Engineering Mechanics
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Statics - 4.5

Often teacher gets this complaint from students. Sir, you are solving very simple problems and asking us difficult problems. For a change I am going to try a little difficult problem. A problem that is usually unusual. We are going to solve a beam problem using virtual work principle. This is a beam problem say I need to find out the reactions at A, that's the problem given here. We have to use virtual work principle. You can use other principles also. Mind you in virtual work principle one of the advantages I will get is I will only have active forces participating as long as I have kinematically admissible displacements. The moment I have kinematically admissible displacements, it becomes easy. Let's do that exercise to find out.

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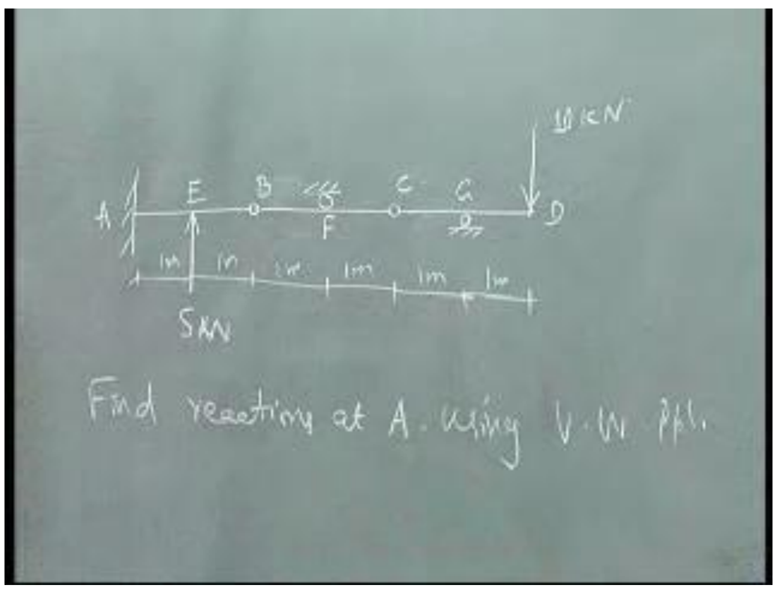

I need to find out the reactions at A using virtual work principle. What would be the first step? It is quite procedural and therefore we will start with each of the steps that we have to carry out. The first step that we have to carry out here is find out what kind of system it is? Is it a system with one degree of freedom, zero degrees of freedom, 2 degrees of freedom. In other words is it a stable beam, is it an unstable beam, is it an over stable beam or in other words is it an indeterminate beam. We need to find that out. To find the degrees of freedom for a beam, we already know lets just have a recap. If I have a beam like this, at this point it is prevented from rotation as well as movement. It's a beam problem which means horizontal displacement is redundant.

If I apply a force here, is it going to be stable? That's the question I am asking. If it is stable, what is the degree of freedom I have? Is it positive 0, positive 1, 0 or negative, each one of this will give me an idea of what this beam is? There are 2 degrees of freedom, forget about x there are two degrees of freedom for this rigid body and if I have two constraints for this rigid body that means total degrees of freedom is equal to 0. Very simple. How many degrees of freedom do I have? I have 3 degrees of freedom for a rigid body. Since this is the beam, one of the displacements, the horizontal displacement is redundant, which means I will take it to be 2 degrees of freedom, y displacement and rotation. How many constraints do I have? 2 constraints which means that imposes 2 constraints to the degrees of freedom which means I have a 0 degree of freedom system. Zero degree of freedom system means it's a stable system. You will not see any movement of this rigid body. If I have to see the movement I either have to remove it completely. If I remove it completely, how many degrees of freedom I have? There are no constraints which means it has 2 degrees of freedom.

Supposing I instead put a hinge over here, this means I have not two but one constraint which means I will get one degree of freedom. Supposing I do like this, please notice it carefully. I am going to release the vertical displacement but will not allow rotation which means what? It is rigidly connected to this guy and this guy can slide only like this. This guy cannot rotate like this. So another way of writing and drawing this is it has something like this where rotation is not allowed, only vertical displacement. How many degrees of freedom will this have? Of course the total degrees of freedom possible are two. There is one constraint in terms of rotation which means minus one, I will get one degree of freedom. This is the simple idea. Now let's look at this particular beam which we need to solve. Now this beam consists of many things.

