Engineering Mechanics
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Dynamics of rigid bodies

Good morning. What we will do in this module is look at planar rigid body dynamics and what we wish to do is to give certain tips that will help solving problems in planar rigid body dynamics. Now I am going to focus on kinematics first. Kinematics as you know is the study of motion and we are just going to look at the geometry of motion in this particular case. We did that exercise earlier for a rigid body both for a point on the rigid body and for a point with respect to a rigid body. Let's just write down the equations first that pertain to rigid body kinematics and then I will give you certain tips that you can use.

One thing if you remember is we always have to have a fixed frame of reference with which we are referring to the kinematics. So tip number one is simple. You ask the question where is the fixed frame or locate the fixed frame that's very important. The other thing that we note is supposing we know the velocity of the point A or the location of point A and looking at the location of point B and we know certain relationships. How do we relate these two, either the location or velocity or the displacement. We did this exercise in the last module. Let me just repeat it for you here. As you know this we call as r_A, this we call as, look at the direction that I have indicated. This is the location of B with respect to A.

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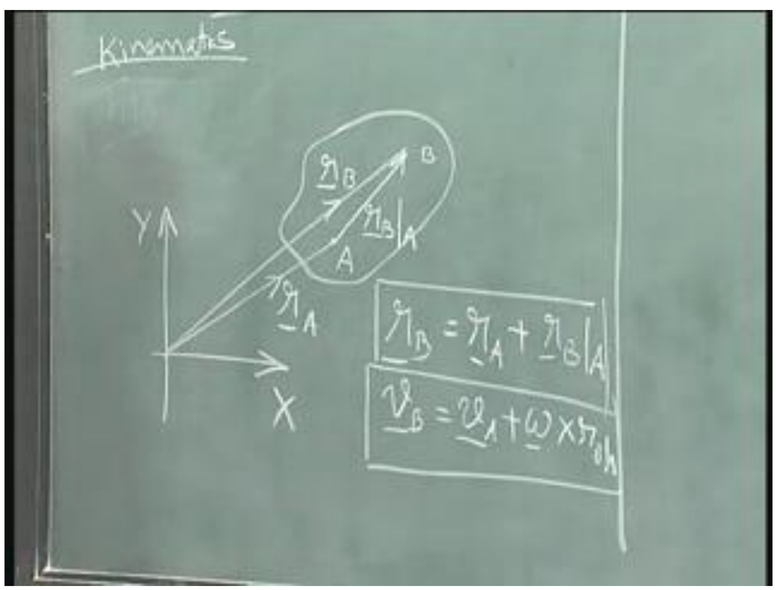

So r_B with respect to this symbol A. If I add these two, what I should be getting is the location of B. All these are vectors. we realized in the earlier module that the relationship between these locations are in terms of the vectors that is the location of B is equal to location of A plus the location of B with respect to A, simple.

When I take the derivative, we understood that the derivative concerns the magnitude and the direction and therefore we found that the velocity of B can be determined using velocity of A plus omega which is the angular velocity at any instant that we are talking about cross r_B with respect to A. Remember each one of these quantities are defined with respect to the fixed frame, very important. How about the acceleration? Acceleration will have acceleration of A and then the acceleration of B relative to A, so we are going to write it over here. Acceleration of B is given by acceleration of A plus let me write down the vectorial notations first and then get an idea of the directions. Omega cross omega cross r_B with respect to A plus, remember all these are vectors plus alpha; alpha is the acceleration of the body, angular acceleration of the body cross r_B with respect to A.

