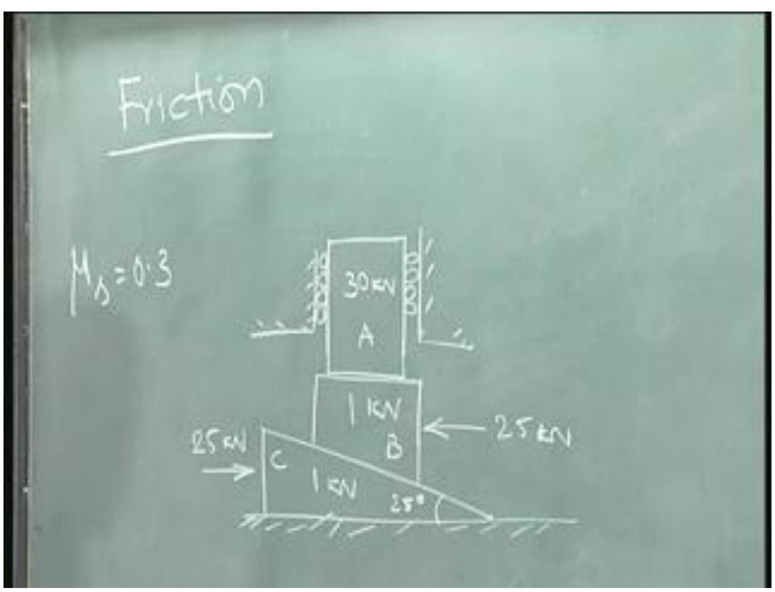
Engineering Mechanics
Prof. Siva Kumar
Department of Civil Engineering
Indian Institute of Technology, Madras
Static's of rigid bodies

We have a problem in friction that needs to be solved. Let me just explain the problem and then seek to find out a solution to it. The problem goes like this. There are three blocks A B C, the top block is 30 kilo Newton. I think this is an engineering problem where I wish to move this block upward or downward whatever the case may be. What they have done is they have two blocks here with a slanting contact between them. Now this is a wedge, this is another wedge but in a block shape. If I push these two this way, this should go up and if I gradually release this, it will come down. That's the engineering problem. We have rollers over here so that there is no friction along this. It's only the normal reaction that come from this sides of 30 kilo Newton.

These are small blocks lets say wooden blocks so they are not heavy one kilo Newton, one kilo Newton I suppose. The angle is given as 25 degrees, it can be 30, 40 also. I am just taking from a textbook here and remember if there is a 25 kilo Newton, 25 kilo Newton. Basically what is happening is there is a guy over there, may be I am just guessing and he is trying to push. So they push this way and they push this way are almost the same, the 25 kilo Newton. Of course 25 kilo Newton is huge, 2.5 tons push is what we are talking about. May be there is some jack and that jack has arms that go towards each other and therefore you have an equal and opposite force coming up. I am just guessing a few things over here as an engineering problem.

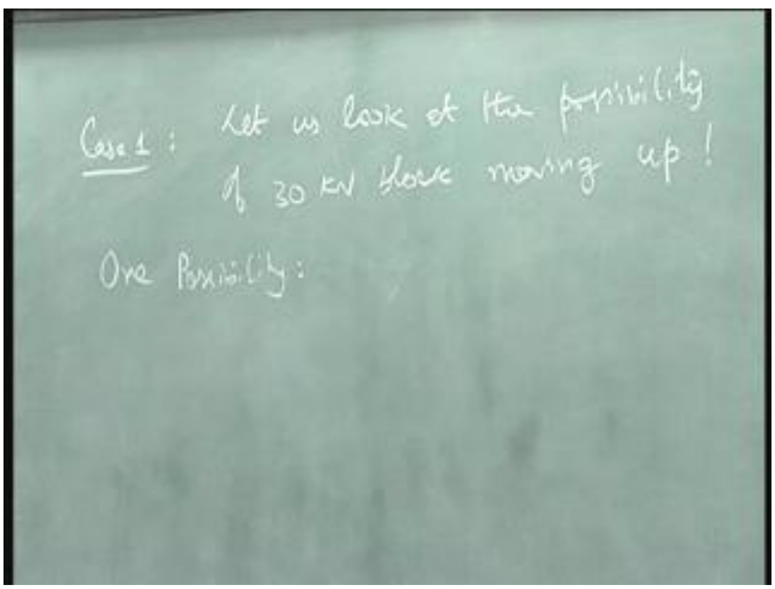
(Refer Slide Time 02:17)



