Engineering Mechanics

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Dynamics of rigid bodies

From this module onwards we will look at dynamics of bodies, especially rigid bodies. In the earlier set of modules there are some deliberate mistakes that I have made. I want you to identify those mistakes. In the following series of modules also there may be some deliberate mistakes just to capture your attention. I am sure you all will be alert in trying to find that particular set of mistakes. With this let's just start understanding dynamics. So I have with me Mr. Venket Rao, I am going to just ask a few questions and start the dialogue. We will try to understand from that what it means to say dynamics of a rigid body, dynamics of a body, from our understanding of dynamics of a particle. Let's say some P, you have a rigid body so let's call this as some beta which is a rigid body. Now the question is how do you distinguish from dynamics point of view, the dynamics of a particle from dynamics of a rigid body.

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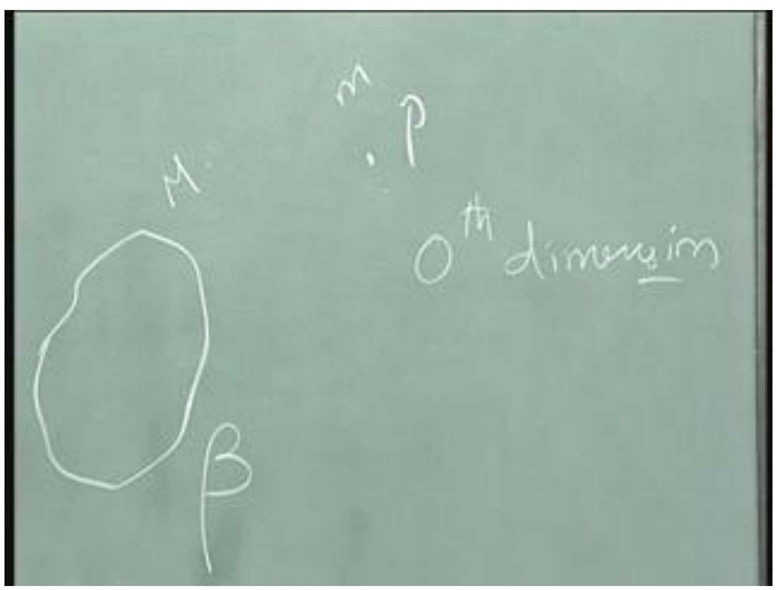

[Student: dynamics of a rigid body, we have different particles in the rigid body and all the particles will have I mean the same motion one of the rigid body moves in a particular direction] [Student: both things will have velocity and acceleration] both will have velocity acceleration. What's the moment of inertia of a particle? [Student: moment of inertia is not] or in other words we don't deal with [Student: moment of inertia with the particle] or [Student: moment of inertia will be considered for a rigid body] or in other words there is something that we don't consider for a particle, when you write down equations of dynamics. What is that? We do consider the motion of the particle.

[Student: yes] What is it that we don't consider when it comes to a particle? Have you heard of rotation of particle? [Student: yeah rotation of a body we have] particle [student: particle, no]. Usually when we consider this, we don't look at rotation of the particle or the angular velocity of the particle, angular acceleration of the particle but the moment we look at rigid bodies we are looking at angular acceleration, angular velocity and so on. What's the characteristic of the rigid body that is missing in the particle, let me consider this. So let me explain that particular aspect. Thanks.

Now one of the important things to understand is when I am looking at a particle, it is a single point dynamics that we are looking at. Usually we look at only the linear accelerations, velocities and so on of the particle. The dimension of this is zeroth dimension. The moment we have a higher dimension, for example we have a length attached to the rigid body. Immediately apart from the movement of this length, we also are looking at the rotation of this length, how it deforms. We are going to look at a general body which is deformable and then bring it down to this particular aspect. Shall we?

Let's think of a body, think of a gel it's easy to understand that the gel goes through a deformation. Let's say there is a gel and you apply some certain forces and it deforms and you move it around. This will deform as well as move. When it moves, it will move in a linear way or in a translatory way also may rotate apart from deforming. It is very difficult to look at the complete deformation from the point of view of a single point. This is assumed to be a continuum of several particles together that this is an ensemble of several particles and we will look at the relative motion between the particles or the points denoting the particles in order to understand the deformation that this goes through.

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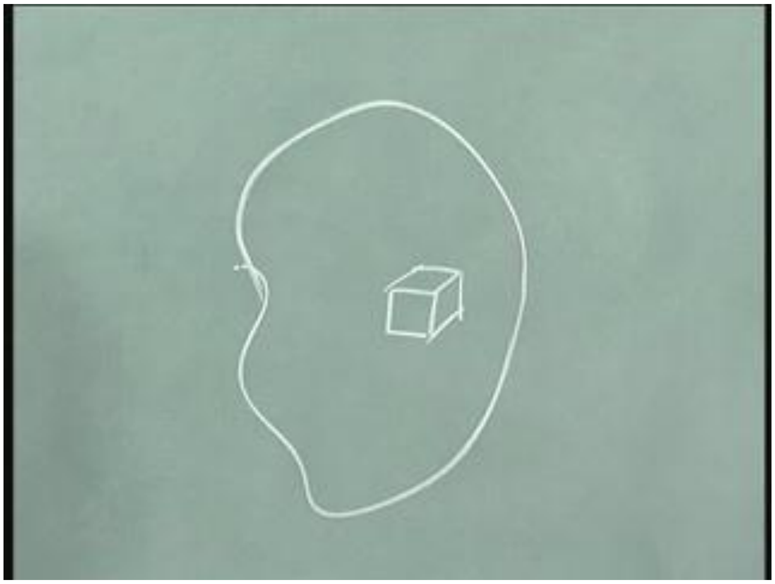

Let’s now look at a small volume of this particular body. If I take a small volume of this particular body, I am just taking a idealistic picture of let’s say a cuboid. If I observe what is happening to this particular cuboid then I will understand, what is happening to the entire body because the entire body is an ensemble of these small bits and pieces. So far is fine? Moment I look at this, what could happen to this particular cuboid? It could probably deform, it could rotate, it could translate. So I can say that there is a translation that is possible. If I draw it as it is like this there is a pure translation that I am looking at or it could probably rotate like this. In addition to this, there will be some deformation that could take place.

