Engineering Mechanics
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Dynamics of rigid bodies

Having understood that part of motion lets look at total motion of a point of a rigid body. Let's say I have a rigid body, I am only going to talk about planar and it's easily extendable to 3 D motion. Let's say we will take two points, as we did earlier there were many points that we took. We can take just two points to illustrate the motion. Let's say this is A, this is B. As we said earlier let me make it a simple rigid body so that I can draw again easily. This has gone through a rotation like this and a translation, lets say this is A, this is B.

As you can see this is the motion that A has undergone, this is the motion that B has undergone. Now whenever we talk about motion, we cannot talk about pure motion unless we know there is one reference which is not moving. In other words I have to have what is called a fixed frame. I am not going to go into details of what a fixed frame is. Assume that this is a fixed frame, any fixed frame we are going to use capital letters say X and Y or I capital and J capital.

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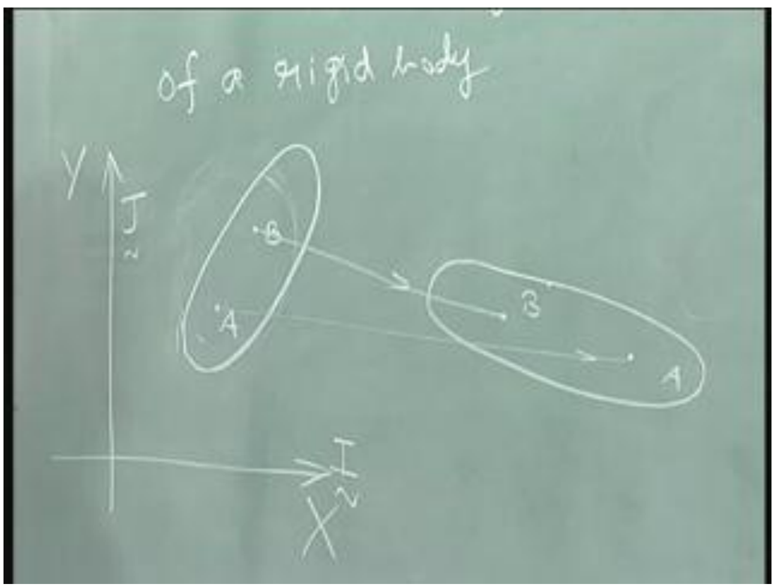

When I put a wiggle here, it means vector. I have a unit vector along this direction which is I, a unit vector along Y which is J. K will be outward unit vector. Now with respect to this it is possible for me to define each of this lets say this is t equal to 0 and this is t equal to t_1 just giving you some example.

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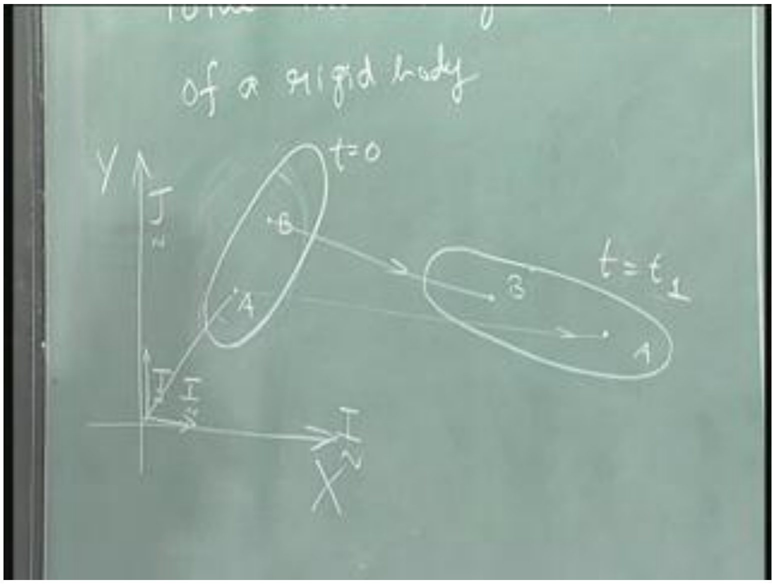