Now there are other parameters given for example the static friction coefficient between all the contact surfaces. I presume the contact surface is between A and B, B and C and C and ground. In all these the static friction is 0.3. The question right now is given that I have applied 25 kilo Newton over here, do you think this will move up, do you think it will move down or it will remain stationary? So it looks like there are few problems that we have to handle it here. Supposing I consider purely static's. What happens? There will be some force over here, there will be some force over here, there will be some force over here which is a frictional force but not exceeding mu$_s$ times the normal reaction that comes out to each one of these blocks.

Now if this block has to remain stationary, there are two possibilities. One is that it is just static, the other could be that these two blocks slide. I am just guessing that kind of a situation may not occur because the reason is you have a 25 kilo Newton here, 25 kilo Newton here which means there is no net horizontal force in order to push it this way or this way. One possibility is that this lets look at one of the cases. Let's take case one and we are looking at a possibility of 30 kilo Newton going up. Shall we do that? Let us look at the possibility of 30 kilo Newton block moving up or down. Let's just consider moving up. If this has to move up one possibility is A and B stick to each other lets say and this C moves to the right, that C moves to the right could push it up that's one possibility. Is there any other possibility? There is another possibility, C could remain as it is and B goes towards the left.

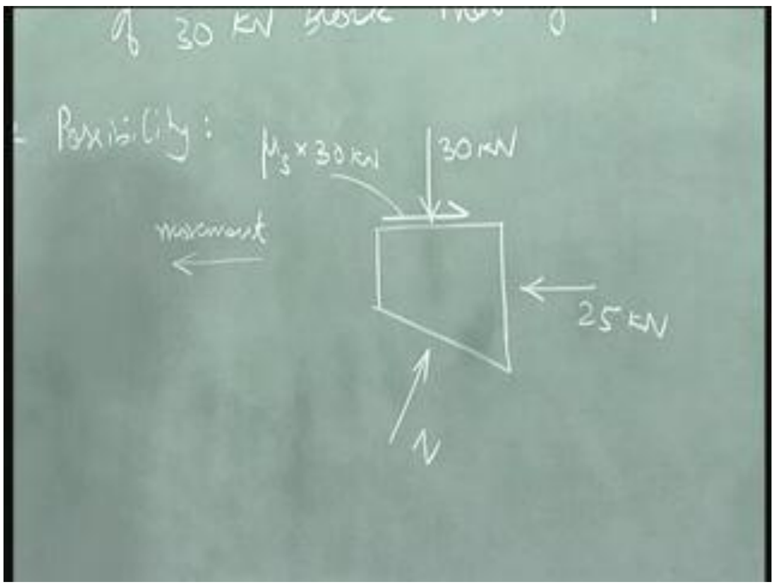
(Refer Slide Time 05:29)



If B goes this way and again I will see an upward movement. The other alternative is that both of them move simultaneously. In either case I see that this block has to move to the left. Let's look at that possibility first. One possibility is what we are exploring. I am presuming that if I look at that possibility, there are many other things that we will understand from that possibility. Let me just draw only that block. I have a slanting contact over here, there is a 25 kilo Newton coming in from other side.

Since there is no sideways friction on this block, there is only one vertical force that is acting on block B from A and that's 30 kilo Newton's. There is no problem about that and since it is moving this way I could have a frictional resistance to that movement. I am assuming that the movement is in this direction. It could be an impending movement but remember in a frictional problem, if I am looking at limiting cases. Remember I am again repeating this, in a frictional problem if I am looking at limiting cases I have to assume a particular kinematics that is impending, very important. Here the kinematics that I am thinking of is that this block moves to the left. That's one important thing that I have to do. Given that it is going to move this way, the static friction will reach its limit equal to mu$_s$ times 30 kilo Newton in this particular case because 30 kilo Newton is the normal force that I know. Here, there will be a normal force (N), of course if there is equilibrium I can find this force. If there is acceleration then we need to worry about it later.

