Engineering Mechanics
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Dynamics of rigid bodies

Let's try and understand the motion of this particular body. Understanding which it is possible to understand how the mathematics related to this works, from the position how do I find out velocity and acceleration? Let's say this is the current position of the body and let's say it has moved to a position like this and I wish to describe the motion of the body altogether, not single point but any particular point to this body, I should be able to describe. How do I do that? If it is a particle, for example if I am looking at only this particular point, the black one here and it has moved to this. It's not very difficult for me to write down the position of this. It is just this translation that has occurred. Basically if this is the single point that I am looking at, it is just this translation that has occurred. But if I am looking at all the points of this body together then I should be able to write down appropriately, a motion that is occurring for the rigid body.

One thing that you can recognize is when this rigid body is moving, what I have primarily done now is a translatory motion. Why translatory? Because there is no rotation involved. What happens to every point of this body? Let's say I move this guy like this. Let me just draw the outline just to give you an idea, this is the black point, red point, yellow point, green point. If you look at what is happening here, the red point has moved to this point, this position, black point has moved to this position, green point has moved to this position and yellow has moved to this position.

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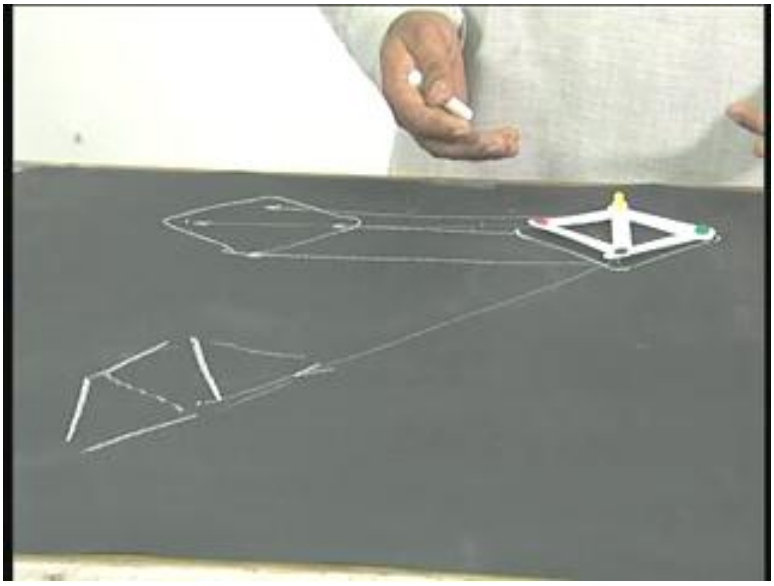

If you find out each one of these movements, the change in the positions, they are exactly the same. But if the motion had been such that the configuration becomes like this, you

will notice that this change in position is not the same as how I described the motion of this point for example the red point. The motion of it seems to be completely different from the motion of this particular black point. I need to find out how to describe motion of every point of the rigid body. How do I do that? It's very simple. Since it is a rigid body, one degree of freedom is gone which is there is no deformation involved.

One thing that I can do is I can take this motion, the final configuration to be such that there is a translatory motion and a rotary motion to accomplish this. The rotation may be about a single point. The point may be inside or outside the body but right now, one thing that I understand is there is a motion involved which is purely translation and then a rotation. It could also be perceived as a rotation first and a pure translation. In other words it is possible to describe the final configuration or the motion by a pure translation plus a pure rotation. Now in this particular example the black point acts as a pivot of rotation. As you can see here it has moved in a translation fashion and then this acts as a pivot while rotation occurs.

One thing that we know here is the black point does not move during the rotation operation. Therefore I can say that this black point goes through a single translatory motion while the other points also move through a translatory point. They also have another motion that is related to rotation about this black point.

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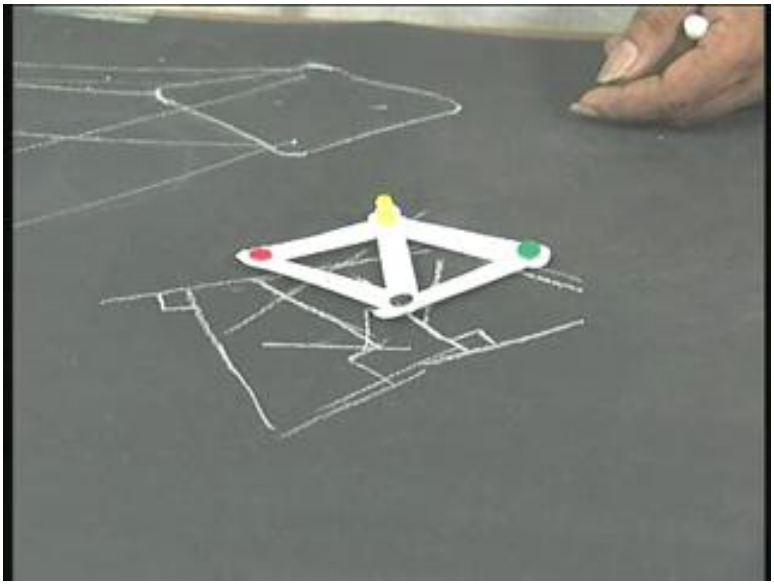

The position of any point can be described as the translation that the black point has undergone. For example the yellow will go through the same translation plus a rotation about the black point. I am going to just draw one like this and one more translation and this is purely due to the rotation that has occurred. If I add the two, I will get the position of this yellow point and that's what we will do as an exercise in finding out the relationship between the positions of a particular rigid body and therefore the velocities of the rigid body, the points of the rigid body and accelerations.

One thing that we also have to understand is this. Let me just do it afresh. Supposing this is the position so let me just draw the outline. Let's say this particular body rotates about the black point and I am going to rotate it by 90 degrees. This particular line, the black to yellow line is going through a 90 degree rotation. Initially it was like this. Now it has become like this and this is 90 degrees. The question I have is will everyone of the point or vectors in this body go through the same rotation. Let's examine. When I have rotated this to this, this is the configuration that I have. Let me just look at the outline alone and verify if it has undergone. If you look at this outline and this outline which is corresponding to the same outline, the angle is 90 degrees. Now this outline has now become this outline and this is 90 degrees. This outline has become this outline 90 degrees.

Similarly this outline has become this outline, 90 degrees. You can see that every line that I draw here, any arbitrary line that I draw here would have undergone a 90 degree rotation and therefore if I rotate by a theta, every line on this would have gone through a rotation theta or in other words this rigid body has undergone a rotation theta. That will hold good if I take the time derivative. that will hold good for angular acceleration as well as angular velocity. If the rigid body is, if one single point that I take is going through a rotation, the same rotation is experienced by every point of this particular rigid body. If one of those points is going through an angular velocity omega, every point of this particular rigid body goes through an angular velocity omega and similarly acceleration. Once we understand this, the mathematics behind will become very simple.