Engineering Mechanics
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Dynamics of rigid bodies

Just to recapitulate in terms of kinematics, we are just dealing with rigid bodies right now. One thing that we have to have in order to define the kinematics is a fixed frame of reference. In this particular set of modules, we will deal with fixed frame of reference and denote them with capital letters, capital X, capital Y or capital I, capital J for the directional vectors. Capital K, capital Z for the other direction. Now there may be many either translating or rotating frames of reference, so let me call this as x_1, y_1. I can say i_1, j_1. There could be a rotating plus translating frame of reference with respect to which something else may be defined. Just to give you an example, the earth is rotating.

So let's say I am stationed in Chennai which is just a little bit displaced from equator. I am moving around and there is movement that I have to define its velocity, its acceleration. But what I know is moment with respect to a particular point in Chennai. If I have to do that first, I should fix a frame of reference which is fixed from the angle of how Newton has defined the equations of motion. That's what we call as fixed frame of reference. Now going to go into the details of what is fixed frame of reference, let's just go with how Newtonian frame of reference has been defined. Given this, once we have an idea of how this frame is moving with respect to fixed frame, we can bring all the motion which are defined with respect to moving frame of reference with respect to a fixed frame of reference.

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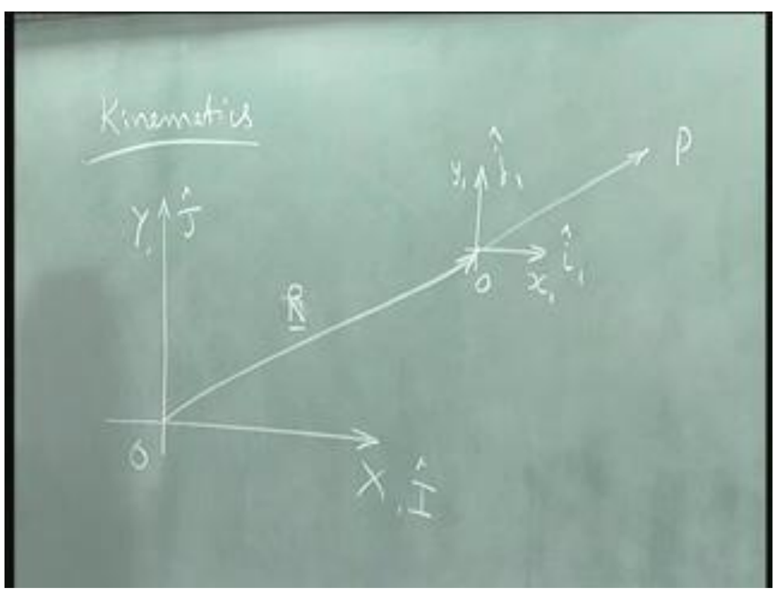

For example if there is some particular body or particle lets say P which is defined with respect to the moving frame of reference and with respect to this moving frame of reference, I see that there is some change that is happening to P. But what I know is how it is changing with respect to this moving frame of reference. If I have to find out, what this movement P is with respect to fixed frame of reference, I have to do the transformations that we did in the last class, in the last module.

Basically one thing that I have to know is lets say this is some small o, this is capital O lets say. This is some r, let me use capital R to denote the origin of this. Then I can write what is P with respect to capital X capital Y, given the movement of this moving frame of reference. The moving frame of reference can do two things. I am just going to stick to those which are Cartesian frames. Now these two could probably rotate. So this frame probably could rotate as well as move. Now the movement of that particular frame, the translation of that particular frame can be easily defined through the change that occurs in R and you have a rotation that takes place about a particular axis. I am just showing it in one plane here, it can be generalized to three dimensions. With respect to that axis there is a rotation that takes place on this moving frame of reference. We use this rotation in order to define what happens to P with respect to capital X and capital Y.

