Engineering Mechanics
Prof. Siva Kumar
Department of Civil Engineering
Indian Institute of Technology, Madras
Statics
System of Planar Rigid Body
Example-1

Let's look at a few problems that we can solve in systems of planar rigid bodies. Let's start with a simple example. We have a platform, I am just drawing a platform here. You would have seen something like this in a swimming pool where the person goes over here and stands. Let's say we want to build a support system for this. One way that you can do is you can have something like this and let's say this is the wall, you can pin these two like this. Let's just call this as A B C is the platform. Let's say this D is the support. We wish to solve for forces in member or in the rigid body A B C and the rigid body B C. Also we wish to find out how much of reaction goes into the wall. That's very important because I need to know how much force, the support here should be able to withstand. So let's try and solve this problem. Before starting to solve this problem, we have to make sure that we have enough unknowns and appropriate equations to solve this problem. One way to go about is to see how many degrees of freedom the body will have. Usually what we do is we start with the fixed frame of reference.

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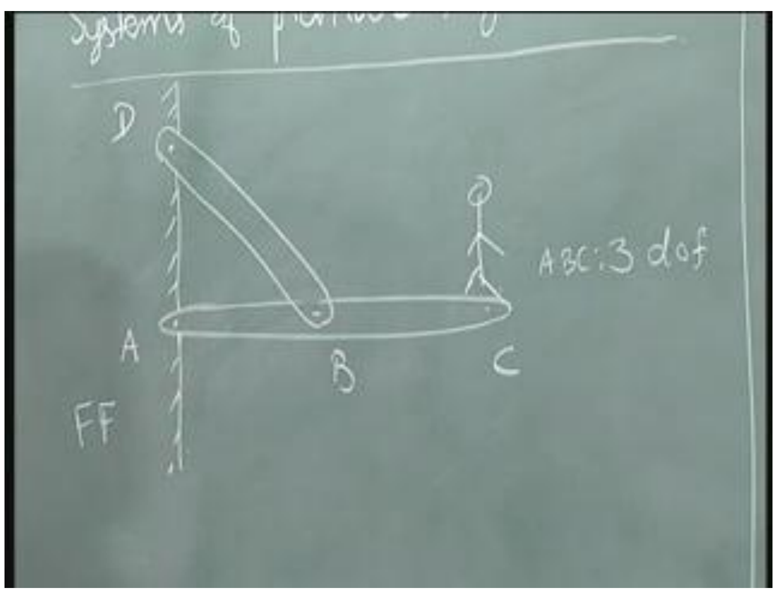

As I already told you if I put a hash, it means it's a fixed frame of reference. Now this particular body A B C will have three degrees of freedom. Let's assume that B D is not there, so this can rotate if I put a hinge over here which means from three degrees of freedom it will become one degree of freedom that this particular platform will have which is just a rotation. I wish to arrest the rotation so that this becomes stationary. The simple thing I can do is to add this particular rigid body which is pinned at B to the body

A B C and pinned at D. If I do that this particular body B D has 3 degrees of freedom. Let's forget about A B C, look at only A B and it is pinned at this particular point to the fixed frame. So the only degree of freedom that it has is rotating about D. While A B C can rotate about A, B D can rotate about D and if these two are relatively arrested from motion, at B you have zero degrees of freedom which means it is stationary.

Now looking at this, it's better to draw the free body diagrams and understand whether we can solve this problem. As you can see here, we have two rigid bodies. Let's just list them down, one is A B C, the other is B D. Mind you I am always using in the increasing order of alphabet A B C, B D so that it's easy to understand. Now if I can draw the free body of A B C and free body of B D then I can solve for unknowns that appear.

