Engineering Mechanics

Prof. Siva Kumar
Department of Civil Engineering
Indian Institute of Technology, Madras

Dynamics of rigid bodies

So having gone through many of those steps, let's see how we can use those steps. I am going to look at different categories of problems. Each category of problem will have a particular way in which we have to solve. I am not going to go through the complete solution methodology but look at the important clues that will help us solve the problem. Once we have those clues with us, it's easy for us to realize how to solve the problem. Now this is one category of problems. I have one rigid body here A B connected to two blocks. One block is constrained to move along this slot and another is constrained to move along this slot. Usually the velocity of this is given and we need to find out the other quantities. How do we proceed in this particular type of problem? In any of those problems it is important that we denote the fixed frame access. So in this particular case the convenient one is an access that runs through like this capital X, capital Y.

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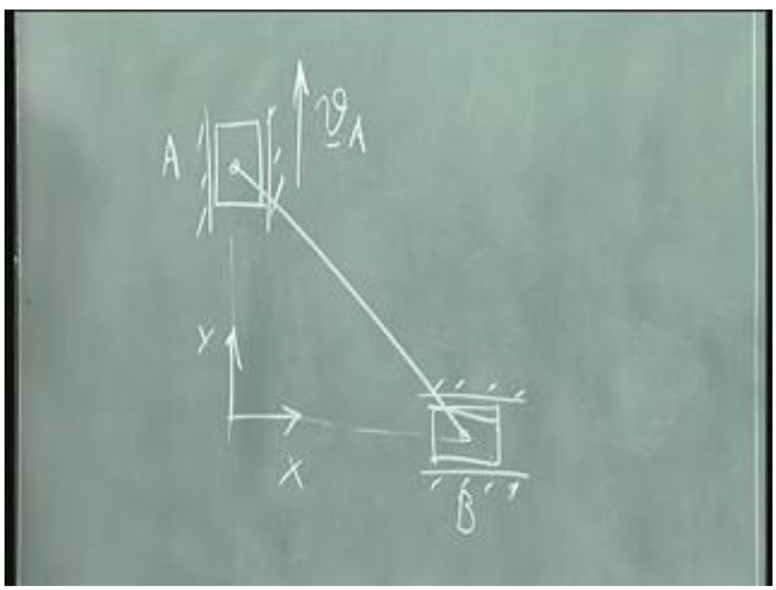

What is known here? The velocity of A. Is it completely known? The answer is yes because the velocity of block A is given and the direction is constrained along A. Now let's say we use this and write down the velocity of B. How does it appear? Velocity of B is equal to velocity of A plus velocity of B with respect to A. Another way of writing is velocity of A plus omega cross r of B with respect to A, either way is fine. What is known here? A is completely known. What else do we know? Do we know r_B with respect to A. The answer is yes. Let's say we have been given what is the location of this X_B and Y_A. As you know X_A is equal to zero Y_B is equal to zero and this location is where we need to find out let's say. Sometimes this is given, sometimes the angle is given. the length is given and the angle is given. Any two geometrical values are given, length and angle or X_B and Y_A. Given this, the geometry of the current location. Remember what I am using here, current location is given. What is this omega? Omega of this it is like this. What is r_B with respect to A? It is like this.

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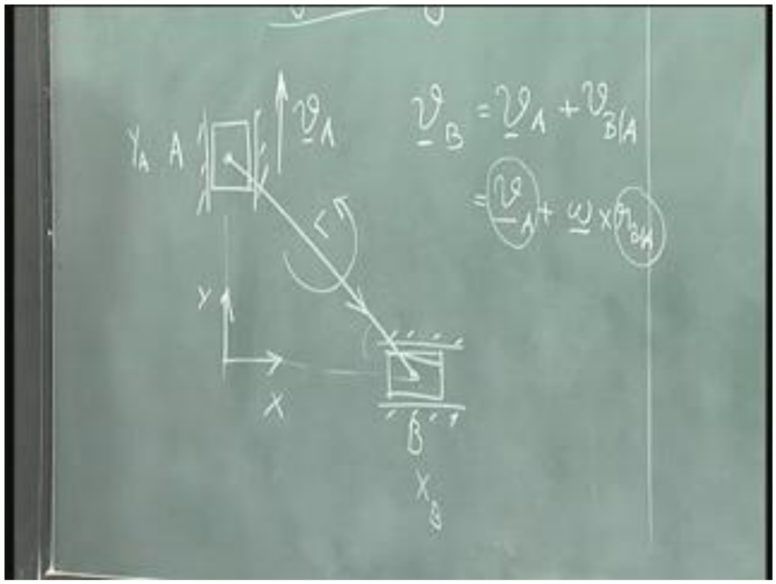