There could be multifarious moving frames, for example, I could be in Chennai and there is a vehicle moving. Inside there is this studio let's say and I am moving around within the studio. I have one frame which is Chennai that is moving with respect to the fixed frame. Within Chennai I have a vehicle in which let's say this studio exists which means I have another frame of reference with respect to this frame of reference. Let me call it as i_2, j_2 and I could be moving around here. Let's call this as P prime which is shiva moving around like this.

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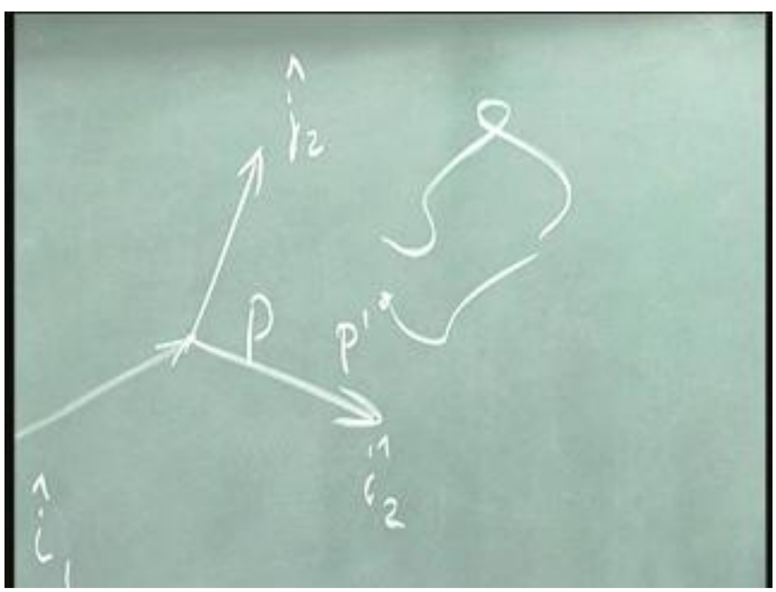

If I have to define this, I'll go hierarchical way. I will go from this to this, find out what happens to this P prime with respect to P and then to this. Alternatively if it is possible for me to find out the movement which refers to translation and rotation of this i_2, j_2 with respect to capital X, capital Y then its easy for me to write down directly what happens to this P prime. Once I have a transformation available between each one of these frames then I am done. That's basically what we did in the last class. We just stopped with just one rotating frame but if I have another rotating frame, it's just a hierarchy of rotations and translations that I have to take care of and this is all there is to kinematics of rigid bodies. The moment we have deformable bodies then we have to look at the relative changes within the deforming body. Let's not explore that right now.

Dynamics of rigid bodies:

Let's having looked at kinematics of rigid bodies, we will look at kinetics. What is kinetics? Kinetics is the one that relates kinematics or the motion to the forces and it relates these two through a law and in this particular case, we use Newton's laws in order to relate them. In most of the laws, you will have a kinematic quantity and a kinetic quantity and the relationship is proportionally large. Some people call it Newtonian principles or Newtonian laws. The difference is a debate that still goes on. I am not going to go into that.

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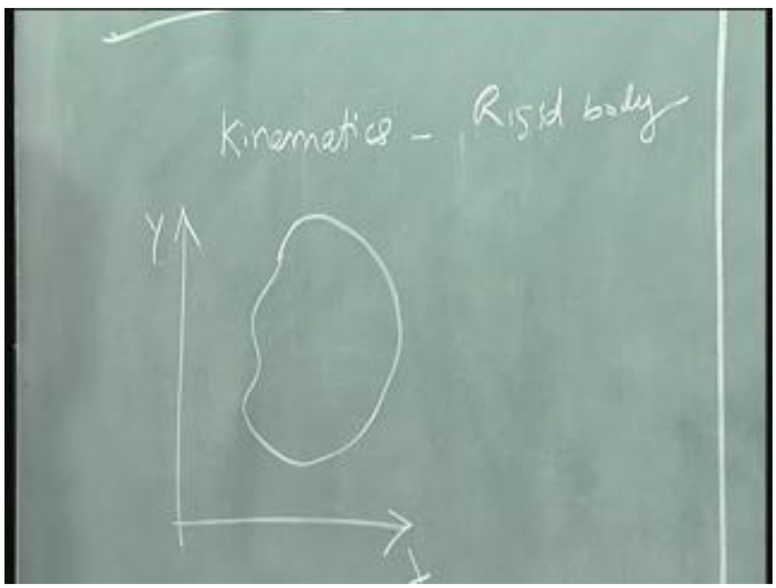

