Engineering Mechanics
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Statics - 4.3

In this case let's say delta B and delta C are the kinematically consistent displacements. Let's look at that particular rigid body. This is B, this is C, this has moved by lets say the position of this is x_c, y_c and in this particular case we know it has moved like this. Let's denote the movement by delta x_c, delta y_c. In a similar way I can denote this point to be x_B and y_B and let the movement be delta x $_B$, delta y_B. Now how do I find out the virtual displacement at any point in between?

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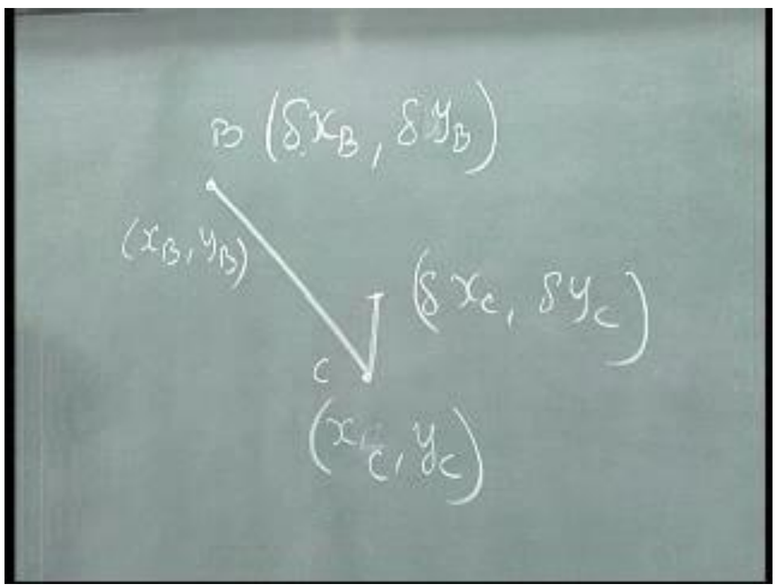

That's not very difficult. This is a straight rigid body. I just have to do one interpolation between these two as simple as that. For example if I take this as anchor, I wish to find out at a particular point. For example in this case I want to find out at the midpoint, let's say this length is L then it is simply delta x_B, I have to add the appropriate difference for example in this particular case delta x_C, so this is delta at x_p is equal to delta x_B plus the gradient of that virtual displacement which is nothing but delta x_C minus delta x_B by L times the appropriate length. For example this is L by 2, I know it is L by 2 and therefore I have to apply L by 2 here. Very simple it is just a linear equation that we are using here. Linearly interpolating it and therefore we will get this to be delta x_B plus delta x_B into L by 2 by L which is delta x_B by L minus, therefore we will get delta x_B by 2 plus delta x_C into L by 2 by L which is delta x_C by 2, this is equal to delta x_B plus delta x_C by 2.

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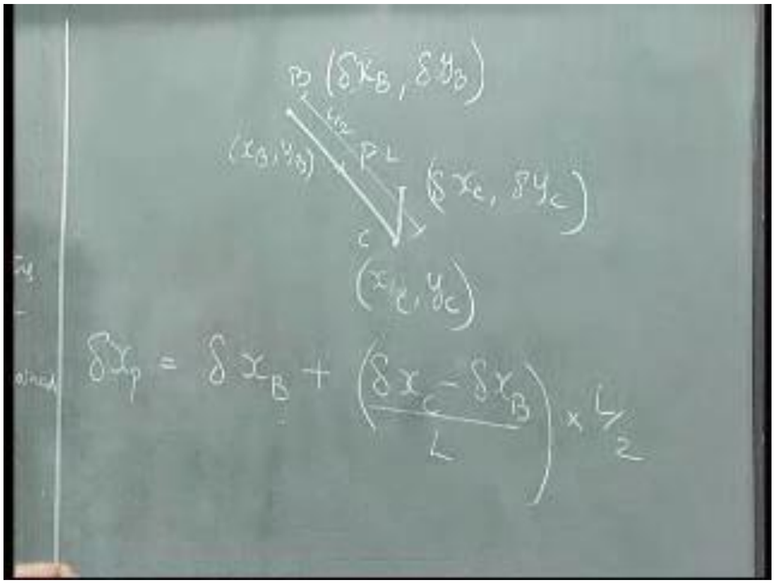

Similarly if I have to find out the virtual displacement in the y direction, I will get it to be delta y_B plus delta y_C divided by 2. Reason why I am telling you this is, it may be worth it finding out the displacements of the ends of these bars.