It is possible for me to track the motion of this particular body. If I have to find out what is this position, I should first know the position with respect to the fixed frame, the initial position. Let’s say this is r_B, let me just put a r_B not (a zero at right hand top and B at the right hand bottom of the letter ‘r’) . If I add that to r, this is B going from 0 to t_1. Then I can find out complete motion of this particular body, this particular point P. Let me call this as r_B at time t_1 and I seem to be complicating it a little bit but the understanding is simple. I describe a point on this body at a reference time t equal to t_0 or 0 with respect to a fixed frame of reference. I have a I and a J, capital I and capital J denoting the fixed frame of reference. Later on we will understand why we are using capital letters for these.

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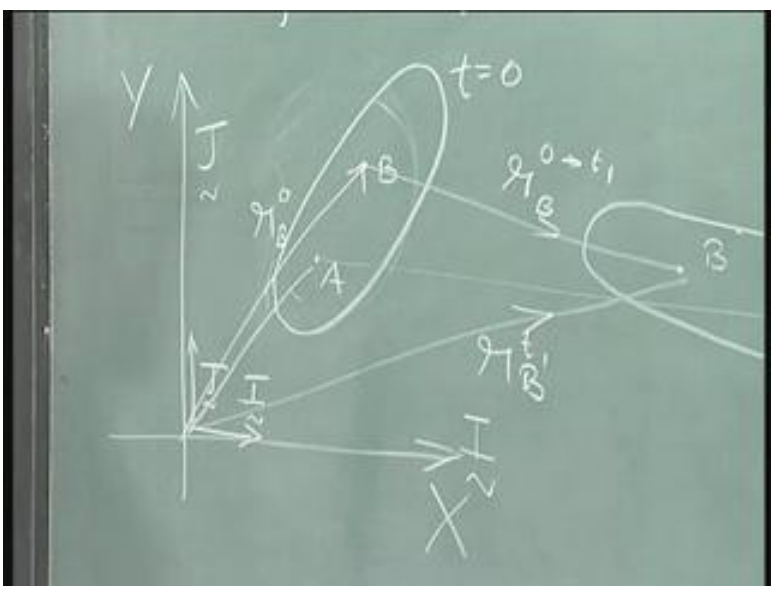

Therefore I can describe this particular point as well as this particular point. This is the motion that has taken place, this is the original, r_B not is the original position. In a similar way I will be able to find out r_A not which is this and r_A taken from 0 to time t_1 so that will represent the position of A at t_1.

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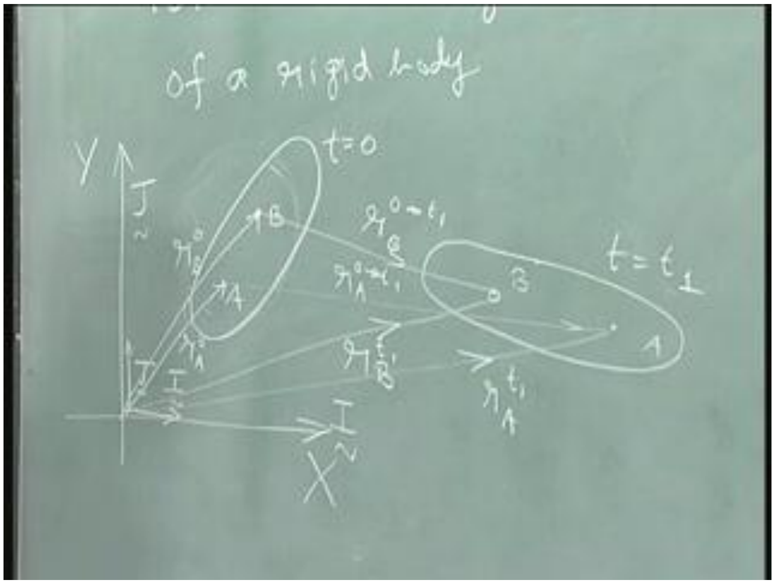

For now let's just make it simple. I just need to know the position of B. From the position of B then I find out the position of A, that's the question I am going to ask. Knowing the current position of B, how do I find out the current position of A? In order to answer this question, we did an exercise over there trying to understand how the motion has occurred. Basically I can look at this as a motion that has occurred, as if there is no rotation. Let's say something like this has occurred. These two are parallel to each other, let me just make it look as if it is the same rigid body.

