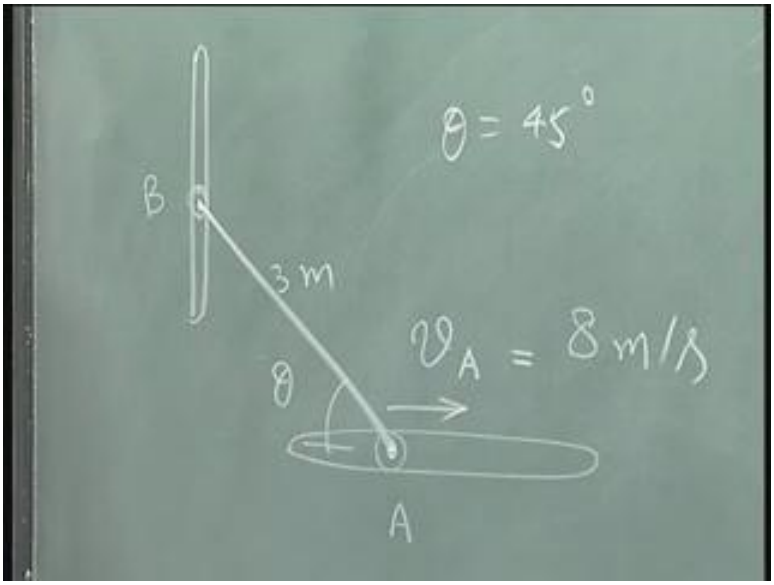
Engineering Mechanics
Prof. Siva Kumar
Department of Civil Engineering
Indian Institute of Technology, Madras
Dynamics of rigid bodies

Let's solve a problem. This is a simple problem. I have a slot over here, a slot over here. This is a vertical slot and this is a horizontal slot and there is one mechanism like this moving. So as this moves, this comes like this. I hope you understand. Now what is asked is if I know the velocity of this, let's say this is at an angle theta, I think theta is given as 45 degrees.

(Refer Slide Time 01:15)

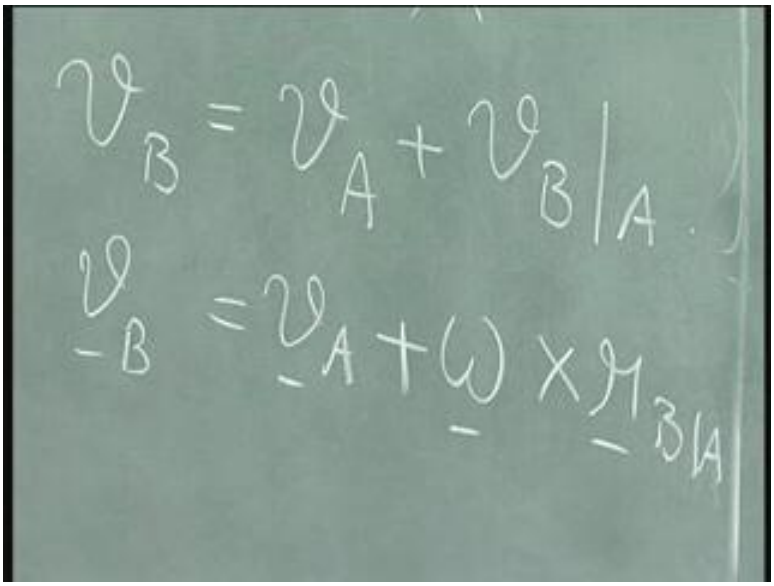


Let's say this is moving at a velocity V_A. This is 8 meters per second. What is the length of this? This is 3 meter length and what is asked of us to find out is, how much is this rotating or in other words what is the angular acceleration of this, angular velocity of this particular rod at this position because depending on the position, the angular velocity may change. So position is given to you. At that instant what is the angular velocity that's the question that is asked. How do I solve this problem let's see.