Now looking at this as an infinitesimal body which basically states that the dimensions are very small. I can retain the linearity of these sides and just show that there may be an expansion as well as shearing that could happen to this body. I can look at the motion of this single volume of the body to be a translatory motion accompanied by a rotatory motion, accompanied by a deformation. I can look at each one of these as one transformation to another transformation to another transformation and if I am putting all these together in a body, I will find out what will be the transformation that this body has gone through.

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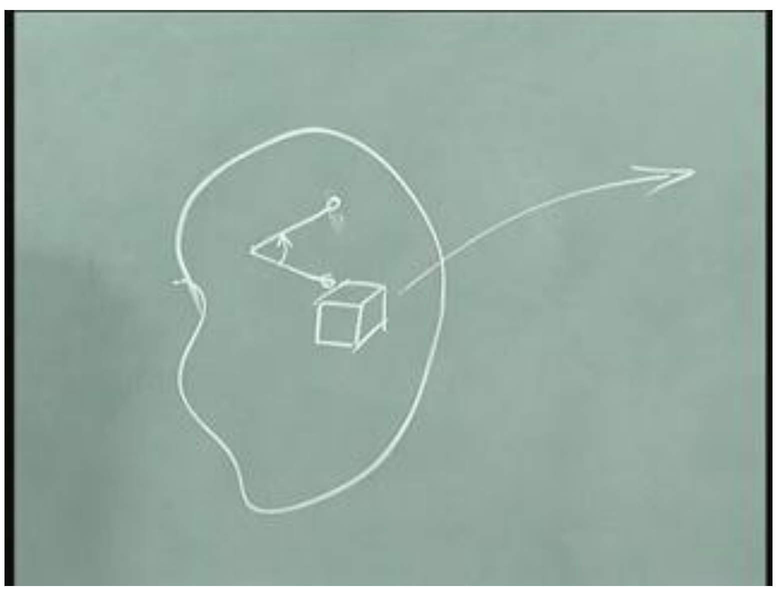

This body in effect would have gone through a translation of the ensemble, rotation of the ensemble and deformation of the ensemble. If this is clear, the first attempt that I can make is let me look at this as a body that is not going through deformation. I can insert this at a later stage. I am now simplifying the problem. What I am saying is let’s look at the rotatory and translatory motion of this body. If I say it is not deformable, what do I mean? If I take any two points of this particular rigid body, the distance between these two points remain the same because deformation involves stretching or relative rotation.

If I have two lines like this, there is an angle between them. This angle could change upon deformation or the length could change upon deformation. I am going to now just discount those two things from happening. If I do that, what I make sure is the distance between any two points remains the same. It is very clear, form here if I draw a line over here, this distance is the same, this distance is the same, this distance is the same, naturally the angles between them also remain the same and that I am going to call as rigid body.

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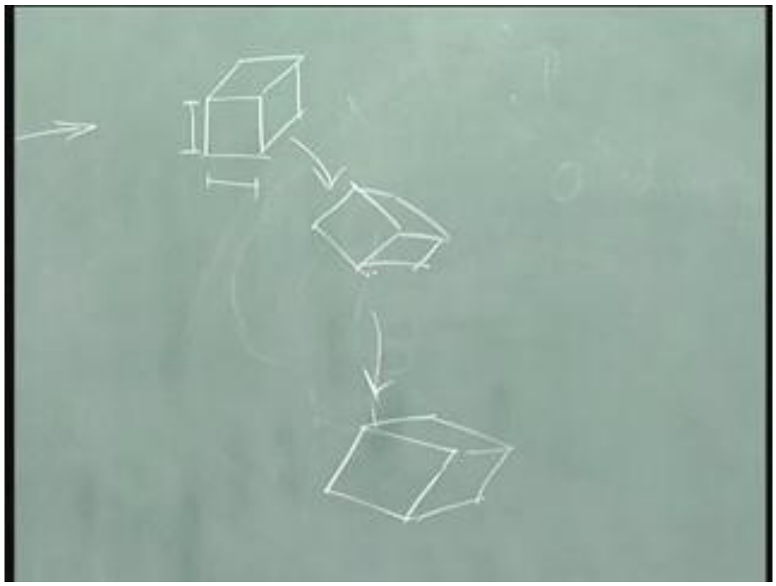

I will look at the rigid body dynamics and I can superimpose the deformation on that. There are theorems that handle this kind of decomposition of rigid body rotations, motions and deformation. So that renders ease of looking at the first step towards understanding the motion which is translatory and rotation. Usually in the first course on engineering mechanics, we deal with translatory and rotary motion of these bodies and therefore we call these as rigid body dynamics. We will look at how this is happening and write down, how we can understand the relations between the position, velocities and accelerations of the body as it is in motion. Now I am looking at body rather than a single particle.