Engineering Mechanics
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Statics
Systems of Rigid Bodies
(Bag of bricks to use)

Now it's time to give you a bag of tricks that you can use in order to solve the problems easily. I am only going to focus on static's of systems of planar rigid bodies. Let's go to the first trick. I am going to call it as first trick, trick number one. Let's say I have a particular rigid body and I have forces all around and there is a moment. Let me just draw so that it is easy. Let's say F_1, F_2, F_3. There is a moment here. I have just drawn some moments that may appear on this. I wish to find out the resultant force and moment. One of the important things to know is one of the problems that we will face is which point should I take in order to find the resultant. The answer is any particular point it can take. There is no difference as such because every point of the rigid body will be stationary. I can arbitrarily choose a point A, find the resultant force F and the resultant moment MA so that I can write these two equations.

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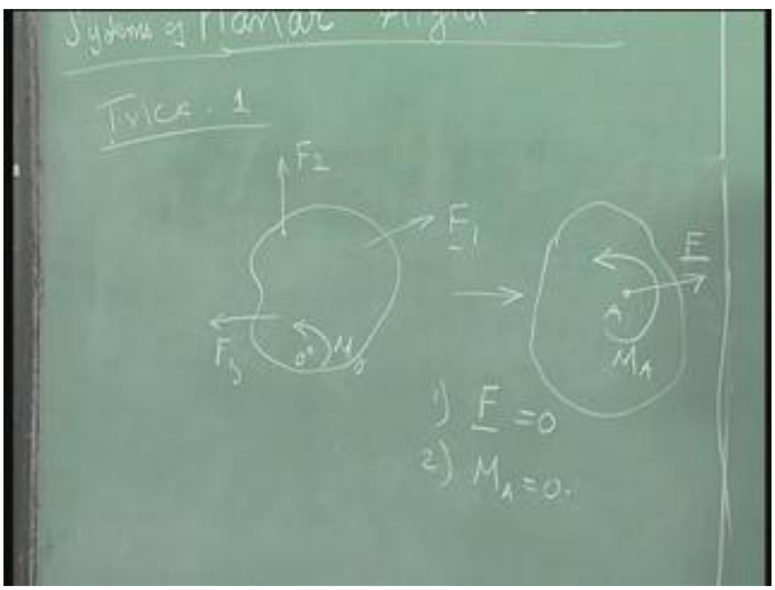

Now some of the points to remember here is can I instead of looking at this, this is force at this particular point is equal to zero and this is a resultant force and the moment about this particular point A is equal to zero. Now the question arises, supposing I take some other point and let's say I found out the resultant moment and that is let say M_B. Can I add the other equation and say M_B equal to zero. Remember there are two equations that I can find from here which is x component and y component of resultant force equal to zero, M_A equal to zero. Supposing I say, I am going to take another point here. I know that this point is also stationary and therefore the resultant force is going to be the same. But moment will be different, M_B; can I add that say I will have the fourth equation. Can I do this that's the equation? The answer to this is very simple. If I look at this particular point B and try to find out the moment given that this is the resultant of forces at B, it is nothing but M_B is equal to M_A plus the moment of this force with respect to B.

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So let's say this is r_A. This is equal to r_A cross F. We already have F is equal to zero which means this will go to zero implies this is equal to zero. This is equal to M_A, I already have M_A equal to zero. This means this is equal to zero or in other words given that M_A equal to zero and F equal to zero, natural result is M_B is equal to zero. This cannot be an independent equation compared to these two. This is one thing that we have to understand which automatically means that I will be able to get only three independent equations that I can solve.

