific zone of a beam, tensile stresses are not generated. This zone is called the kern of a section. For a section symmetric about a vertical axis, the kern is within the levels of upper and lower kern points. The kern zone is limited within two levels: one is called the top kern level, and the other is called the bottom kern level. Hence, to calculate the kern zone, we need to find out the kern levels, also called the kern points.

(Refer Slide Time: 11:31)



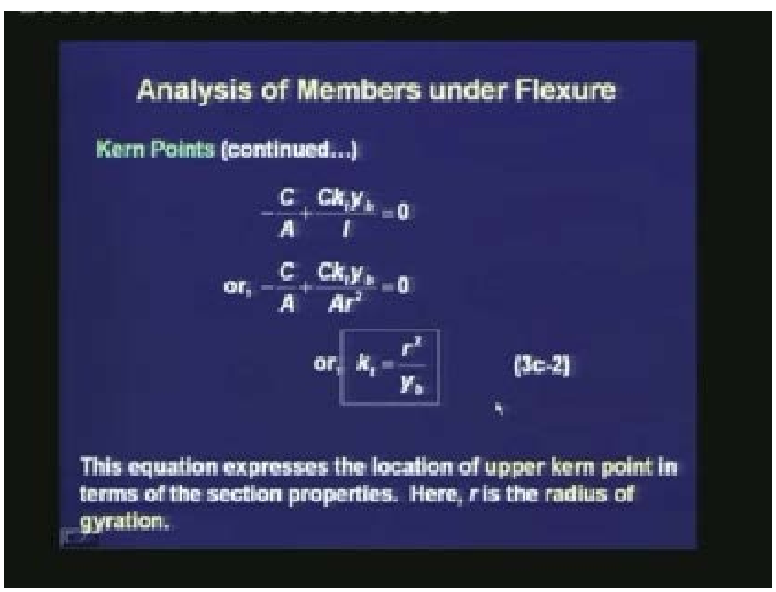
How do we define the kern points? The condition is that when the resultant compression in the concrete (C) occurs at the upper kern point under service loads, then the bottom edge will have a zero stress; this is the condition of the upper kern point. Similarly, when C at transfer of prestress is located at the bottom kern point, the stress at the upper edge is zero. The levels of the upper and lower kern points from the CGC are denoted as k_t and k_b, respectively.

(Refer Slide Time: 12:58)



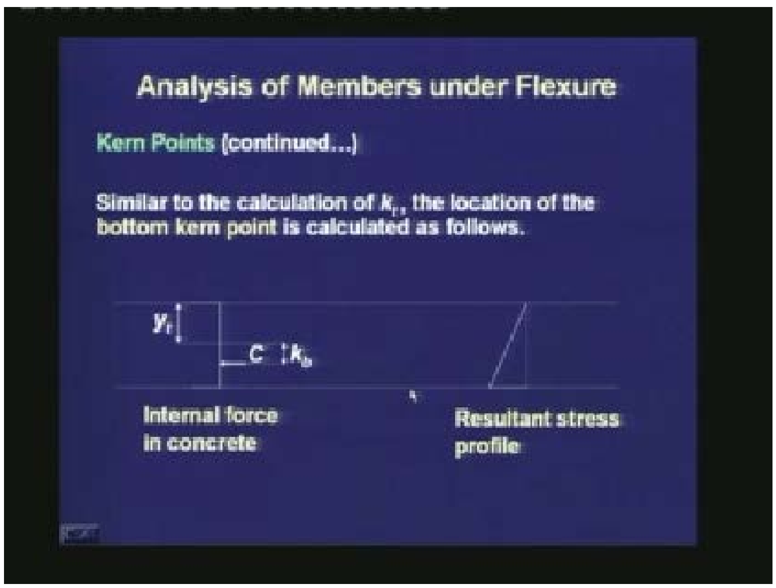
Based on the stress concept, the stress at the bottom edge corresponding to C at k_t above CGC is equated to zero. This is the way we are calculating the kern point or the kern level. When C acts at the upper kern level, then we have a zero stress at the bottom. On the right hand side, we see the resultant stress profile when C acts at the upper kern point.

(Refer Slide Time: 13:41)



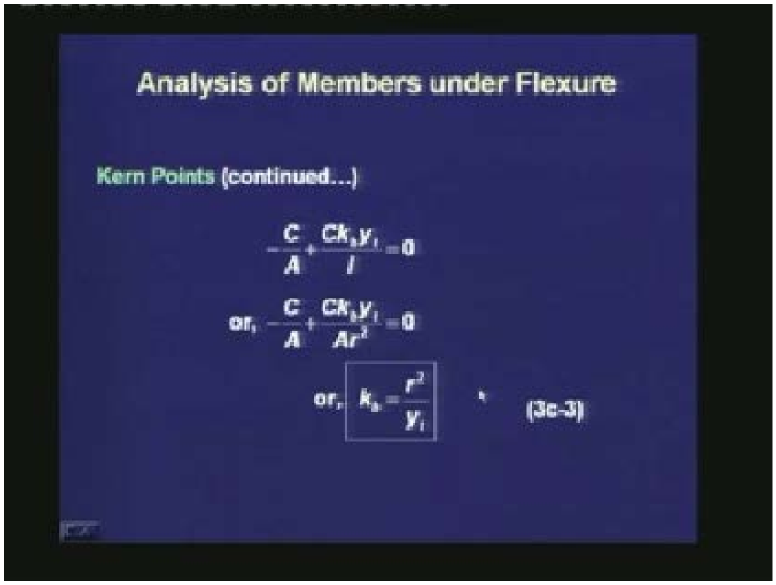
If we write the expression of the stress at the bottom corresponding to this location of C we find that, the first term is the uniform compression caused by C and the second term is due to the eccentricity of C from the CGC. Here, Ck_t is the moment due to C, and y_b is the distance of the bottom edge from the CGC. The resultant stress is equal to zero. We are substituting $I = Ar^2$, where r is the radius of gyration. Once we substitute that and we transpose the terms, we can find out an expression of the upper kern point. $k_t = r^2/y_b$. Thus, this equation expresses the location of upper kern point in terms of the section properties.

(Refer Slide Time: 15:07)



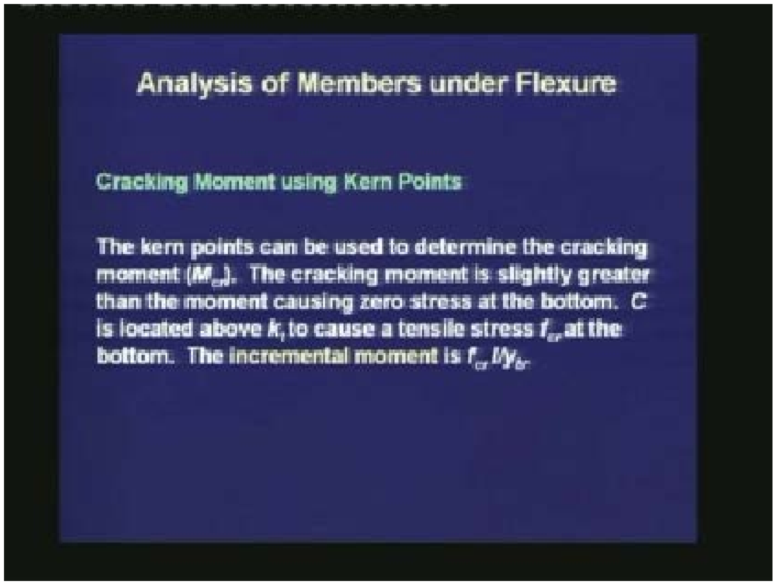
Similar to the location of k_t, the location of the bottom kern point is calculated as follows. When C occurs at the bottom kern point, which is at a distance k_b from the CGC, the resultant stress profile is shown on the right side. There is zero stress at the top.

(Refer Slide Time: 15:37)



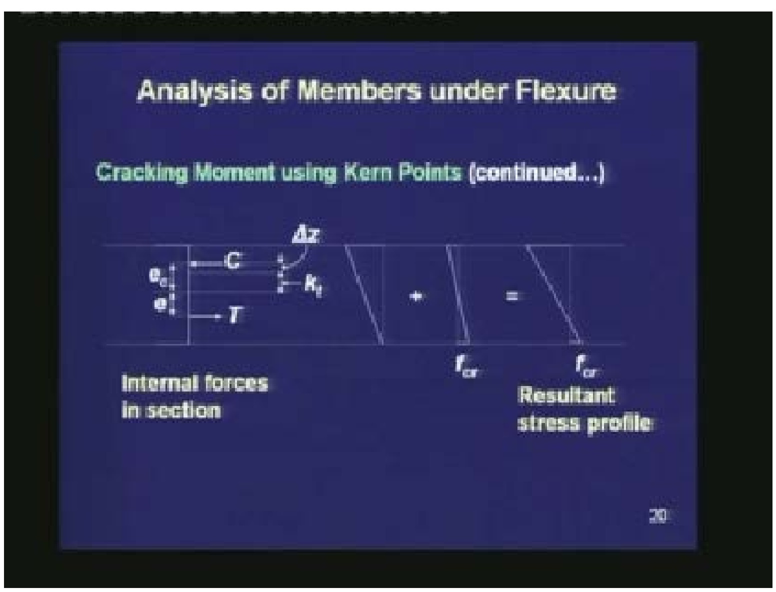
If we write the expression of the stress at the top, then the first term is the uniform compression, and the second term is the stress corresponding to the eccentricity of C, which is equal to $Ck_b y_t/I$, and the sum total is equal to zero. Again, substituting I equal to Ar^2, we can find out an expression of the bottom kern point which is given as $k_b = r^2/y_t$. Again, here r is the radius of gyration and y_t is the distance of the top edge from the CGC.

(Refer Slide Time: 16:33)



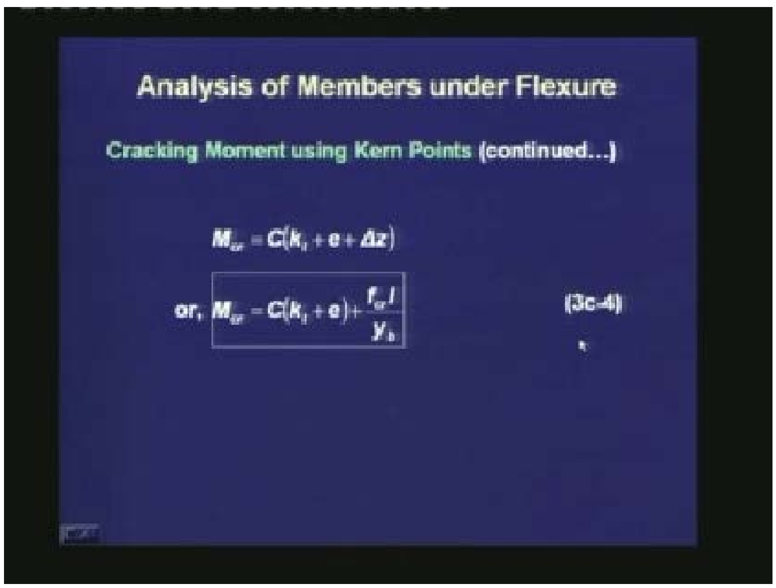
Once we know the kern points, we can also determine the cracking moment using these kern points. The cracking moment is slightly greater than the moment causing zero stress at the bottom. C is located above k_t to cause a tensile stress equal to the modulus of rupture f_{cr} at the bottom. The incremental moment is given as $f_{cr}I/y_b$. Let us understand this from the following stress diagram.

