Engineering Mechanics

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Lecture No: 1.3
Statics
Planar Rigid Body
Equations of Equilibrium

Today we will see on systems of rigid bodies, supposing there are forces and moments acting. How do I find out equations of equilibrium? I will focus mainly on planar rigid bodies. I am going to give a few tricks that you can use in order to solve problems. From a problem solving angle only, I am going to look at this particular session. To start with let's examine on planar rigid body subjected to various forces let's say F_1, F_2, F_3, F_4. Let's say there is a couple here M_A and so on. We know that we can write an equivalent of these four systems on the same rigid body as resultant force acting, so let me call this as F. All these are vectors and resultant moment lets say this is point o, M_o. Now over this we have to apply the condition of what is happening to this particular body. Is it moving or is it stationary?

Let's look at the first question of it being stationary. If it is being stationary, we say that the acceleration is equal to 0. It is a principle of static's that we used right now. It is not rotating which means the angular acceleration is 0, it is not moving in the x or y direction which means those 2 accelerations are equal to 0. If we apply the principle F equals m a, mind you that I am writing this as vector. This m is the mass of this particular rigid body. If this is equal to 0 we get F equal to 0 and this is one equation.

The other equation has to do with the moment is equal to I times alpha where alpha is the angular acceleration of this rigid body. I am just writing it as scalar. It is something that is about an axis that is perpendicular to the board and this is also equal to 0 in a condition of static's which means we get these two, mind you these two vector equations. This is only one particular direction and therefore we just we are going to make it as scalar equation. This will have two components, the moment we give an x direction and a y direction.

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Therefore this will simplify to F equals zero will imply its first component F_x equal to zero and the component along y direction being equal to zero and the moment about o equal to zero. F_x and F_y are components of this vector F which is a resultant of all these forces put together which you have already seen. This is a simple procedure, we can find the relationship between the forces and moments by solving these equations. We can solve for these values, once we know the forces acting on the body.

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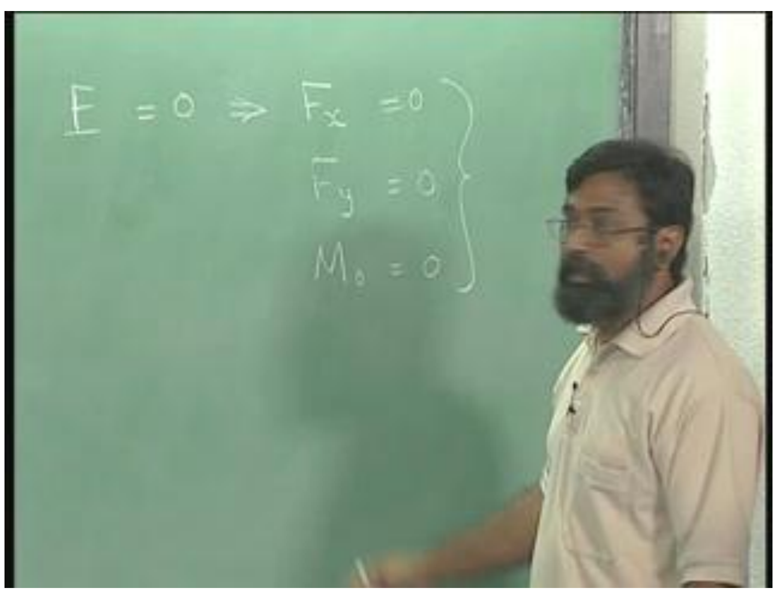

Let's look at a simple example. Supposing I have a body which has a force F acting like this, along horizontal and there is a particular point here which is hinged. There is another point over here which is a roller support. I wish to find out what will be the resultant force, resultant reaction that will come on to this particular rigid body. Very simple problem. The first exercise I have to do here is I have to now represent all these constraints that I have here which is the hinged constraint and the roller constraint has forces that take part in the constraint. We call those as reactions.

In this particular rigid body this hinge will offer two reactions. I am going to represent this as x and this as y. So I will have one reaction like this, let say this is A, this is B. This is point C which will have a horizontal component of this so let me call that as A_x. There is a vertical component of this reaction A_y and there is a particular direction to this, so let me just represent the direction. Let's say this angle is theta with respect to vertical. I can represent this as a force B. Mind you in this particular case, I know at what angle the force is acting which means only the magnitude is an unknown. Here the two unknowns are A_x and A_y. So if I complete it, this shows a free body diagram. This free body diagram is essential in order to solve for unknowns using system of equations. Let's write it down.