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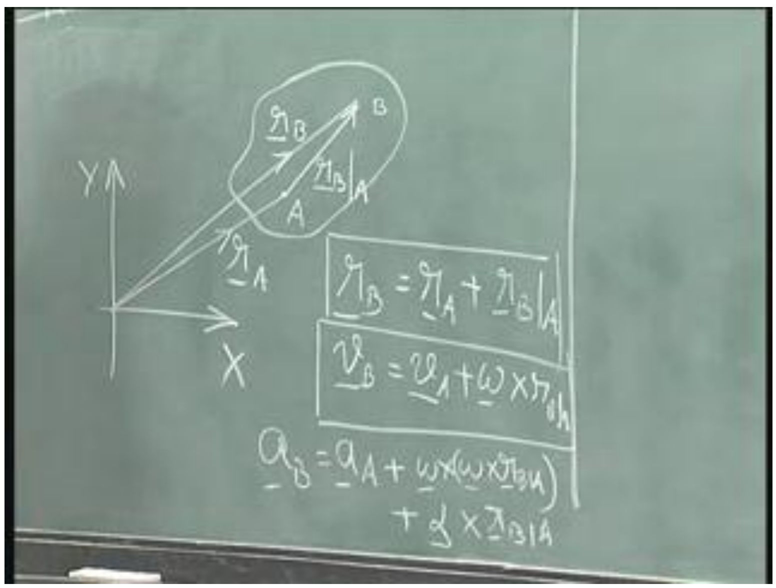

Tip number two which is usually forgotten, I have already talked about this earlier. The entire rigid body, if I am talking about one single rigid body, the entire rigid body has a single angular velocity or acceleration. This is a redundant information but this is very important because many of us have a quandary. We don't know how about if I take another point on the rigid body etc. Remember the body is rotating with one single angular velocity or acceleration. The other tip concerns getting an idea of the directions. Let's look at this particular case, I have an A over here point A, point B. I am just looking at these two points on the rigid body.

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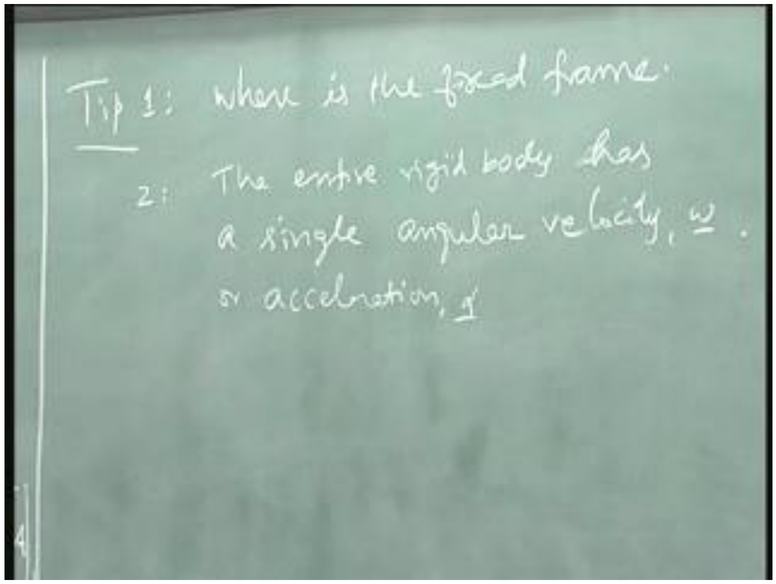

What is this? This is r_B with respect to A and how about omega? Omega is directed outward from the board. If I have to write omega, omega is actually omega times K vector where K is outside. If this is I, this is J, K is outward usually represented like this, outward with respect to the board. So omega is taken as magnitude of omega times K. Now this is the direction that we have for r_B with respect to A. What will be the direction of omega cross r that's the question. Look at it, omega is like this, r_B with respect to A is like this. These two are perpendicular, omega and r are perpendicular to each other which means I will have a direction that is perpendicular to both this and omega and therefore it will be in this direction.

Now omega is K, r_B is in this direction, let's say this is I prime; K cross I is J. I usually write like this, we have to write in a cyclic form I, J, K and a direction like this. When I cross I with J, I should get positive K. J cross K is positive I, K cross I is positive J. But if I go the other way for example J cross I is minus K, so in this particular case if this is I prime, this is J prime and K cross I has to be positive J. It will be in this direction. Are you with me? This is the direction of the component of velocity that we are looking at. In other words we have V velocity of B is equal to velocity at A, whatever is the velocity of this point plus velocity in a direction as shown here.