This rigid body has undergone a translation. What do I understand by translation? This particular vector that represents the translation, if used here will tell me where the point A is. Let me call this as A prime, B prime. B prime is the same as B because I have used B as the reference for translation. If I use the same vector that takes it from this time t equal to 0 to t_1, for A I will get to A prime which is describing a translatory motion. Then as we did in the earlier exercise, if I pivot about this particular point and rotate I will reach this particular final conclusion.

Now I know how to find out the position. First I will locate the position of B. Once I locate the position of B then I will find out what is the position of A with respect to B. How do I write that? The position of A can be found out by position of B plus position of A but now with respect to B. What I mean is I can go from this point through this, through this to this or I can go to this point, go to this point and then translate to this. That's what I am going to do. That's what this means.

If I can locate the motion of point B and if I know what is the relative location of A with respect to point B, I have solved the problem of finding out what is the position of A.

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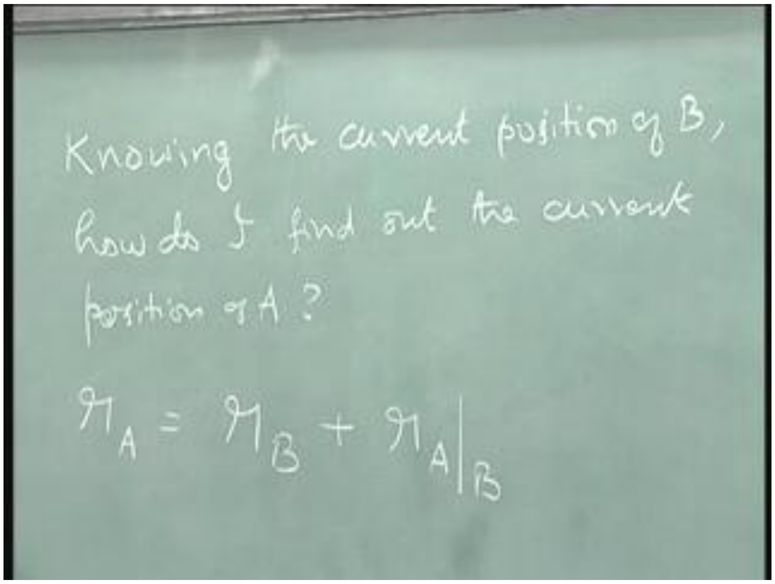

Now what is this going to be? I have already done a translatory motion, so this will not involve translatory motion, this will involve only a rotational motion. Is that clear? So once I introduce the rotational motion over here, I have solved the problem. Pictorially if this is the location of B, r_B I have put a vector symbol over here because this is the position vector. Given the position vector of B and the relative location of A with respect to B, I can find out the location of A.