(Refer Slide Time: 17:20)



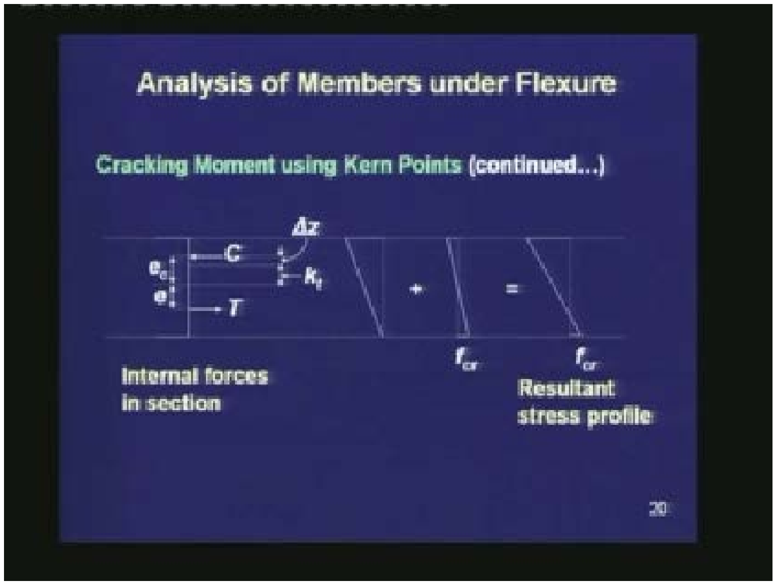
In this diagram we can observe that, if the compression occurs at the top kern point then we have the first stress diagram as shown with zero stress at the bottom. If we shift C slightly above the top kern point by a distance Δz, then there will be an additional stress profile with the modulus of rupture showing up at the bottom edge. The resultant stress profile will have a bottom stress of equal to f_{cr} and which is tensile in nature. This additional increase in the moment due to the shift of the C from the top kern point to a level which is Δz above the top kern point causes this additional tensile stress in the section, which corresponds to the cracking moment of the section.

(Refer Slide Time: 18:39)



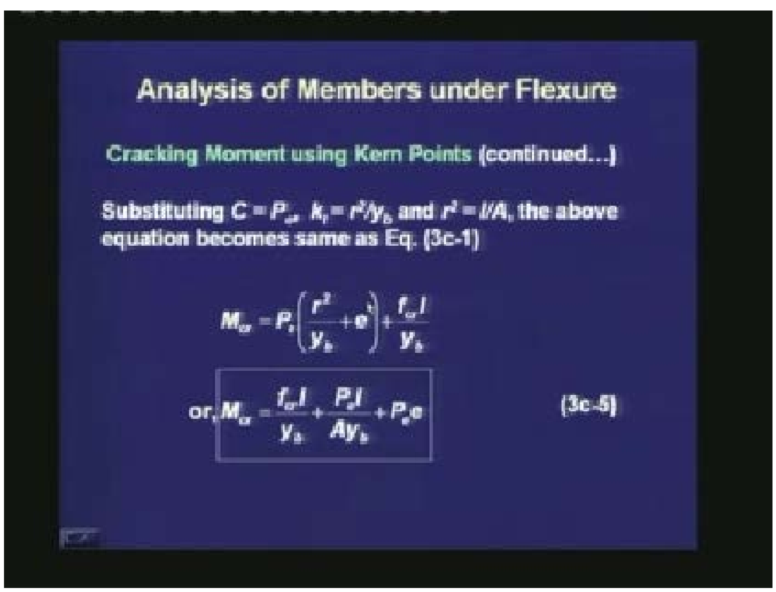
The cracking moment is thus given as C times the total lever arm.

Refer Slide Time: 18:50)



If we go back to the figure, the total lever arm is equal to e + e_c, which is equal to e + k_t + Δz. M_{cr} is equal to C(e + k_t + Δz). Next, we are equating CΔz to $f_{cr}I/y_b$ because that is the incremental moment, beyond the moment causing zero stress at the bottom. This incremental moment causes cracking at the bottom. This is another expression of the cracking moment which is in terms of the kern points and the modulus of rupture.

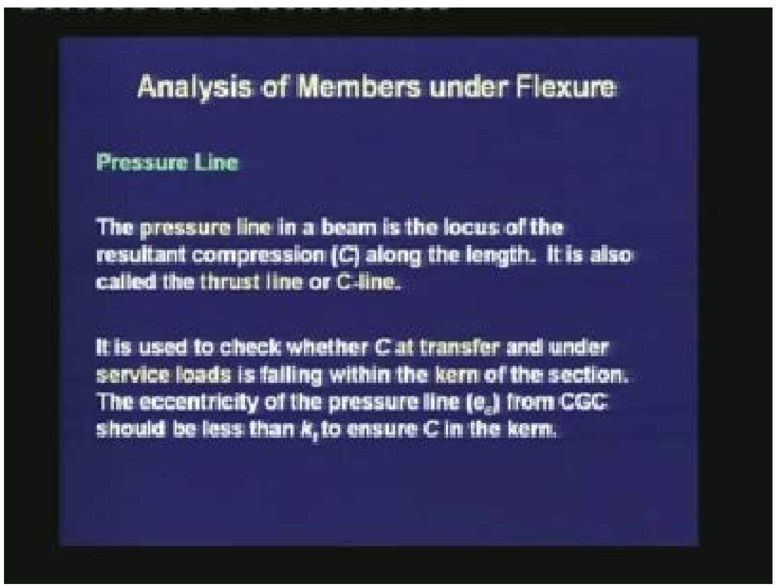
(Refer Slide Time: 19:45)



The expression of M_{cr} that we had seen earlier and this second expression, they are in fact same. We can prove this by substituting $C = P_e$, $k_t = r^2/y_b$ and $r^2 = I/A$. Once we substitute these variables into the expression, we can get back the first expression of the cracking moment.

Thus, we have two approaches to calculate the cracking moment: the first one is from the basic definition based on the stress concept, and the second one is based on the location of C above the upper kern point. Both these approaches will give the same value of the cracking moment.

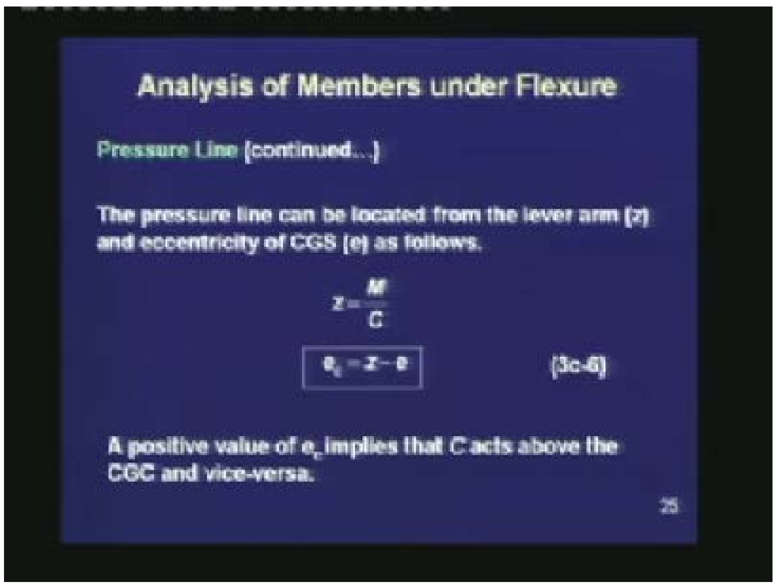
(Refer Slide Time: 20:56)



Next, we are studying the pressure line. The pressure line in a beam is the locus of the resultant compression C along the length. It is also called the thrust line or C-line. As we move along the span of a beam, the profile of the tendon changes, the external moment changes, and the shift of C from the tendon also changes. If we plot a line connecting all the points of the location of C along the span of the beam, that line is called the pressure line for the beam for that particular given load. The pressure line is used to check whether the C at transfer and under service loads is falling within the kern zone of the section. Thus, we are using the pressure line to ensure whether there is any tensile stress in the section or not.

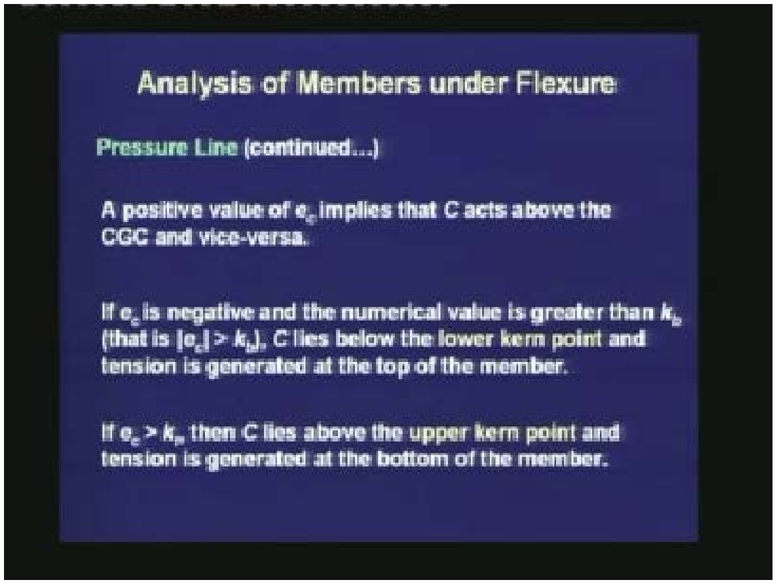
If the C at transfer is within the kern zone, and if the C at service loads is also within the kern zone, then we can say that for any type of service loads, we can expect that C will always be within the kern zone, and the section will always be under compression. At service, the eccentricity of the pressure line which is represented as e_c from the CGC, should be less than k_t to ensure C is in the kern zone. The way to make sure that C is within the kern zone is that the distance of the pressure line at any point of the beam from the CGC should be less than the top kern level under service loads.

(Refer Slide Time: 23:26)



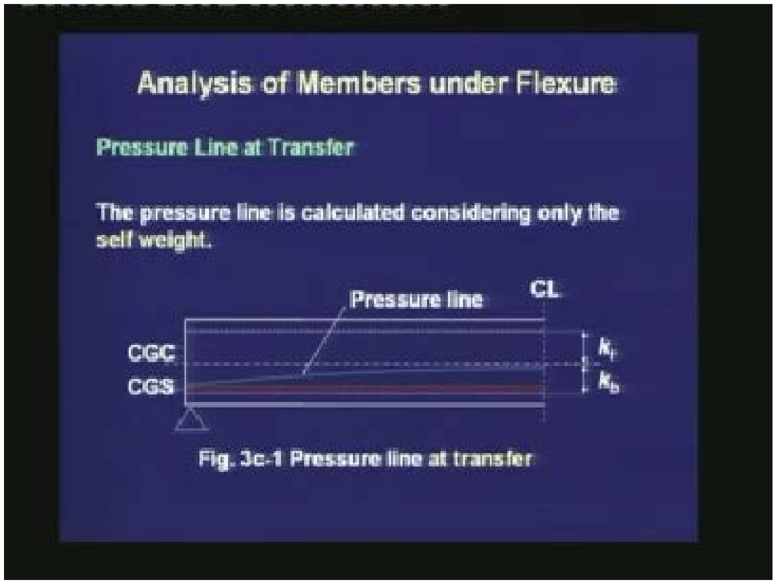
The pressure line can be located from the lever arm (z) and eccentricity of the CGS (e) as follows. The lever arm is the distance between the tension in the prestressing tendon and compression in the concrete. It is given as the moment divided by the compressive force. Then, the location of the pressure line is given by the variable e_c, which is equal to z – e. A positive value of e_c implies that C acts above the CGC. Based on this definition we can say that if e_c is positive, then the compression acts above the CGC and if e_c is negative, then the compression acts below the CGC.