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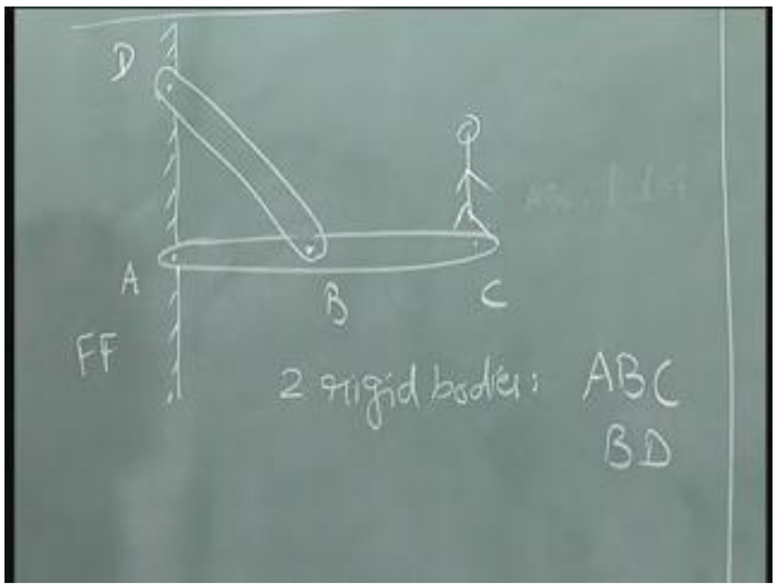

So let's just start the exercise. Let's look at A B C. I am going to go here, draw this particular body. This is A, this is B and this is C. The first exercise we will do is we will remove this particular body system of rigid bodies from the fixed frame of reference. So remove from fixed frame of reference. I am going to use FF like we have done earlier. What happens? Let's look and examine. If you look at this particular point A, at this particular point it is pinned to fixed frame of reference which means it doesn't allow movement in this direction and movement in the y direction which means there are two forces that will appear.

Let's say we are going to take these directions for now. If there is a convenience in using some other direction, we will change it later. This is a reaction because of the constraint in the vertical direction at A. This will be a reaction because there is a constraint in the horizontal direction. If you look at D again I have a constraint like this and a constraint like this D_y, D_x because this point D does not move with respect to the fixed frame of reference.

So looking at this, now I have removed from the fixed frame of reference. We also have a man standing on this, let's say we have this platform which is mass less or negligible weight. The person's weight W will be acting on this.

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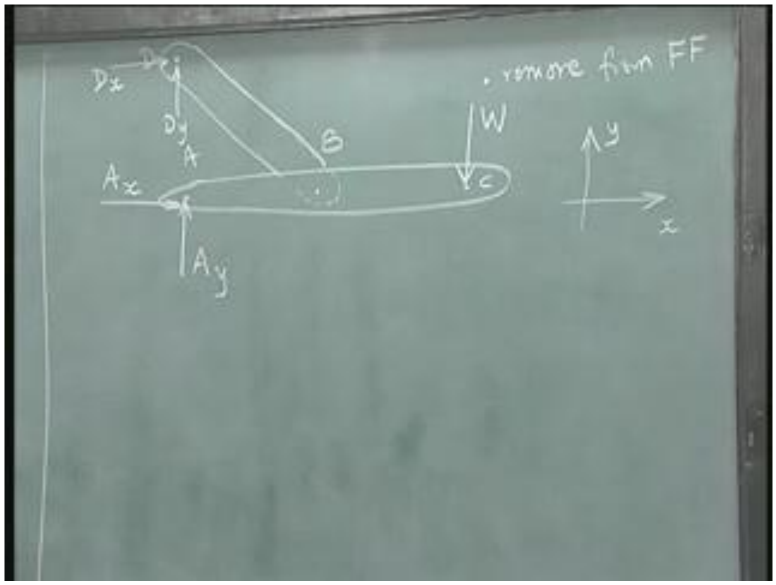

There are no other forces here. Mind you this is a system of rigid bodies that I have drawn and this free body is complete now. Again I will repeat it. I just have to remove from the fixed frame of reference so that I get this reactions, add the external forces to the system. One of the mistakes people do is to add the reaction that occurs on A B C due to B D. Please do not do that because this is a system that we are looking at. Now let's examine this particular free body. The unknowns that we have, that we need to solve for are A_x, A_y, D_x and D_y which are the reactions with respect to this.