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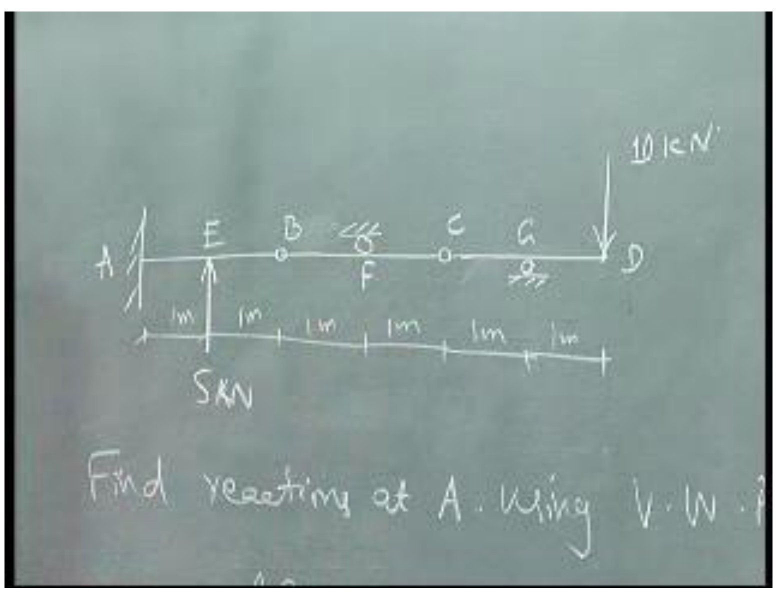

So its complex to look at to start with. Let's just break it down so that it's easy for us to understand. There is one part AB, so I am going to write AB. There is another part BC, another part CD so I am breaking it down into the three rigid bodies which are connected through hinges. Let's come back to this A later, between AB and BC, what do I know? Supposing I fix AB, the displacement B of BC is fixed. It offers a constraint at B. Whatever is the displacement of AB at B, it is the same displacement of B at BC. Correct? Therefore can it rotate between them? The rotation between them is allowed. As it is we already know, each one of them can have two degrees of freedom so I am just writing that here.

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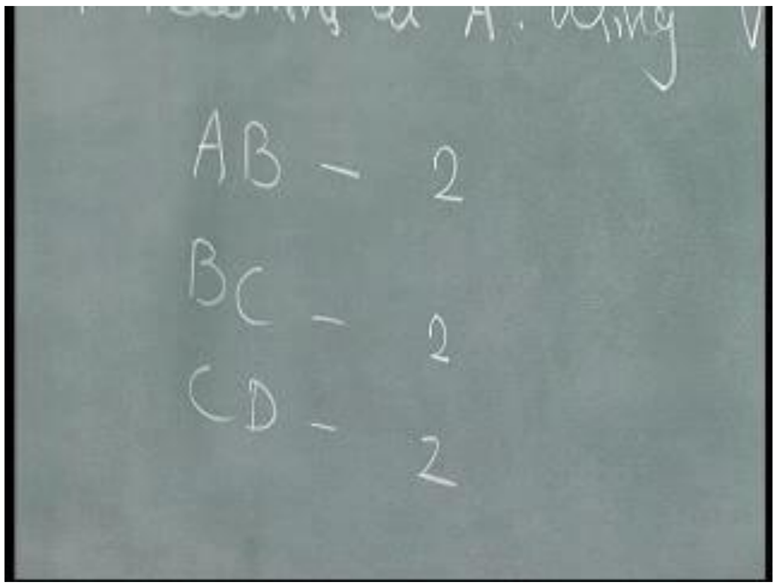

This is free degrees of freedom. It doesn't make sense but just understand it this way. If I completely free it, as a beam which means horizontal displacement is redundant two degrees of freedom is possible for AB, BC and CD. For AB and BC together there is one constraint that is offered at B. I have a minus 1 between AB and BC. Between BC and CD another constraint is in the form of hinges so one more is gone. How about the support at F? It vertically constrains the motion. support at G again vertically constrains the motion which means one constraint each at F and G. So F offers minus 1, G offers minus 1. We have considered F, we have considered C, we have considered B, we have considered G, D is free. How about A? A, I have arrested like what we have done here. We have arrested both rotation and vertical displacement at A which means A offers minus 2. So let's total them; minus 1, minus 1, minus 1, minus 1, minus two, total is 6. I have 6 degrees of freedom, constraints of 6 which means I am not left with any degree of freedom.

For the system total degrees of freedom equals zero which means it is a determinate system which means what? I can use equations of equilibrium to solve it. Mind you it's this way of doing should be done with caution. There are certain problems you will face but I am not talking about it right now. For example if this support had been over here, it is a completely different story. I am not going to talk about it in this particular problem. For now this holds okay and we have a zero degree of freedom system which means that it is a determinate system and I can solve for that. Mind you I have to do this for each one of these separately and each one of these should have the proper constraints. Then only it is possible to state that it is a determinate system. I have just over simplified it and said that here.

Now if it is a determinate system with a degree of freedom equal to zero, what do I know? I can't apply virtual work method directly. I have to release at least one degree of freedom in order to apply virtual work principle. What is asked is find the reactions at A? Correct? What are the reactions here? There is a moment reaction and a vertical reaction. I may probably opt to release one of them and do the problem. Let's do that. Again in this exercise please remember it is possible that you can free both the rotation as well as the vertical constraint here which means you can make it free and have two virtual displacements taken care of it. For now let's make it simple. We will convert this into a single degree of freedom system by releasing the vertical displacement. If we do that I am going to draw this line diagram again.