Given this let's say I know kinematics of the rigid body and I now wish to relate the forces acting on the rigid body to the kinematics and that's what I want to do. Let's look at how we can do that. Let's say there is a rigid body, a fixed frame of reference that I always use. Let's assume for now, with respect to fixed frame of reference, I know all the quantities especially the acceleration quantities. The first exercise that I may do is to look up the acceleration quantities with respect to fixed frame of reference. Given that I can now proceed to look at the equilibrium of this.

Now when I say equilibrium here, I am talking about satisfying Newton's laws. What does Newton's law state? F is equal to m a. There are two things that happen here. This is okay for a particle because there is no resultant moment that we are looking at. In a rigid body however we have a moment as well as a force that appears. The force is directly handled through f equals m a and the moment has to be handled appropriately. I am going to look at the result first and then we will go back and find out what the quantities are that we have in the law. Let's say I have a rigid body here and whose mass is already known which is m. I wish to find out the relationship between acceleration and force. Let's say for now there are some external forces acting on the body, so let's say F_1, F_2. Let's say there is a moment M_1, let's say F_3 and so on. In static's we already looked at this particular set of forces and found out how to write down the static equilibrium equations.

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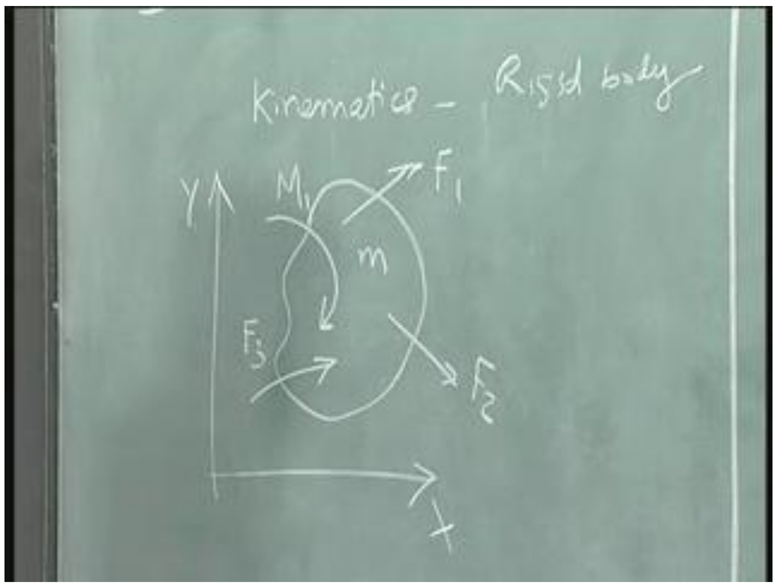

What we did there is we looked at all these forces. Finally we put them all together and said there is a resultant force and a resultant moment at a particular point that I can take where this is sum of all the forces and this is sum of all the moments and the moment of the forces. So M is sum of all the moments plus sum of all the r_i cross F_i. This is basically what it is. We did this exercise in the statics problems. Given a particular point we did this, found out these two and said since there is no acceleration, this should be equal to zero and this should be equal to zero so that we have equations that solve for a static equilibrium requirement of the rigid body. Now it's a natural extension that I want to bring in which is, it is no more in static equilibrium, it is in motion.

