Engineering Mechanics
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Dynamics of rigid bodies

We had five tips that we understand. Let's go to other tips, so tip number five. This is a very important tip, they will be very useful, tip number six? Okay, tip number six. I didn't realize five are over. Now sixth one has to do with what is called compatibility. When I look at system of rigid body, for example we looked at only the rigid body earlier in the first five tips. Sixth tip we are looking at systems of rigid bodies. I am going to explain to you, certain things that we have to understand related to that. For example this is a simple set of rigid body, two rigid bodies that we already saw in static's.

Now if each of these bodies are rotating like this, you realize; assume they are pinned all the time. They are pinned all the time at this particular point, this yellow point and they are moving. This goes with some angular velocity, this goes with some other angular velocity and they have different accelerations let's say. Now if we have to connect them, we connect through this point of the two rigid bodies which is going through the same motion. Supposing I have one rigid body like this, another rigid body like this. So let's say this is point A, this is B. They are pinned here B and this is C.

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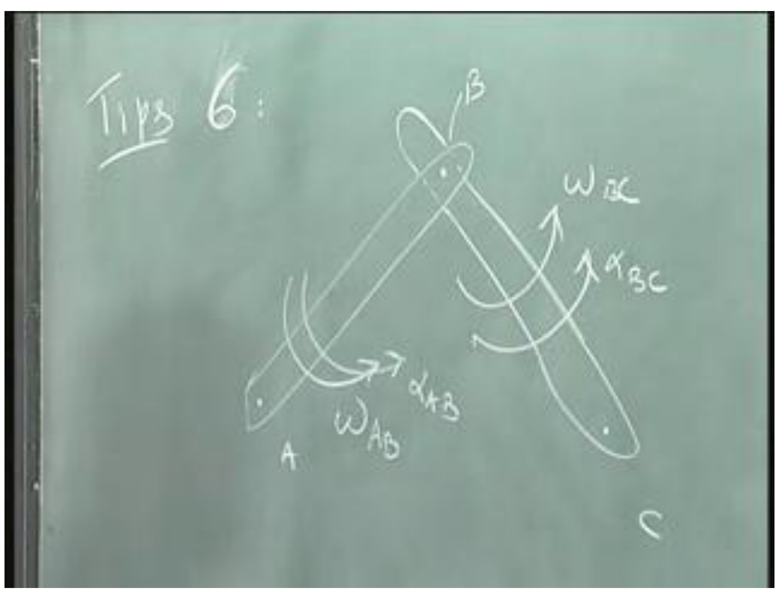

Let's say this is going through say omega$_{A\ B}$ and acceleration alpha$_{A\ B}$. Let's say this is going through omega$_{B\ C}$ and alpha$_{B\ C}$. Now what I know about this particular point is whether it belongs to this body or this body, the velocity of this has to be maintained the same. Velocity of B as I see from the body A B should be exactly the same as velocity of B with respect to the body B C. This is true of course also of $r_{B,}$ the position because that is the condition that we used in static's. This is the condition that we used in static's.

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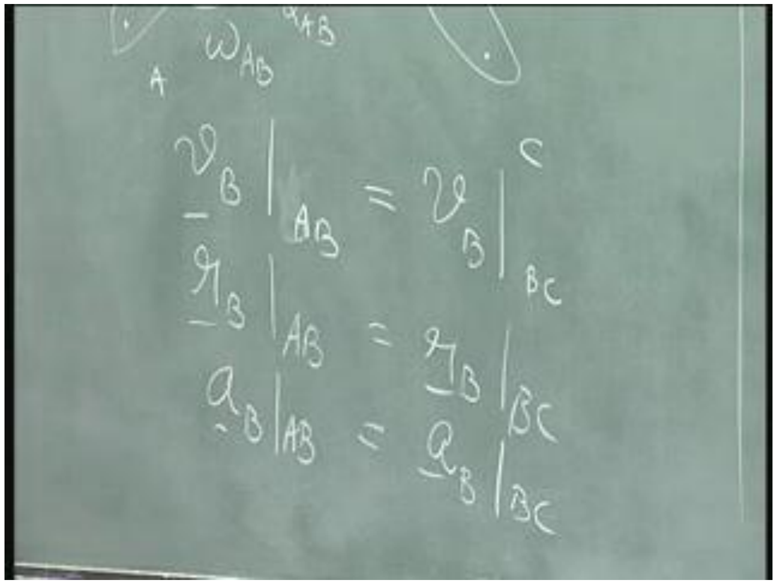