(Refer Slide Time: 24:40)



To summarise, a positive value of e_c implies that C acts above the CGC and vice versa. If e_c is negative and the numerical value is greater than k_b, then C lies below the lower kern point and tension is generated at the top of the member. Similarly, if e_c is greater than k_t, then C lies above the upper kern point and tension is generated at the bottom of the member. Thus, this is the way to check whether the compression lies within the kern zone by the use of the pressure line.

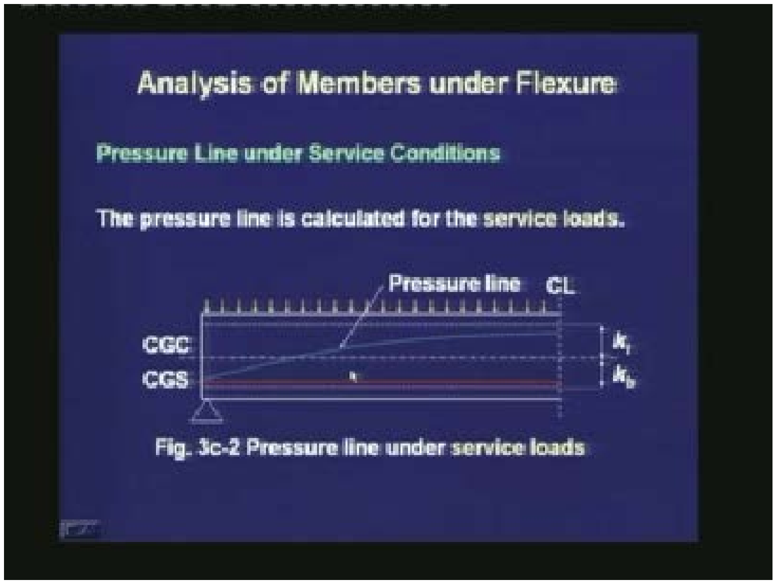
(Refer Slide Time: 25:29)



In this sketch, the pressure line is calculated considering only the self-weight of the beam. Here, the tendon is at a constant eccentricity throughout the member. Due to the self-weight, the compression (C) has shifted from the location of the tendon towards upwards. At the centre, the moment is maximum for a simply supported beam and the shift of C is also maximum. The blue line shows the locus of the C for the various locations along the span of the beam and hence, this is the pressure line at transfer. That means, at transfer, only the self-weight is acting and we are considering that this pressure line is due to the moment from the self-weight.

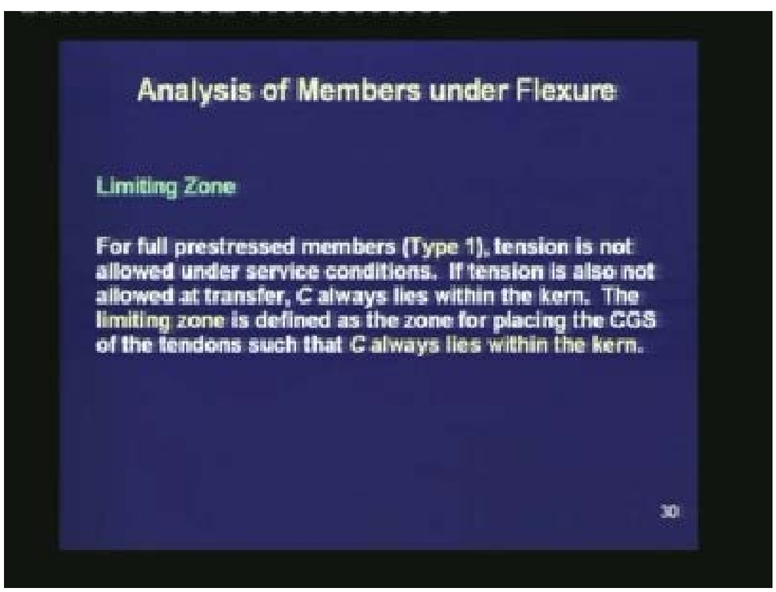
The next sketch shows, the location of the pressure line at service.

(Refer Slide Time: 26:44)



At service also, for a uniformly distributed load, the maximum moment is at the centre and hence, the maximum shift of the pressure line from the CGS is at the centre. Here also, the blue line shows the locus of the points of C along the span of the beam. What we ensure for Type 1 (fully prestressed) members is that the pressure line should lie within the kern zone which is limited by the top kern point and the bottom kern point.

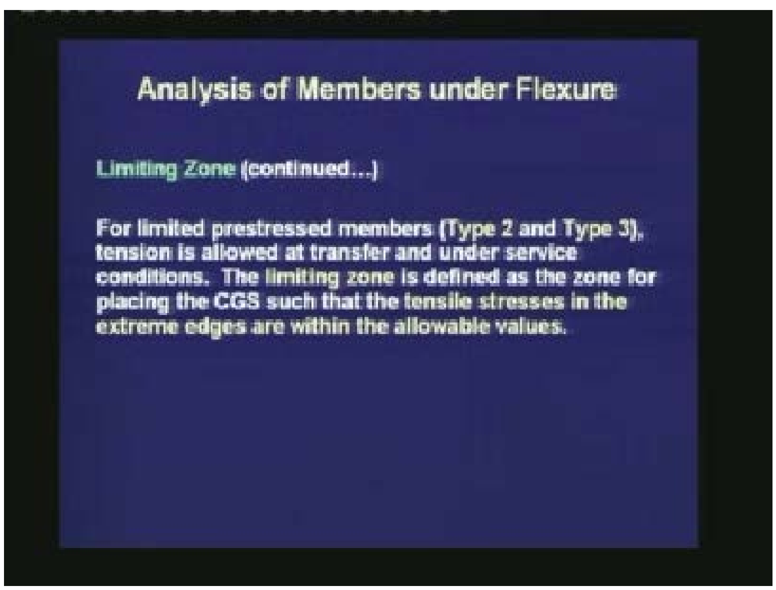
(Refer Slide Time: 27:42)



There is another concept which is used in the design of prestressed concrete members, and this concept is called the limiting zone. For Type 1 members, tension is not allowed under service conditions. If tension is not allowed at transfer also, then C always lies within the kern. The limiting zone is defined as the zone for placing the CGS of the tendons such that C always lies within the kern. To summarize, in a fully prestressed member, where we do not want any tension at the bottom under service loads, C lies within the top kern point. During the transfer of prestress in such a member, if we ensure that C lies within the bottom kern point due to the self-weight, then throughout its service life we expect C to lie between the bottom and the upper kern points.

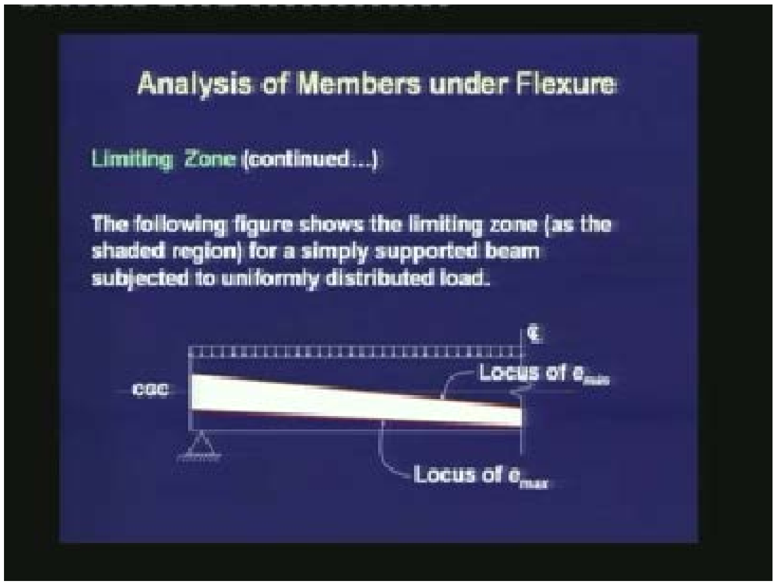
In order to ensure that C will be lying within the kern zone, we can place the CGS only within a certain zone and that zone is called the limiting zone. The limiting zone is used in the design of prestressed concrete members to place the CGS of the tendons, such that C is located within the kern zone for a Type 1 member.

(Refer Slide Time: 29:49)



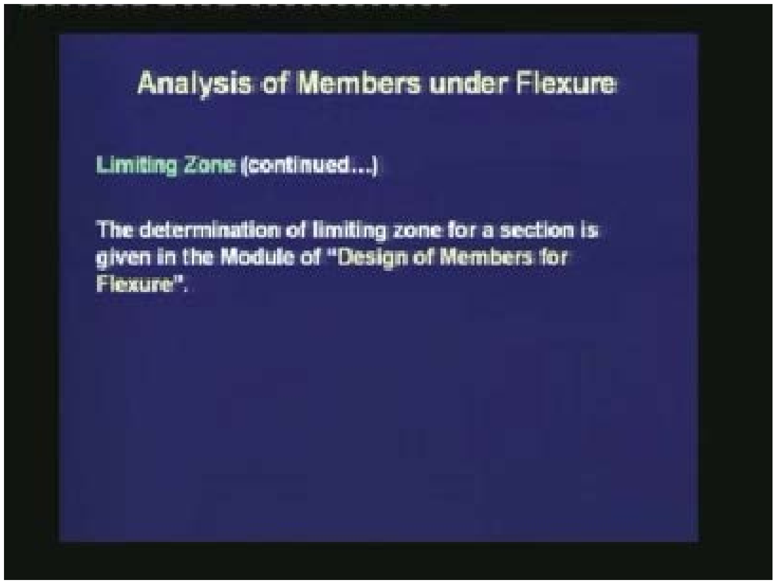
For limited prestressed members (Type 2 and Type 3), tension is allowed at transfer and under service conditions. The limiting zone is defined as the zone for placing the CGS such that the tensile stresses in the extreme edges are within the allowable values. The limiting zone for a Type 2 or Type 3 member is a bit more relaxed than a limiting zone for a Type 1 member. C may be outside the kern zones such that the tensile stress at the extreme edges is within the allowable values.