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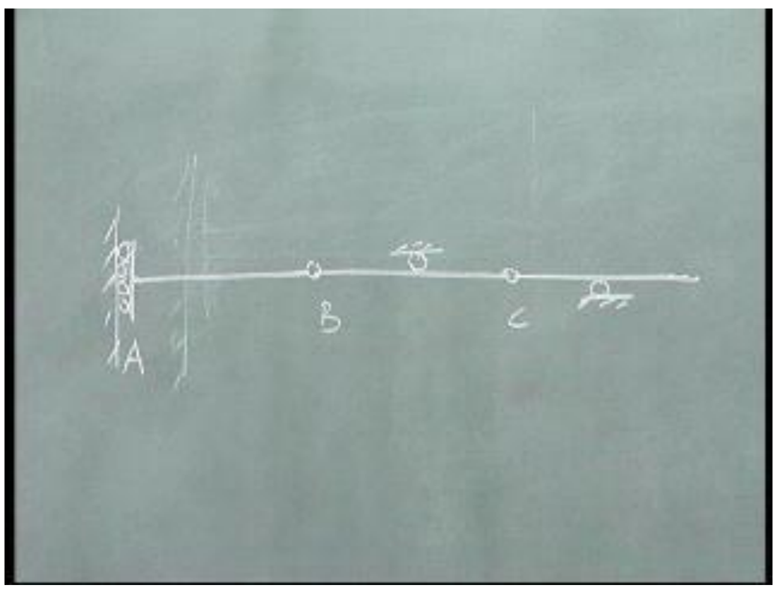

What do I have? I have a support over here, I have another support over here. I am just drawing those supports. At A, I am releasing the vertical displacement yet rotation is not possible. This is at A. How many degrees of freedom does it have? d.o.f equals one which means if I specify one vertical displacement or rotation at any one of these points, I can find out the virtual displacement at any other point. The only condition is what I have to impose is a kinematically admissible virtual displacement. For example I can't opt to make a rotation over here. I can't say "I will put a virtual rotation and find out". Please remember it has to be consistent with the support conditions as well as the notion of rigid body. So kinematically admissible displacements is what I have to find out means: consistent with support conditions and the rigid body motion. Simple, as long as I maintain this, it is kinematically admissible function. What is the best thing I can do? I can probably opt to move this guy.

How will it look like? If I move this, the entire guy will move, entire AB will move. So let's just draw. It's easy to draw, this will remain horizontal the reason is I am not allowing any rotation. It is only possible to move it. So I have moved it and let's say this displacement is delta, delta A but uniquely specified, so I am just going to call it as delta. At this point what should happen? This will not allow movement which means the only way possible is going along this.

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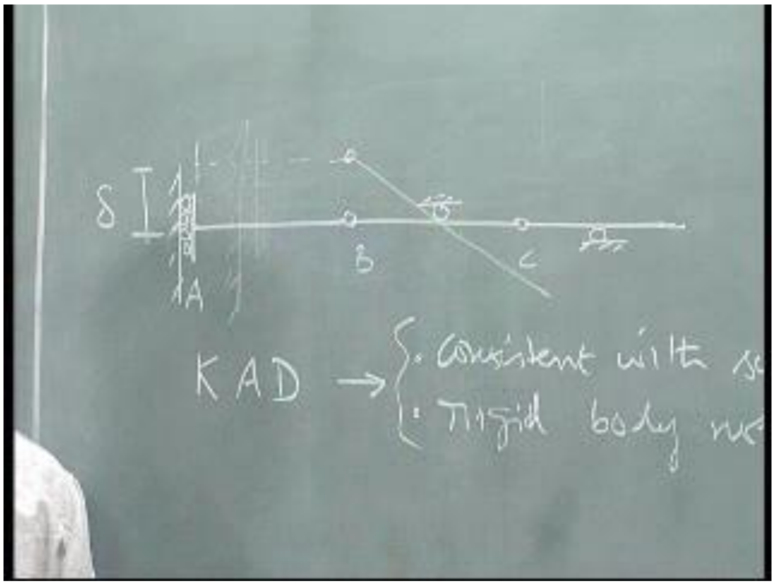

The point C has to move down like this. Now at point C again, this guy will not allow vertical movement which means the only way possible is a displacement like this. Very simple. Geometrically if I draw it, it will make it simple for you. The moment I know this I have solved the problem. Now let me draw the active forces. Remember I have not drawn any active forces yet, having released the vertical displacement here I have to apply a vertical reaction. Let's say this is A_y, again it's an option I have used vertical this way. Let's just have a notion of what would be the reference. The reference is the initial position and let's say this is the y direction I am looking at. I need only one reference here, reference could be the initial starting point of this.