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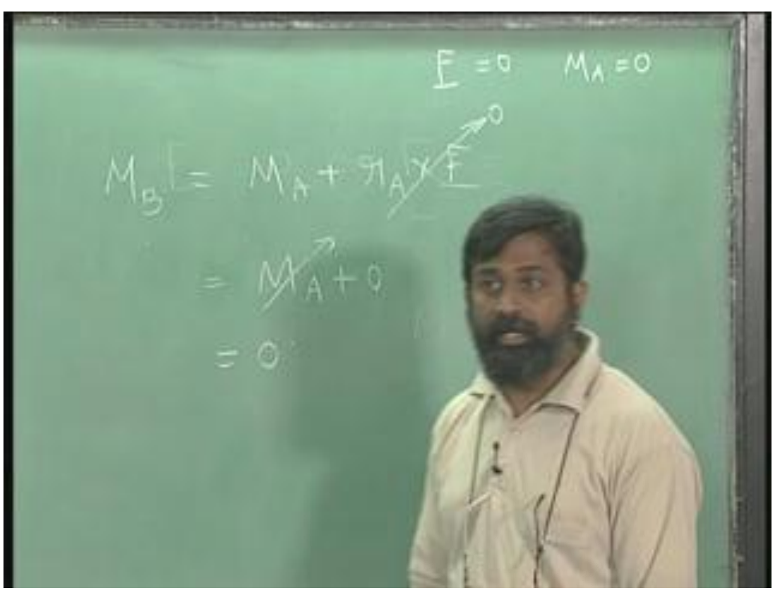

Now let say I have a force on a body and I seek to find out the three equations that are needed in order to solve this problem. Let say these are the forces that are acting on this. Like before we can choose a particular point and go about doing it. We will get F equal to zero and let say there is a moment over here. We have M_A equal to zero. Now if I have to ask this question, can I take lets say M_A equal to zero, I will choose another point B and say M_B equal to zero. These two together, if I take only these two equations is this going to be valid. The answer is not fully, I need to take one more equation, lets say point here c and then find out the moment of the resultant force and then said each one of them equal to zero. Can I reproduce the solutions that I get for this or in other words, let me just repeat this. Can I rewrite these set of equations in to this set of equations? The answer is yes provided if I just connect, let me just erase all this M_A, M_B, M_c so that it is easy for you to understand, provided these points A B C that I choose are not co-linear and that this resultant force is non-zero.

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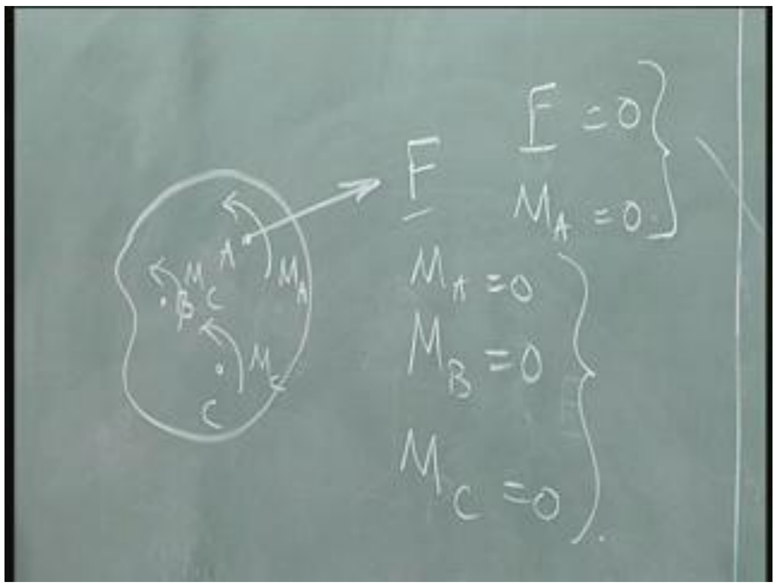

If it is zero then what will happen is some of these equations will give zero equal zero which is the same that you will get here also. If this resultant force is equal to zero, we will get zero equal to zero or in other words, if I have to replace an equivalent set of equations, it is possible to use only moment equations provided you choose three points that are not lying on a single line like this. Suppose take a point here C prime which is co-linear, I will find that I will reproduce the equation that I got for B and C, a combination of A and B. As long as you do this it's easier. Sometimes it may be beneficial to use these equations compared to these equations. I will show those in the examples.