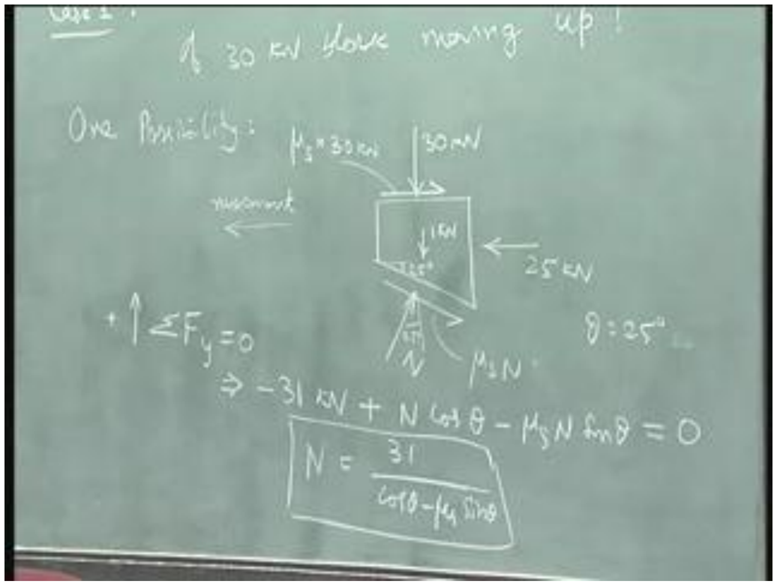
(Refer Slide Time 07:24)



Again since the movement is in this direction, there is a resistance offered in the opposite direction to the movement and therefore this will be mu$_s$ times N (the normal force between the contact surfaces of block B and C). Let's just draw the angles. This is 25 degrees, so let's say this is a 25 degrees, so this is 25 degrees and there is also a weight coming on to this which is one kilo Newton. Anything else that I have missed? I have almost done everything. Now this is the free body diagram. Remember I have assumed a static case that assumption itself may not be holding true but let's say for the given condition that I have, it is in statics to start with and that say the movement is impending only.

In which case mu$_s$ times 30 and mu$_s$ times N makes sense. Now after that it is very simple. All I need to do is the equilibrium equations, vertical components 25 does not take part, mu$_s$ N and N will take part, 30 kilo Newton will take part, one kilo Newton will take part. Again I will write. I can take upward direction to be the direction positive. This implies that I have 30 plus 1 minus 31 kilo Newton downward plus N times cos 25. Let me just assume theta first and let's say theta is equal to 25 degrees. This will help us later so that in case I have to change the angle. The direction of mu s times N is in the downward direction, so it is minus mu_s N sine theta and that should be equal to 0.

(Refer Slide Time 10:24)

