**Engineering Mechanics**
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**Statics - 5.2**

Now what we want to do is given a surface which is let's assume the surface is circular in nature. I am just going to draw a little bit of it and fix it, so that it's easier for us to understand that it is not rotating. Let's say there is a belt that runs like this. As you can see between this point and this point, it is in contact with the surface. Let's just draw that. This angle is say theta.There is a force that is acting here is let's say $T_1$, there is a force acting here say $T_2$. This $T_1$ is such that it is trying to pull this cable so that it slides this belt or cable, so that it slides on the surface or about to slide on the surface. Therefore there will be a $T_2$ that can act in this direction.

Now the question is what is the force $T_1$ that has to be applied in such a way that I have a $T_2$ over here? I am just going to pull it with $T_2$ and ask what is the $T_1$ that I have to use in order for it to just start to move. So that's the idea. Let's take the angle to be starting from here to here. What do I know about these two points? At this point the tension in the belt is equal to $T_1$, at this point the tension in belt is equal to $T_2$. Should $T_1$ be greater than $T_2$? The answer is yes, because this has to pull with an additional force that is offsetting the resistance offered by the surface.

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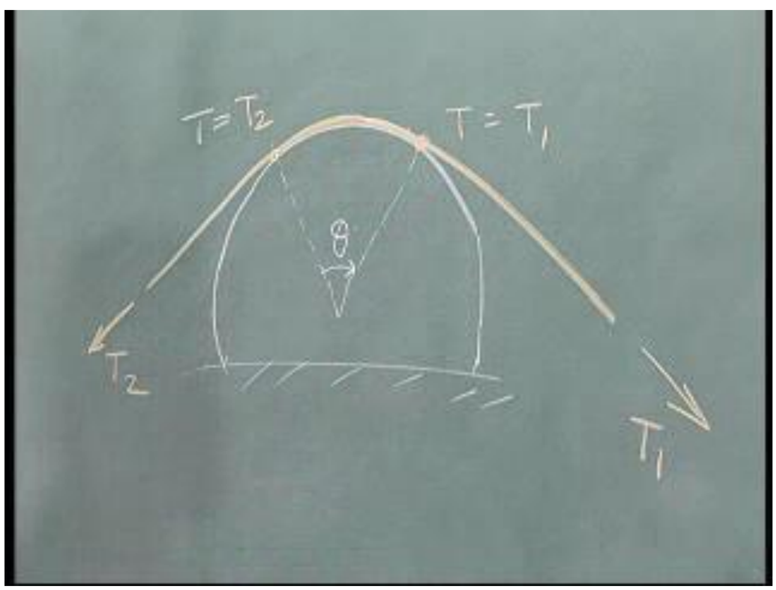


How do I find out this relationship for the impending belt motion? That is the question. Let's just do one thing. Since we know that the tension is varying over here from $T_1$ to $T_2$, there will be a frictional force that will also be varying from this point to this point. In a static case I will not be able to find out but in a case when there is impending motion, it is possible for me to use the fact that the frictional force (f) is related to the normal force by this relationship mu times N. I should be able

to solve this problem for the relationship. For that what should I do? Like what we have done in the earlier cases, we have to examine a small portion of this particular belt for equilibrium and that's basically what I am going to do in the next couple of months. Let's say I am just taking this small belt region that I have shown over here and I have separated the surface. The moment I separated the surface there is a normal reaction that acts on it. There is a tension over here and a tension over here. As I move from left to right, the tension keeps on increasing from $T_2$ to $T_1$. Let's say this is T and this is T plus delta T. Where is this region? This region is let's say, from this an angle phi from where the tension is equal to $T_2$, phi will vary from 0 when it is $T_2$ to phi equal to theta when it is $T_1$. I need to find out equilibrium of a portion which is in between.