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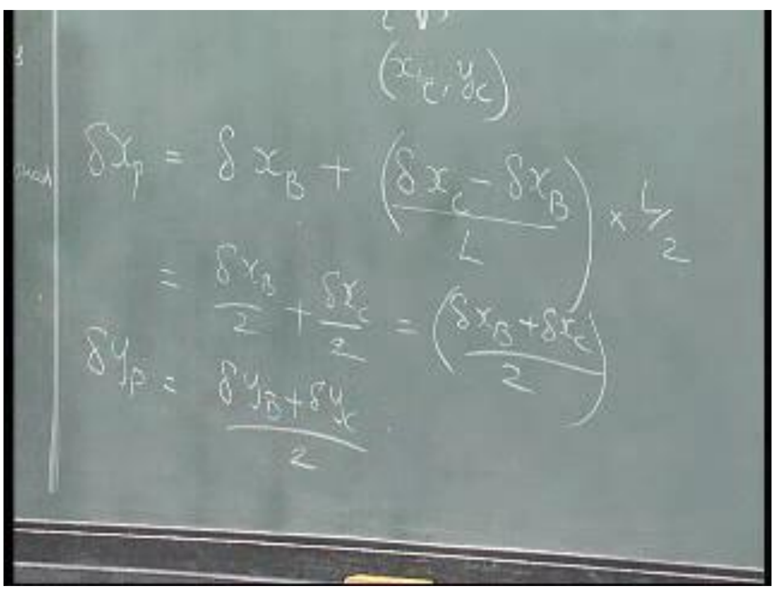

Once I find out the ends, end displacements, virtual displacements it's possible to find out at any point in between. If there is a force acting here like this, if I know this length with respect to the total length, it's very simple to find out the virtual displacement at this point and I can use that for this active force. This should be clearly understood.

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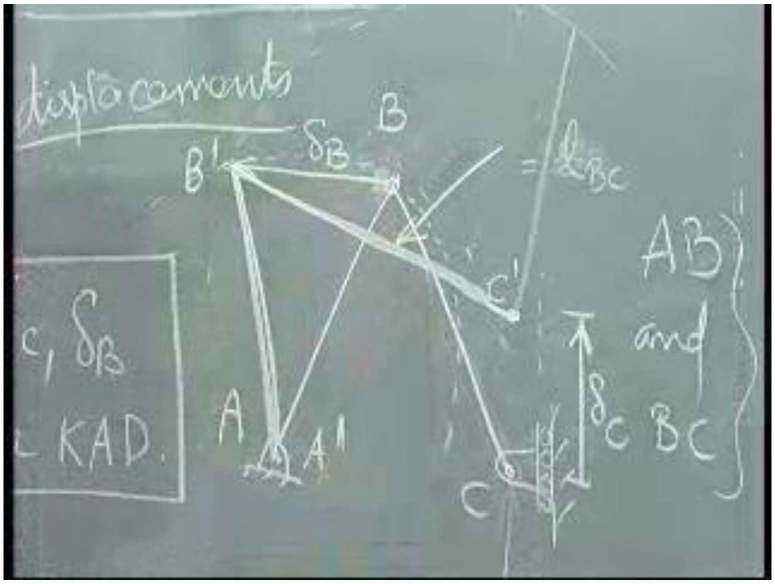

Therefore the bottom line is I need to find out the displacements at each of the salient points of the rigid bodies. Sometimes I may have a hinge, this may be extending like this. So it's better that I find out here because this is connected at this point. As long as this is clear we should be able to solve this problem.

One more important thing before we solve the problem that we started up yesterday. Let's say in this exercise I wish to find out the reactions at C. How many reactions are possible? Let's just start with the problem as such. We have two pin supports here, it is a pinned joint over here, A B C as we know already. I am just not denoting the external forces here. If I have to find out the reactions here, there are two reactions one reaction I can say is vertical. If I remove this pin joint, I have C_y and C_x, I need to find out both these reactions. If I need to find out both the reactions what can I do? In the earlier exercises that we have carried out, we only said you release one of them. In this particular case I have released two of them. What shall I do? The answer is not very difficult. Well, apply two virtual displacements, so one virtual displacement along this direction and another virtual displacement along this direction.