Using this particular rigid body since it is stationary, if I ask the question how many equations can I generate? The answer is I can generate three equations from this stationary object that horizontal force, net horizontal force is equal to zero, net vertical force is equal to zero and net movement about any particular point that I take is equal to zero. It makes sense, but the thing is I have only three equations that I can get. The reason why I am getting three equations is if you look at this particular system, this system is not a system which is rigid by itself. It has its own internal degree of freedom. I will demonstrate in a moment how this appears. If you look at this carefully for this particular rigid body, if I take this particular point and ask the question can B D rotate relative to A B C? The answer is yes.

Therefore there is one degree of freedom that I have to take care of when it comes to solving the system and therefore it is important that I draw the free body of other two rigid bodies. I am going to now separate that. I am going to separate these two objects at B. How do I do this? I will fix all the other rigid bodies except the one that I am considering, so this is B D.

I am looking at this particular rigid body. When I am examining the forces on this, I am going to fix apart from the fixed frame, I am going to fix this A B C. It is not moving at all. If I do that like what we have done for fixed frame of reference, this particular point is fixed to A B C and it is not moving which means A B is prevented from moving at B.

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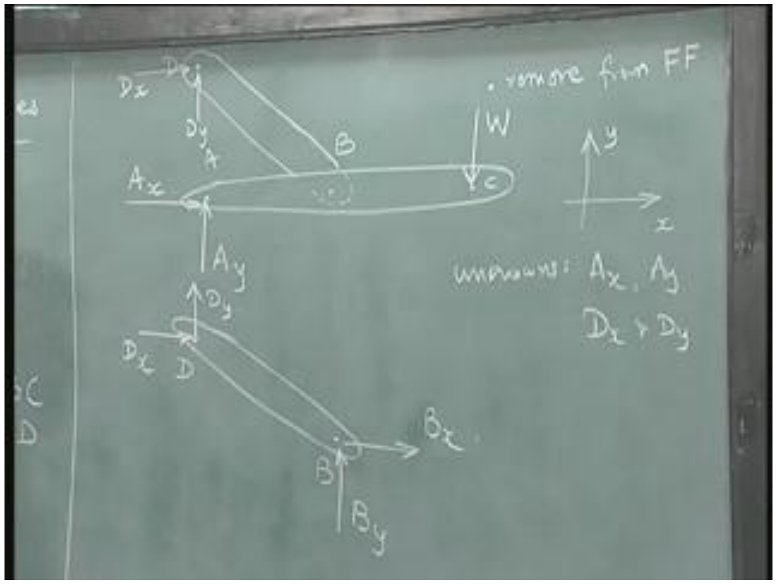

Therefore there will be a reaction that will appear which is the restraining force that appears in restraining moments. I can add the other two forces that appear due to the fixed frame of reference D_x and D_y. We will come to what we can infer out of these things, once we draw all the free body diagrams. There is one system of free body diagram that I have drawn. I have drawn for B D, let's draw for A B C, let me switch over to the next. So A B C, I will do the same exercise. I will do the same thing that I did earlier which is I will look at this particular body A B C, fix all the other rigid bodies to the fixed frame. So which means what, this is immovable.

There is a fixed frame that is immovable, only A B C is detached. So what will happen? At A like before I will have the two reactions A_y and A_x that appears due to fixed frame of reference. There is an external force acting here so I am just going to insert it. At B since B D is holding this A B C in place, I will have the restraints offered by B_y and B_x. In order to avoid confusion I am just first going to take this as B bar x and B bar y. In a while we will discuss about these points. This is the free body that I have drawn for rigid body A B C. I split the system into sets of rigid bodies and I have drawn this.

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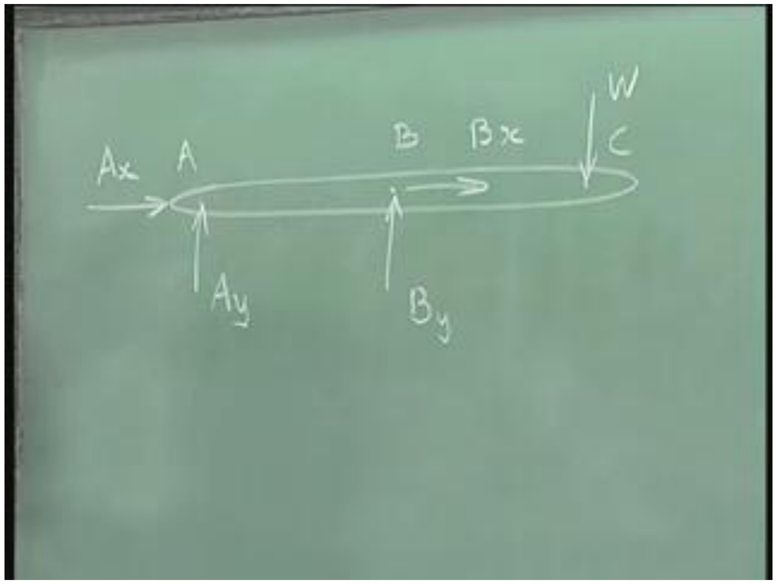