The first thing I have to do in all these exercises is fix fixed frame of reference. It's like biblical statement, you have to fix that's the geetha. I will fix a frame of reference, this is x and this is y. The fixed frame of reference is now given. This is B, this is A, the length is given I need to find out the angular velocity, very simple. If I know the movement of this with respect to the fixed frame, what is the movement of B is another question that is possible. Let's look at that. The movement of B or the velocity of B should be equal to the velocity of A plus velocity of the line connecting them. Correct?

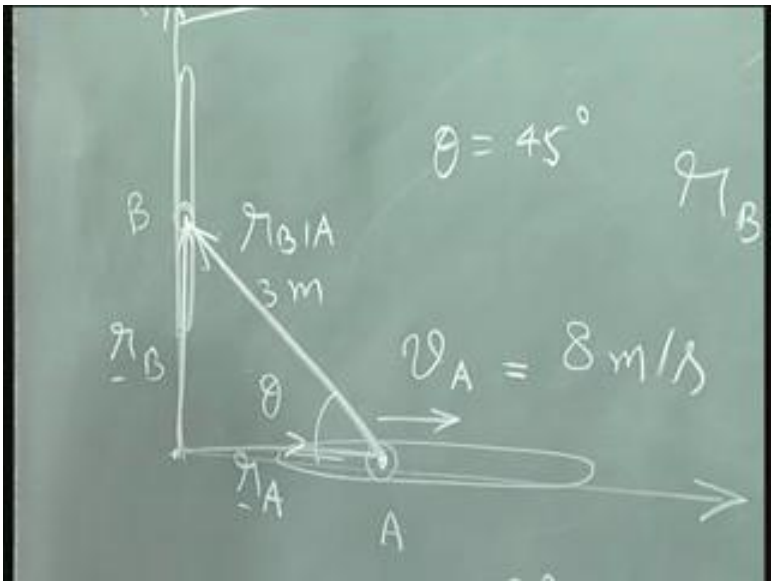
Velocity of A plus velocity of B with respect to A. What is velocity of B with respect to A? We have already done that exercise. This is equal to V_A plus omega cross r of B with respect to A and this is the equation I have.

(Refer Slide Time 03:38)



Now what is the next step that I have to do? What is asked for is, what is omega equal to? I have this equation, the next thing I have to do is examine what quantities I know and what quantities I don't know. Can we do that? Now is it possible to find out r B with respect to A? Given that there is a position that I have already shown you that it is at an angle theta say 45 degrees.

(Refer Slide Time 04:59)



Is it possible to find out r of B with respect to A? Yes, r of B with respect to A is nothing but r B minus r A. If this is the position of B and this is the position of A, then it is simple, r B minus r A will give me r of B with respect to A. So this is pretty simple. Since I know this particular r B with respect to A because the length is also given, this I know, complete vector I know. What is the direction of omega? Since this is on the plane, the direction of omega is perpendicular to the board. I know the direction of omega. What do I know here? I don't know the magnitude, I know the direction. Is that clear? This I am just going to circle so that you know this is something that is to be found out. Do I know V_A? Of course yes, V_A is given by 8 meters per second times I, I am using a capital I here to denote that it is a fixed frame of reference. Because later on when you have a rotating frame of reference, you could use a small i.

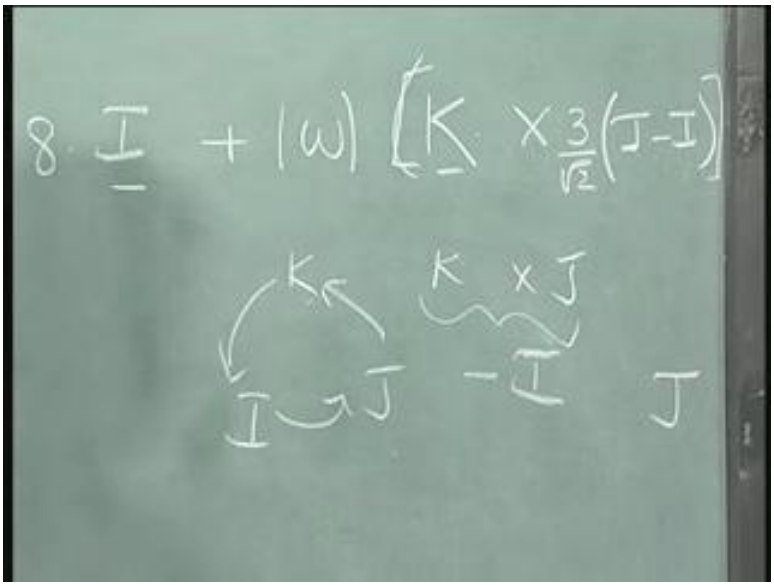
I am going to use capital J, capital I. Therefore here do I know the direction of V_A? The answer is yes. Do I know the magnitude of V_A? The answer is yes. It is 8 meters per second. This is completely known, this is completely known. The direction is completely known. The magnitude of omega is not known. Let us look at V_B? Do I know the direction of V_B? The answer is yes, it is along this direction. I can take it to be upward so that the magnitude can be negative and so I know the direction of this is along the J direction which means only magnitude is not known.