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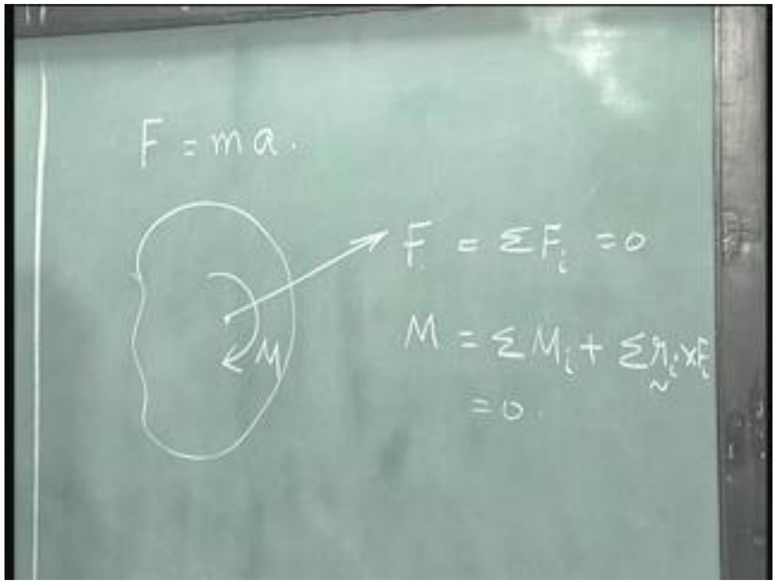

The motion could be constant velocity which means acceleration is zero or you could have constant angular acceleration etc. Now as far as the force resultant is concerned, I can look at this particular rigid body and focus on this particular point. Let's say I am going to call this point as P.

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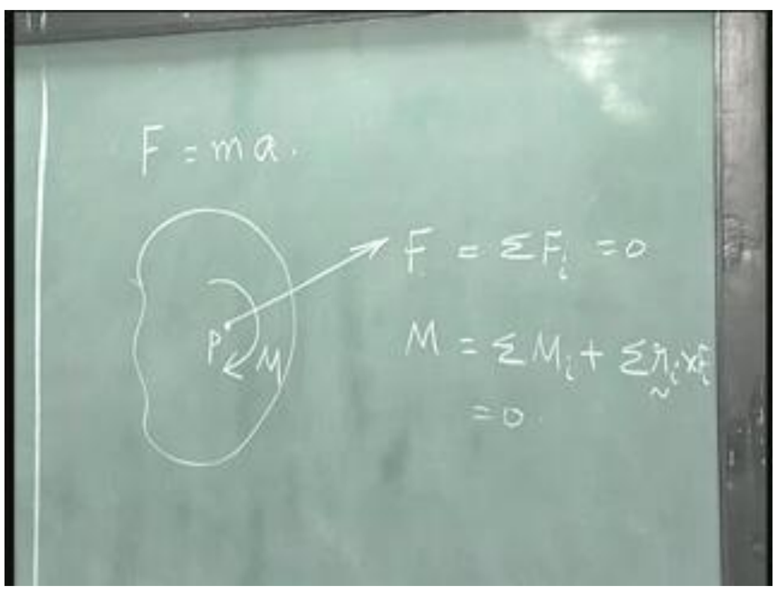

I have looked at the resultant moment and resultant force at this particular point P. Now what could be happening to this rigid body? This rigid body could be moving so that P moves along and it could be rotating with respect to P, pivoted about P. As we have already talked about, we can take any point on the rigid body and describe the motion of the rigid body through a single translation of that particular point in question and a rotation about it. Supposing this rigid body has taken another shape over here, we did this exercise earlier and if this is the point that we have as focus. The first thing that we could have done is do a rigid body translation like this so that you get something like this and a rotation to accomplish the configuration that we have here. Given this I can describe the motion of this rigid body with respect to this reference point P as a pure translation and a pure rotation. Therefore, I can attribute the translation to F and the rotation to M, simple. What should I write? If the translational acceleration of this rigid body at point P is, let's say a_p and the angular acceleration, so I am just going to say this is a_P, it's a vector. Let's say there is an angular acceleration. Is this a vector? Yes, this is a vector.