Now let's examine the first reduction that I can make. So what are the simplifications that I can make? This is next question I will ask. I already know there are four unknowns. One simplification that I can do is I know if I combine these two at B, the rigid body A B and B D if I combine them there is an equal and opposite reaction taking place at B so that the external force is equal to zero. Therefore B_x plus B_x bar is equal to zero. This implies B_x bar is equal to minus of B_x. So one thing I can do is given it is this way, I will change the direction here and remove this B. By a similar argument I can show that B bar y equals minus B_y and therefore I will change the sign over here, the direction over here and remove this bar, the first simplification. Now if you look at it, it remains the same except now I have to add two more reactions here B_x and B_y are the two other reactions that I need to find out. Are you with me?

The next simplification derives from what we did for a body like this where forces are only at the ends or at two points only. The force B_x B_y appear at B, D_x D_y appear at D and we call this particular type of rigid body as a link. We already observed that for a link ~~the forces can be~~, the free body can be drawn to be like this and like this. If I can call this as say B force, you can just put a vector here, there is a B force here. If I take the other force, this is D bar where this is the resultant force. From this if I take equilibrium I get B bar or B vector is equal to D vector. Let's not do the simplification immediately. The next simplification as we have found out is BD is a link. Therefore forces appear only along the line joining two points B and D. I am making it more general. If it is a straight member I will just say it is along the axis.

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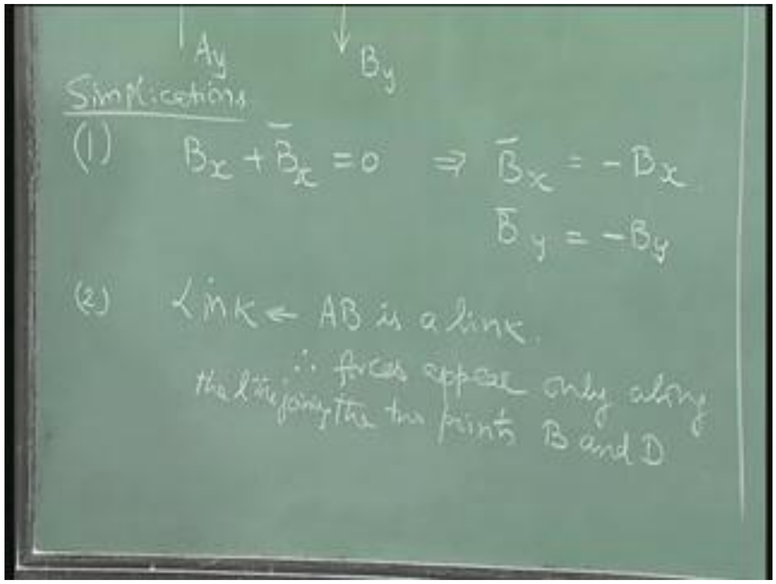

Now is there any other simplification that I can do? I can say looking at this immediately, I can find that the force B is equal to force D. I am sorry I made a mistake here this has to be B D, sorry about that. B D, we have B equals D or the force at B is equal to force at D. Going back to this, can I have any other equation that can be written on this? The answer is no. I have already exhausted all the equations necessary for B D. The only thing that is left, the only set that is left is this particular rigid body and this particular system of rigid body. Having done this, one thing that I know here is instead of drawing B_x and B_y like this, since I have a B vector like this equal and opposite force can be applied over here. What's the direction of this? The direction of this is the same as the angle between the A B and B D which means I need to know only the magnitude of this.