(Refer Slide Time: 30:32)



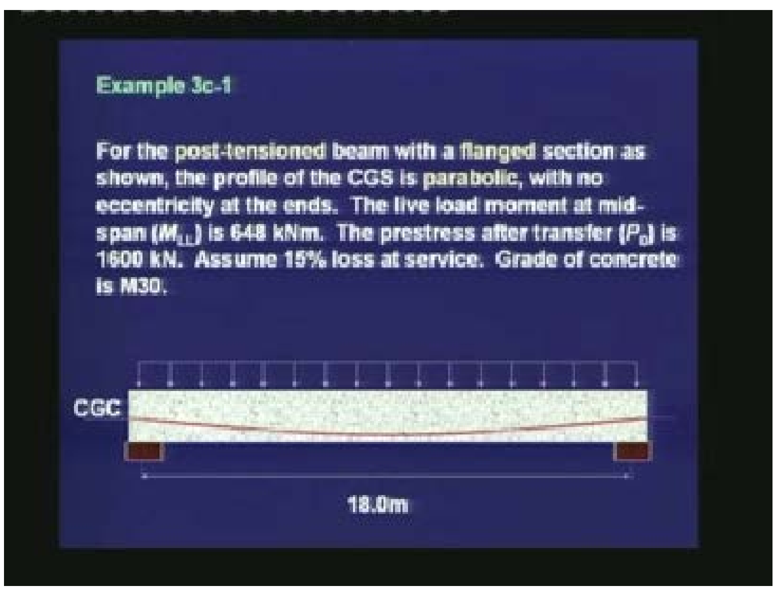
The following figure shows the limiting zone as the shaded region for a simply supported beam subjected to uniformly distributed load. Here, the top line is the locus of the minimum values of the eccentricity of the CGS along the span of the beam, and the bottom line is the locus of the maximum values of the eccentricity of the CGS along the span. We have shown the sketch only for half the length of the beam and the other side, it will be symmetric. If we place the CGS between these two bounds, then we ensure that the compression (C) will lie within the kern zone for a Type 1 member, or it may lie outside the kern zone for Type 2 and Type 3 members, but the tensile stresses that are generated in the extreme fibres will be within the allowable values.

(Refer Slide Time: 31:47)



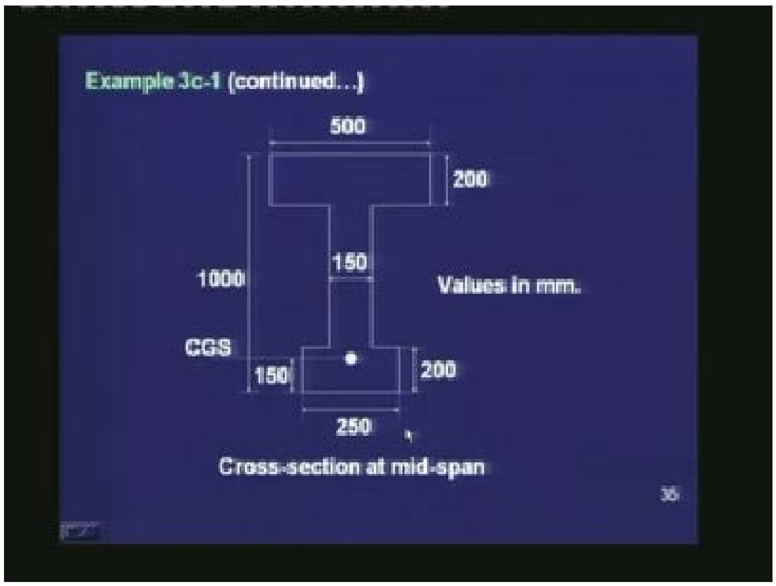
The determination of limiting zone for a section will be given in detail in the module of design of members for flexure. In this particular lecture, we are not further going into the determination of limiting zone.

(Refer Slide Time: 32:09)



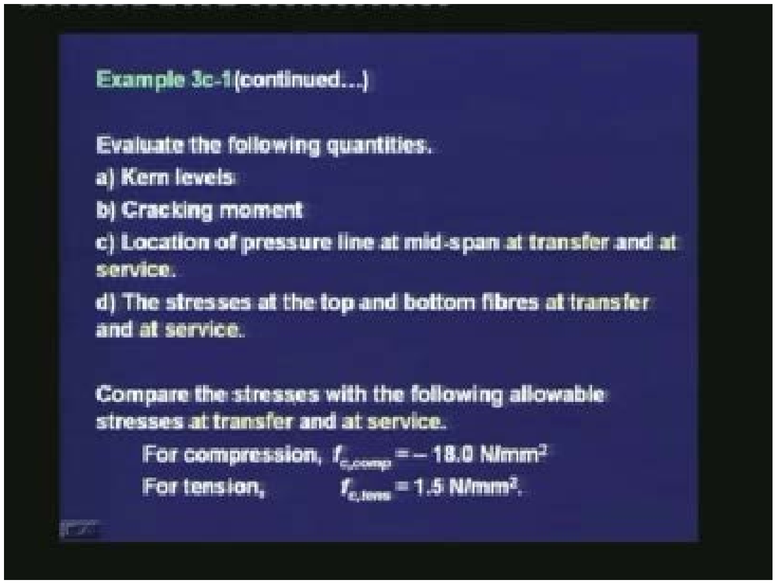
Next, let us solve a problem to determine the cracking moment, the kern points and the location of the pressure line for a particular member. For the post-tensioned beam with a flanged section as shown in the next slide, the profile of the CGS is parabolic with no eccentricity at the ends. The live load moment at mid-span due to service loads is 648 kNm. The prestress after transfer which we have been able to measure from the jacks is 1600 kN. Assume 15 % loss at service. The grade of concrete is given as M30. The span of the beam is 80 m, and the tendon is parabolic with zero eccentricity at the ends and maximum eccentricity at the middle.

(Refer Slide Time: 33:18)



The cross-section of the beam has a top flange with width 500 mm and depth 200 mm. We have a bottom flange with width 250 mm and depth 200 mm. The width of the web is 150 mm. The total depth of the section is 1000 mm. The CGS is located 150 mm above the soffit of the beam.

(Refer Slide Time: 33:55)



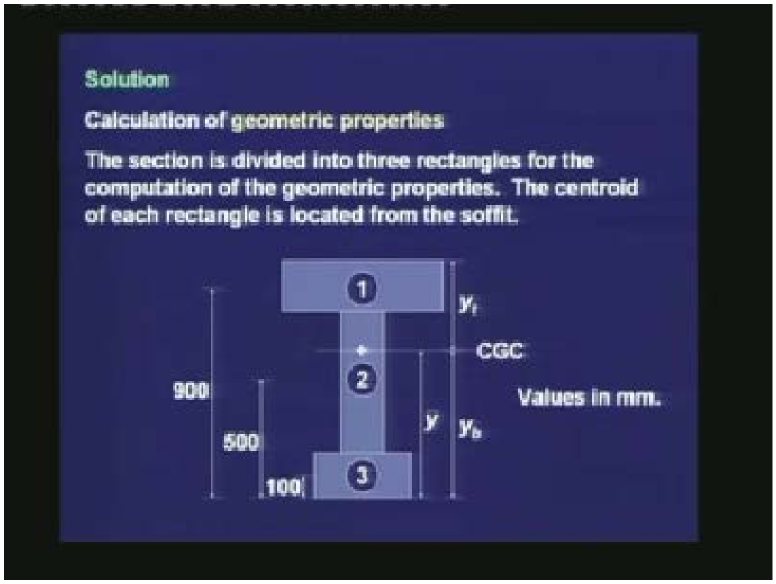
For this member, evaluate the following quantities: a) the kern level, b) the cracking moment, c) the location of pressure line at mid-span at transfer and at service.

Once we know the location of the pressure line at mid-span, we can draw the complete pressure line, because at the ends, the location of the pressure line is at the CGC. Since, the CGS does not have any eccentricity at the ends, and at the end the moment is zero, there the CGS and the pressure line coincide at the CGC. Once we know the location of the pressure line at the mid-span, we will be able to draw a parabolic line between the end and the location at mid-span. Hence, the calculation only at mid-span is sufficient to draw the pressure line.

d) Calculate the stresses at the top and bottom of the member at transfer and at service, and compare the stresses with the following respective allowable values.

Here, the allowable stresses are given to be same for transfer and service. In real situation, these values may be different. For compression, the allowable stress is – 18 N/mm^2. For tension, the allowable stress is 1.5 N/mm^2.

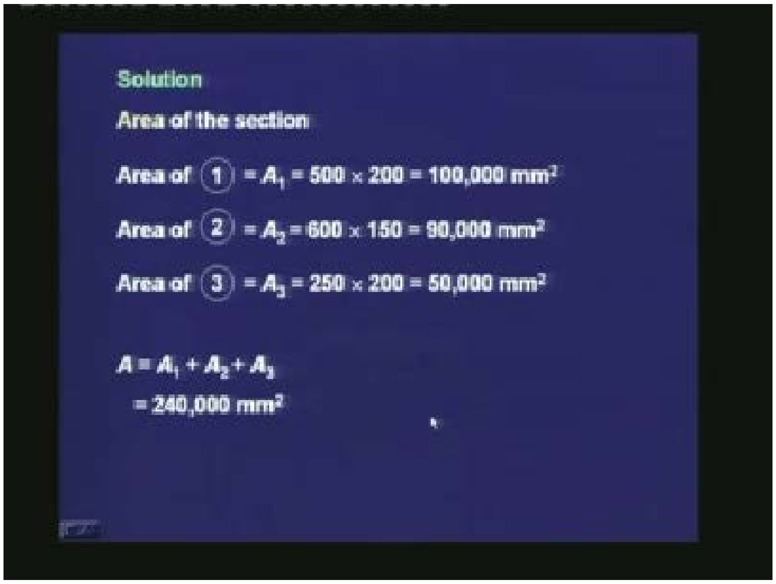
(Refer Slide Time: 35:51)



In our solution, the first step is to calculate the geometric properties. The section is divided into three rectangles for the computation of the geometric properties. This is the essential difference between a rectangular section and a flange section. In our last lecture, we had solved a problem with a rectangular section. There, the calculation of the geometric properties was simpler with standard formulas. But here, we are decomposing the flange section into component rectangles from which we are calculating the geometric properties.

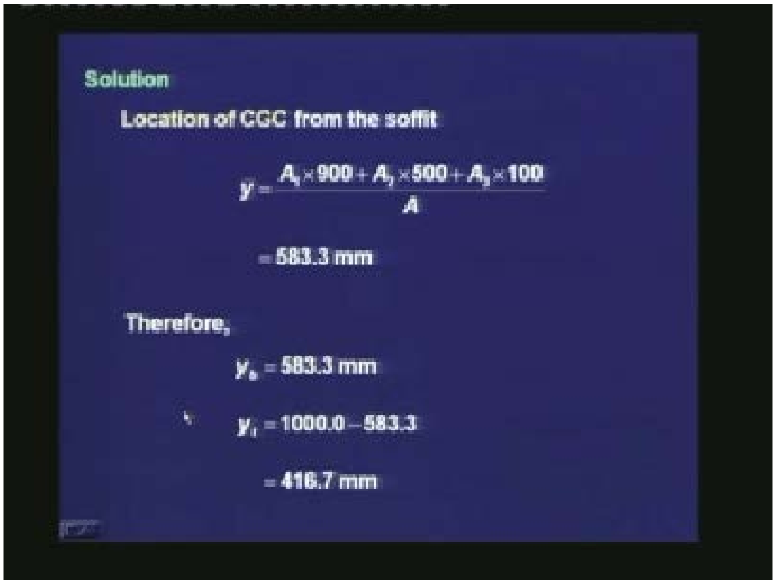
The centroid of each rectangle is located from the soffit of the beam. That means, the top rectangle which is denoted as 1, its centroid is located at 900 mm from the bottom. The second rectangle which represents the web, its centroid is located 500 mm from the bottom. The rectangle which represents the bottom flange, its centroid is located at 100 mm from the bottom. Given this data and given the dimensions of each rectangle, we can find out the location of the CGC from the soffit of the beam, which we shall represent as $\bar{y}$. The distance of the top fibre from the CGC will be denoted as y_t and the distance of the bottom fibre from the CGC will be represented as y_b. Note that y_b will be same as $\bar{y}$.