Therefore the y virtual displacement that has occurred is in the positive y direction. What are the other active forces? Let me draw these active forces in this. Then also indicate the appropriate virtual displacements. There is a 5 kilo Newton here and a 10 kilo Newton here, that's all. I am going to apply that.

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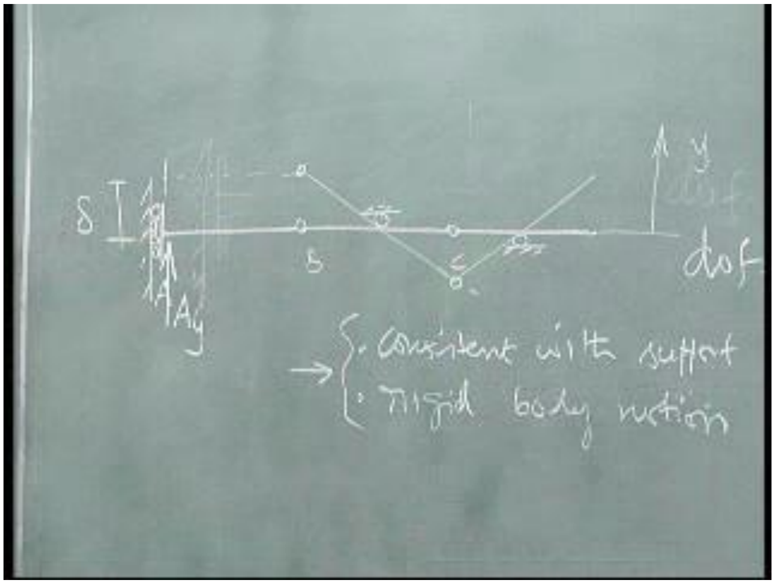

There is a 5 kilo Newton applied here and this is the virtual displacement. I am going to now represent this as delta$_E$, I have to write delta$_E$ in terms of delta, if I have to solve for this. Do I need to draw anything else? Only here. Here I have a vertical force acting like this, equal to 10 kilo Newton and this is the virtual displacement delta$_D$. Is this clear?

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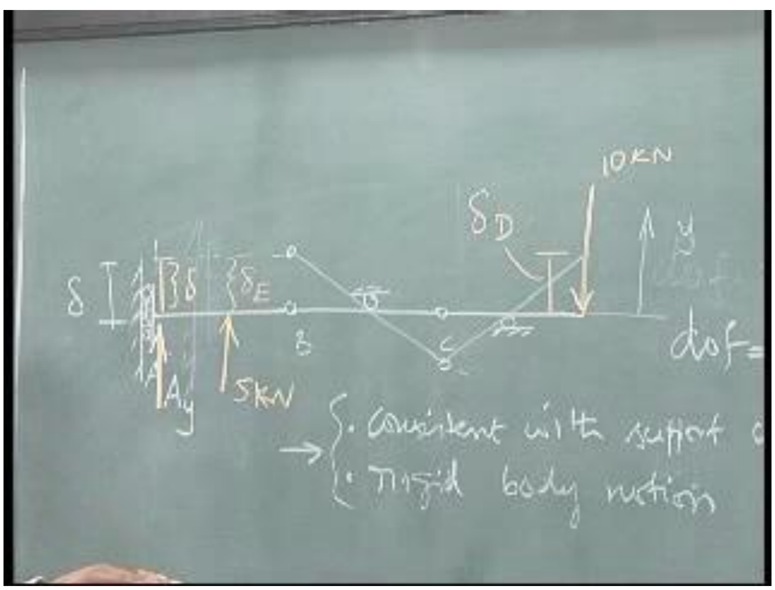

Let me first write down the virtual work equation and then get the relationship between these through K A D kinematically admissible displacements. What does virtual work equation principle tell you? A_y dotted with delta in the same direction, here since only one direction is existing I just have to write times delta positive quantities plus 5 times delta$_E$ plus delta$_D$ is upward. Please remember, I should have put the directions of these displacements, 5 kilo Newton upwards, delta$_E$ upwards so that is why 5 plus 5 times delta$_E$. Here delta$_D$ is upwards, 10 is downwards so I have to have minus 10 times delta$_D$ which is minus 10 delta$_D$ and this is equal to 0.