Let's now change the unknowns that I have to find out. Instead of B_x and B_y now we have just B, instead of D_x and D_y we have already shown from this that B is equal to D which means I can remove this. Is it B force that I need to find out? No, its direction is already known. It is only the magnitude of this that I need to find out. If you examine now what are the unknowns to be solved? A_x, A_y and magnitude of B that I need to find out. Let's look at this particular problem and this and ask the question, can we solve it now. The answer is yes, let's just do that particular exercise.

Now remember there are three simplifications that we did, two of them related to link. The other related to equal and opposite reactions. Now if I need to find out the value of b from this particular rigid body, what is the equation I can write? The answer is very simple. There are three unknowns A_x, A_y and B that are existing. I can write the three equations and solve for these. But if I write the equation of moment equal to zero at M_A or in other words A, A_x and A_y will not take part. I can immediately find out what will be B.

Further simplification is since I know this is the angle at which it is acting. I can now take the components instead of B as B sine theta and B cos theta are the two components. Taking moment about A, we already know that this force will not take part in the moment equation. This is the only force that will take part in the moment equation related to the unknown, the other one is W and I can write the equation to solve for B. If I ask the question can I solve for A_y? The answer is very simple. Once I find this out, I can do sigma F_y equal to zero in order to solve for A_y. In order to solve for A_x, I already have this particular magnitude available and therefore I can find this out and I can solve for A_x.

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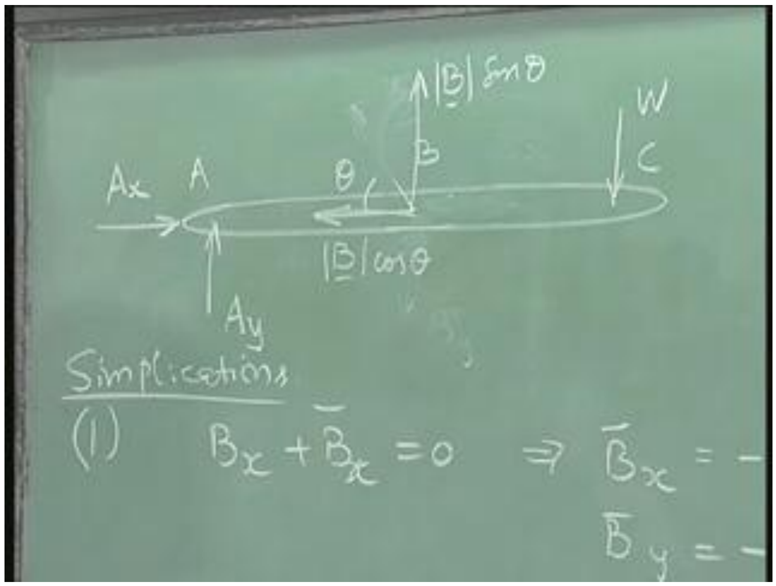

Just to complete it, the first exercise I will do is sigma M_A equal to zero means moment about A equal to zero. Let's just be cautious. Let's take sign convention, let's say it is anticlockwise positive. As I mentioned earlier, how do I find out the terms that appear in that equation? Very simple, I will pivot about A and look at what will be the direction in which these forces will push this bar. W will push it clockwise which means its contribution to moment at A will be negative. This particular B sine theta will push it anticlockwise which means its contribution is positive.

I can now write in a very simple way. It is B sine theta minus W, I need to multiply by the length let's say this is occurring at half the length of A B C. This is L by 2 times this minus W times L and this is equal to zero. Immediately it is possible to solve, I can take out L, B sine theta is equal to two times so I will take 2 W by sin theta. I have now solved for the magnitude of B. The next equation I can take is either F_y equal to zero or A_x equal to zero? So let's do complete it equals zero. Mind you I will do this introducing sign convention here. It makes it easy for you to figure out. A_y will have a positive notion, B sine theta will have a positive notion, W will have a negative notion and therefore this will be A_y plus B sine theta minus W equals zero. Therefore taking all the others to the right hand side A_y equals W minus, I already have B sine theta is equal to 2 W minus 2 W that is equal to minus W.