(Refer Slide Time: 37:43)



For the area of the section, first, we are calculating the area of the first rectangle which is $A_1 = 500 \times 200 = 100{,}000\ mm^2$. The second one is the area of Rectangle 2 which is $A_2 = 600 \times 150 = 90{,}000\ mm^2$. Area of Rectangle 3 is $A_3 = 250 \times 200 = 50{,}000\ mm^2$. The total area is given as $A_1 + A_2 + A_3 = 240{,}000\ mm^2$.

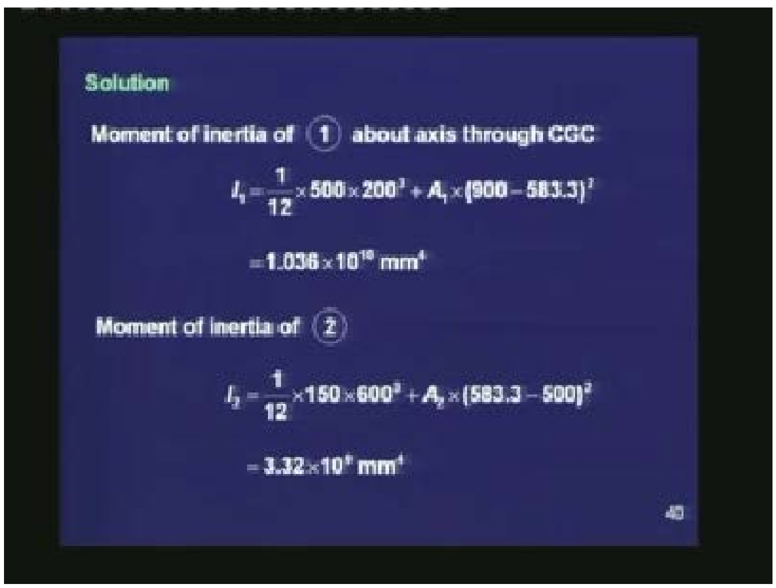
(Refer Slide Time: 38:25)



The location of the CGC is based on calculating the first moment of each of the areas about the level of the soffit, and then sum of these first moments divided by the total area. This is equal to $(A_1 \times 900 + A_2 \times 500 + A_3 \times 100)/A = 583.3$ mm. Thus, the CGC is located at a distance of $\bar{y} = 583.3$ mm from the soffit of the beam. From here, we can calculate the value of y_b which is same as $\bar{y}$ and equal to 583.3 mm. The value of y_t is equal to 1000 (total depth) – 583.3 = 416.7 mm. Thus, we know the distances of the two extreme edges from the CGC.

Next, we are calculating the moment of inertias of the individual rectangles and then we shall add them up to get the moment of inertia of the total section about the CGC.

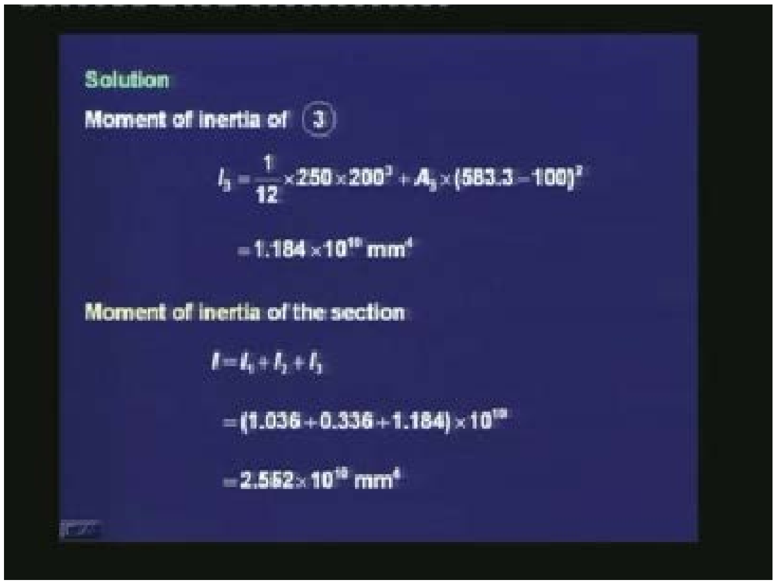
(Refer Slide Time: 39:55)



Using the principle of parallel axis theorem, the moment of inertia of Rectangle 1 about an axis through CGC is equal to the sum of the moment of inertia about its centroid, plus the area times the square of the distance between the two parallel axes. The moment of inertia about its centroid is given as $1/12 \times 500$ (breadth of the section) $\times 200^3$ (depth of the section). The additional term is $A_1 \times (900 - 583.3)^2$. Once we substitute the value of A_1, we get the value of $I_1 = 1.036 \times 10^{10}$ mm^4.

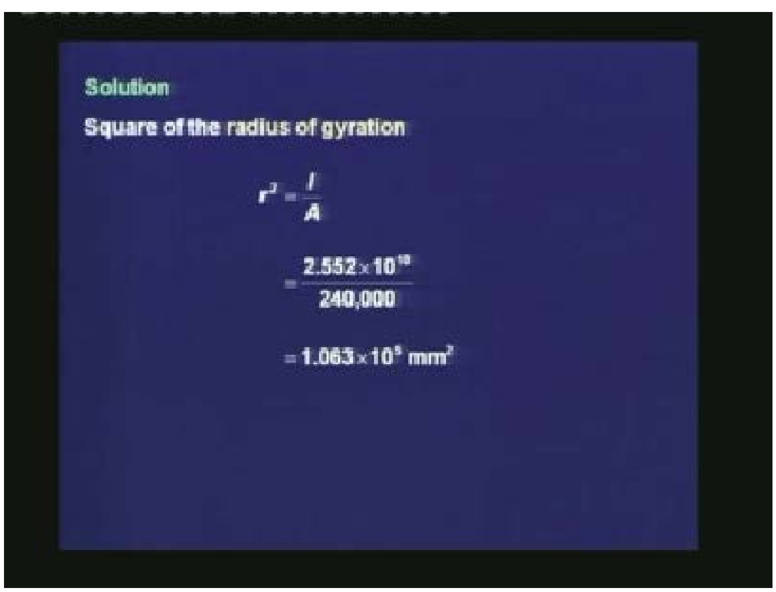
Similarly, we are calculating moment of inertia of Rectangle 2. We get $I_2 = 3.32 \times 10^9$ mm^4.

(Refer Slide Time: 41:25)



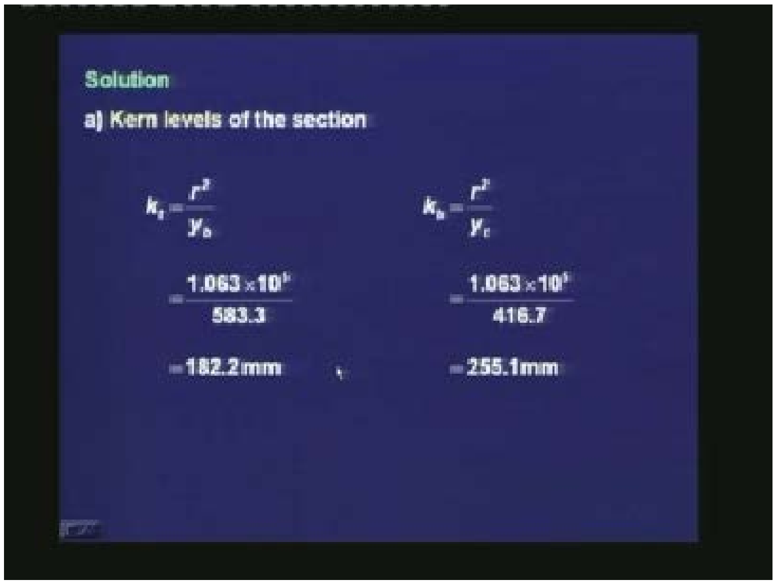
Similarly, we calculate moment of inertia of Rectangle 3 which is $I_3 = 1.184 \times 10^{10}$ mm^4. The moment of inertia of the total section is $I = I_1 + I_2 + I_3 = 2.552 \times 10^{10}$ mm^4.

(Refer Slide Time: 41:55)



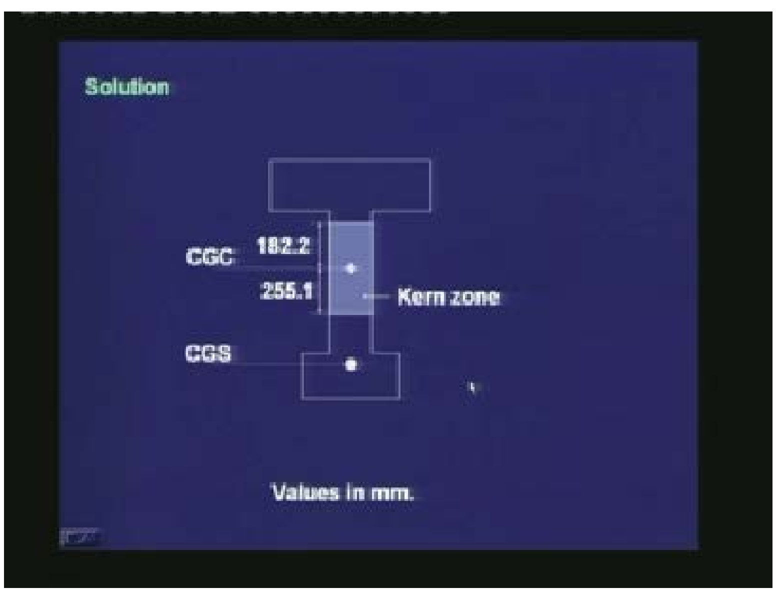
We calculate the square of radius of gyration. $r^2 = I/A$. Once we substitute the values of I and A, we get the value of $r^2 = 1.063 \times 10^5$ mm^2.

(Refer Slide Time: 42:17)



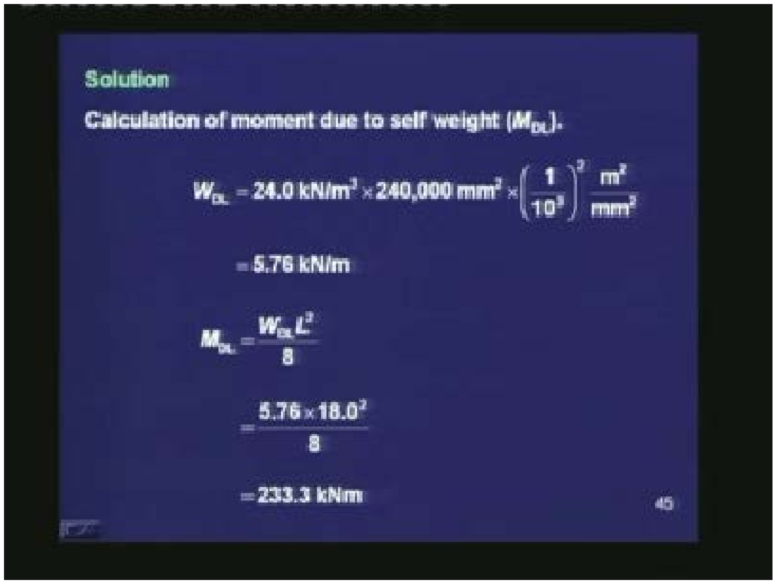
Now, we have all the variables to calculate the kern levels of the section. $k_t = r^2/y_b$ = 182.2 mm. $k_b = r^2/y_t$ = 255.1 mm.