I am looking at static's, I know that each of these resultants components have to be equal to zero. I have to convert this in to same rigid body this is the same rigid body I am drawing. I have force here, force here, force here, force here and a force over here. Let's say the resultant force is acting like this. This is vector F. I am taking a point over here which is let's say o. We will come to what point to take at a later stage and a moment m at o. If this is stationary, the force should be equal to 0. The vector itself should be equal to zero or F_x F_y should be equal to 0. I mean just change to P so that we don't confuse between this F and that F. This M_0 which is the moment about an axis that is coming out is also equal to 0.

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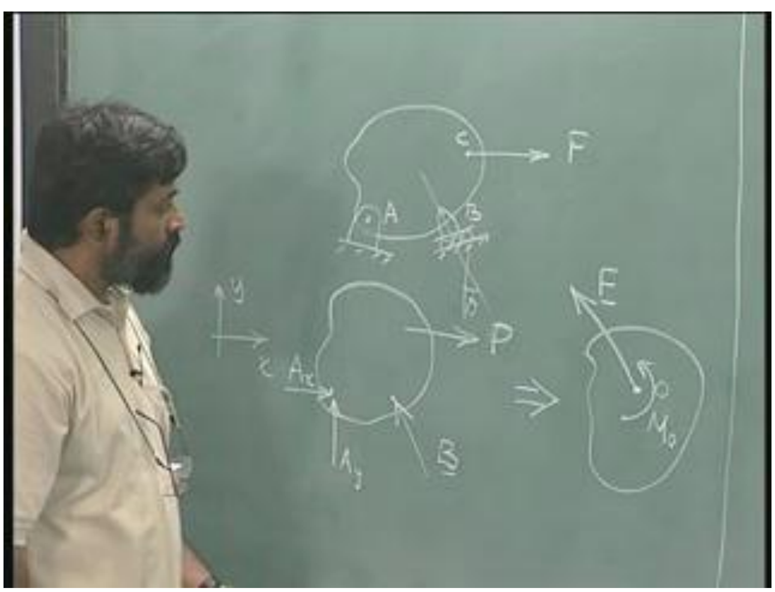

How do we write F? F is nothing but we have these forces A_x acting along this direction plus A_y acting along this direction plus B which is acting along the direction theta two vertical plus p that is acting along horizontal direction. Mind you it's a vector equation. I have to now separate this in to horizontal components and vertical components. The other equation, so if it is stationary I have to say this is equal to 0. The other equation tells me that M_0 should be equal to 0, M_0 I can find out by lets just do that exercise. I have force P over here, if have to find out the moment of this p about this o, what is a relationship? Let me call this point as one and let say this is a vector r. Then the moment about this o power of force p can be given as M is equal to r cross p.

In this particular case since it is just a planar problem, if I find out the perpendicular distance here let's say I am going to call this as d_1 then this is simply, the direction is going to be like this. If I hold some object and pull it this way, it is going to rotate like this, as an understanding that I can have and therefore I can say this is minus d_1 times p. p is the value of this force. I know p is along the horizontal direction. So primarily x component of p. So like this I can find out, this is M_o for the first point. We will have to do that for the other force B, the force A_y and the force A_x and then insert it over here, in order to get relationships between these forces.

One more important thing to understand here is I wish to tell clearly which is a direction that is assumed to be positive. If you go back, we have used x positive direction to be like this, y positive direction to be this way and if I take vector outward to the board, it's a positive direction. If I have a vector outward, the moment is going to be anticlockwise which is taken to be positive. One way of doing this is I can write F_x or the component of this force along this x direction which is this positive equal to zero. This is a positive quantity equal to 0. I can do this same thing over here, M at o taking anticlockwise to be positive is equal to 0.