The question is can I? The answer is yes because the number of degrees of freedom for this system is two. I have released two of them here, originally the degrees of freedom were zero. When I released one of them degrees of freedom became one. When I released two, it has two degrees of freedom which means I can specify two values, one independent of the other as displacements at any single point. Is this point very clear? So in which case, in this particular body BC, I can specify a displacement in the x direction lets say this is delta x_c and delta y_c independently and seek to apply the virtual work principle so that I can write equations of equilibrium. I have two of them, what shall I do? We will answer that question subsequently but to just give an answer immediately, I would only say that if you group the terms related to delta x_c and terms related to delta y_x, you have two terms that can individually go to zero because this is independently specified and this is independently specified.

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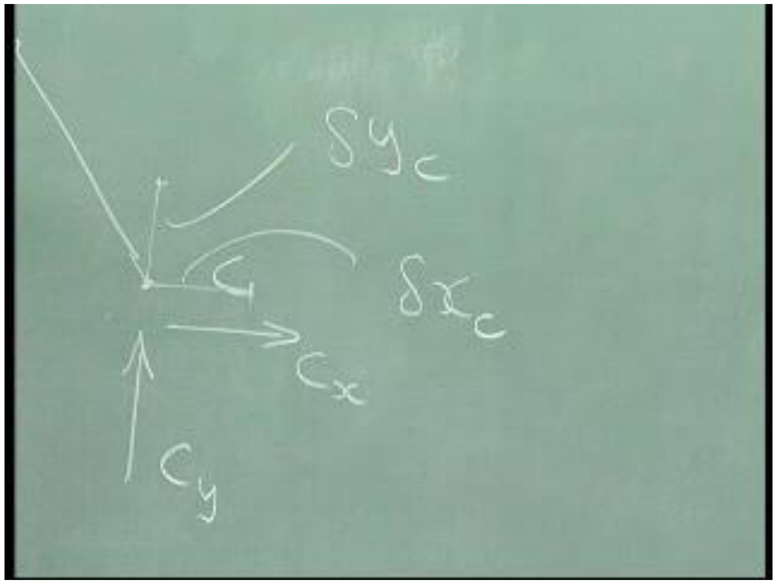

If I have something times delta x_c plus something times delta y_c equal to zero as a result of virtual work principle, this means since this is arbitrary and this is arbitrary independently, each of these terms have to go to zero which means I will get two equations of equilibrium from which I can solve the entire problem. Therefore depending on the number of degrees of freedom, you will be able to apply that many virtual displacements. Is this clear? Usually it is a little cumbersome if you go to too many number of degrees of freedom but it is possible to do and that's what is done in the popular methods such as finite element methods.

We will go back to the problem that we were looking at. I am going to use only the line diagram, now that you are familiar with the line diagram. If I remember correct they had self-weights like this. The self-weights were far homogeneous state members for this and this, the problem was to find out vertical reaction at A. That's the problem am I right. What should I do naturally? Since I need to find out this vertical reaction, I have to release the vertical displacement of A and substitute with the reaction. Mind you this is a notion that I have inserted. The vertical reaction could be this way or this way. If I have assumed like this just as in equilibrium equations, I will get a negative value in the actual direction of the opposite. Since I should not allow movement in this, I will have the horizontal displacement arrested at A.

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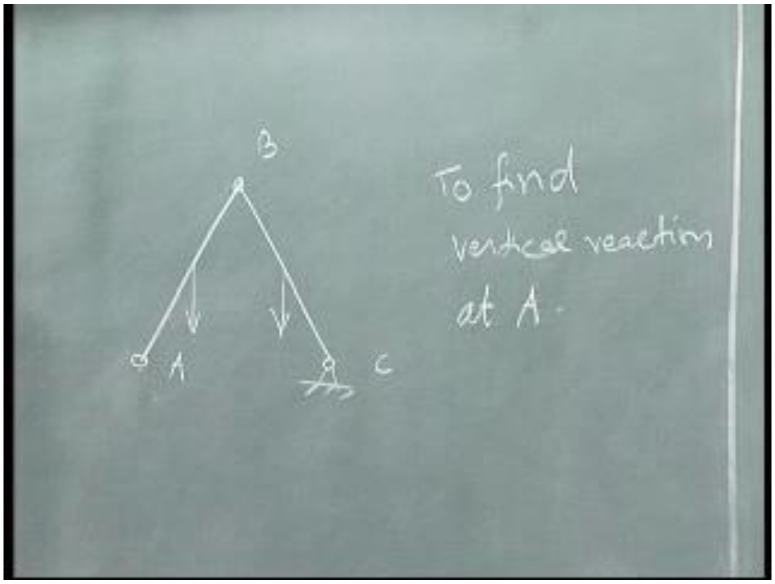

I need to now give a virtual displacement. It is not necessary I have to give a virtual displacement here, I can give virtual displacement at any point. This is a difficulty you will face, you can give it at any point, and it does not matter. You will get the same equation of equilibrium. For now let me just give a vertical displacement here and find out the appropriate virtual displacements.