(Refer Slide Time: 42:46)



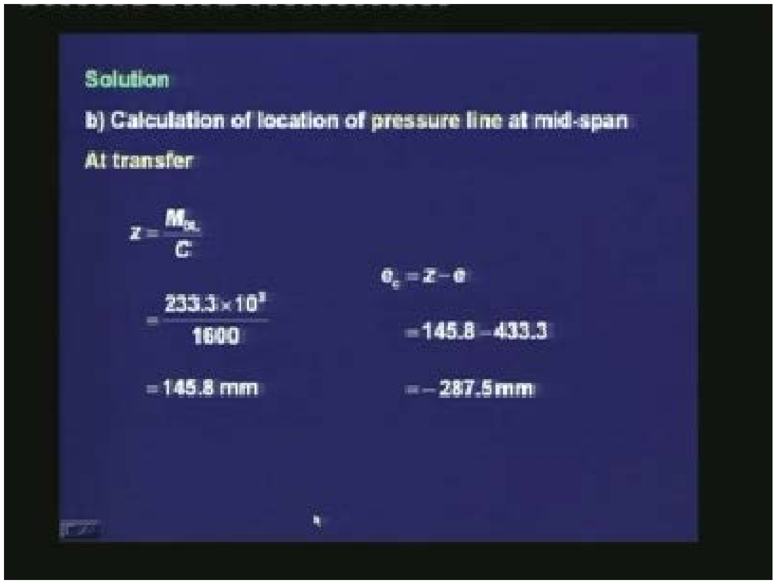
If we plot these kern levels or the kern points for the section, we designate the intermediate zone as the kern zone for the section. Thus, if the compression under any stage of load lies within this kern zone, then there will be no tension in the cross-section.

(Refer Slide Time: 43:17)



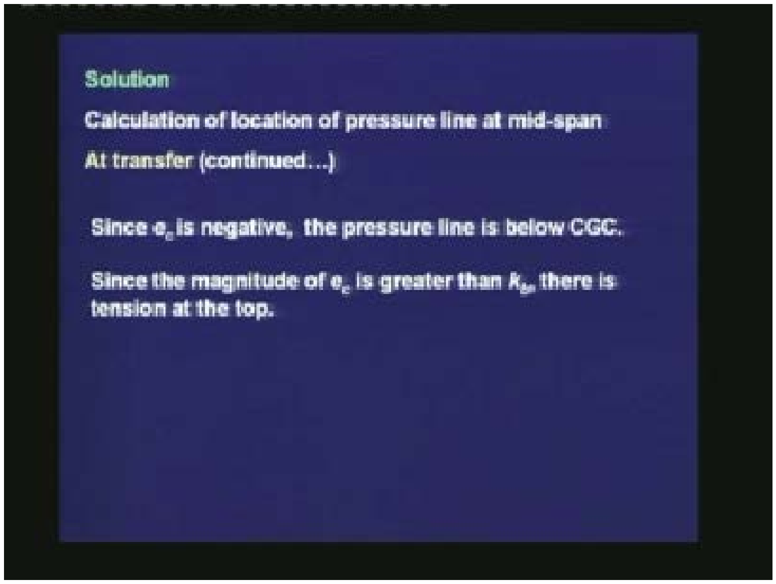
We move on to find out the location of pressure line. At transfer, we are calculating the moment due to self-weight. We are calculating the weight per unit length of the beam as the unit weight of the concrete, which is assumed to be 24 kN/m^3 times the area, and then a factor to convert the mm^2 to m^2. We get the weight per unit length equal to 5.76 kN/m. From that, we can calculate the moment due to self-weight, which is equal to $W_{DL}L^2/8$. The span is equal to 18 m. Once we substitute the values of W_{DL} and L, the moment due to the dead load is M_{DL} = 233.3 kNm.

(Refer Slide Time: 44:27)



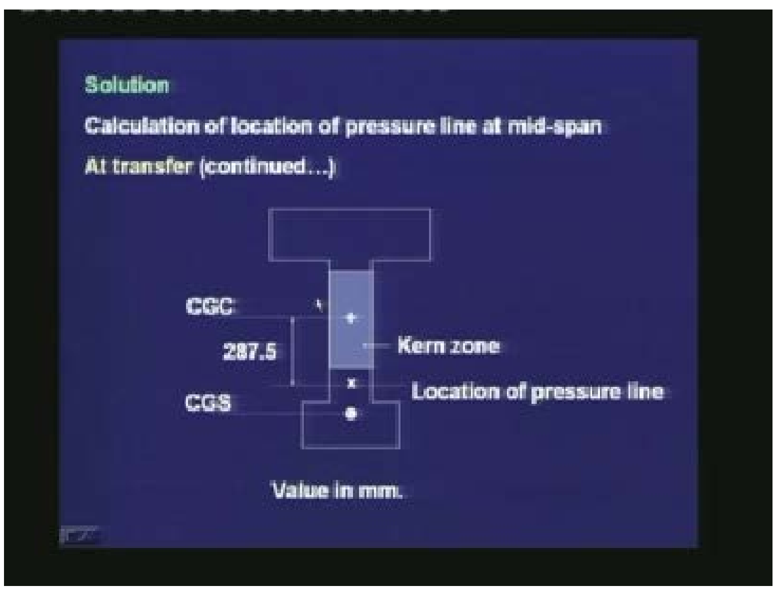
At this point, we are calculating the location of the pressure line at mid-span, for transfer. The transfer is the first load stage, when only the prestress is acting without the long term losses, and the self-weight is acting. At transfer, the lever arm (z) is given as the moment due to the dead load divided by C, which is equal to $233.3 \times 10^3/1600 = 145.8$ mm. That means the C shifts from the CGS by a distance of 145.8 mm at the centre of the beam. Thus, the location of the pressure line at the mid-span is equal to the lever arm minus the eccentricity of the CGS, which is 145.8 – 433.3 = –287.5 mm.

(Refer Slide Time: 45:50)



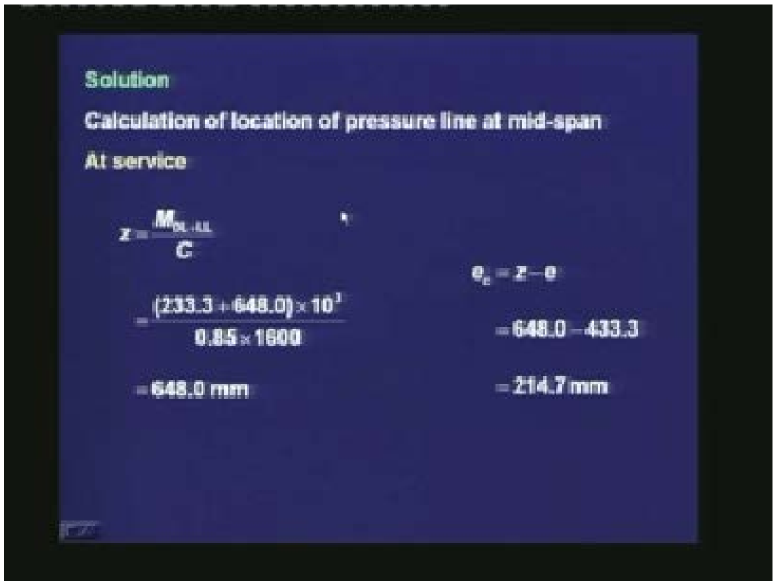
Since e_c is negative, the pressure line at transfer is below the CGC. Since the magnitude of e_c is greater than k_b, there will be tension at the top for this member at transfer.

(Refer Slide Time: 46:14)



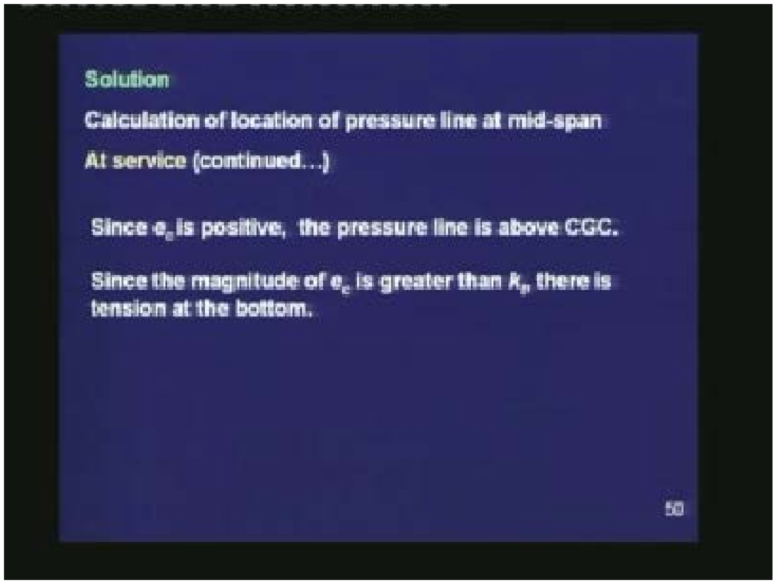
In this sketch, we are plotting the location of the pressure line at transfer from the CGC. We know the bottom kern point, and we have found that the pressure line is located outside the kern zone. Hence, we expect that there will be tension at the top of the beam at transfer.

(Refer Slide Time: 46:44)



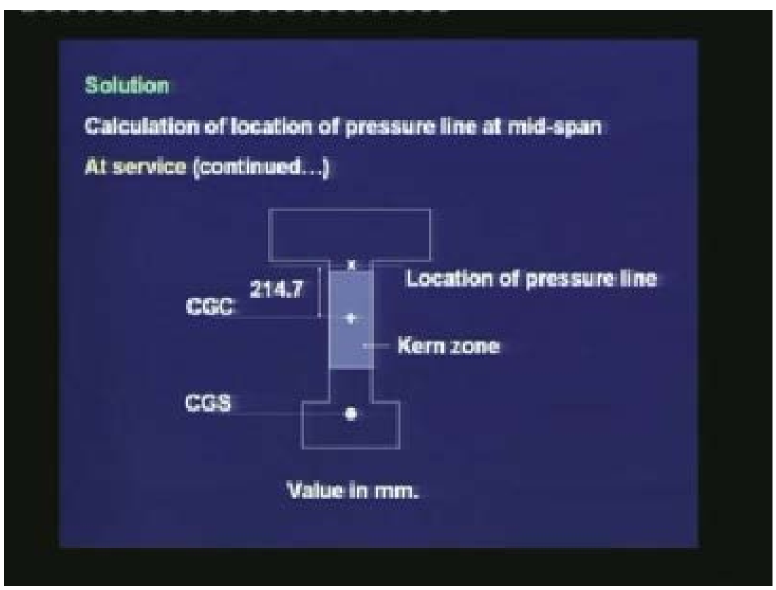
Next, we are calculating the location of the pressure line at mid-span, at service. For service, the lever arm is calculated from the total moment, which is due to the dead load and live load, and from the compression which is now equal to the effective prestress. The total moment is given as the moment due to the self-weight plus the live load moment, which has been specified as 648 kNm. The effective prestress is 85% of the prestress at transfer which is 1600 kN. Once we substitute these values, we get the lever arm equal to 648 mm. Thus, at service, the C shifts from the CGS by a distance of 648 mm at the mid-span of the beam. The location of the pressure line, which is equal to the eccentricity of C, is given as z – e = 648.0 – 433.3 = 214.7 mm.