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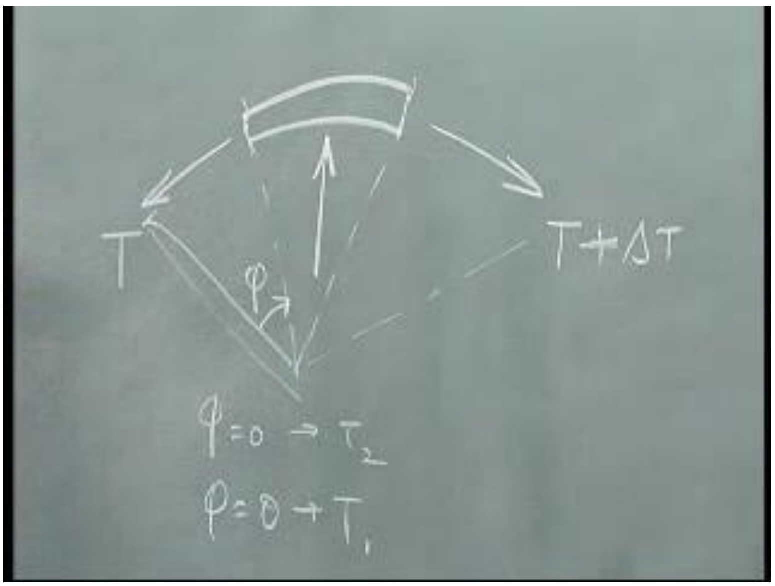


There will be a T plus delta T, I am going to call this as T at phi, T at phi, the normal reaction. The normal reaction is nothing but the pressure that is provided by the surface on to this belt, the resultant of which is shown over here. If the size becomes smaller and smaller, the normal reaction is also smaller and smaller. So let me call this as delta N, this depends on R times delta phi which is the angle that this subtends over this particular point. Now what else is missing? There is a frictional force that is resisting movement in this direction. So the movement is like this, what should be the direction of the frictional force? It should be resisting which means the frictional force is like this equal to f. Since it is an impending motion, I know f is related to delta N by mu times delta N. Let me call it as delta f so that it makes sense.

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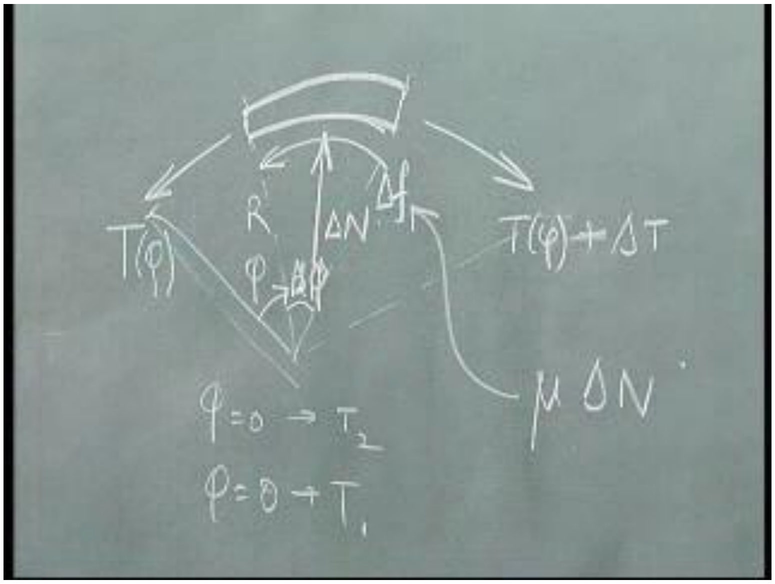


Having drawn this, now I just have to apply static equilibrium condition because it is just an impending motion, in order to find out the relationship between all these that you are seeing. Now since this is very small, I can take a tangential direction. Let me call this tangential direction as t and a normal direction which is n and take the equilibrium along t and along n or in other words the total force along t is equal to 0 for this body. This implies as you can see here there is an angle that this has with the horizontal or tangent which is equal to, if this total angle is delta phi, this is delta phi by 2. Similarly this is also delta phi by 2.