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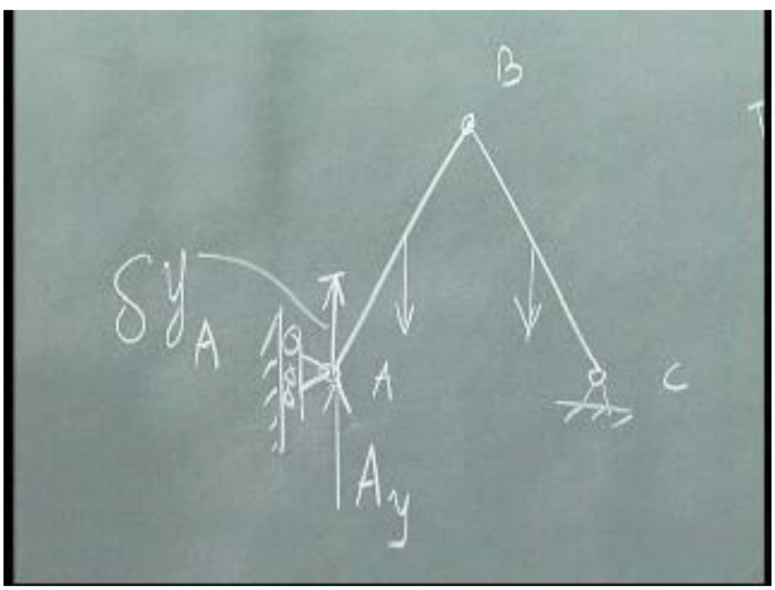

Let me give a virtual displacement here, I am going to give a direction and this is delta y at A. Now I am not done something. What is that? I have to fix the reference coordinates, I know this is a fixed frame of reference and therefore I will use this as a positive notion, it does not hurt using this. This is x and this is y. What's the displacement of C? It is always zero. We have to be consistent with the way in which they have been arrested, displacements have been arrested and the movements are possible. Delta x_c is equal to delta y_c is equal to 0. The virtual displacement whatever displacement I give anywhere, the virtual displacements at C are equal to 0. It's a nice reference to use because these are 0. I have given delta y_A, what I need to find out are the virtual displacements at these two points, especially vertical virtual displacements.

Once I have the virtual displacements of these two points, let me call them as P and Q then I can write the virtual work equation. The aim is to find out these two. Please remember as I had mentioned earlier, this is one rigid body, this is another rigid body. I should seek to find out what is the virtual displacement of B so that I can find out the virtual displacements of P and Q. That's the way we have to go about. What is the virtual displacement of A along the x direction? It is equal to zero why because it is not allowed to move, this is already known. Delta y_A is specified, delta x_B and delta y_B to be found out, this is the problem.

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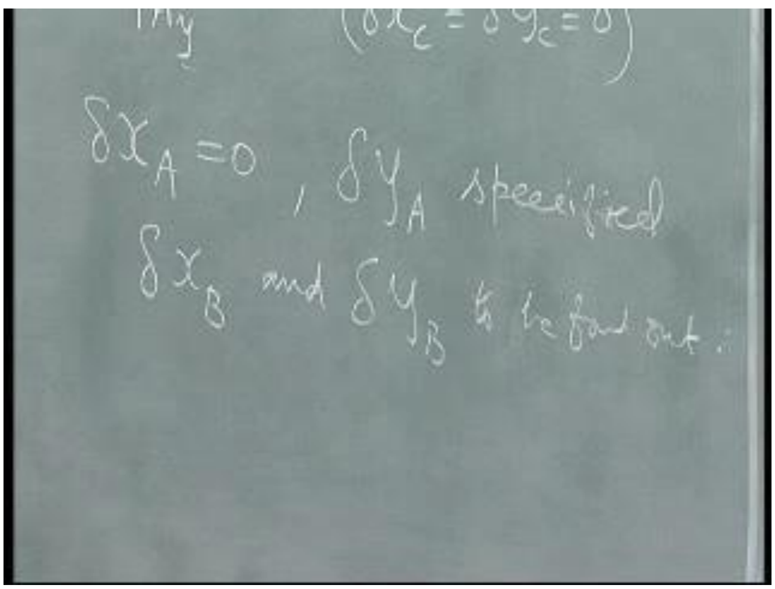