(Refer Slide Time: 48:20)



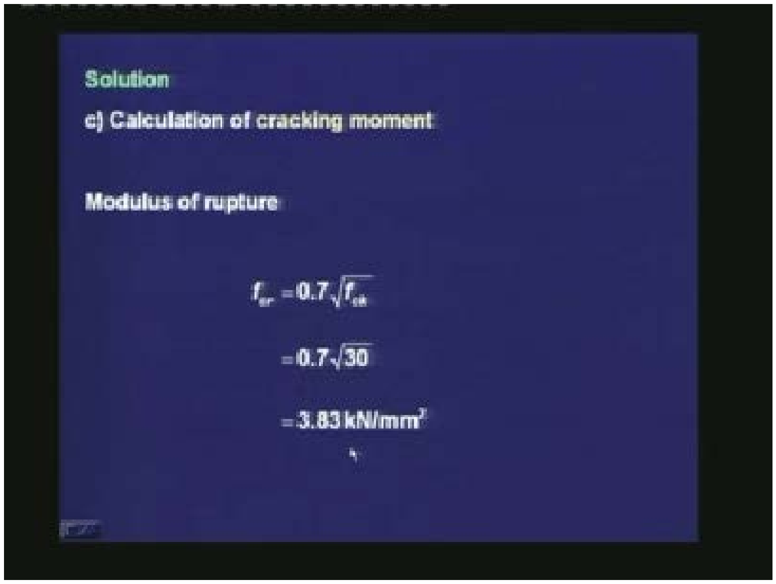
Since e_c is positive, the pressure line is above CGC. Since the magnitude of e_c is greater than k_t the upper kern point, there is tension at the bottom.

(Refer Slide Time: 48:34)



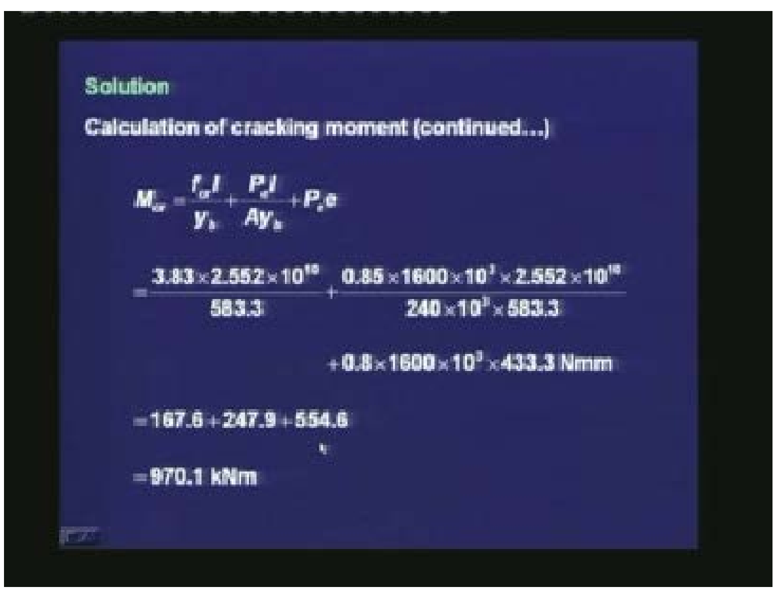
In this sketch we are plotting the location of the pressure line at service, and we find that this is located above the upper kern point at a distance of 214.7 mm from the CGC.

(Refer Slide Time: 49:05)



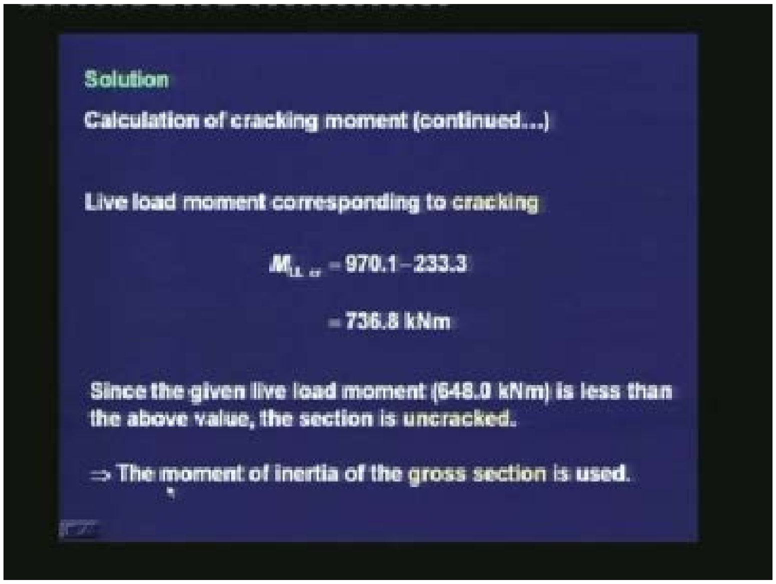
Next, we are calculating the cracking moment. For that, we are first calculating the modulus of rupture: $f_{cr} = 0.7\sqrt{f_{ck}}$, where f_{ck} is the characteristic strength equal to 30 N/mm^2 for M30 grade of concrete. The modulus of rupture is equal to 3.83 N/mm^2.

(Refer Slide Time: 49:38)



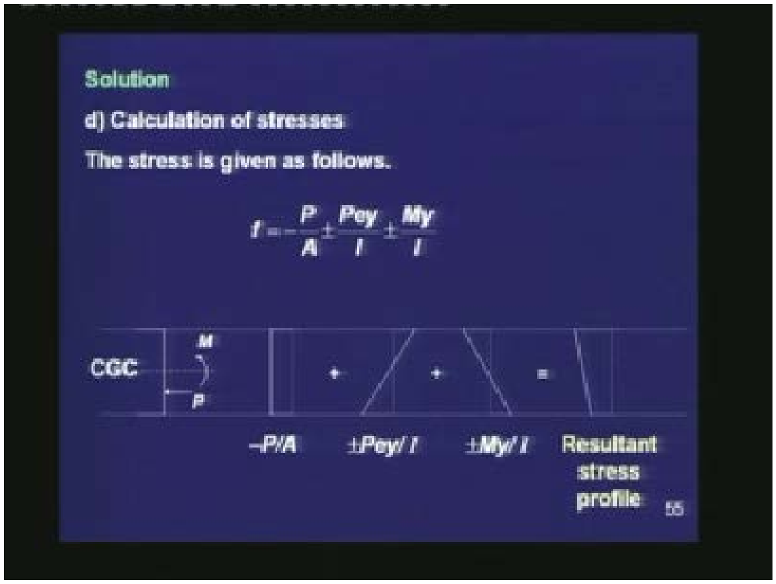
Substituting the value of the modulus of rupture, the sectional properties and the effective prestress, we can evaluate the cracking moment. The cracking moment comes out to be 970.1 kNm.

(Refer Slide Time: 49:59)



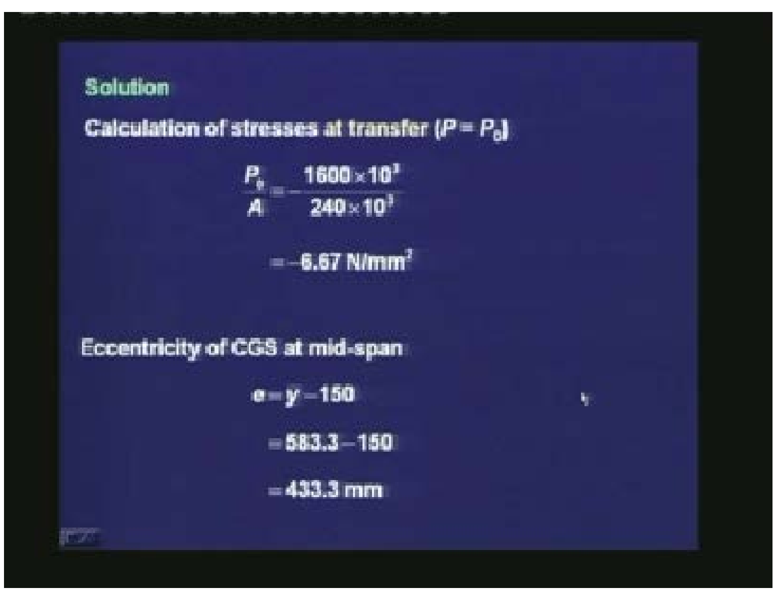
Hence, the live load moment corresponding to cracking is M_{LLcr} is equal to 970.1 – 233.3 = 736.8 kNm. Since, the given live load moment 648.0 kNm is less than the above value, the section is uncracked under service loads. It is a Type 1 member. Hence, we can use the moment of inertia of the gross section.

(Refer Slide Time: 50:41)



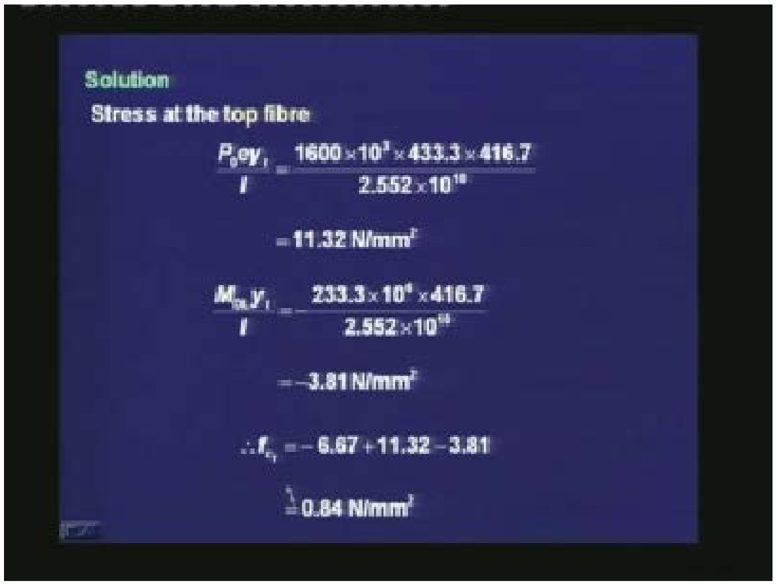
Finally, we are calculating the stresses in the member under the transfer and service loads. The expression of the stress is given as follows: the first one is the uniform stress; the second one is the stress due to the eccentricity of the prestressing force; and the third one is the stress due to the external moment. We expect a resultant stress profile as shown in the right.

(Refer Slide Time: 51:07)



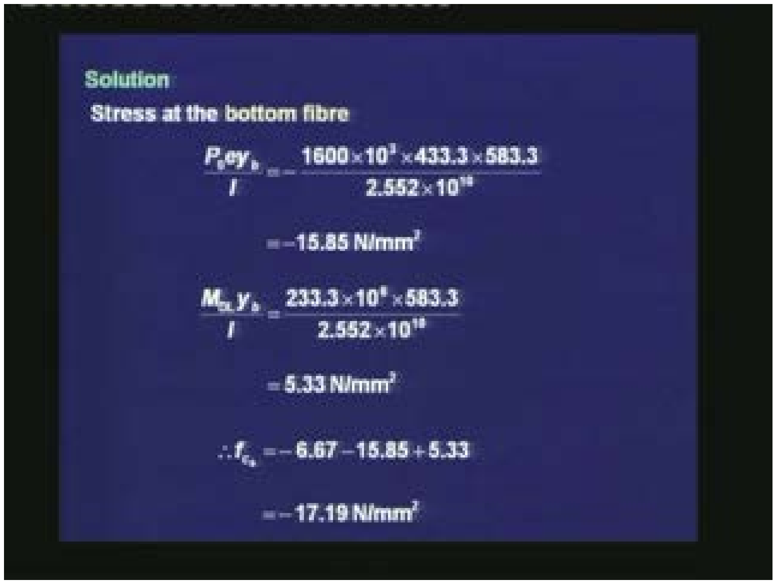
At transfer, we calculate the stress due to the prestressing force P_0. We know the value of e = ȳ – 150 = 433.3 mm.