Is this clear? B sine theta is equal to 2 W, I insert that over here W minus B sine theta which is 2 W therefore it is minus W. So A_y equals minus W is what I get. In a similar way if you do A_x, again the same thing I will do. This is a positive sign, A_x will be positive, B cos theta will be negative, A_x equals B cos theta is what I will get. I just jumped one step here, B is already found out here which means this will be 2 W cos theta by sine theta and that is equal to 2 W cot theta.

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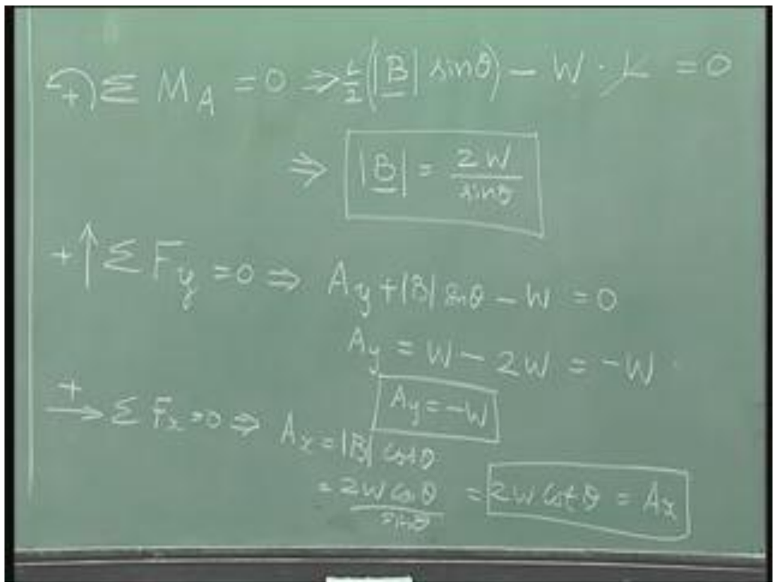

I need to find out which is the best configuration I can have. For example let's say this is the platform. At what angle should I fix this B D in order to optimize my design? That's the next question that we can try to answer because we know what all forces act on which rigid bodies. Whether it is a fixed frame or A B C or B D. Thank you.

Let's get a physical feel of this particular problem. Let's say this blackboard is the fixed frame of reference. Supposing I take this particular body A B C and I fix at A to the fixed frame with respect to the fixed frame, the body can only move. I am talking about only planar rigid body, the body can only move by rotating about A. It's very simple, you can have a look at it and it's not very difficult for you to make. Just make scripts of these sheets and get an understanding.

Now in order to prevent this rotation, what one can do is add this particular link. Let me just do this pinning, one end of this link B D to this particular central point. The problem has to do with the central point but if it is some other point also a similar understanding can prevail. So let's say this is the horizontal that I want to make. Now if you look at it this can move and this can move independent. If I fix this particular point to the fixed frame, this cannot move and therefore let me do that exercise of fixing. So I have fixed A and D now to the fixed frame. Remember it doesn't allow movement either in x direction or in y direction.

If I take I am going to call this as x direction and this as y direction. I can just say that this is the notion of x and y. This is along x direction, this is along y direction. These two are along y direction so it doesn't let this point D move in this direction as well as in this direction which means there has to be a force that is offered by the fixed frame in both x and y direction, similarly at A. If I remove these two then there is a freedom to move or in other words, I need to apply two forces, the fixed frame has to apply two forces at A and two forces at D in order to keep it stationary, the system stationary. Now like what we did earlier, if we have to examine the free body of B D, what you do is we already have D fixed to the fixed frame, A B pinned to A B C here. If I hold D D fixed to the fixed frame and examine what movement this can have. Now if I remove this, it will have a movement of this sort. Now it cannot move at all, if I remove this you can see that it can move like this or in other words the body A B C offers a resistance for movement of B on B D which means we have to have two restraints that we need to place at B. That's what we did in the exercise.