How about accelerations? It's the same. The motion of that particular point is same irrespective of which body we are looking at. This is very important and this is the one that connects the motion of this and the motion of this. In other words there is a constraint in terms of location velocity and acceleration when it comes to this kind of frame. We have different kinds of constraints. Let me show one more constraint. I may have a constraint like this for example let's say, let me do it better here. Let's say I have a block frictionless here, moving along a particular slot as shown here and let's say this is connected through a particular rod, say the rod is like this. Let's say A B. what have I done here? There is a restraint offered by this kind of slot in such a way that the velocity of A will be along this direction and that's important to note. That's very important.

One more thing if this is clear, the other thing that we already knew from a single rigid body is, so let me say these are about constraints. Supposing I have a body which is pinned here let's say something like this and let's say it is having an angular velocity and angular acceleration. What do you know about the motion of this point? Lets say this is A and this is B. I know two things, motion of A is zero. Whenever I put a hash like this, it is fixed to the fixed frame of reference at a point here and therefore the point A is not moving at all. Velocity is zero, acceleration is zero also the location is some constant location. How about B? Knowing that this is equal to zero, it is easy for me to write down the directions of velocity and acceleration of B. Velocity will naturally be like this. Acceleration as such will have a component like this, so this is velocity of B. Acceleration will have two components, one is the tangential acceleration in this direction and another like this which is in the radial direction or normal direction. What is this equal to? Omega square. If this length is r, it is omega square r times this direction. Similarly this is alpha times r times this direction.

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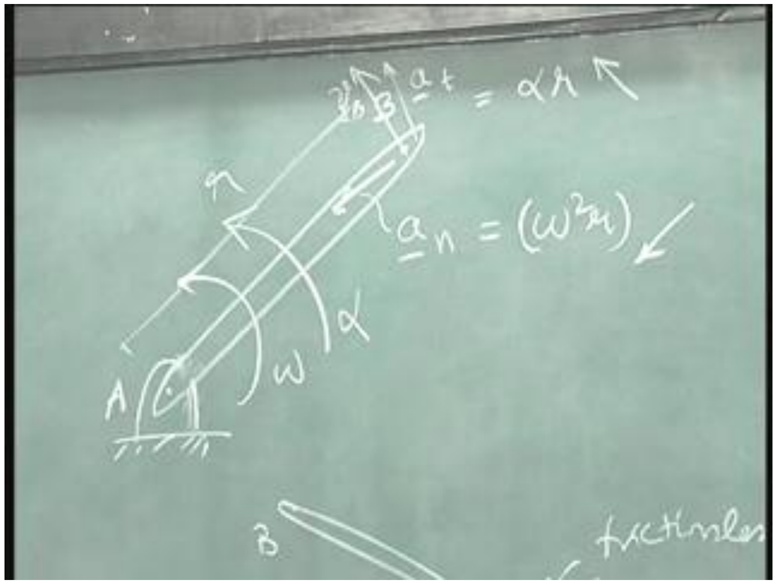

This is omega times r times this direction. So it's possible to immediately find out the directions of acceleration and velocity in this kind of situations. This is a special situation because most of the times you will have a motor that is rotating. The axis of rotation is what this is and there may be a rod or something that is transferring the power. Now this is often used in order to translate a rotatory motion to a linear motion. As you can see the velocity of A is fixed along a particular direction. It can be something like this, it can also be some; let me just do a little differently. It can also be something like this where this point A of this rigid body A B is restricted to move along a slot that is given. So once I know the slot, I know the direction of the motion, the geometry of motion is already specified. I can use the geometry of the motion in order to solve the problem but these two are very critical. Usually you will see a rotation like this with an angular acceleration or velocity or translation of rotary motion to a linear motion. Most of the reciprocating type of terms will have this kind. These two are very important to understand, this is the tip related to this.