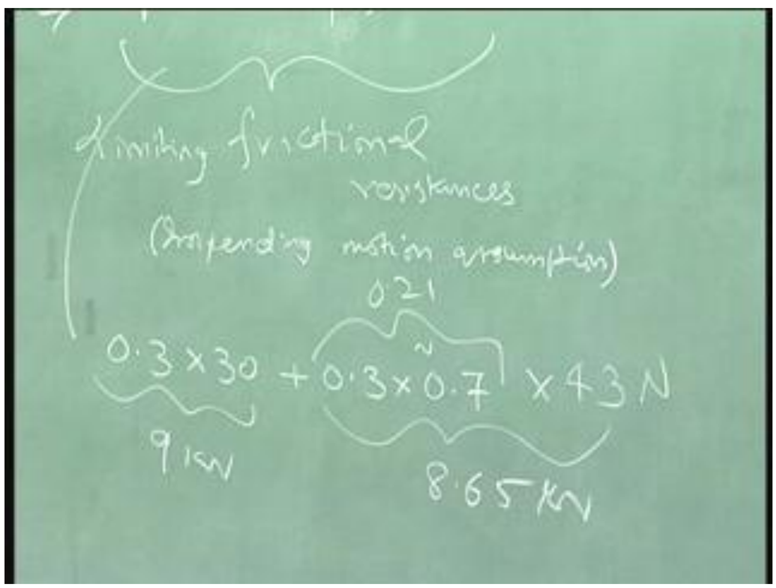


This is pretty simple. I can find out N from this directly. What is N? N equals 31 divided by I have a cos theta minus mu_s N. So cos theta minus mu_s sine theta. Let's substitute values and see what we get. If theta is 25 I think you don't have a calculator right now, no problem. If it is 30, cos 30 is root 3 by 2, sine 30 is 1 by 2 and mu_s is 0.3 and roughly what do I get here root three minus 0.3, root 3 is 1.7 odd minus 0.3 is around 1.4 by 2 which is 0.7, roughly around 0.7 is this quantity. 31 by 0.7 works out to around 30 % more. So it may be shooting up to around 40 kilo Newton's. I am just looking at a ballpark figure here just to understand. Remember I am just doing it extempore.

Having done this now it's possible to find out whether this possibility exists. The question that we have right now, is there a possibility that this could exist. Now I have a force, resistance due to friction towards the right hand direction. Similarly resistance due to friction in the right hand direction and there is a 25 kilo Newton which is offsetting. If I have to write horizontal equilibrium, I can write sigma F_x equals 0. Let's say this direction is positive, this implies I have mu_s times 30 kilo Newton's this one and this one is mu_s, so it is also plus mu_s N and that's a cos 25 or let me just write it as cos theta minus 25. If this is equal to zero implies impending. Please remember what I have written here. It is a very important thing that we have to remember. If there is an impending motion that's when I take it to be zero. If it is in motion lets say there is an acceleration, remember it is not equal to 0 it will have mass times that acceleration.

If it is not moving, if it is not impending these assumptions that I have made are not Correct. Remember so many assumptions that I have made but what is true here is this value mu_s times N or mu_s times 30 are the highest values that are possible, if it is in static condition. Therefore if the values are lesser, we will find out what is happening. Let’s look at this, these two are the frictional resistance. I am going to add this word limiting. I am adding this limiting because this is an impending motion assumption. This is an important thing to remember because when we do this which is assumed statics and just go on, don’t bother about what is happening to the structure. This is again impending, this is impending. This is only due to the frictional resistances and this is what is applied. Let’s just estimate what could be this. I am going to do a rough estimate here, mu_s is 0.3 so this is 0.3 times 30. This happens to be 9 kilo Newton’s and I have a cos theta times mu_s that is say plus 0.3 times cos 30. If I take theta to be roughly 30, cos 30 is root 3 by 2 which is around 0.7 let’s say. This is roughly I am just writing. This together will give me 0.21 times N, N is given by this which is around 43. This is roughly around 43 kilo Newton’s. For want of calculators I have just put this.

(Refer Slide Time 15:56)

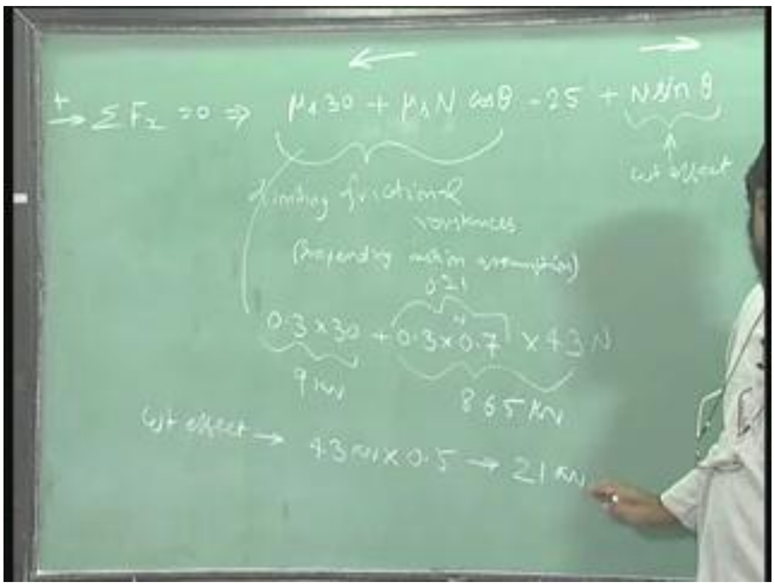


So its 0.21 times 43, roughly it is around 8.6, 8.7. So roughly around 8.65 kilo Newton’s. Have I done this correctly? This is an important step, have I done this correctly? Most of the times we make a mistake here. Have I done this correctly. Let’s assume this, if mu_s is equal to zero these two are gone, only this exists. How come? Because I have missed out this N, N has a component. What is the direction of that? That is also in the positive direction so it is plus N times sine theta and this is due to the weight that is coming on to it, this is due to the weight of it. I am separating them, one is the weight of it.