These two will have components in the x direction. What else will have component? This delta f, delta f in total may be taken to be tangential equal to mu times delta N. Therefore if I write all of them together, I have a T phi plus delta phi or T plus delta T times cos delta phi by 2 minus T cos delta phi by 2 because this direction is positive, this direction is negative and delta f is also in the same direction, so minus delta f, but delta f is equal to mu times delta N. That is equal to 0, this is the equation we get.

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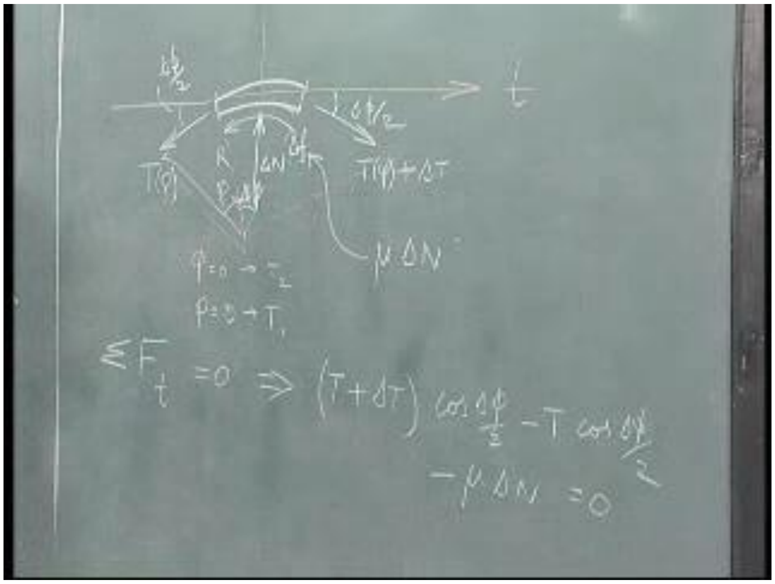


As you can see here T times cos delta phi by 2 and T times cos delta phi by 2 will cancel each other. So these two will go off. The net result we get is delta T cos phi by 2 minus mu delta N is equal to 0. Let’s just write it down. Let’s write down delta N, delta N will turn out to be delta T cos phi by 2 divided by mu. How did we get this? We took the tangential equilibrium. Now we will resort to normal equilibrium or radial equilibrium.

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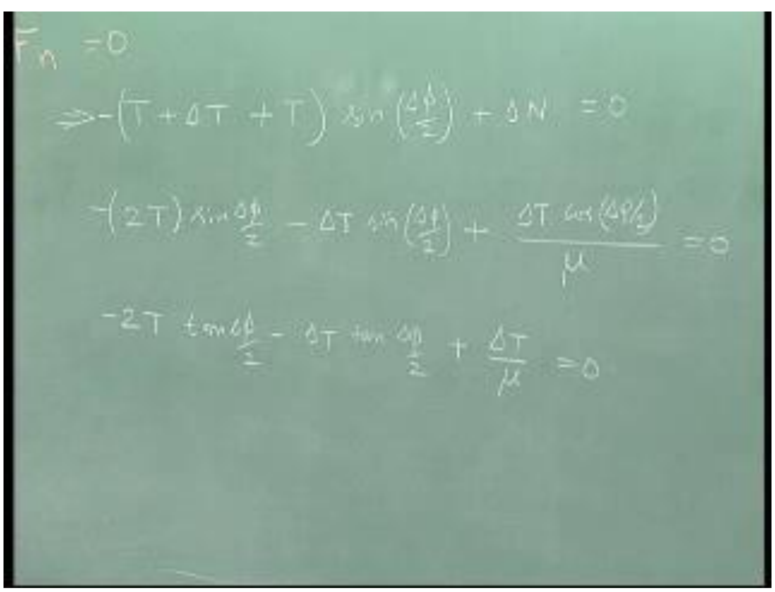