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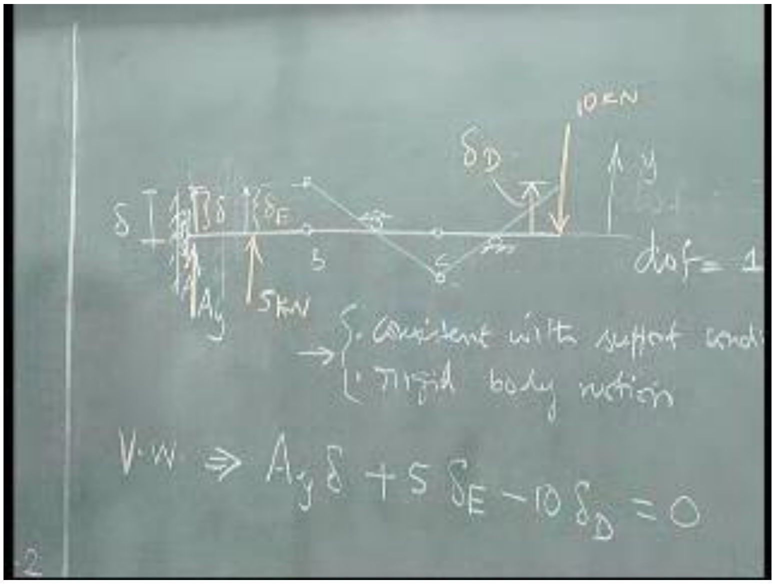

Now if I find out delta$_E$ equals something times delta and delta$_D$ is equal to something else times delta, if I substitute over here delta will separate out, I will get the equilibrium concerning A_y 5 and minus 10. We can solve them, it's as simple as that. We will come back to that after finding out the relationships between these. I am going to draw this again over there. delta$_A$. So let's start from here. I know geometry is very simple. So I am just going to go fast. delta$_B$ if I have to find out, delta$_B$ is equal to delta$_A$. We also have delta$_E$ in between, delta$_E$ is also equal to delta$_A$. delta$_E$ is in this direction, delta$_B$ is in this direction. How about delta$_C$?

Since this is exactly in the middle, the amount of moment here will coincide with the moment over here. Very simple. Please remember we are looking at small changes, I have just drawn in such a way that these changes are too big. Please remember the rotation is such that this displacement is small. For all practical purposes I am not going to consider the horizontal displacement. This is delta$_B$.

delta$_C$ will be not the same but minus delta$_B$. I am just going to be consistent with the directions. I will take upward to be positive, downward to be negative. Again looking at this, delta$_D$ magnitude will be the same as delta$_C$ because this is at the middle but sign change.
I will have delta$_D$ equals minus delta$_C$ that is equal to minus of minus delta$_B$, that is equal to delta$_B$ and given that delta$_B$ is equal to delta$_A$, this comes out to be delta$_A$. Beautiful, simple.

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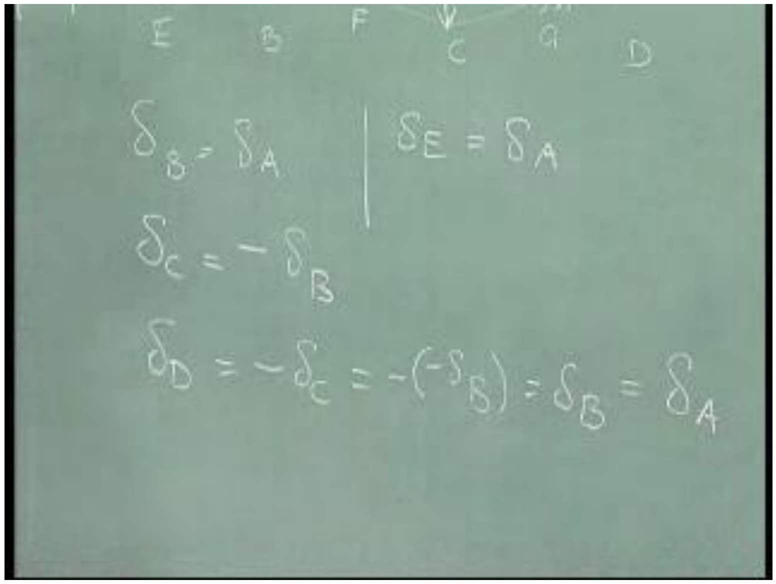

Therefore what I have here is upward direction delta$_D$ is equal to delta$_A$. This is what I get. Upward notion is positive, so delta$_D$ happens to be delta$_A$. delta$_E$ happens to be delta$_A$. Do I need anything else? Over. Now I have to write down the equation. I come back here, what do I know about this? delta$_E$ is equal to what times delta? One times delta. What is delta$_D$ in terms of delta$_A$? It is one time delta. Remember if this is positive, this is positive. Now it's very simple, I just have to take this to be delta, this to be delta and I get the answer immediately.