The other is frictional effect, of course I cannot separate the two directly but just to get an idea. What is the rough value of this? This N is around 43 so let’s just write that also. Let’s say this is weight of it, just to get an idea and what would be the weight of it? this is 43 kilo Newton’s times sine 30 is around 0.5 and that gives me around 21 kilo Newton’s. Supposing mu_s were zero then you find that this is around 18 to 20 kilo Newton’s force that is offered in terms of sliding, in terms of moving this way. Is that clear? Therefore if

I look at all of them together, this 9 plus approximately 18 kilo Newton friction is offered.
Roughly almost the same amount of force is offered by the weight of it and if I have to think of a resistance, resistance will come in terms of for example if I have to push this all the way like this, the resistance will come from this and at the weight of it. I have to offset 18 plus roughly 18 again or let's say 20 plus 20, I need to have at least 40 kilo Newton's in order to push this guy. I have done a very very rough calculation but it's very useful to me now and remember what I have done. I have looked at the limiting case of resistances, I have looked at the weight of it.

(Refer Slide Time 21:11)

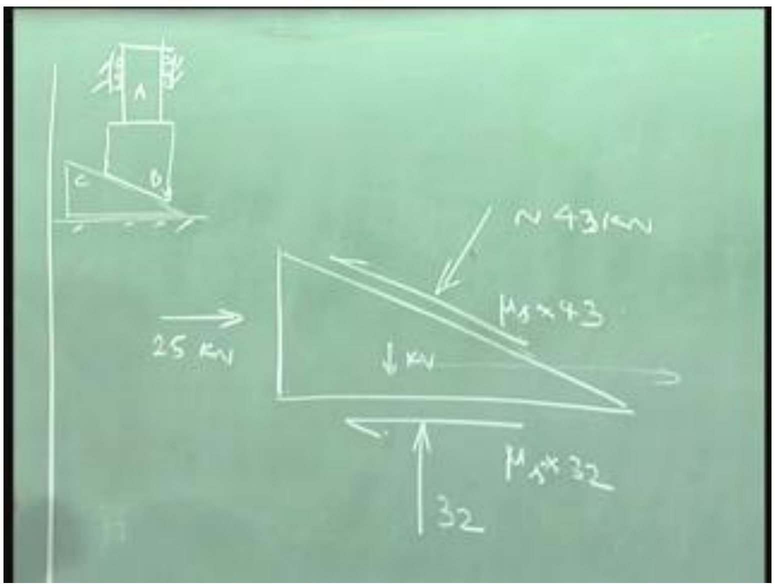


Supposing there were no friction at all and I have to hold it in place, I need to apply around 20 kilo Newton's in order to keep it in place but the frictional force is enough to keep it in place. Is this clear? Remember this is impending motion in what direction? Leftward. Let's just make it a little different. Let's say the movement is this way. When will that occur? When will the movement occur this way? When this is pushing it and this is sliding down, correct. Let's look at that particular possibility. So this is sliding, what all will change here? The direction of frictional force are the only ones that will change. The direction of frictional forces will affect to a certain extent, this normal reaction but remember mu_s times sine theta is roughly around 0.5 times 0.3 around 0.15 or around 15% is what its contribution is and cos theta is roughly around 0.7 and therefore the major effect is due to cos theta.

Now the reason why I am putting this here is there will be a plus sign coming, the moment it is the other direction. Now look at this carefully. This is the resistance that we have now. This is due to, the direction only has changed now. How about this direction? Will it change? It will not change, this direction will remain the same and what is the direction of this? Like this. What is the direction of this? Like this. Forget about this right now, let's look at these two effects. This guy needs 21 kilo Newton's to approximately to keep it in place.

