Engineering Mechanics

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Statics
System of Planar Rigid Body
Example-2

Let's look at another problem here. Let me just explain the problem first and then we will first draw the free body and seek to solve the problem. I will just explain to you, how you will go about solving the entire problem. There is a particular rod here A B, let me call this as E. So A E B that is pinned at A to the fixed frame, as you can see here it is hashed. This is a fixed frame. It is pinned to a body which is L in shape, B C D pinned to A B at B and it is pinned to a wheel which is resting on A E B at E and this wheel is just standing at E. Now this B C D is prevented from moving at C and this is the problem. We wished to find out all the unknowns related to this. When we say we have to solve the problem, we need to solve for the reactions from the fixed supports and the reactions that appear as an interaction between the rigid bodies. How do I go about doing this? The first exercise is to understand that there are three rigid bodies in this.

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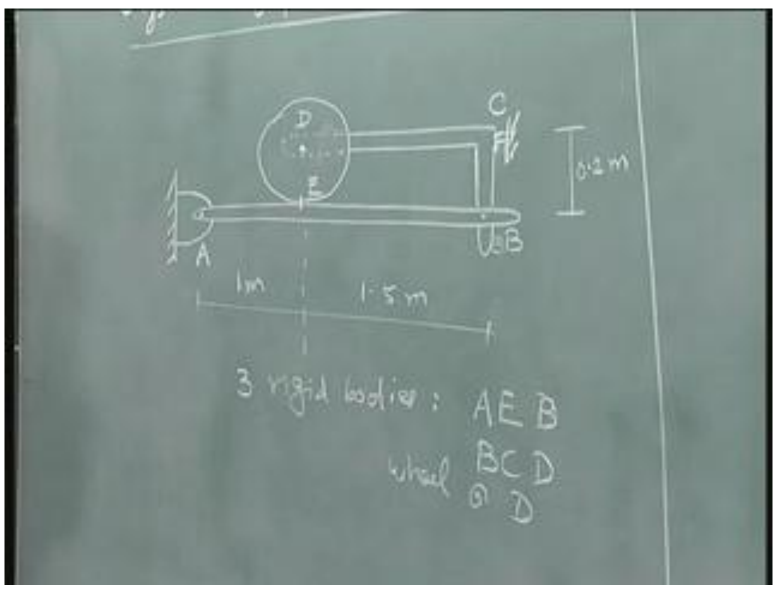

One rigid body is this A E B. The other rigid body is B C D and the third rigid body is this particular wheel which is hinged about D. There are three rigid bodies, this will help me draw the free body diagrams appropriately. One is A B, you can either call it as A E B or A B. The other rigid body is B C D and the wheel D, the three rigid bodies that we have. What is the first exercise that we need to carry out? We need to remove the restrains that appear due to fixed frame of reference. So let's do that particular exercise to start with. Now I am going to just shift to simple line diagram. I am just going to draw this A B as a line, B C D as an L and a circle for a wheel. This is a point A B, we have like this which is BCD and you have a wheel. Now what all have we removed here? We have removed the support that restrains motion in the horizontal direction which means I should have a reaction from the fixed frame on to this system of planar rigid bodies. So let's call this as C or C_x along the x direction. So we are going to take this as x direction and this as y direction for our own convenience.

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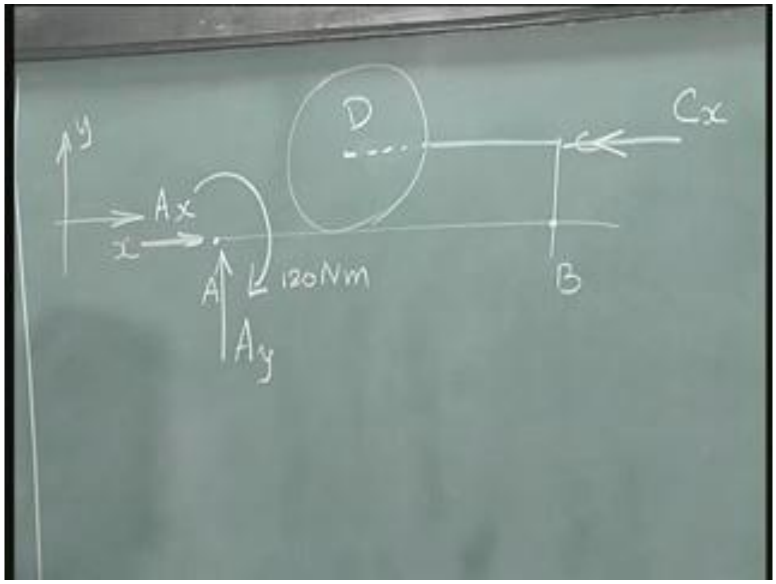