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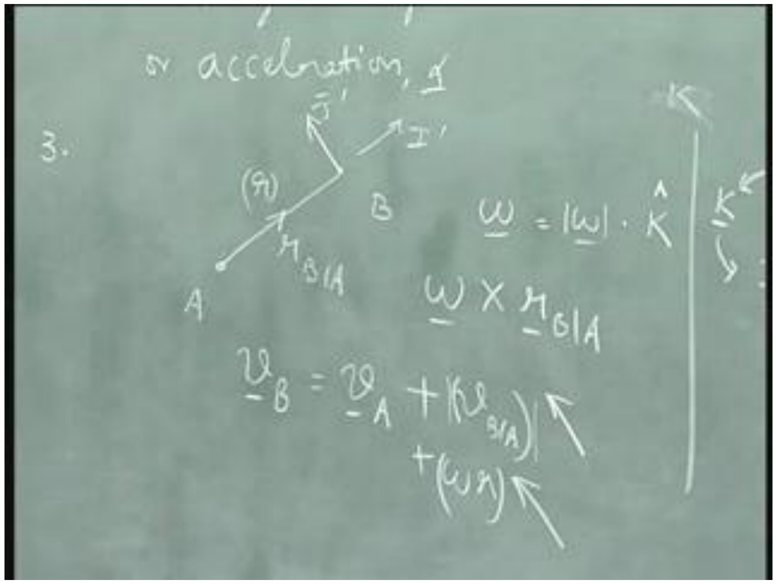

Let's say velocity of B with respect to A, I am just going to write it like this, times a direction like this. This is the magnitude. Now V_B with respect to A is magnitude of this times, this is what we are doing. Let me just take this as r. So let's say this is r, the value is r and this is omega, the value is omega. So omega times r is what I get here, so this is nothing but omega times r magnitude and the direction is like this. Are you with me on this?

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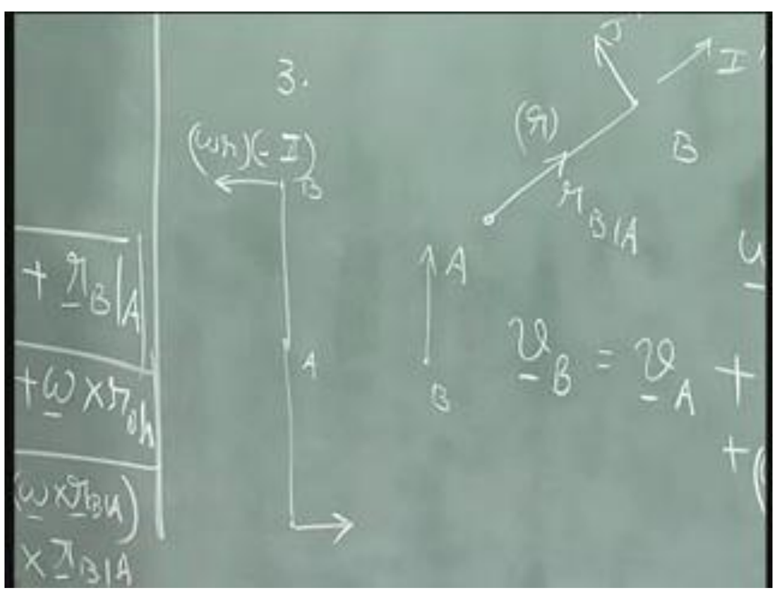

Whenever we are looking at planar kinematics, it's very simple we can visualize what the direction will be. You have this direction and the K direction, automatically it will be perpendicular to that in a direction that it is going in an anticlockwise direction, K cross I is J. Is this clear? This is a tip that was very useful because many times, we will have a difficulty understanding what the direction is. So it may be worth writing it like this. For example if I had something like this A here and B here, I know the direction of velocity is like this, if omega is the angle. This is omega times r times, if this is the direction lets say this is minus I. Supposing B were here then it would have been like this. If b were here it would have been like this. Remember how it is going, it is an anticlockwise direction. So in a way that I understand is I pin it over here, if I circle it in the anticlockwise direction that's the direction of the velocity. It's a very simple notion because you will have confusion, is it minus I plus I and so on.