Sigma F along n direction is equal to zero. For this, this will play a role and the components of these two (the tension forces) along the normal direction will play a role. This implies we have a T plus delta T minus T times sine delta phi by 2. Is it minus or plus? The answer is plus. We have written the component of the tensions in the tangent direction. What is remaining is the normal ,and that directly acts on it. Which direction is this? This direction is negative due to tension and the other one is positive; plus delta N is equal to 0. Now we can simplify this, but before simplifying what did we get as result over here? We have delta N equals delta T cos phi by 2 by mu. I am going to substitute delta N in this equation. I have minus, this is 2 times T sine delta phi by 2 minus delta T sine delta phi by 2 plus delta T cos delta phi by 2 divided by mu and this is equal to 0.

Now what am I going to do? I am going to divide throughout by cos delta phi by 2. If I do that, I get 2 T sine delta phi by 2 by cos delta phi by 2 which is tan delta phi by 2 minus delta T tan delta phi by 2 plus delta T by mu. Now when delta phi is small, tan delta phi by 2 is equal to delta phi by 2. Using that relationship we get: minus 2 T delta phi by 2 minus delta T delta phi by 2 plus delta T is equal to 0. Here 2 and 2 cancel off. Let’s divide throughout by delta phi now, to get minus T divided by times delta phi by delta phi is minus T minus delta T by 2 plus delta. Have I missed a mu here? (Refer Slide Time: 13:42) plus delta T by mu by delta phi is equal to 0. Is this okay? Minus T minus delta T by 2 plus delta T by mu times delta phi. If I take delta phi tending to 0, realize this doesn’t have any delta phi. This is delta T by 2, delta T goes to 0, delta T by delta phi goes to d T by d phi. Therefore I will get from this equation delta T by delta phi is equal to mu times T limit as delta phi tends to 0, implies d T by d phi is equal to mu times T. This is an important relationship.

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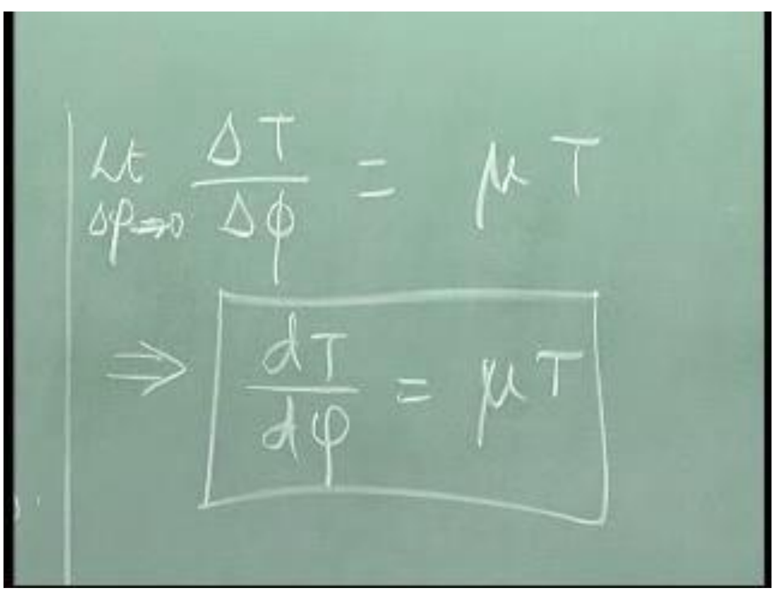


Now this relationship can be used because this is coming from equilibrium. There are two conditions that we applied, one is static equilibrium, the other is impending motion. Along what direction? Along $T_1$. These two are important because along $T_1$ tells me that

the friction is opposite to $T_1$ direction. Applying these two we get d T by d phi equals mu T.
This is an important relationship that we will be using in order to understand the friction on a circular surface.