At K it is pinned here which means the fixed frame of reference doesn't let it move with respect to the fixed frame of reference at A, both in the vertical and the horizontal direction which means there will be two forces that will be needed by the fixed frame of reference in order to restrain the motion. Let's call this as A_y and A_x. So we have 3 unknowns A_x, A_y and C_x which are to be found out from equilibrium equations. In addition in this particular problem it is given that there is a moment acting here. Think of some motor that is applying a torque of 120 Newton meter. I need to transfer that over here. There is a 120 Newton meter that is acting external to this. Assume all the rigid bodies are negligible mass compared to the 120 Newton meter that is applied. The first free body concerns this entire system without the fixed frame of reference and it contains all the three. Then we seek to draw the free body of each one of them separately.

Let's start with the wheel because that seems to be the simplest one. This is the wheel D that we seek to draw. What happens here as we have discussed earlier, in order to draw the free body of this wheel, I need to make sure that I fix A E B and B C D the two other rigid bodies fixed to the fixed frame of reference which means A E B and B C D are not moving at all. In which case this point D of B C D will restrain this wheel from moving in x as well as y direction which means I will have a reaction that appears on the wheel as D_y and D_x that is responsible for preventing motion from x and y. Apart from that you already notice here that the wheel is resting on A E B which means there is a normal reaction equal to, let's just call this as N E or since it is in y direction, we will just call it as E_y. We have a question here. Will I have a horizontal reaction over here? I don't know. Let's say it is not frictionless, it has friction. Let's just introduce E_x for now.

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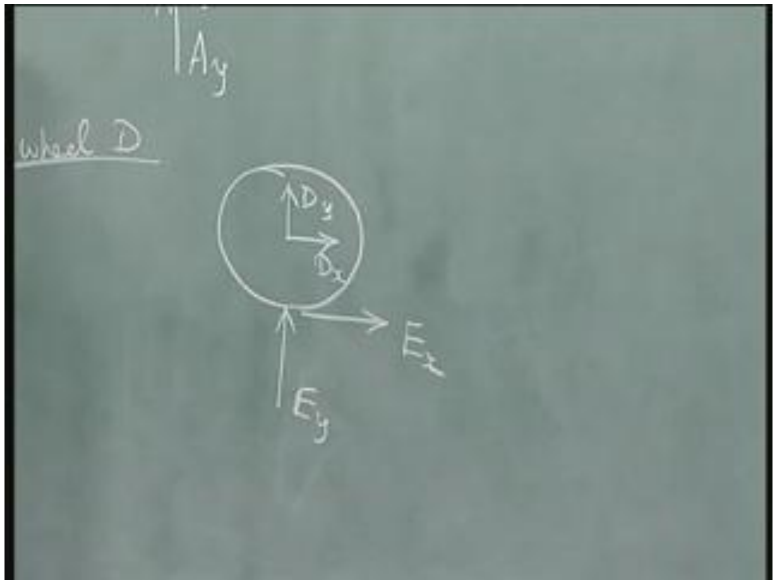

There is a resistance in the x direction. we will come back to simplification as far as this wheel is concerned in a while. Have you missed out anything else? the mass of this particular wheel is negligible which means there are only two points that i have to consider. One point of restrain coming from B C D and the other was one is due to resting at B. so this is complete as far as wheel D is concerned. Let's seek to draw the other two free bodies. I am going to use line diagrams in order to make it simple. Now let's just look at this particular B C D. If you look at B C D, we have to fix A E B and the wheel to the fixed frame of reference and have a look at what will be the restrains that will be offered. There are three points at which restrains will be offered on this B C D, one is at B by A E B. The other is at C due to fixed frame of reference and the other one is at D due to this wheel remaining unmoveable and therefore I will have a reaction.