(Refer Slide Time 06:52)

Now it is very simple. All that I don't know is the magnitude of V_B and the magnitude of omega. These are the two unknowns. How many equations do I have? Remember this is a vectorial equation. I'll have, if I write it completely, this is the magnitude of V_B times the J vector is equal to magnitude of V_A, is given by 8 I because it is in the positive direction plus magnitude of omega. This is n direction, this is r B A. So n is nothing but K cross… what is r B with respect to A? This length minus this, this is 3 meters and therefore this is 3 cos 45 and this is 3 cos 45. Correct?

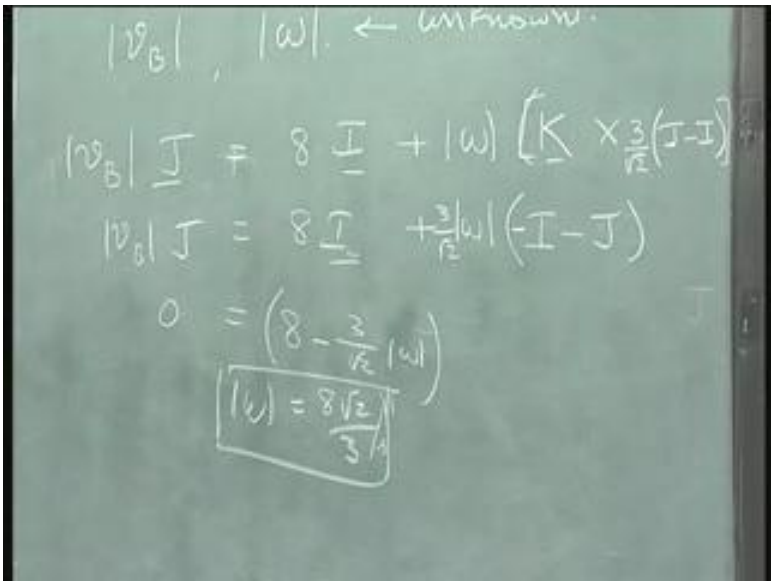
It is crossed with 3 by root 2 times J minus I. Is this clear? What is K cross J? K cross J will come in here that will be nothing but? How do I find out? I, J, K if it is in the cyclic order, I will get a positive when I do a cross product.

(Refer Slide Time 09:09)



So K cross J will be a minus I and I have a K cross I, K cross I is positive so I will get a J. Therefore here I should be getting plus omega, I will take this out. So 3 by root 2 times K cross J is minus I minus J. How do I now separate into two equations? Very simple, I will take all the I terms separately, all the J terms separately and equate them. That's very simple.

(Refer Slide Time 10:42)



There is no I term here, so it is 0. The component of I here is 0, is nothing but 8 minus 3 by root 2 omega. I don't need the other equation because what is asked for is the value of omega and value of omega is simply 8 times root 2 by 3. What's the unit of this? Radians per second, that's it over. This is what is very important I need to know. Look at this as a vector equation and therefore I will have 2 unknowns that I will solve for. Whichever unknown I need to solve for, I will appropriately take the component in relation to that. In most of the problems this is how you will understand solving.