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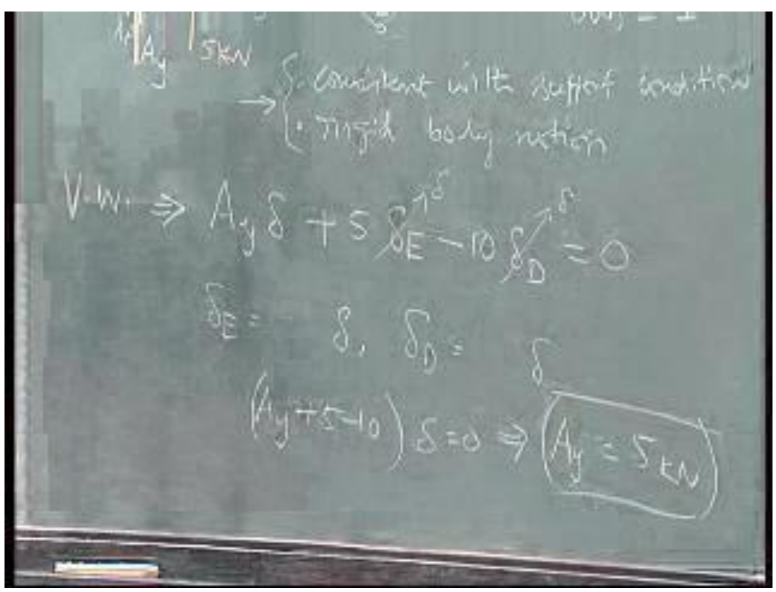

I have A_y plus 5 minus 10 times delta equals 0 implies A_y equals 5 kilo Newton.

Now, there is a doubt, let's see if we can clarify. Supposing I have a rigid body, let's say a straight member like this. If I tell you that one single point is constrained from rotation. Can you tell me what is the rotation of any other point? Please remember, rotation of any other point is zero which means if I constrain a rotation of a single point of a rigid body, every point of the rigid body will be constrained from rotation. That's a fact that we use in the previous section of the problem.

So when we had something like this, in this beam problem when we released only the y displacement and kept the constraint of rotation, we had something like this till B. When it moved up, remember this particular point is prevented from rotation with respect to fixed frame of reference. When I move this up, this point cannot rotate. If one point of the rigid body cannot rotate, none of these points A B can rotate and therefore it has to only go up like this till point B. That's the only possibility and therefore we did this way, consistent with again those two assumptions of should be able to be consistent with the support conditions and the rigid body notion.

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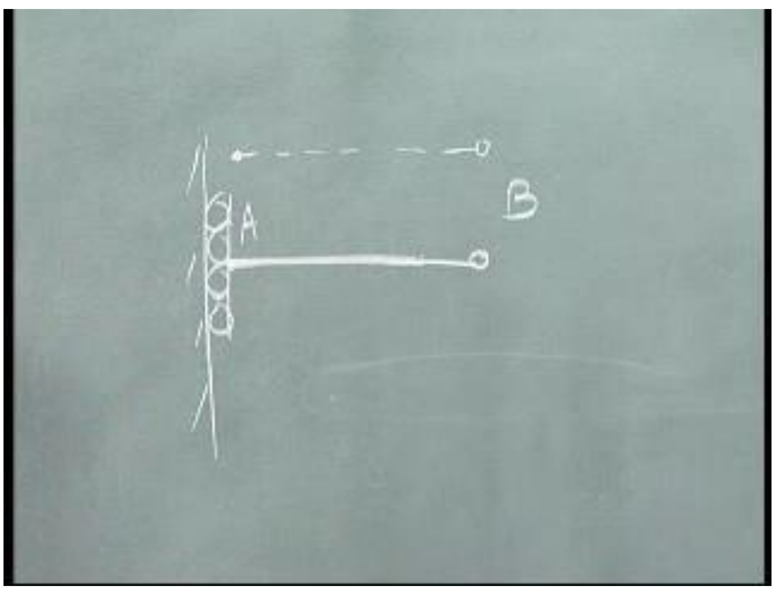

Now, I want to find out the support reaction. If I have to find out the support reaction, we have already found out the vertical reaction. We need to find out the moment reaction. How do I find out the moment reaction? Very simple. I will start with the same kind of notion. The total degrees of freedom for this is equal to 0. I have to release the rotational degree of freedom at A which means it should be possible for it to rotate about this particular point A which means from a fixed, if I have to allow only rotation it's very simple. All I have to do is convert it into a hinge and apply a moment. We need to find out this moment A. In this particular case I have… let me just denote the points. I have one 5 kilo Newton acting here, one 10 kilo Newton acting here. This cannot move, this cannot move, these two are pinned with each other. So AB, BC and CD are pinned with each other, we already saw that in the previous example. Now in order to find this out, what should I do? For the virtual work principle M_A into the rotation that has taken place at this particular point is the virtual work.

So let's say I provide a rotation here equal to θ_A so that the virtual work let me call it as delta θ_A. For now let's just assume that the virtual rotation itself is θ_A, just to avoid too many symbols. If I am rotating like this, what should happen to B? B will start to move up. I am just using the simple geometry. Let me draw with some other color. It would have moved by this. How do I find out the displacement at B? That's not very difficult. This rotation times the length will automatically give me the displacement. Once this point is moved to this, we are left with the similar concept that we used earlier. This point cannot move, the only way that this displacement can occur is by movement like this. This is the upward moment.