Do I get that 21 kilo Newton's from here? Roughly 18 kilo Newton's is what I am getting from here. The remaining I can supply through a force 25 kilo Newton because it is opposing the motion towards the right. So 25 is more than enough for me to keep it in place. So two things that I have found out now. The movement is not to the left, the movement is not to the right because I have already shown that 25 kilo Newton is more than enough to keep it in place and if 25 kilo Newton has to push it to the left hand side, the possibility does not exist we have seen that. So left hand side movement is not possible, right hand side 25 kilo Newton is more than enough for it to keep it in place which means it is in the static condition as well as this block is concerned. So far so good.

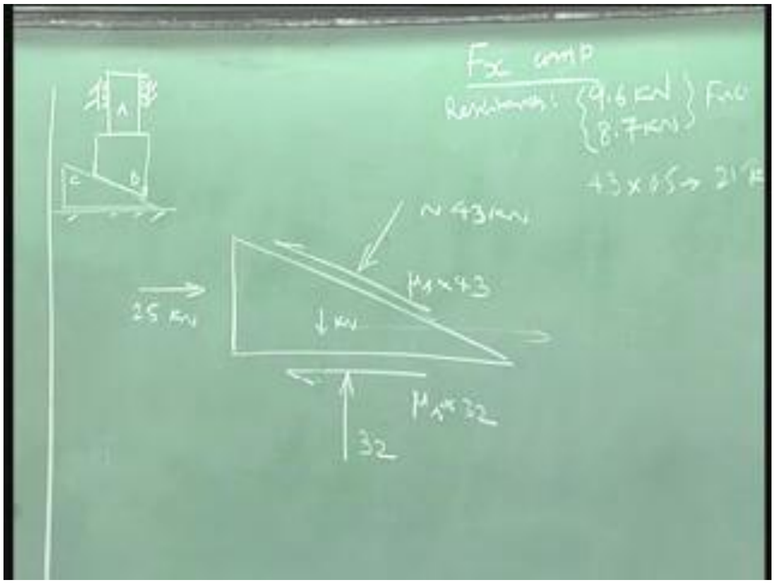
(Refer Slide Time 22:59)



One of the good things that we have seen is that this guy is in stationary condition, this and this together are now occurring. There is one possibility of pushing this. If I look at that possibility and discount that then I know the entire thing is in static equilibrium. We have seen that A and B don't move relative to each other because of the earlier exercise that we did. Now let's see if only wedge C can move towards the right, in which case it can go on. There is an up or down possibility, so let's look at those possibilities.

Now if this goes towards the right hand direction, the resistance is offered by number one the frictional force that is coming on because of B C contact and the frictional force offered by ground on to this wedge. The other resistance is due to the component of normal reaction. Let's just write those. I am just going to write F_x components. First let's look at the resistances to this application of force. One resistance is due to the friction below that is 0.3 times 32 is 9.6 kilo Newton's. This we already found out is mu_s times 43 times 0.7 or so that came out to be around 8.7 kilo Newton's. These are the frictional resistances then the other resistances that comes in is due to this normal reaction and that is 43 times sine 30, roughly around 0.5 that gives me 21.5 or let's say 21 kilo Newton's.

(Refer Slide Time 25:06)



There is a resistance in terms of the normal reaction which is 21 kilo Newton's. Remaining 4 if is taken by the frictional force then the block cannot be moved by 25 kilo Newton's. I need to overcome the frictional forces in order to do that. But the frictional forces themselves account for around 18 kilo Newton's. So in which case if I have to move this, apart from this 21 adding 18, roughly I should have 40 kilo Newton force in order to move the block up.

Let's look at the other possibility. Can this wedge move towards the left hand direction? If it is moving towards the left hand direction, the possibilities are it has to resist this 25 kilo Newton's, it has to resist the two frictional forces now in the opposite direction. Therefore 21 kilo Newton's should be able to push this to the left but the resistance by friction itself is 18. The remaining 3 or 4 kilo Newton's is automatically offset by this 25 kilo Newton's which means that this wedge C is also in the static condition. Therefore there is no movement that occurs from what we have found out. I have done a rough calculation, specific calculations you can do in order to understand this completely but as we have seen already here, looking at the limiting conditions it is within the limits of movement in one direction or the other direction and therefore it is in the stationary condition.