In similar way if I have to draw for A B C, there is one restraint here which does not let A move in this direction or in this direction. If I fix B D to fixed frame of reference, it doesn't let A B C move in both vertical and horizontal direction and therefore I have to apply two restraints over here and two restraints over here. That's what we did while drawing the free body diagram. This simple understanding will be good to have when you are drawing the free body. Another thing to note is if there is a load occurring here, let's say a man is standing on this, this is A B C without B D will try to move like this. If B D is preventing it, it is pulling point B from rotating downward or in other words B D will be pulled by A B C. Therefore the force that I have to get on B C will be such that it is a pulling force. These are some understandings which will help in designing structures.

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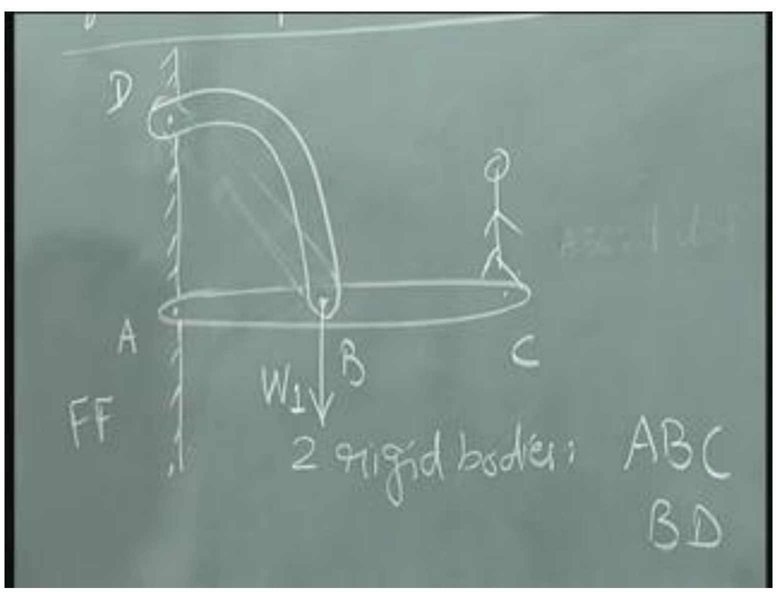

We have understood how to solve this problem over reactions at A and B and the forces in the rigid bodies A B C and B D. Let me twist the problem a little bit now and see if we are understanding it. Instead of this link being straight let's say I have like this. I have like this now, same configuration except this is now curved like this. What will change in your solution? Very simple question. What will change in your solution? That's the first question. The question number one, what is the difference in the solution between the previous configuration and the current one that I have drawn here. Try and answer this question. Question number two or challenge number two, let us say I have something like this, I am going to hang a weight over here, let's call this as weight one. How will I draw the free body of A B C and B D such that you account for this force acting on it?

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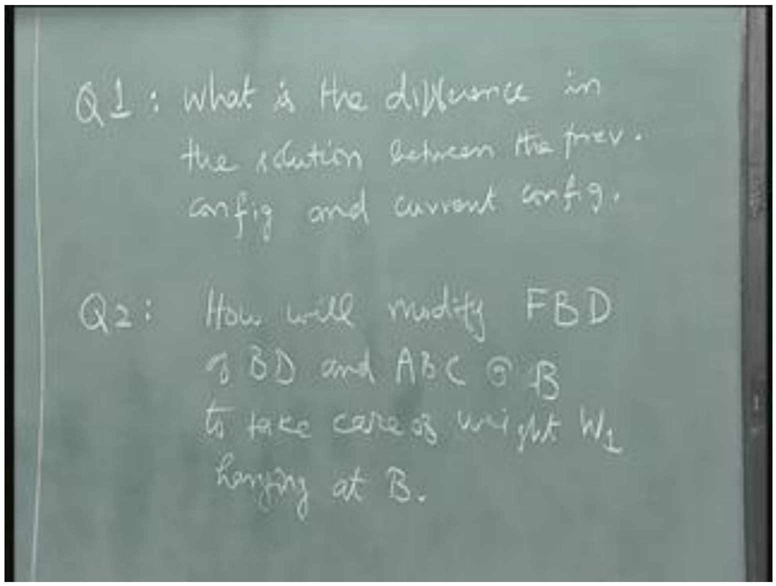

I am just going to write it, how will you modify the free body diagram. I am going to call by acronym F B D of B D and A B C at B to take care of weight W_1 hanging at B. That's the second challenge that I want you to answer.
Thank you.