For now since we already know that we have drawn D_x and D_y in this direction. We will take equal and opposite directions for this point D, this is D_y D_x. We have finished this. Looking at this there is a restrain offered here. So we will just stick to that. At B we will have a restrain in the vertical direction and therefore a force appears. There is a restrain in the horizontal direction which means there is another force B_x that appears. Are we complete? The answer is yes. The mass we have here is negligible which means we have completed the free body of B C D.

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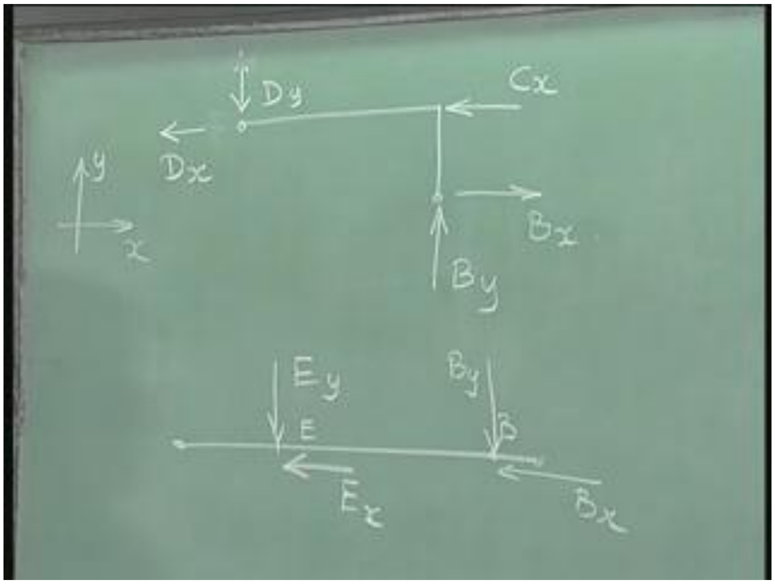

This is x direction, this is y direction. What remains now is A E B. Again I am going to resort with line diagram, it makes it simple for drawing. At B remember I am going to fix B C D as well as the wheel E D which means at this particular point, I will have an equal and opposite reaction which means I will have B_y here and B_x like this. Simple. At E there is an interaction with the wheel and therefore I should take the opposite signs of this E_x and E_y and insert those as the forces, so E_x and E_y. This way we can avoid adding more number of unknowns, we take equal and opposite reactions automatically in this. At A we have already drawn for the system of rigid body. I will have A_x and A_y appearing. We have drawn all the free body diagrams here that of wheel, that of B C D and that of A E B.

The next exercise is to find out what all unknowns do we have to solve. Let's look at that. The simplest one is this. I am going to start with this particular wheel. Let me draw that again here. Now we are looking at the unknowns. For the wheel we have D_x and D_y, E_x and E_y. We already accounted for this, we have C_x that needs to be solved and we have B_x and B_y.

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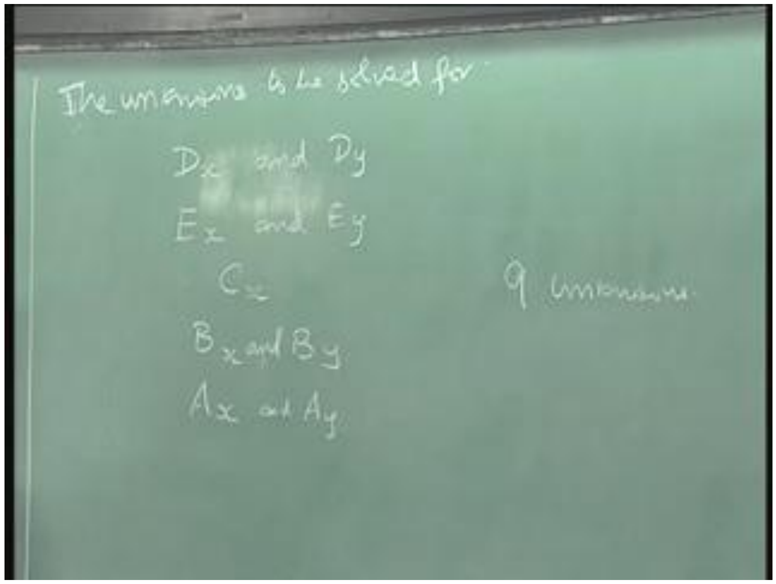

For this body we have to solve for A_x and A_y, we already accounted for E_x, E_y, B_x, B_y which means A_x and A_y. In all how many? 2 plus 2, 4 plus 4, 5 plus 4 is 9 unknowns. Can I write down 9 equations? The answer is yes. Because I have 1, 2, 3 rigid bodies. Each can generate 3 equations and therefore I can solve for the 9 unknowns. Now if I do it this way, it will be a long procedure.