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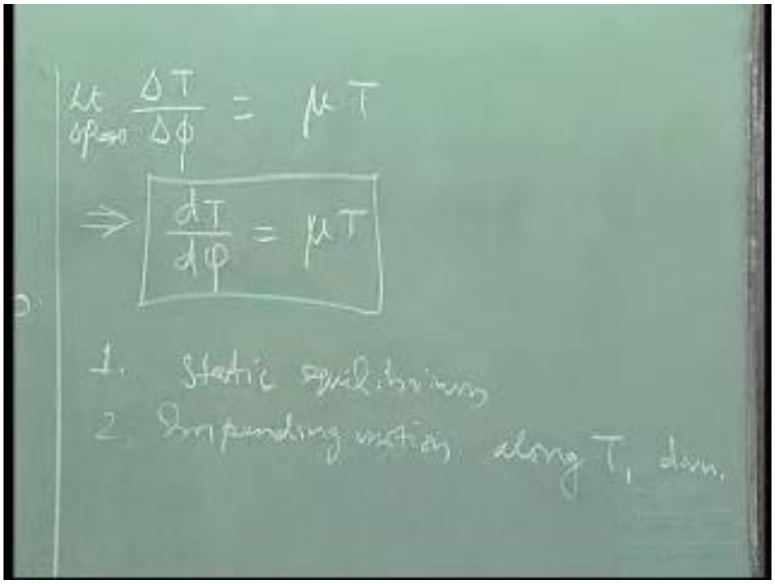


Let’s go back to this. We have therefore d T by d phi equals mu times T or if I have to find out variation of T, I have to integrate from phi equal to 0 to phi equal to theta from $T_2$ to $T_1$.

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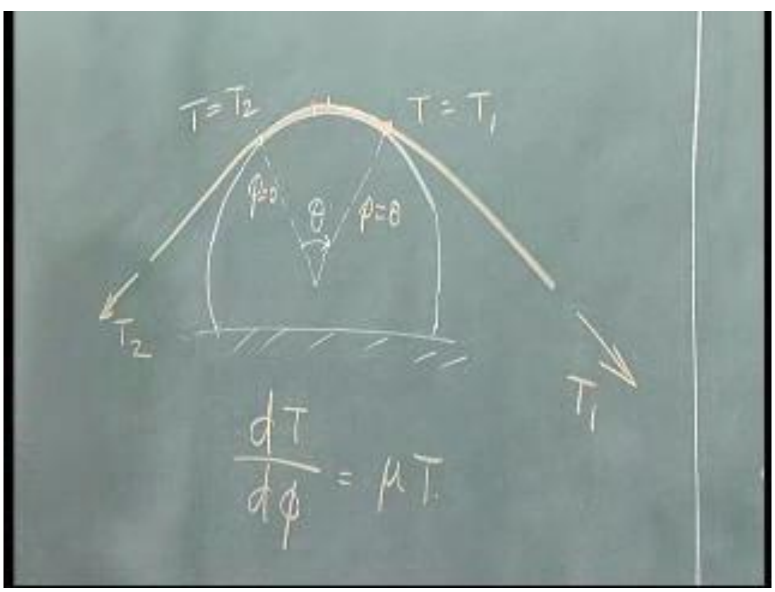

Let's do that exercise here. We have started from here and we are going to this, as you can see over here. Let's do that. I can take d phi over here, get T over here so that I get d T by T equals mu times d phi, integrating this limit is from phi equal to 0 to theta. At phi equal to 0 it is $T_2$ and this is equal to $T_1$, d T by T integrated will give me ln T, natural logarithm of T. So natural logarithm of T ranging from $T_2$ to $T_1$ is equal to mu times theta minus mu times 0 which is mu times theta. This implies logarithm, natural logarithm of $T_1$ by $T_2$ is equal to mu theta. I am just going to extend this and write implies $T_1$ by $T_2$ is equal to exponential of mu theta. Therefore in the impending motion along the $T_1$ direction, the ratio of $T_1$ and $T_2$ is determined through e raised to mu theta.