Now, how do I go about doing this? Let's just look at the geometry. For now let's assume small displacements. So small variations is what I am trying to find out. This is x_B, y_B, this is x_C, y_C for now I am going to take as 0 0. This is the positive notion, this is the positive notion and this is x_A and y_A because this point is A. What do I know about length here? This length is equal to 1 which is already specified 1 meter and this is also 1 meter. The x displacement of this particular x_A is equal to 0 and the distance between this is also specified as 1 meter. Let's look at those which are unchanged during the virtual displacement. What are the unchanged or unchanged lengths? This is very useful to start with. What are the unchanged lengths? Length AB, length BC and the other one also is something I know length AC. I can make use of these in the geometry in order to write down relationships between these points (x_A, x_B), (y_A, y_B).

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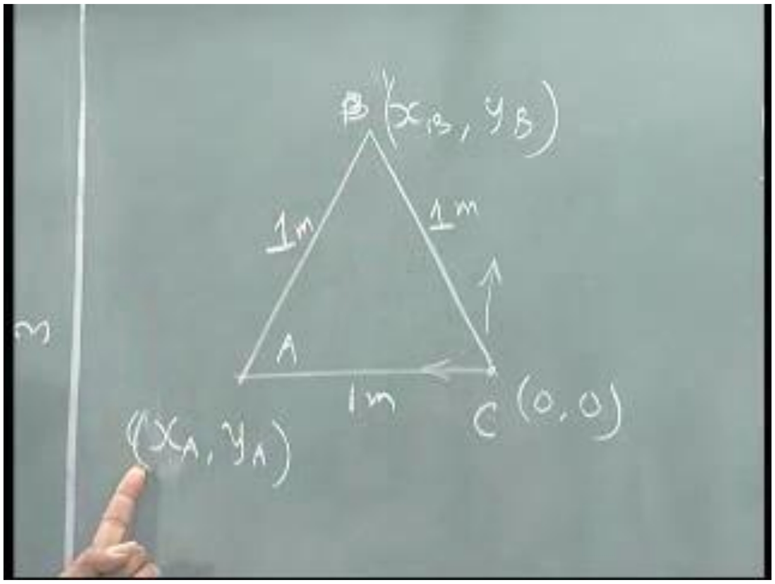

The simplest thing I can do is use the geometry here, let's say this height is h. What is h equal to? If I take with respect to this, y_B is given by h directly. What I am doing right now is this right hand side, let me call this as D. I am looking at the triangle BCD. I know height is y_B, it is a positive value. Please remember this is the direction y_B. How about this length? This length CD is equal to x_B greater than 0, greater than 0. Clear? Now what is this length BC? The length here is root of x_B square plus y_B square that always remains the same. Let me use that fact.

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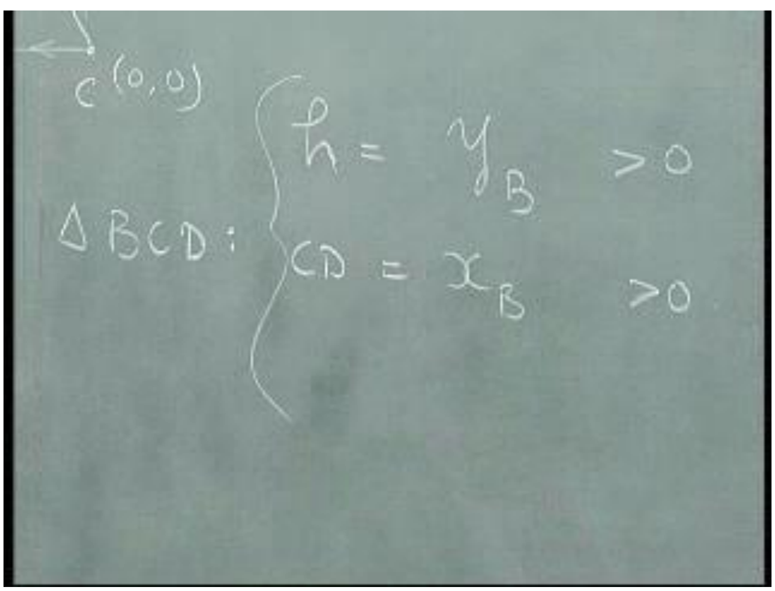

What do I have? Since this is a right angled triangle, I have right angled triangle BCD implies x_B square plus y_B square is equal to 1. Whatever I do, you notice that there is a right angle that I can maintain, this is x_B, this is y_B and therefore this relationship is going to hold good irrespective of whether there is a movement that has occurred to B. You have made use of that particular fact, I also know that I am going to move by a very small amount, virtually move by a very small amount.