Let me look for certain simplifications that I can do. So simplification one. If we examine the wheel, let me draw it over here. You have D_x, D_y, E_x, E_y. These are the only forces that are acting. If I take moment about this particular point and immediate result is that E_y is along the vertical direction that passes through D. D_x and D_y don't take part in that moment equilibrium which means only E_x takes part in the moment equilibrium. Therefore if I take moment about D lets say positive, there is only one force that results in that. Let's say this radius is r, we already know that this is 0.2 meters. We get it is anticlockwise in its motion E_x times 0.2 should be equal to 0. It immediately gives me E_x equals zero.

There is no horizontal force that appears on this. This is clear? Now if E_x is equal to 0, sigma F_x equals 0 will immediately tell me that D_x is equal to 0. You get this? I am just going to erase it to make it simple, this is not there, this is not there. Like what we did earlier, for a rigid body where there are only two forces two points at which forces are acting, immediate answer that we get here is D_y and E_y are opposite to each other.

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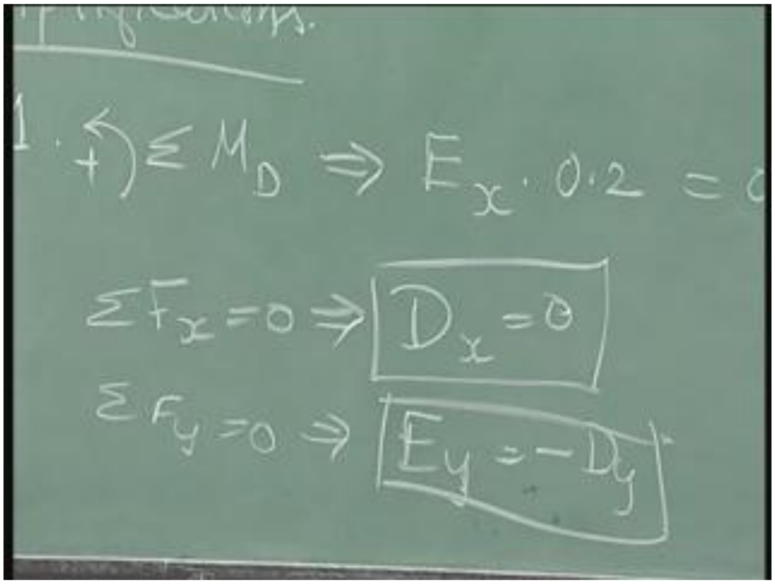

So sigma F_y equals 0 gives me E_y equals minus D_y. I need not now look at all these four. I just need one unknown, either E_y or D_y and that will reduce the unknowns in the other free body diagrams. You have D_x and D_y here, you have E_x and E_y over here. So let's seek to now remove those so that we simplify. Please remember these simplifications will help us solve in a much better way. I am going to remove this, I am going to call this as E_y equal and opposite. So I am going to remove this and shall we just take this instead of E_y, we will just put it as normal reaction. That's the reaction that we see from the ground for a wheel. There will be N acting on this. Here also I don't have this E_x, this is nothing but the N acting from the wheel. Now we have reduced it.

We don't have this anymore, this anymore, instead it is replaced by a single unknown N. Now instead of 9 equations, we have 1, 2, 3, 4, 5, 6 equations to be solved. Mind you in doing that we have already used up all the three equations necessary for the wheel. Going back to this, there are 1, 2, 3, 4 unknowns present in this. 1, 2, 3, 4, 5 unknowns present in this. In order to solve for this, what would be the points we take in order to write down the moment equations.