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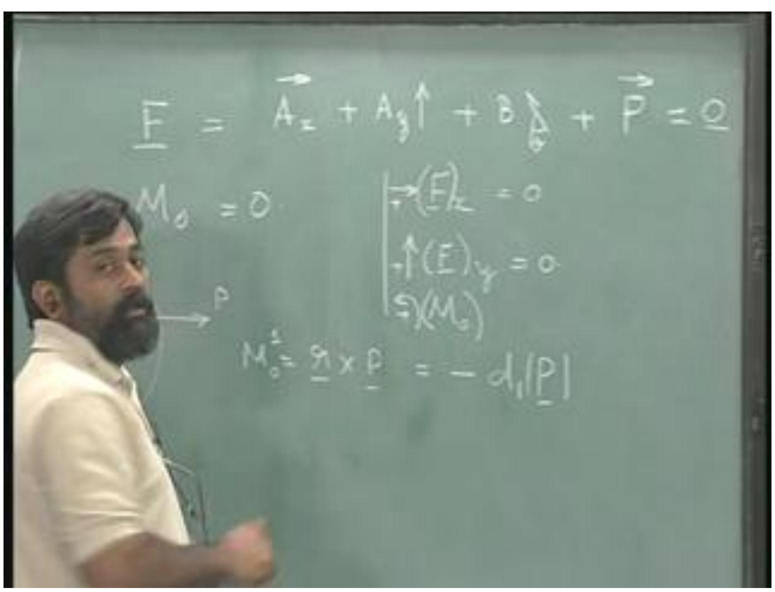

By doing this we will not miss out on the sign conventions. For example in this particular case, this particular force p is trying to rotate about this point o in a clockwise direction which is negative of anticlockwise and you can see here d_1 is a positive number, p magnitude is a positive number which means we will have a negative sign over here indicating that it is actually a positive clockwise value represented by anticlockwise. You can do the same exercise for each one of these. One of the advantages of planar problem is all I need to do is to find out the distance of the vector direction to a particular point in question.

For example if this is the force B, I can extend this line. Find out what would be the perpendicular distance of o with respect to this B? Let me call this as d_1. Again if you notice, the force is such that acting to rotate the body in the clockwise direction. I will get a negative value when I insert it in this particular thing and this is how I can solve the problem. In the examples we will have a few problems that you will tackle. But before ending there is one important trick or principle that we should know. Remember whether the force is over here or the force is over here. A moment of this force about o will remain the same.

Even if I had put the force over here, let's say this point is two, I would have gotten the same moment. So it's only the perpendicular distance of the force with respect to particular point that is important to us. In a similar way here, it is simpler if I write it as A_x and A_y because the distance that I can find out for this.

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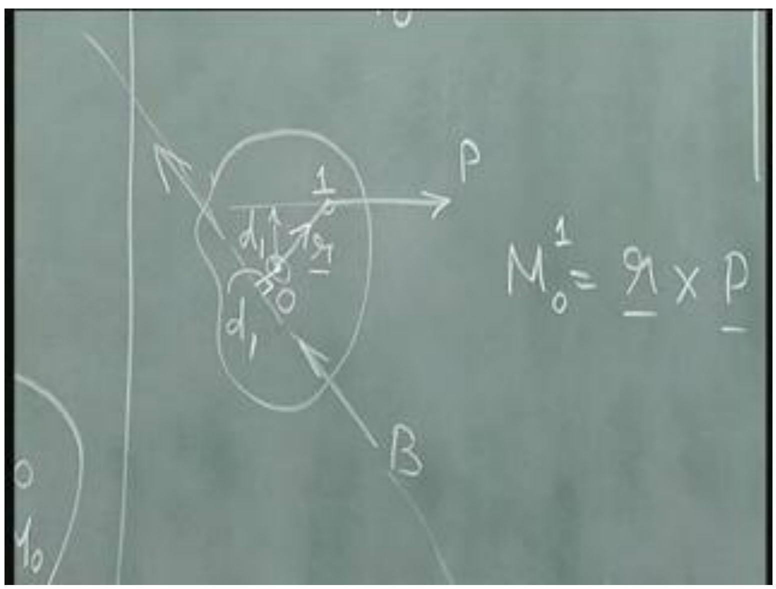

For example I am just going to erase this and draw fresh so that you can understand, how easy it is if I write it in terms of components. If this is the body, if I take this force p, I know this is a horizontal force. All I need to know is to find out the vertical coordinate difference between this point o and this point one. This y coordinate one with respect to o will immediately tell me that is the lever arm that I have to use to multiply with the magnitude of p.

Similarly if I have to split this reaction in to A_x and A_y, it becomes easy because all I need to do is I had to find out the difference between the y coordinates of this point. Let me call this as three between the y coordinate of three and y coordinate of o or in other words this is y_3 minus y_o. Then I take this magnitude, multiply with A_x. Again to make it very simple, I will physically look at what the direction is. If this is the positive distance and this is the way, the forces acting, it is going to go anticlockwise which means in this particular equation I will have it as positive.