Because you will have an axis about which it is rotating and there will be an angular acceleration. Knowing these two it is possible for me to now, I am ready to write down in Newton's equations. Looks simple but there are many other derivations that are involved in order to arrive at this particular thing. Let's just move on and write down. This is no more valid. What I will do is I will chop of this and chop this of. Since it is an acceleration and it has to obey Newton's laws, it is equal to total mass times a_P because, the force resultant in question that I have found out is with respect to P and I find the acceleration of that particular point in the rigid body that is a_P and i get an equation.

Remember this equation, F equals m a_P is a vectorial equation. How many components? F has three components, a has three components which means three equations I can write x, y and z in a Cartesian system. I am just going to stick to Cartesian system so it's easy to talk about all these things. The other is moment. What result do I get? Again a very simple result we get M happens to be, the same thing I can take but the right hand side now will be some I times alpha. What is this I? I is the polar moment of inertia about P, along the axis of alpha. I will repeat this again.

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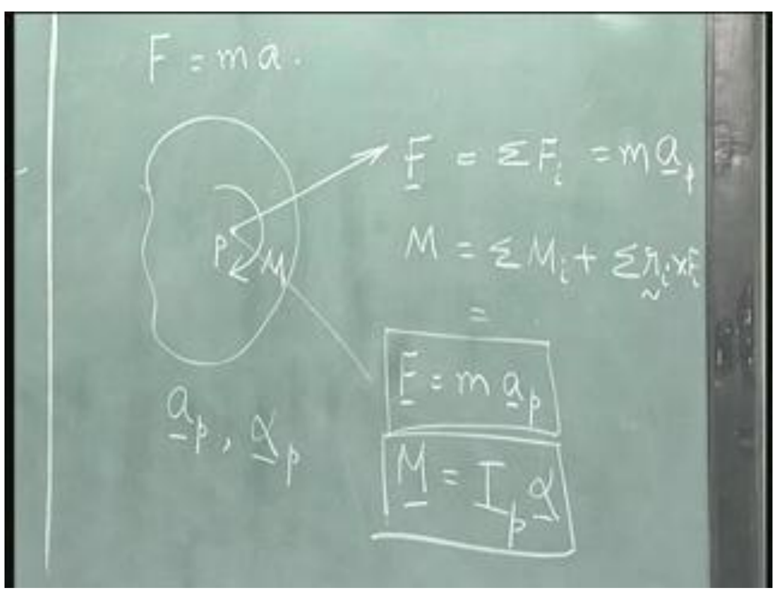

Supposing it is having an angular acceleration about an axis and I know the axis, I will take that axis passing through this P and find out the polar moment of inertia I_P and that will give me, M is a vector, alpha is a vector. If you look at this, I will get an understanding of how it is occurring. Remember in this equation these two are coaxial in the sense that direction of a_P relates to the direction of force, similarly here. Now how many equations do I get? I get one equation here, another equation here, a vector equation and a vector equation which means 3 plus 3, 6 equations. I have 6 equations just like in static equilibrium. There may be unknowns here, unlike here which is right hand side equal to zero in the static condition. Here, the right hand side quantities also may be unknowns.

Essentially the problem related to kinetics which involves basically unknowns to the right hand side or a left hand side. Some of the literature talks about this as an inertial force. I would avoid that, I would go for F equals m a directly because this completely defines the law that I am using. Is this clear? Knowing this I can do kinetics directly. What are these a_P and alpha? They are accelerations and angular accelerations and linear accelerations with respect to a fixed frame of reference that is described by F equals m a, as very very important to know. You have done this exercise earlier but I am just stressing this over and over because many times we make a mistake from this particular angle. Fixed frame of reference, fixed frame of reference that should get fixed in your mind.
Thank you.