So r_B with respect to A is known, omega is known. The answer is no. We know the direction of this. Do we know the direction of this? The answer is yes. We know the direction of this. Do we know the velocity of this? Answer is no. So we don't know the velocity of this. We don't know the speed of B, we know the direction of velocity of B. We know the angular, the direction of angular speed but we don't know the angular speed. How many unknowns are there? Two unknowns. How many equations do I have here? I have something like this V_B equals V_A plus omega cross r, how many equations? Remember I can form two equations from this, one with respect to capital X direction. The other is with respect to, so if I write this in terms of capital I and capital J vectors, it becomes easy for you. What is V_B? V_B is a direction, direction of the velocity is already known which is say V_B which is the value times the direction is known to be along I direction.

Now I am just going to assume this direction. If I get a value that is negative for V_B then it will be in the other direction whereas for this I know the direction. Given V_A is some quantity that is given to me. So this is V_A that is given and it is directed along J, positive J that is already known plus omega cross r the direction of which is already known to be like this. Do I know this direction? The answer is yes because I know this direction and I know K direction which means this direction is known. So let's assume in this particular case if this is X_B and Y_A what will be the direction? What will be this direction? The direction of this if this is X_B and this is Y_A, the direction of this is given by the reverse of the ratios.

For now let me just put it as some I plus some J and this is the unit direction. I am going to call this as say, shall I call this as n direction where n direction is known. So this is a known direction time's omega times r; r is known, omega is what is to be found out. What else is known here? This has to be found out. Everything else is found. Now it's a matter of supposing I write this as just as let's say y_A I plus x_A J divided by y_A square plus x_A square. Is it plus or minus? We will get it to be a minus. If this is a minus direction, if this is the case I have some I component coming here omega r times y_A along I direction there is no I direction over here V_B. So I will get V_B equals omega r times y_A divided by root of or let me just call this as L. The total length is L.

Now y_A is known, L is known, r is known, V_B in terms of omega is what we have here but on the other hand if I had taken the other equation, remember J component is missing over here which means I will be devoid of B. So the best one I could have taken to solve would have been the direction along the J direction which means left hand side is equal to zero, right hand side involves V_A minus omega r X_A divided by L. This is a quantity that I already know, this is already known which means omega can be found. Once I find out omega I can find out V_B. I can do that. There are simpler procedures I am just giving you an idea that is in general solvable.

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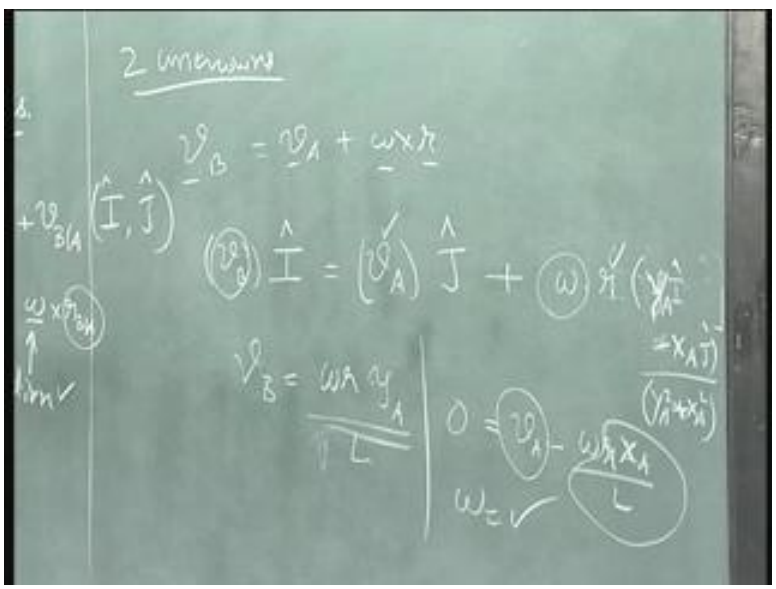

The simpler procedure is here in this particular case, remember the direction of this velocity is along x direction and this is along y direction. Do I know the relationship between X_B and Y_B? The answer is yes. Y_B and X_B are related as X_B square plus Y_B square is a constant equal to L. I can use that to find out the relationship between X dot B and Y dot A, simply by doing a differentiation and the relationships can be immediately found out. As you know it will be dependent on X_B, X_B dot, Y_B, Y_A, Y_A dot; Y_A dot is already known which means I can find out X_B dot immediately. That's the simplest solution.