Mind you I am just using the upward moment here. This rotation has given counter clockwise that is the given virtual displacement, in this particular case virtual rotation. Consistent to this virtual displacement I need to find out. This is an upward displacement δ_B, this is a downward displacement δ_C and again consistent to this I should get this which is δ_D, simple. Now, once I find out δ_B, from that I will find out δ_C from that I will find out δ_D.

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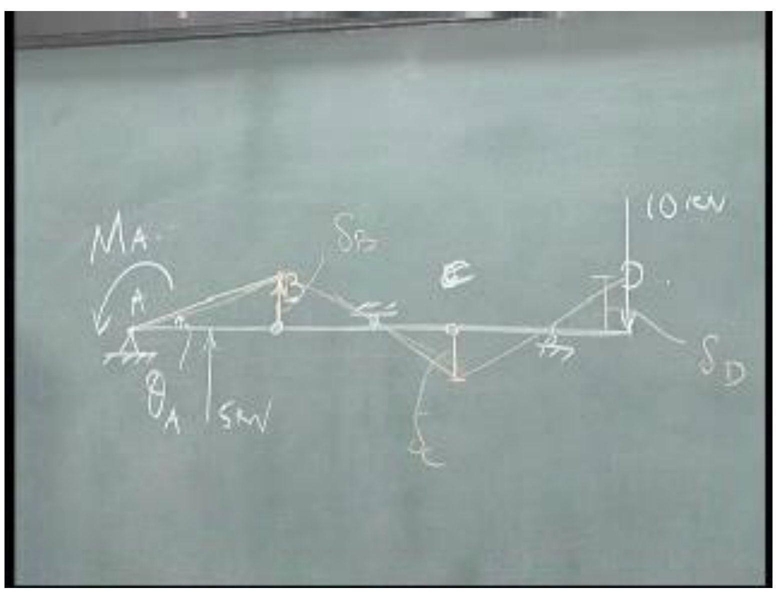

What I am going to do now is find out kinematically admissible virtual displacements because now we know we have to do that in order to provide the virtual displacements at 5 kilo Newton and 10 kilo Newton apart from the virtual rotation that is already given.
At E the distance from that support A to E is one meter and therefore delta is it upward? The answer is yes, δ_E is equal to one meter times θ_A upward.

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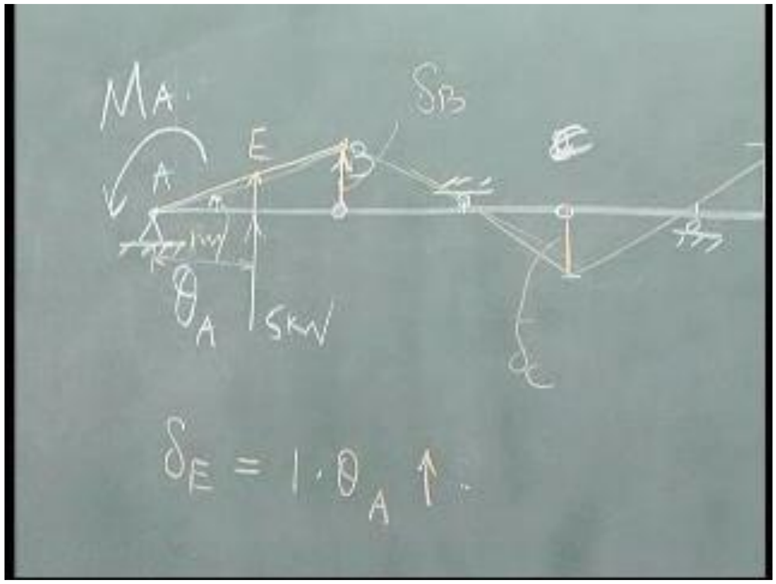

What is delta$_B$? delta$_B$ equals 2 meters times theta$_A$, again upward. Can I find out delta$_C$ in terms of delta$_B$? Since it is at the center it's very simple. delta$_C$ equals minus delta$_B$, delta$_D$ is minus delta$_C$ and that is equal to delta$_B$. What is delta$_B$? It is equal to 2 times theta$_A$.