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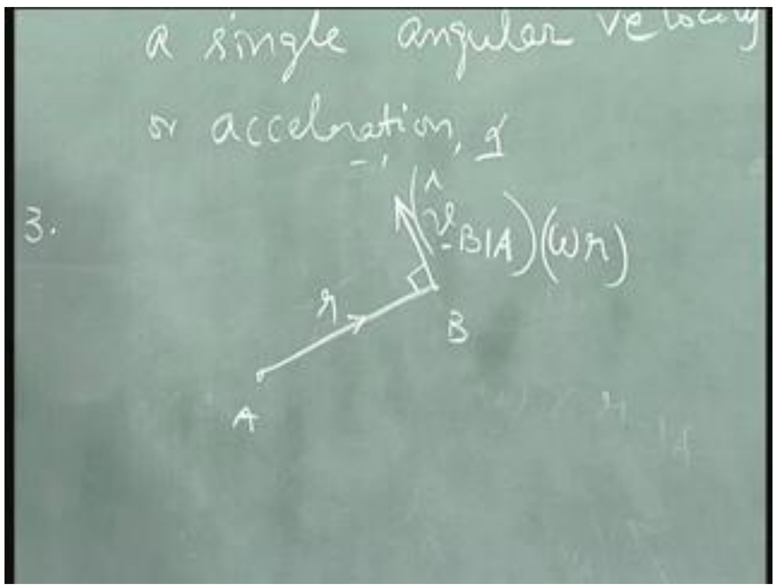

This is an easy way of understanding. So let me just erase this and write it to you in a very simple way. If this point is A, this point is B, the velocity of B with respect to A will be in a direction that is taking B in a anticlockwise direction and tangential to this radial line or in other words this is perpendicular and this is V. If this direction is given by this omega times r where r is this distance is the magnitude. This is another tip that will be very useful while solving the problem. If you have questions you can ask me. Let's look at the fourth benefit. Let me just go to the other board so that it's easy for you to see.

Tip number four. Let's go back to this unlike velocity which has V_A plus one term, acceleration at B has two terms. This is one term, this is another term. This is usually called the radial acceleration and this is usually called the tangential acceleration. I am just going to explain it to you so that it's clear to you. This is point A, this is point B. One component that we have is omega cross omega cross r. It is a vectorial triple cross product. What we can do is we can determine this and then take the cross product again. To make life simple, let's just explore this for different cases. Let me take along I direction.

Let me take r_B with respect to A as r times the direction is I. Explore what will be the direction of this component? I will do the same thing with respect to this point, this point and this point. Just to make it clear to you, what would be the direction. Let's understand, if I take omega cross r, what is omega cross r? It is nothing but the tangential velocity. We have already seen that. What will be the direction? I know it is anticlockwise direction and this is omega cross r here. This is omega cross r here, this is omega cross r here, this is omega cross r. Is this clear?

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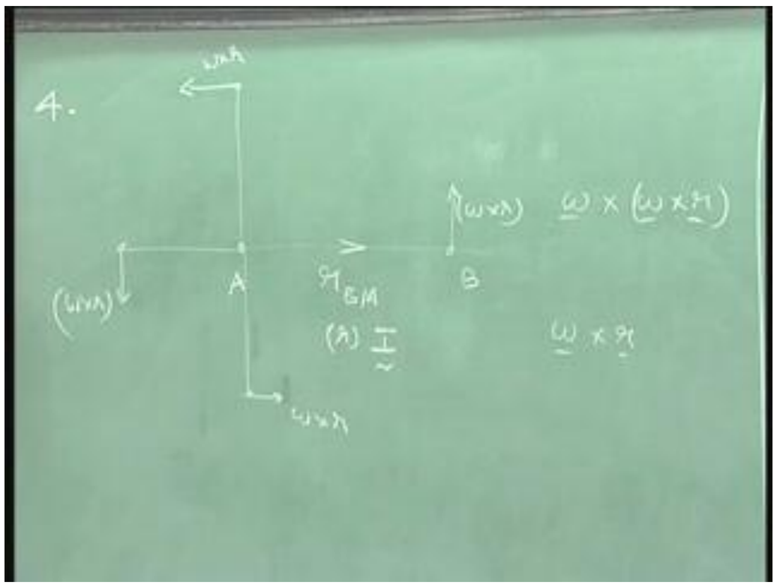