Let's go to another tip. This is also a related tip but it may be worth understanding it separately. And that is about what? Wheels. We did this exercise to a certain extent in friction. We are revisiting in this particular case. Let's say I am going to have a purely rolling wheel. Means what? I have something like this and I have a wheel over here. This wheel let's say only does rolling. There is no; what does that mean? That means that if I take this particular point, let's say this is A prime and this is A. A belongs to the surface on which it is rolling and A prime belongs to a point on the wheel. At that point of contact, the velocity of A will be the same as velocity of A prime. Velocity of A will be the same as velocity of A prime. Vertical velocity depends on the velocity of the surface. Supposing if I take two components, one component like this and one component like this. If it is a rigid surface, velocity in the vertical direction is equal to zero. If it is a surface that is moving, then the velocity of these two points is the same as the velocity of this surface which is moving or this wheel that is moving.

The other velocity of this surface and this surface will match at that particular point. Not only velocity, it will also be the acceleration. But acceleration I am going to write, acceleration of A component along this direction which is perpendicular to the point of contact is the same as acceleration of A prime. Supposing this is a rigid surface. The acceleration of the rigid surface is zero, so this is equal to zero. The component which is perpendicular to the point of contact. So this is the perpendicular direction, that direction acceleration is equal to the direction acceleration of A prime and these are the conditions that we use in order to solve the problem. What is the special case? Special case is when the surface is attached to fixed frame or in other words, the surface is not moving which automatically means that $V_{A\ prime}$ equals zero.

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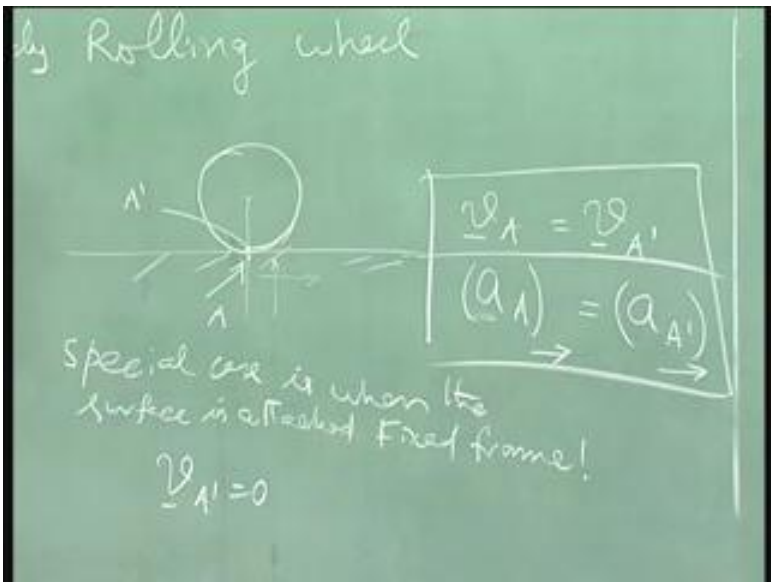

The velocity of this contact point of the wheel is equal to zero, I am sorry, A prime. A prime is the point of the wheel, A is the point of the rigid surface. Since the point of the rigid surface is not moving at all, V_A prime is equal to zero. Remember the vector that I have indicated here. Acceleration of A prime along this direction, this component is equal to zero.

So when is this valid? When there is a pure rolling and when this surface is fixed to the fixed frame of reference. If not if this surface is moving, if you know the velocity of this, acceleration of this perpendicular to this then you can enforce these conditions. These are again constraints. As you can see these are the constraints for a system of rigid bodies. You can see another constraint over here, this is another constraint that we have introduced over here for a wheel. Most of the problems that you encounter will belong to any of these categories or a mixture of these categories. Knowing this is now very easy for you to cast the equations which you can use to solve for the kinematics of different sets of rigid bodies. Now there are few more tips.

Let me just go to one more tip before we enter into solving problems. This is also an important tip which has already been told earlier but I will repeat this over here so that it's easy for you. Tip number eight, it's a simple tip. I am going to look at only one rigid body. If there is a rigid body like this, this is point A, point B. Supposing this is the direction of the velocity of point A. The same is the direction. So let's say this direction is let me call it as n direction. Let's say this is along n and let's say this is also along n. We can make a few conclusions from this which are very useful when solving problems. The first conclusion is A B undergoes a pure translation along n. it's not very apparent here unless I state the next one because these two are connected. What is the angular velocity of A B?