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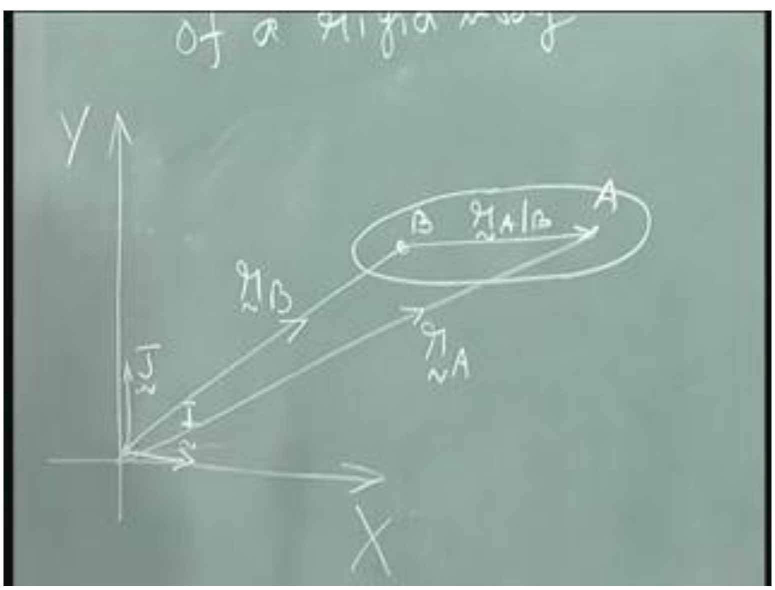

That's basically what this means. Now how do I find out the velocity of B? It is nothing but the time derivative of r_B. What is the velocity of A? It is the time derivative of A. So velocity of A equals time derivative of A. It's very important that I look at this as a time derivative of the vector. Similarly velocity vector of B is nothing but time derivative of the position vector of B. If I can find out the relationships between v_A and v_B, again I will be able to write down, given the velocity of B what will be the velocity of A. In this particular exercise if I do that r dot of A is nothing but r dot of B plus r dot of A with respect to B. This can also be written as v_A equals v_B plus v of A with respect to B.

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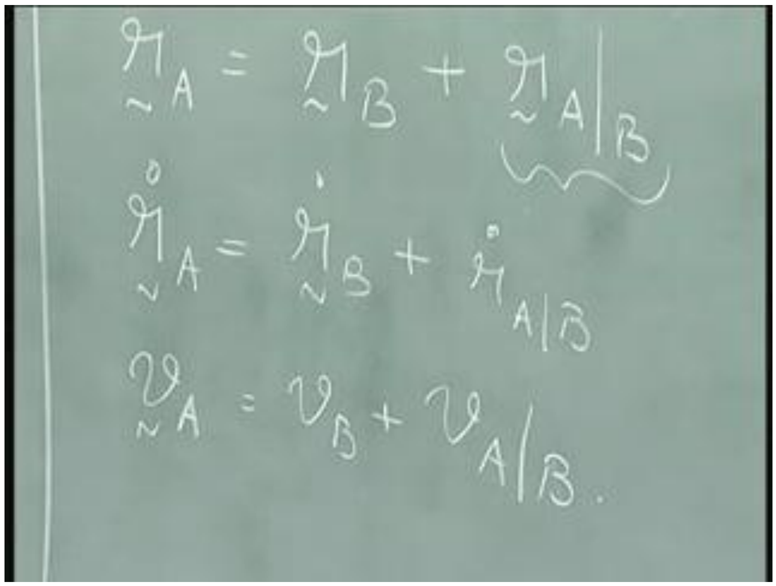

If it is a pure translation what do you know about this? Is A translating with respect to B? Let's look at that. Supposing if it is a pure translation which means that it remains the same. This is B, this is A, let's say it has gone through this by going through some other configuration. Let's say it's going like this and moving to this. What do you see as this vector? Remember this vector is not like a position vector. Is this vector the same as this vector as same as this vector? The answer is yes because it is a rigid body that we are looking at or in other words if it is going through a pure translation. The derivative which shows that there is a change that has occurred in this particular vector has to be equal to zero or in other words, the velocity of B and velocity of A will be exactly the same, if it is a translatory motion. Is this clear? Therefore this indicates that I first do a translatory motion.

Supposing if this were nonexistent, v_A is equal to v_B implies that there is a pure translation involved. Automatically that means this is nothing as to do with… this is just pure translation and this involves, supposing this were not the case it was something like this, like what we have done earlier. If I had taken all through like this and reach this particular point, it would have been something like this. Then after this I have to do a rotation. You can see that this involves just angular velocity. You have an angular velocity here and a translatory velocity over here. That's how we understand this.