Now what we are having here is omega cross this or in other words, I take this, I have K vector this is pointing upward.
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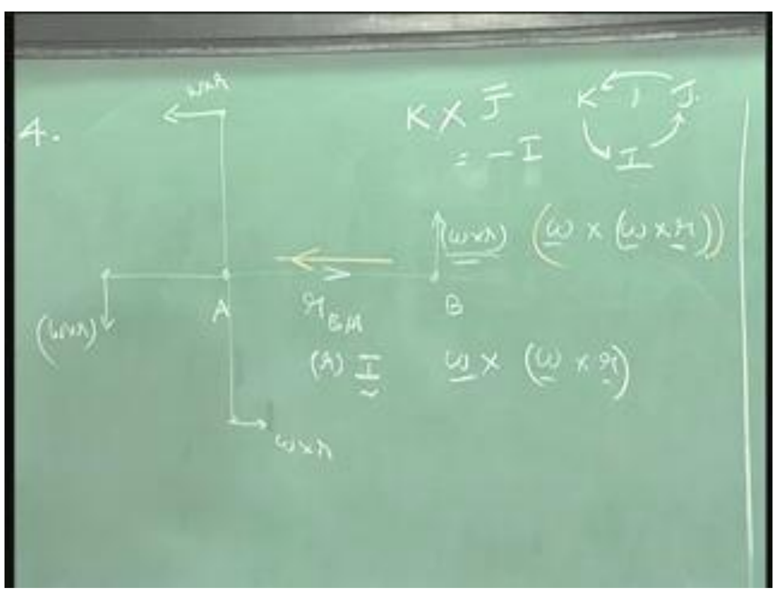

So this is the unit vector along this direction is J, correct. Omega is along K. As I said the cyclical notion I put it this way; I here, J here, K here and it is a anticlockwise direction that gives me positives. I cross K is I cross J is K, J cross K is I, K cross I is J. What I have here is K cross J. K cross J which means it is clockwise which will be minus I. Is this clear? So what would be the direction? The direction of this will be along this direction. Do you agree with me? Let’s explore this. What is this direction? It is along minus I. correct? Minus I right? Omega cross this is K cross minus I. K cross I is J, K cross minus I is minus J, so the direction is like this. If you do the same thing for this, you will get the direction to be this way. For this you will get the direction this way or in other words, even if I take some other point over here, I know the direction of this component will be along a direction that is towards the centre of this which is A or in other words it is towards minus of this direction. That should be understood clearly.

So one way that I can write is the magnitude of this is omega, omega, r so I will have omega square r which is the magnitude and the direction is given by whenever I put a cap, it means only the direction unit vector along the direction. Unit vector along the direction which is negative to r_B with respect to A. Again this is a usual tip. All you have to know is that this particular component is a component that is directed towards the centre of the rotation or in other words here in this particular case is A.

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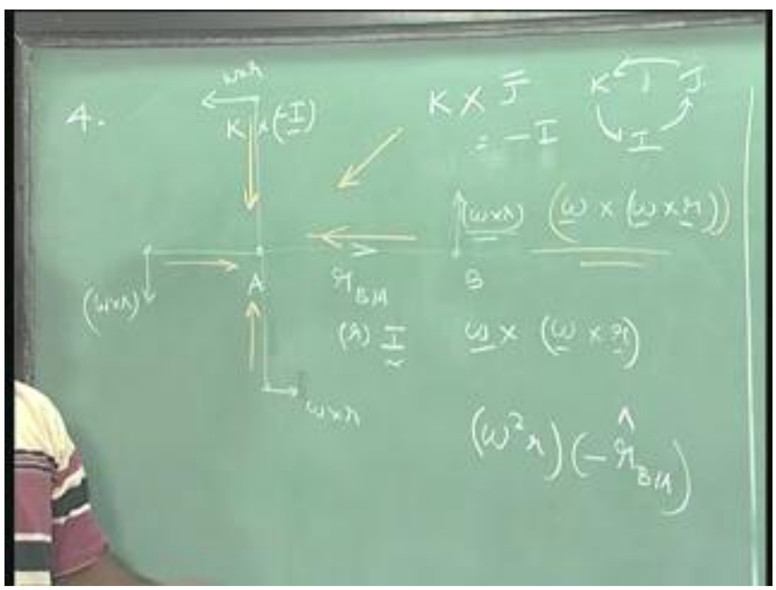

It is pointing from B to A. Is this clear? Let me erase this and write it in a very simple formal way. I am going to call this as a radial component because as you know, it is along the radial direction. So radial component of acceleration of B with respect to A which is omega cross omega cross r will be such that if I have A here and B over here, this is the direction of r_B with respect to A. This is the direction of omega cross omega cross r. Clear? So it is actually omega square r times minus r_B with respect to A direction, cap showing a direction.