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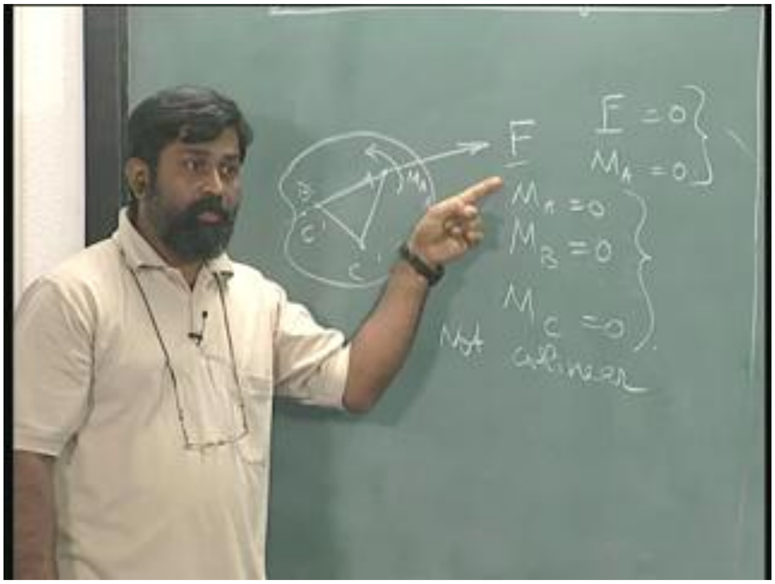

Let's take a simple example meaning simply supported beam let say. Simply supported usually is given by this. There is a pin support here or hinge support there and a roller support here. Let say this length is L, there is a force acting on this let call this as P. The aim is to find out the reactions say at A and at B. We will use this system of equations that we get from equilibrium equation in order to solve this problem. The first exercise that we have to do is draw the free body. Remember this is a single rigid body that we have here. So it is not going to be difficult for us to draw free body of this. Let say this distance is 2 L by 3 and this distance is L by 3, removing this I will have two reactions. One reaction like this and the other reaction is like this. Let call this A_y and this as A_x. Another reaction over here, since it is a roller support it does not restrain moment alone x direction and therefore let say this is B or B_y and this is a force p.

Let's examine the number of unknowns. We have A_x is an unknown, A_y is an unknown, B_y is an unknown. Given p we need to find out this. How many equations can we generate? There are three equations we can generate which means that we will be able to solve this problem completely. Let's try to solve this problem. The first exercise is if you look at this carefully there are three vertical forces. The next thing that we do usually is examine the types of forces that are present in this particular rigid body. We have three vertical forces, there is only one single horizontal force. The first exercise I can do is I can write the summation of all components on the rigid body along x direction is equal to zero that implies A_x equal to zero. This is the immediate result that I can get.

What is the next step? I could probably go for the net force in the y direction is equal to zero that will involve A_y and B_y. This is where we use a simple trick. Instead of going for vertical component being equal to zero of the net force on this, let's look at any other option. We have the moment equation that is possible. I can take a moment about this particular point which means it will involve A_y and B_y because they have a distance from this particular point. But instead if I take a point here on A or on B, I know that the moment of A_y at A is equal to zero or if I had taken this particular point, the moment of b_y is equal to zero at this point. So I will use either of these points and write the moment equation. Let me write it over here.