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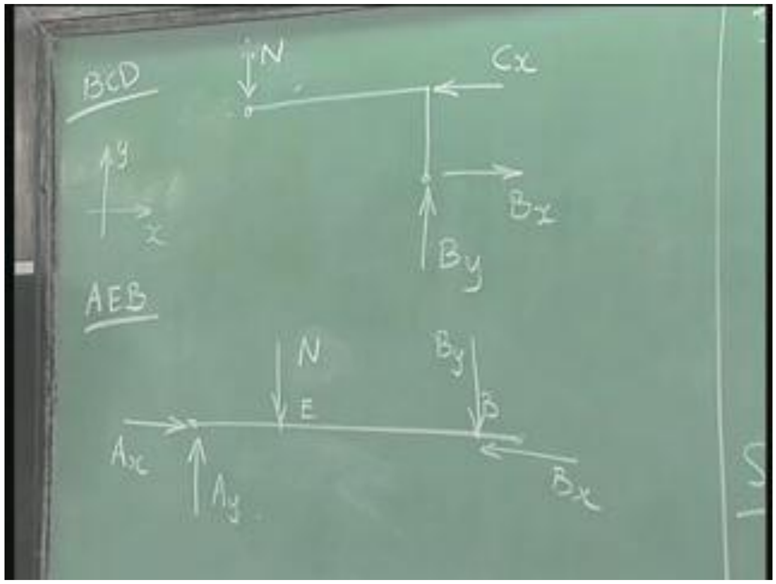

Is there a simplification possible in this? There are simplifications that we can think of but mind you, I don't have to look at the wheel anymore. That's already taken care of. How do I solve for the other unknowns? We already saw 5 here, 5 there. If I take moment about this point, what all unknowns can I avoid? I can avoid A_x, A_y as well as B_x which means I will have an equation that is in terms of N and B_y. Is that clear? So N and B_y will take part, if I have equal zero. This will have N_1 and B_y taking part. Go to the other free body, what is the equation that gives me a relationship between N and B_y only? That's not very difficult, it is the vertical force equilibrium.

So far this, I will do this. I will do sigma F_y equal zero that it will not involve B_x as well as C_x, it will involve only these two N and B_y. This involved N and B_y, this involved N and B_y, I can solve using these two equations N and B_y. Once I have solved for N and B_y, I am now left with A_x, A_y, B_x. in this case B_x and C_x. If I take the vertical equilibrium here, A_y can be solved for because N and B_y are already known. So I can solve for that by simply taking sigma F_y equals zero here.

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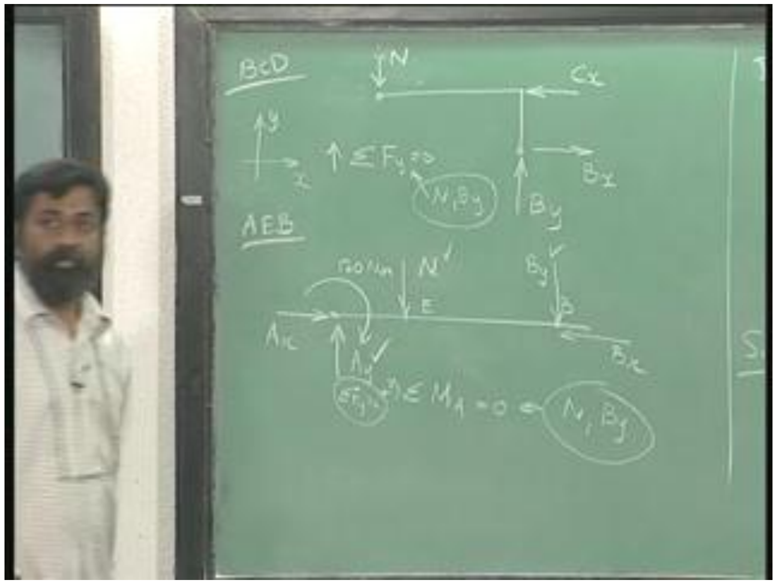

I am left with the horizontal equilibrium there that will relate A_x and B_x. If I take the horizontal equilibrium there, it will relate B_x and C_x. I am sorry, I should have inserted 120 Newton meter here. Taking moment about A, I should involve this 120 Newton meter. So A_x equals B_x, C_x equals B_x are the two equations I get. I need three equations because I have A_x, B_x and C_x. How do I get the third one? I will go back to this guy and if I take moment about this, I avoid A_x and A_y. There is only C_x that needs to be solved here.