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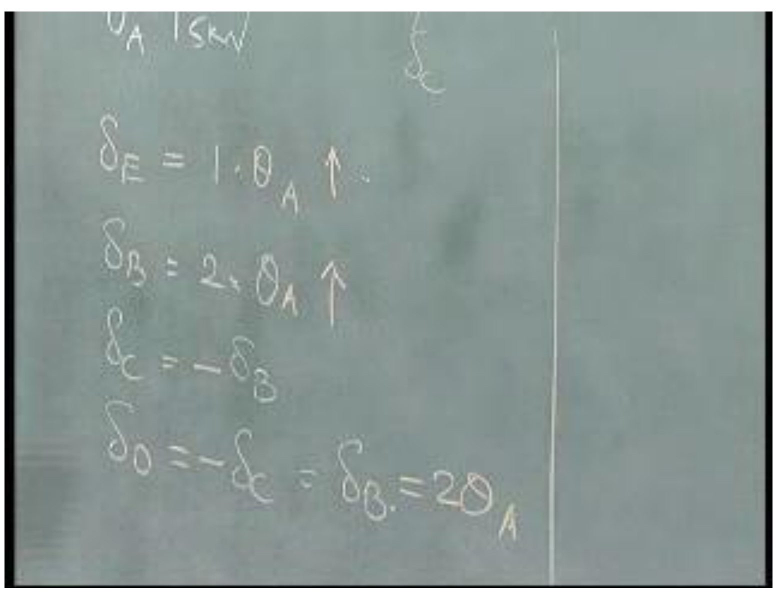

Do I know delta$_D$ in terms of theta$_A$? The answer is yes. Do I know delta$_E$ in terms of theta$_A$? Yes, now I am ready for writing the virtual work principle. The virtual work

principle now is M_A times theta$_A$. So M_A times theta$_A$, remember the direction of M_A is counter clockwise, theta is also the rotation in counter clockwise direction. So this is a positive quantity plus 5 kilo Newton's times delta$_E$, delta$_E$ is equal to theta$_A$ times one. How about this? This is a positive delta$_D$ but a negative sign so it is minus 10 times, what is delta$_D$? It is 2 theta$_A$, so it is two times theta$_A$ and that is equal to zero. Separating theta$_A$, I have M_A plus 5 minus 20 which is minus 15 equals 0 or M_A equals 15 kilo Newton meter.
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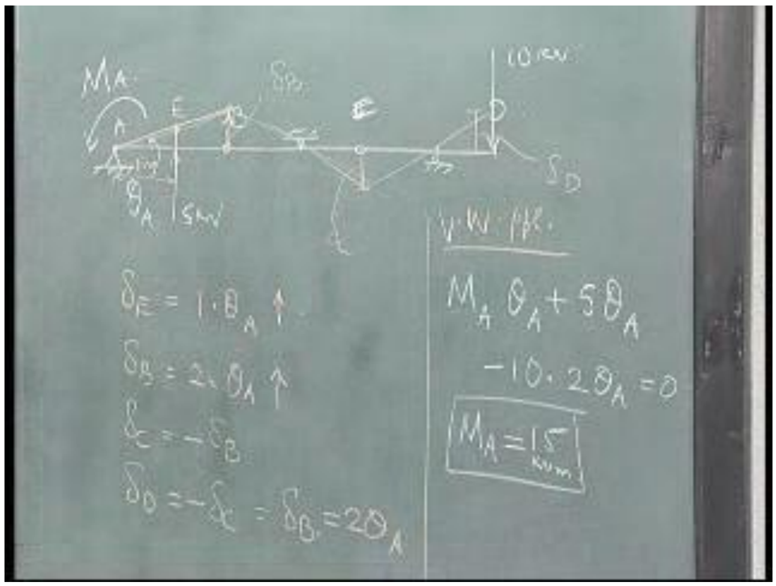

Why is it so? Because theta$_A$ cannot be 0, it is arbitrary. If it is 0, it doesn't make any sense and therefore M_A should be equal to 15 in order to satisfy this equation, that's it. One of the advantages of this is, if kinematically admissible displacements are easy to find out nothing can beat virtual work displacement. Now as I said earlier, it is possible to do vice versa supposing we need to find out kinematically admissible displacements from a set of equilibrium forces, it is possible to find out. We will take up that example in the next module. Any questions? Is this clear? Now as an exercise, I want you to apply both the virtual displacements, vertical and rotational and then consider what is going to happen. I will give you a clue.

The first thing that you have to do is at this point A, if this is trough this is B, this is C, what have I released? I have released this support completely, I will have A_y and the moment M_A acting on it. There will be this active force at the middle equal to 5 kilo Newton, here 10 kilo Newton. I have these supports still present. This is the body for which I need to find out. What should I apply? I should apply a vertical displacement and a rotation. If I do that then I will be able to find. What will happen? I can apply vertical displacement and a rotation which means I will be applying this mainly. This is the rotation theta$_A$, this is the displacement delta$_A$ and find out consistent to this. So you know how to find out this from which you will find out this, from which you will find out this. Mind you it will be in terms of delta$_D$ and delta$_E$ will be in terms of delta$_A$ and theta$_A$. Separate the expressions, each of the coefficients of the expressions in terms of

$delta_A$ and $theta_A$ should be equal to 0. You will get the two equations simultaneously, you can solve that. I will leave this exercise as a homework for you. Thank you.