I am going to write sigma or summation of all the moments at this vertical point A is equal to zero. It automatically eliminates use of A_y as well as A_x, only P and B_y will come in to picture. Let me take anti clockwise to be positive. This helps in writing it out clearly. Again this exercise is simple. I just hold this, look at what kind of moment this will create. p will try to push it down and therefore will introduce a clockwise rotation. Anticlockwise is positive and this distance I already know is 2 L by 3 and therefore it will be minus P times 2 L by 3 is the contribution of P for moment at A. The other one is to do with B_y and that is at a distance L. I hold again pivot at this particular point A and c, what is the direction contribution of B_y? It is anti-clockwise like this and therefore this is plus B_y times L and that is equal to zero because of the condition that the rigid body is stationary. Remember in this particular case this is known, these are known which means I can immediately find out what is B_y. L will get cancelled, it is nothing but 2 P by 3.

The next step is to find out A_y. I can do using vertical forces equal to zero or moment about B equal to zero either one of them will give the answer. But the key point that I am trying to make out here is try and find out a point about which you can take a moment in such a way that you have the least number of unknowns that take part in the equation. If you do that then it is easier to solve for reactions and that is very simple. If this is 2 P by 3 and this is p, vertical equilibrium will automatically give you A_y equals P by 3 because A_y plus B_y equal to p. Thank you.

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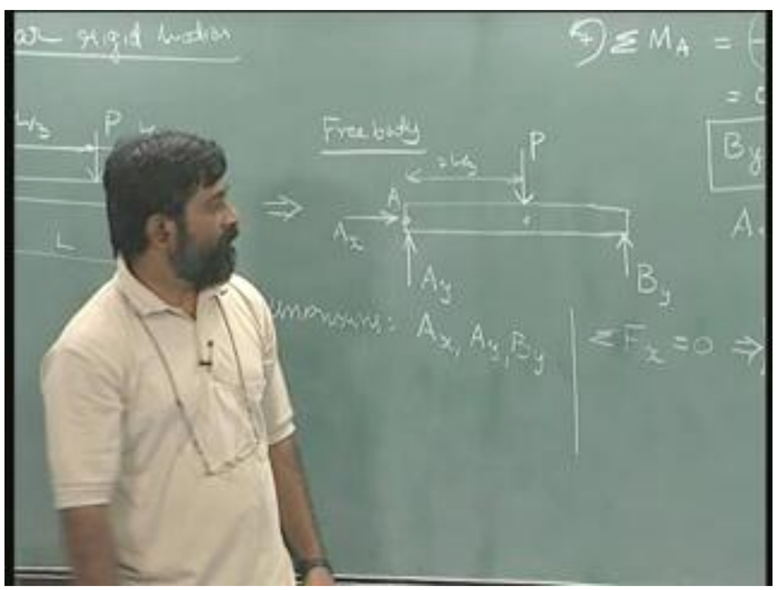

Now let’s look at one more trick or tip that you can use before solving for systems of rigid bodies. Supposing I have rigid body, I just make it a rigid body of this sort and lets say these are the two points this is A and B on which some forces are acting. Let say this is F_1, this is F_2.

Now one of the immediate simplifications that can be done as far as this particular rigid body is concerned is like we did earlier, if you look at it we have force here and force here and if we want to find out the relationships or to solve for let say F_2 or F_1, we should take moment about A or moment about B and solve the problem. In this particular case let's say moment about A is equal to zero, if this has to be stationary. Then what do we get here? We are going to just change this problem a little bit and see whether we can get some simplification. Instead of taking this let me take a component of this which is F_{2x} and component like this which is F_{2y}. Let's look at the moment contribution of these two components of this force. The immediate thing that we find is since this x, let say x is along this direction, if x where along AB if I take AB and set that to be a x direction then I know if I take moment about this, this particular F_{2x} will not take part in moment. The only force that will take part in the moment is F_{2y}.

Therefore this will result in let say this length is AB, this will result in F_{2y} times AB equals 0 where AB is the length. This automatically means that this is equal to 0. By similar argument if I take moment about this, it is possible to show that the horizontal component or the component along AB which is F_{1x} and F_{1y} are the two components that we can split and we can immediately say that this is equal to 0 or in other words I can simplify this as a problem where if this is A, this is B, I will have forces acting on this way and this way let say this is F_1, this is F.