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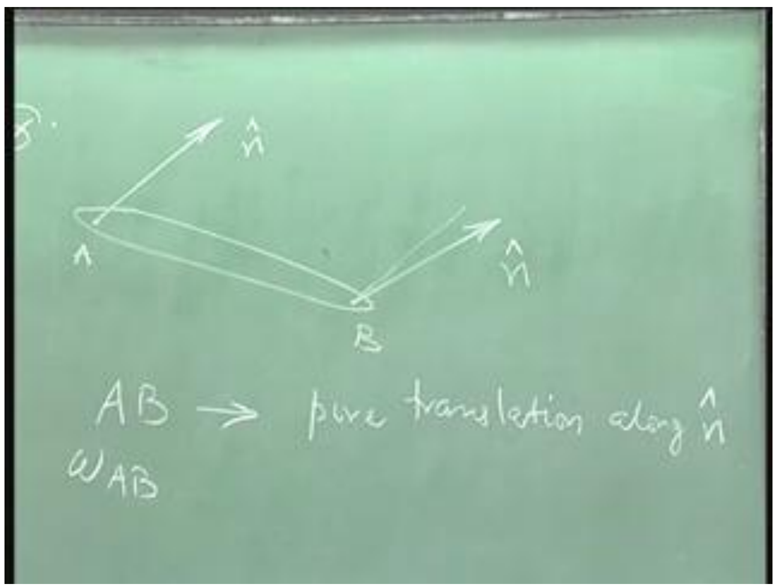

Remember velocity of B is equal to velocity of A plus this. In order to understand this, I have the other tip that is useful; velocity of A and velocity of B have to be equal for a pure translation. They have to move like this. This is very important. If it is a pure translation, you will notice that the directions are the same but velocity of this and velocity of these, directions are the same. Number one, not only that; the velocities also will be the same because it is a pure translation. But because of this pure translation, $\text{omega}_{A\ B}$ is equal to zero. It's a very important conclusion because this is used in many of those problems that you will encounter in your exams. So from an exam point of view this is very useful. Understand this very clearly. The direction of this velocity and the direction of this velocity should be the same. In that case, I can do make this conclusion that the translation is the only thing that happens and remember in many of the problems, this may be momentary. At a particular moment the translation may go through only a pure translation, at a particular time t and therefore you cannot do the same thing for acceleration.

Remember if the acceleration of this and the acceleration of this are the same, there are two components that will go to zero. If the directions I am talking about are the same, there is one direction that is along A B and another direction that is perpendicular to A B that could act as an additional contribution and therefore the acceleration of B and acceleration of A could have same directions but may have different magnitudes. This holds good only for the velocity. Please remember this clearly.

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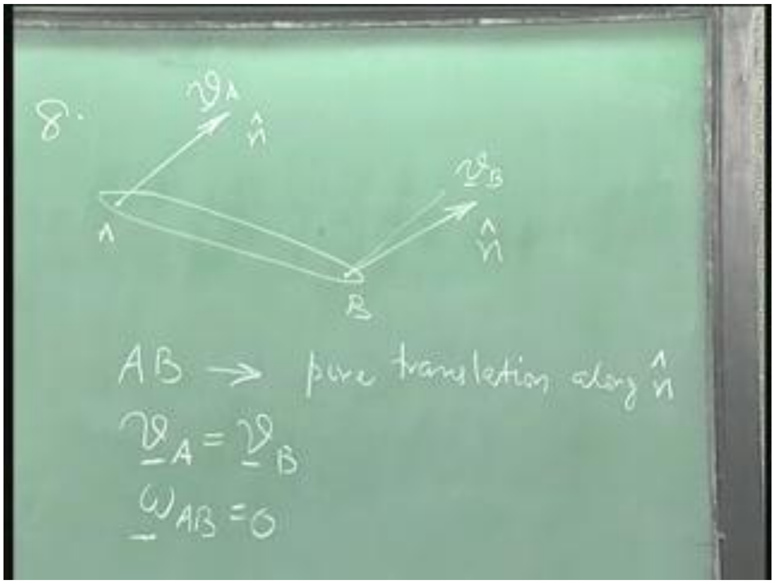

I think it was vithugito who asked the question. Tell me what is it that I should know so that I know everything. So like that tell me what are the tips I should know so that I can solve all the problems in planar kinematics. I think I have exhausted all the tips that you will need in order to solve problems in planar kinematics. Planar kinetics is a very simple translation and we will take it up in the next module.