Now just let's check when theta is equal to 0, I have no contact here, very small contact. It tells me that the friction has to be equal to 0 which means $T_1$ is equal to $T_2$. Do I get that here? The answer is yes. $T_1$ by $T_2$ equals e raised to 0 which is equal to 1 means $T_1$ is equal to $T_2$. Otherwise there will be a friction that I will consider. So far this is what we understand. Now remember the ratio is an exponential of theta, so as theta increases gradually $T_1$ increases exponentially with respect to $T_2$.

Another way of writing this, is instead of writing like this, $T_1$ equals $T_2$ times e raised to mu theta. Think of it like this. Supposing theta is some let us say mu is, let's give it say 1 by pi which is approximately around 0.3 or whatever. I am just going to give an example over here. Let's say theta equals pi. What do I get? e raised to 1 by time pi times pi is equal to 1 which means $T_1$ equals $T_2$ times e, e is 2.7 times 2.71 or whatever which means roughly $T_1$ is 3 times $T_2$, a very high number.

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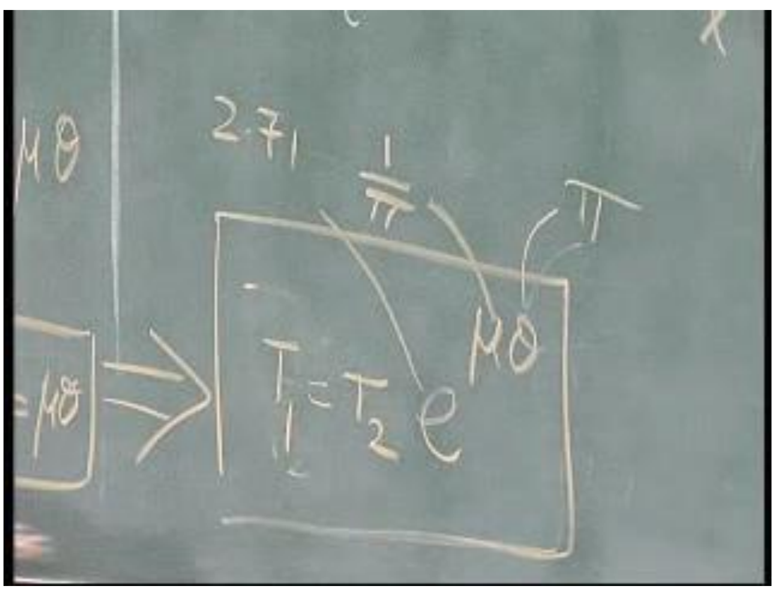


What does pi mean? In this particular diagram if it goes like this and like this so that there is an angle of pi over here. The ratio of $T_1$ to $T_2$ will be as much as 2.7 times. If I take one more circle before going like this for example I take one full rotation and again bring it back over here then this theta will now be 2 pi plus pi. This is 2 pi plus pi which means 3 pi which means e raised to 1 by pi times 3 pi which is e raised to 3. e raised to 3 is approximately about 8 or 9, this is 2.7 raised to 3 is roughly around 20 to 25 which means $T_1$ is 20y times $T_2$.

Supposing I am pulling this with a force and expect that I pull this with some other force. If I have to make a motion out of this, I have to pull it with a force which is 20 times the force that is offered over here. Think of some small rock that is subtended over here. In order to pull this, I have to apply 20 times this and this is an important application. In mechanical engineering this is what is used quite often in order to make sure that there is no slipping occurring. When I have a pulley like this, I will just make one more rotation like this and take it and guaranteed for most part of it unless mu tends to very small value. It is possible for me to keep in contact with the pulley.
Thank you.