Remember this system of planar bodies free body will help me solve for C_x directly. This is an important point you have to note. If I do sigma M_A equals zero here, I will get this directly. Having got on this directly I can find out B_x, having found out B_x I can found out A_x which means I have solved for all the unknowns in this particular problem. Please use this kind of an approach in order to solve the problem, else you will end up with 9 equations with 9 unknowns that need to be solved and it will be too cumbersome for you to solve.

It is important to look at simplifications that you can do, you will always have to look at, is there a simple way to solve for one unknown or the other. If you do use this approach, it will be possible for you to solve many problems in a very simple way. You can draw, you can make these strips in order to understand the problem that we discussed. This is that A B, let me draw this point of interaction as E. Now this is a point fixed to the fixed frame, this is the point A fixed to the fixed frame and remember this is not fixed to the fixed frame but it fixes B C D and A E B. Now we will remove each restrain and insert a force. As before this has x direction this has y direction.

This restrain essentially does not allow the points C from moving in this direction. I am going to remove this and insert a force like this. Nice to do this kind of an exercise, this gives us understanding. In order to make this body free from the fixed frame, I need to remove A. What does A do? It doesn't allow the point A to move either in vertical or in horizontal direction. I am going to insert the two restrains A_x and A_y. Now this is a free body that you have for the entire system. Supposing I have to remove this particular planar body from the system, I need to fix A B and fix B C D. The pin at point D does not allow it to move. So what I am going to do is I am going to remove this and insert two forces that prevented movement at D. In addition at this point E, there is horizontal and vertical restrains so I can add those two. An equal and opposite reaction has to be inserted over here.

I am just going to remove this and insert it over here. There is a restrain this way equal and opposite restrain as inserted over here, let me just move it over here like this. There is one more like this that I have to insert, an equal and opposite reaction at D. Here we have made a simplification to find out that these two are zero. We might as well remove them which mean this is also zero. You can do this kind of simplification in order to understand, how to draw the free body diagram.

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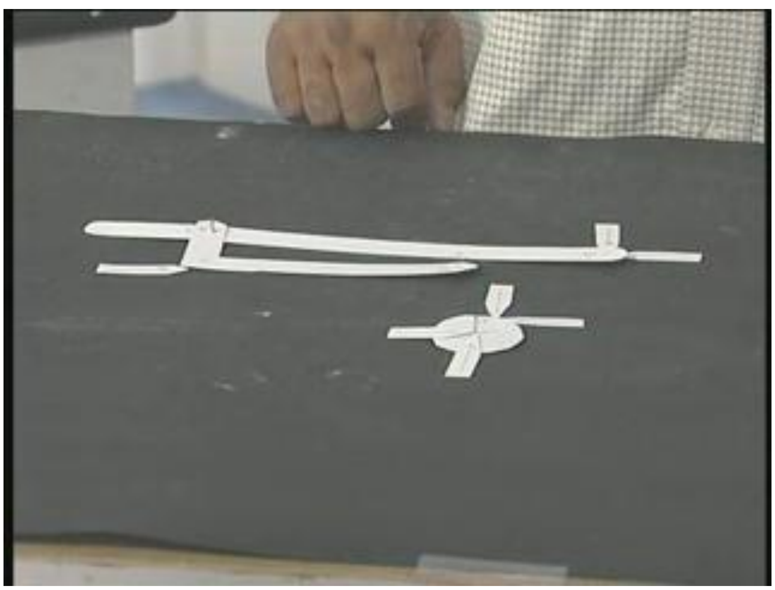

Now if you look at it, I have a force over here let me retain that force. There is another force on this. If I have to remove A E B now, I have to remove the restrain here. When I remove the restrain here, I need to add the two restrains related to that and this is the free body diagram of B C D.

In a very similar way you can do the same thing. This point is restrained by B C D and therefore let me just take this out. It will have a reaction like this. A is already restrained
So let me just take those two. These are the two other restraints. At E also I have one restraint that comes from the wheel. I have used something like this, so this is like this

(Refer Slide Time: 27:03). This is E upward equal and opposite reaction occurs here and this is the free body apart from the moment that occurs at A. This should be very clear to you. Try this exercise a few times to get a physical feel of how to draw the free body. Once you become a master in free body diagrams then lot of things can be accomplished in a very simple way.

Thank you.