(Refer Time Slide: 51:26)



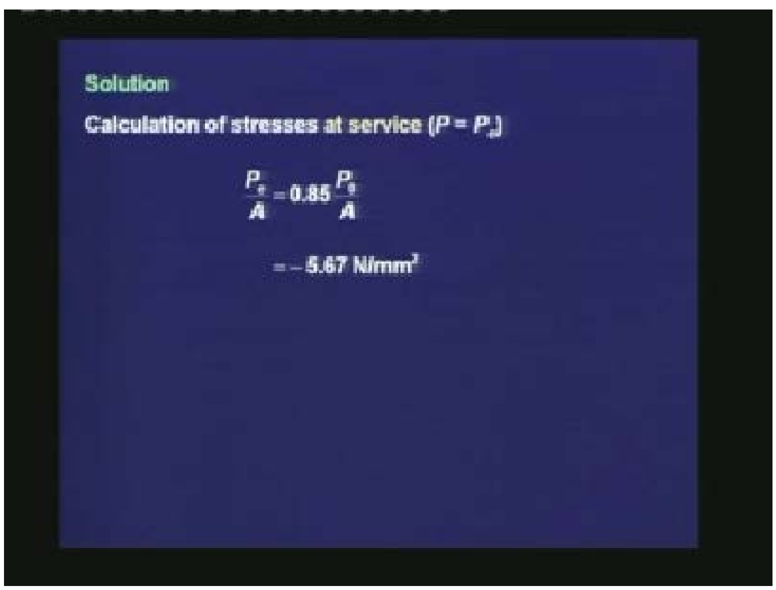
When we substitute the values of all the variables into the individual stress components, we find that the stress, at the top is given as – 6.67 + 11.32 – 3.81 = 0.84 N/mm^2. Thus, we have positive tensile stress at the top, which we had earlier observed from the location of C at transfer below the bottom kern point.

(Refer Slide Time: 52:03)



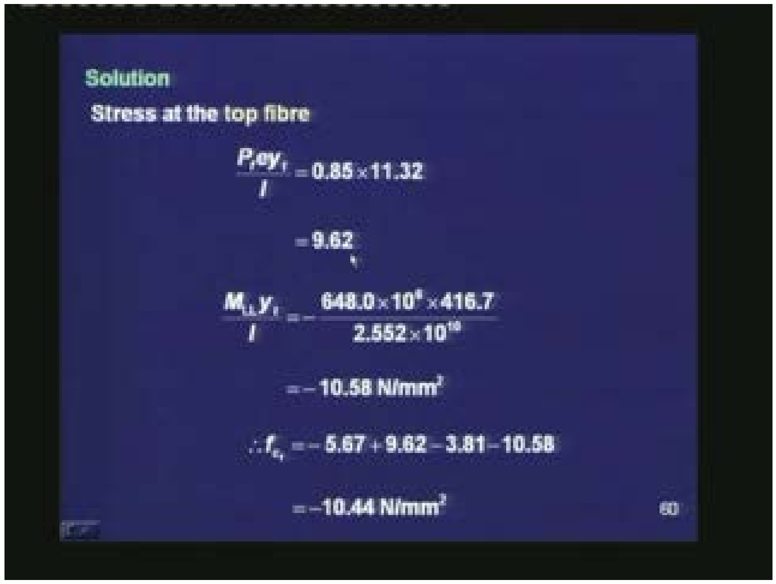
For the stress at the bottom, the value is – 17.19 N/mm^2.

(Refer Slide Time: 52:13)



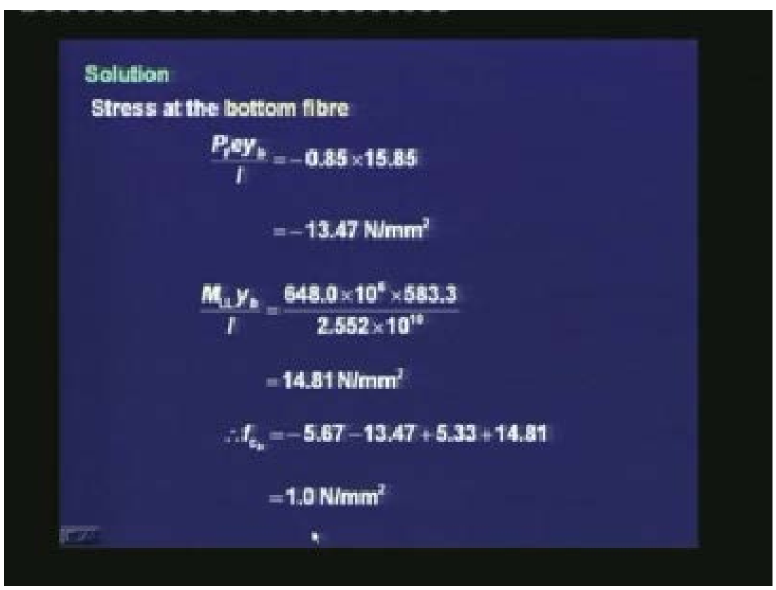
Next, we are calculating the stresses at service. The uniform stress is given as 85% of the stress at transfer, due to the 15 % losses.

(Refer Slide Time: 52:27)



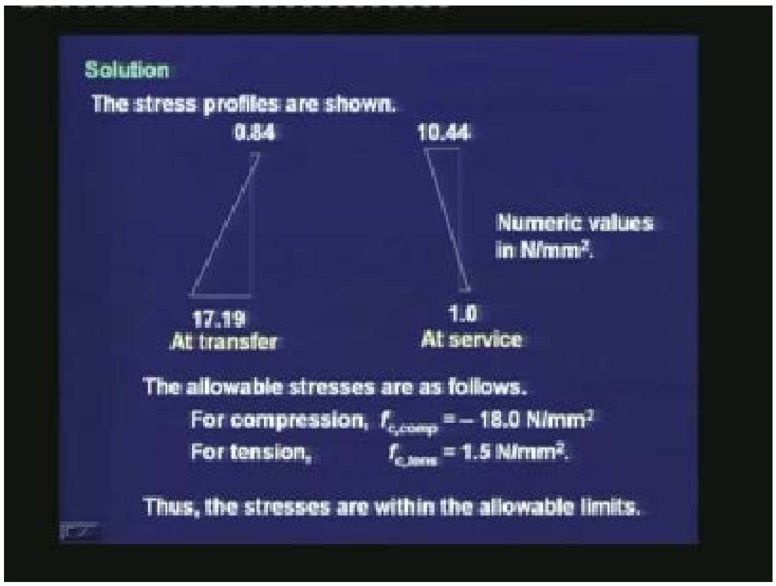
For the stress at the top fibre, calculating the individual values we find that the stress is –10.44 N/mm^2.

(Refer Slide Time: 52:41)



With similar calculations, we find that the stress at the bottom is equal to 1.0 N/mm^2. Remember that, in this expression here, we knew the stress due to the self-weight moment which we have retained, and we have just added the stress due to the imposed live load moment. The uniform component of the stress is – 5.67 N/mm^2. Then we have – 13.47 N/mm^2 due to the eccentricity of the prestressing force. 5.33 N/mm^2 is from the dead load moment and 14.81 N/mm^2 is from the live load moment. We observe that at the bottom, there is a resultant tensile stress under service loads.

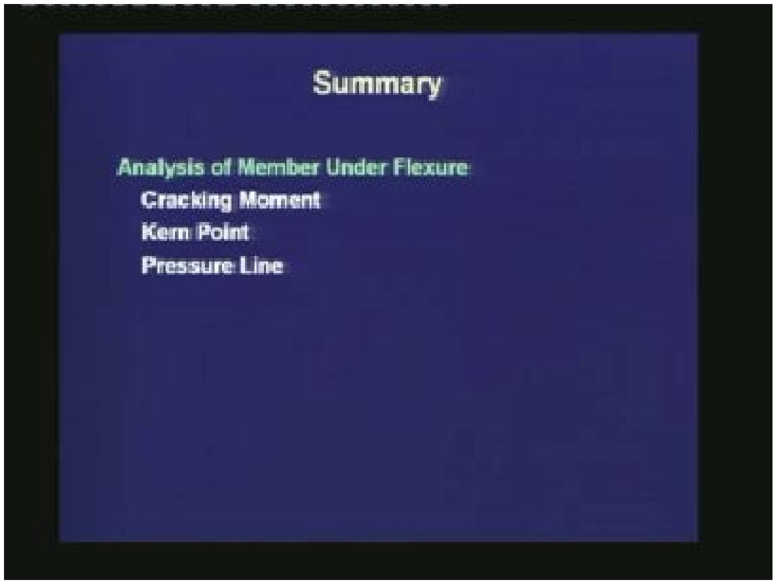
(Refer Slide Time: 53:42)



The stress profiles are shown. At transfer, we have a positive tensile stress at the top and compressive stress at the bottom. At service, we have compressive stress at the top and positive tensile stress at the bottom. Let us now compare these stresses with the allowable values. Both for transfer and service, the allowable compressive stress is –18 N/mm^2. We see that the magnitudes of the compressive stresses both at transfer and at service are lower than 18 N/mm^2. Hence, the allowable compressive stresses have been satisfied.

Next, the allowable tensile stress is 1.5 N/mm^2. Here also, the top tensile stress at transfer and the bottom tensile stress at service, are lower than the allowable value. Thus, the stresses are within the allowable limits. Our observations from the kern levels and the locations of pressure line, are consistent with the observations of the calculated stresses.

(Refer Slide Time: 55:15)



To summarize today's lecture, we studied some specific portions of analysis of members under flexure. First, we studied the cracking moment. We have defined the cracking moment as the moment due to the external loads, when the stress at the bottom is equal to the modulus of rupture. We have seen one expression of cracking moment based on the stress concept, and the other expression of the cracking moment based on the location of the kern points. Second, we moved on to the definition of the kern points. If the compression is located in the kern zone, then there will not be any tension in the section.

Next, we moved on to the pressure line. Pressure line is the location of the resultant compression throughout the span of the beam. We observed that for a simply supported beam, if we know the location of the pressure line at mid-span, we will be able to draw the complete pressure line throughout the span. From the pressure line, if the C lies within the kern zone at transfer and at service, then we ensure that there will not be any tension in the prestressed concrete member during its service life period.

Finally, we solved a problem, where we saw the analysis of a flanged section. First, we calculated the geometric properties by decomposing the section into individual rectangles. We calculated the kern levels. Next, we found out the cracking moment for the particular section. We have found that the given live load is lower than the live load corresponding to the cracking moment. Hence, the section will not crack. However, since the pressure line is outside the kern zone, there will be some tension at the extreme edges. Same observations were obtained by the calculation of the stresses from the stress concept.

From the stress concept analysis, we found that at transfer, there is some tension at the top, and at service, there is some tension at the bottom. Both these tensile stresses are within the allowable value. Also, the compressive stresses at transfer and at service are also within the allowable value. This ensures that this member will not crack, and it is satisfactory under service loads. With this, we are ending the analysis under service loads, where we ensure that the stresses are within the allowable values.

Thank you.