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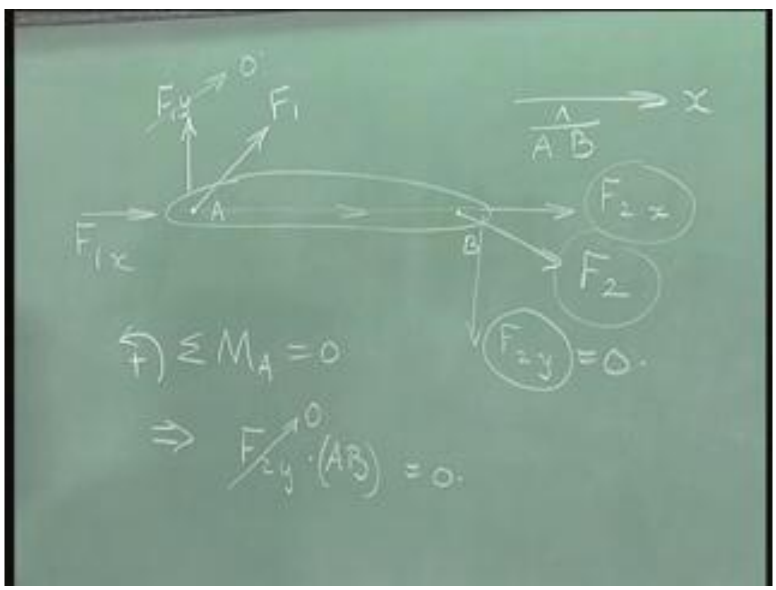

If I use horizontal equilibrium along AB, I can write this to be minus 1. I have used all the three equations in order to come up with this particular conclusion. In other words this is equivalent to saying that if I have a body where forces are acting only on two particular point AB then the forces will act along that line AB and let's call this as F_{AB}. Now this body that I am talking about need not be one single direction like this, one straight member like this.

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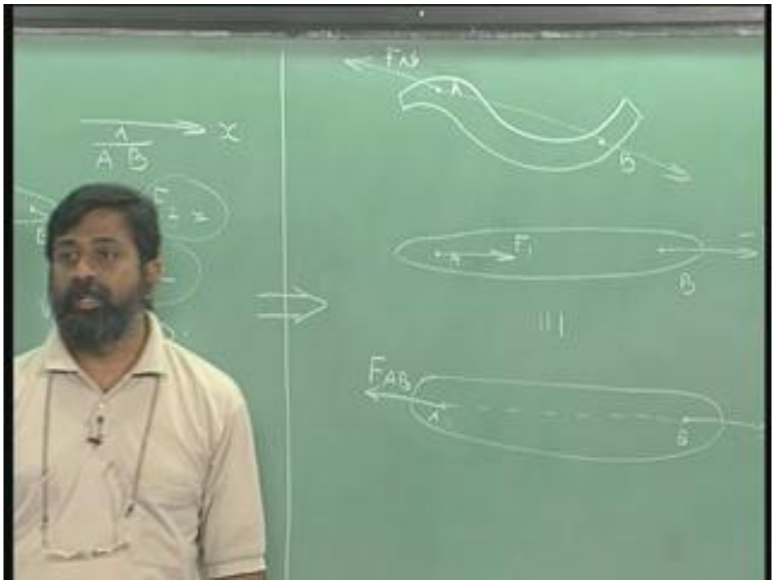

Let me draw it over here, I can also have something like this, let say this is point B, this is point A. If I know that the forces are acting only at A and B, I can just connect these two and say that the resultant forces on this particular rigid body will be acting along this direction. If I have straight member like this, straight body like this, we call this as an axial member. If we have something like this, the conclusion is that it is along AB that the force resultant will be finally acting. This result is very useful when we come to systems of rigid bodies and examples of system of rigid bodies being trusses, beams and so on.