I am just showing this because in general, it is easier to solve it. Now we looked at this particular problem in two different ways. One was writing it as V_B equals V_A plus omega cross r. The other one is looking at this. Since V_B direction is known, V_A direction is known. This is nothing but X_B dot and this is nothing but Y_A dot. I am looking at only magnitudes. I also have X_A square plus X_B square plus Y_A square is equal to L square and remember this length is maintained constant, it's a rigid body and therefore there is a relationship between X_B and Y_A which can be exploited directly in order to take the derivatives. So you have 2 X_B X_B dot plus 2 Y_A Y_A dot is equal to zero.

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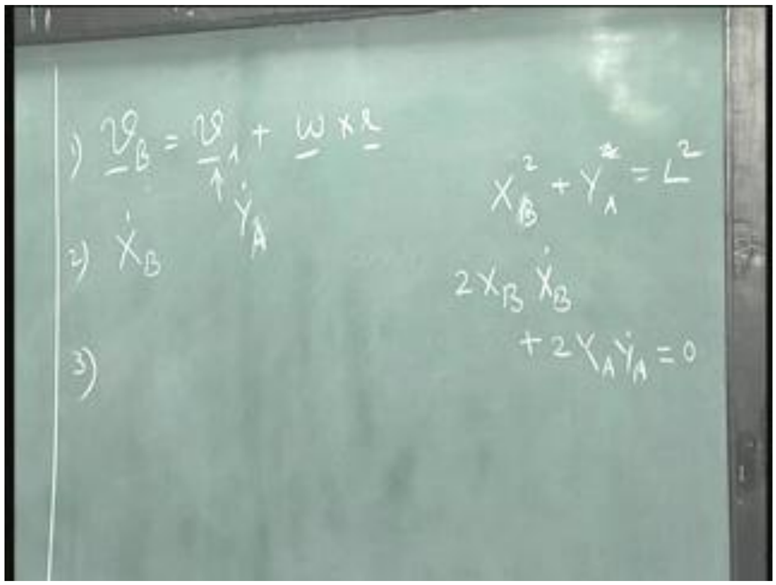

Now you can use this in order to directly get, what will be this particular thing. So that's the other method. There is also a third way of doing it which is often seen in many of the textbooks. Do I know the direction of velocity of A? The answer is yes. Do I know the direction of velocity of B. The answer is yes. So let's say this is the direction of A. Geometrically speaking this is the velocity of A. So I have this which is V_B. Remember this is perpendicular to this line A B. So if I have that perpendicular line drawn here so that say this is the line A B. If I draw the perpendicular line here V_A plus V_B with respect to A equals V_B.

Now this is perpendicular. I can simply do a projection in order to find out what is V_B. It becomes very simple. If I take the projection of V_B with respect to A on to V_A, the components have to match. That does not involve V_B means I can solve for omega cross r here. It's similar to what we saw here but looking at it as a pictorial set of quantities.

So this is the method of triangle of vectors. It can be triangle of vectors or method of vectors. We use the geometry in order to solve this problem. This is a vector method and this is a geometry method. Sometimes people use this method also especially when you know the directions of each one of those velocities, I know the direction of this, I know the direction of this and I know the direction of this and I know the angle. The moment I know these, it is possible for me to use triangle in order to solve the problem.

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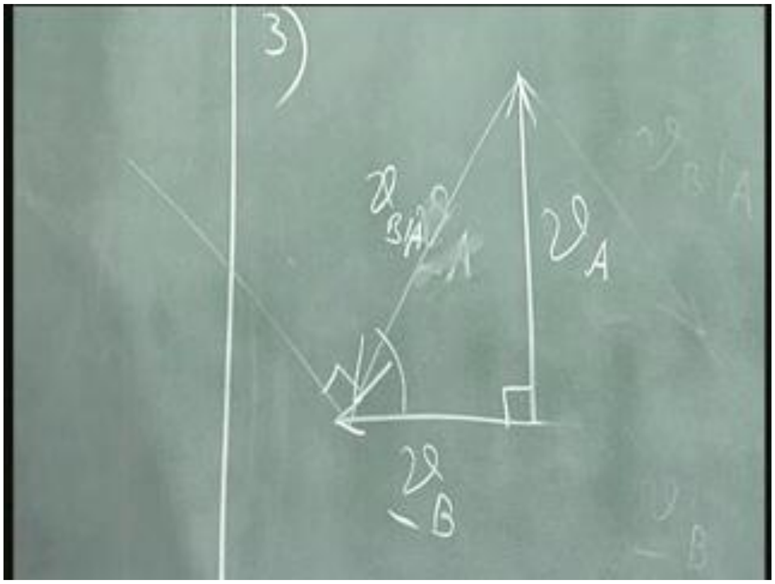

For example if I know the angle of this, I need to let's say I have the angle of this. Then I can say V_A by sine of this angle is equal to V_B by sine of this angle. This is equal to V_B with respect to A by sine 90. If I can get easily quantities that I need. Usually these are the ways in which people solve the problem.