Of course naturally the next step is with respect to the other component. Other acceleration component that is very simple. What is that? Alpha cross r. What's the direction of alpha? The direction of alpha is the same as direction of omega which is K direction and this is r_B with respect to A. We did this exercise earlier for this.

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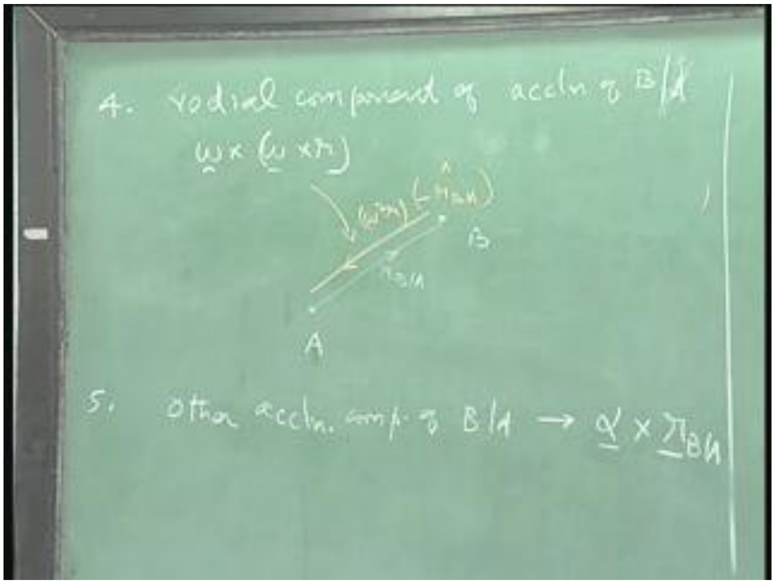

This is what? This is nothing but omega cross r_B with respect to A and the direction is anticlockwise pivoted over A and therefore this is very simple. If I have point A over here, point B over here, the direction of alpha cross r_B with respect to A will be in this direction.

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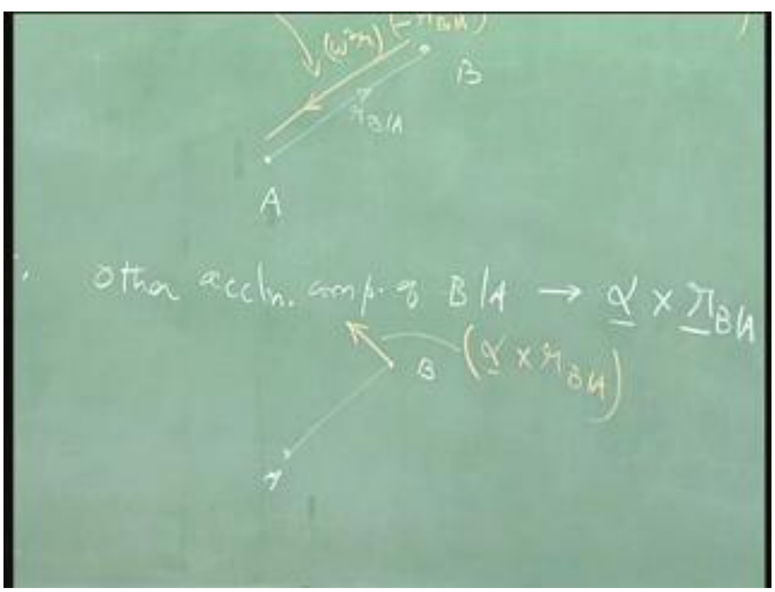

This is alpha cross r_B with respect to A, simple. So given the acceleration at A and let's say the other values over omega, alpha and r_B with respect to A, I can find out acceleration of B by adding one component this way, another component this way. So that I can get r_B. Any questions so far? Having understood this, let's proceed a little further and see what are the things that we have to bother about while solving some problems.