So in that sense if I write, this will turn out to be A_x times, let me call this as y_{03} distance between o and 3. This is a positive sign because it is going anticlockwise. On the other hand if I take this one, it is going to go clockwise so I will have a minus A_y times… right now I have to look at the distance in the horizontal direction which means this is o, this is x_{o3}. Here I am not going to bother about whether 3o or o3. I will just take the magnitude of the distance. This negative sign will take care of all the other list.

The notion of directions that we have used is here have to be retained. Let's complete this by adding these. There is this force B that is acting, let's say that is acting like this at an angle theta, again I can find out the perpendicular distance from it and use that particular thing. Alternatively I can also write this force as two components like this. This is B_y and B_x. In this particular case given B, I can write B_y should be equal to B cos theta and this to be B sine theta where theta is already known. By writing this, if I take this coordinate let's say this is 4 I can use the coordinate and write the moment with respect to B_y separately and B_x separately. So I will add that particular part to it and then I will take. I have completed finding out moment for everything and if I said this is equal to 0, I will get a relationship, an additional relationship. How many unknowns do I have here?

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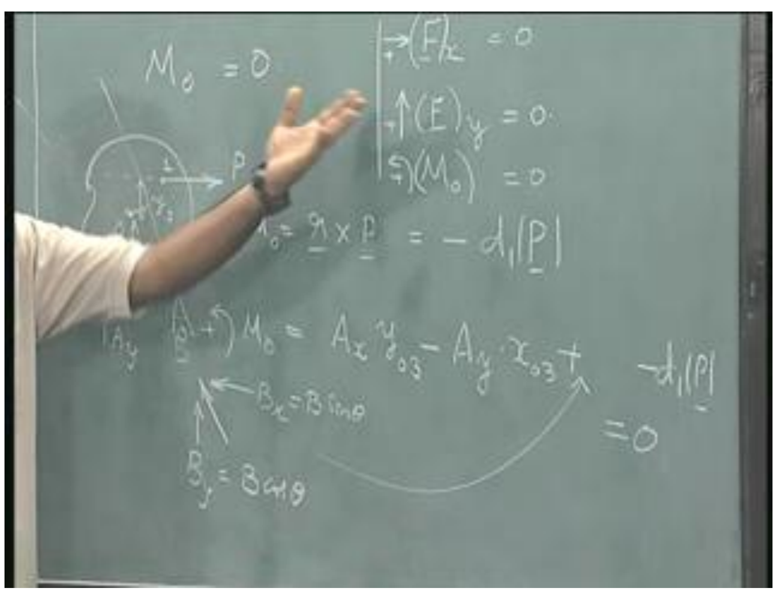

Going back to this particular diagram, this is something I don't know. This is something I don't know. The magnitude of this is something that I don't know. 1 2 3, I should have 3 equations here, 3 independent equations that I should be able to solve. This force equation which translates in to two component equations will give me two equations and the moment equation together will give three equations which will help solve these three unknowns. This is all to it solving a problem with a single rigid body.

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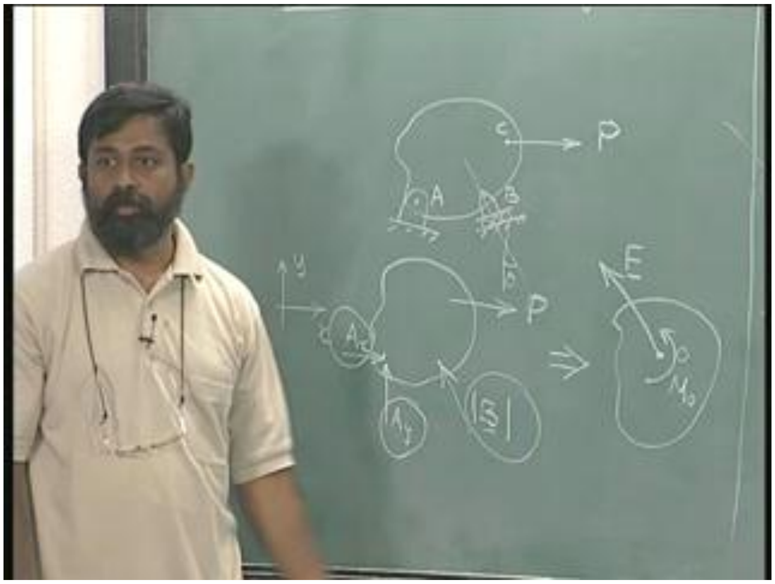

If it is a multiple rigid body or a system of planar rigid bodies, we will have to do a few more amendments I am going to talk about it.

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