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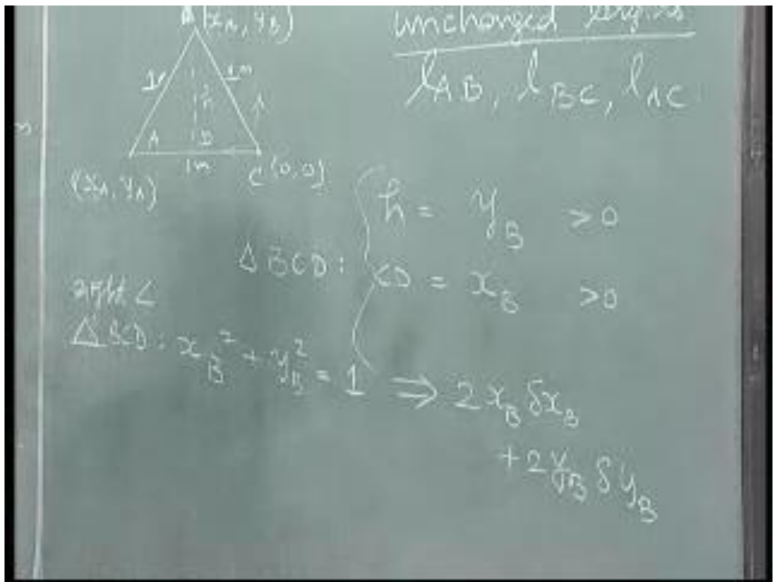

Therefore if I can take a derivative of this, I will get 2 x_B delta x_B where that is the small variation, plus 2 y_B times the variation of y_B is equal to variation of one. Variation of one is zero and I get a relationship like this. This tells me that the variation of x_B is related to variation of y_B through this relation. Immediately we can write this to be minus of y_B by x_B times delta y_B. That's simple in this particular case, when I start from this particular configuration, I know the height and the base, the base is half of 1 meter which is 0.5 meters and the height is root or immediately I will get it to be root of 3 by 2. I can use this over here directly. What do I get here? It is equal to minus root 3 times delta y_B. What does this mean? This means if I start from a configuration like this where this height is given by root 3 by 2 times the hypotenuse and this being half of one meter then this is the relationship I get. Please remember that very clearly.

Supposing I had used some other configuration to start with, this will change. Therefore this holds good all the time. This holds good only for the configuration I start with here which is given by y_B equal to root 3 by 2 and x_B equal to half and that's how we get this relationship.
Please remember that it is only applicable for small displacements. If it is not so I have to use this. I still can use virtual displacement equation but if I use this, it is for even larger displacements. Now having found out this, we have found out only the relationship between x_B and y_B virtual displacements. We need to some how connect the x_B and y_B to

delta y_A which means we need one more relationship between delta y_A and x_B delta y_B. Once we get that we are done with that.
Let's look at this configuration. Again if we take the left hand side right angled triangle, AB remains unchanged and in this particular configuration that we see, the height remains root 3 by 2 and this base remains half. Let's write down the relationship, so I have B here, A here and this is 0.5, this is root 3 by 2. Let me write this as half meters and I have x_B here, x_A here. Is x_A greater than x_B? The answer is yes. If I write x_A minus x_B that's a positive quantity which is this length and how about this? This is y_B which is greater than y_A, so I will write it as y_B minus y_A so that I am handling only the positive values.

I will do the other way and show you that you will get a wrong relationship and you have to be careful that you take the positive quantities. This square plus this square for right angled triangle should give me the hypotenuse square which is equal to one square. Just like the previous example of geometry, we now for the triangle ABD, we get this. Again if we need to find out the relationships between the virtual displacements, I need to take the variation of this which is nothing but two times x_A minus x_B. Mind you this x_A minus x_B according to the geometry is a positive quantity times delta x_A minus delta x_B plus I have two times y_B minus y_A times delta y_B minus delta y_A is equal to 0. What have we achieved here?

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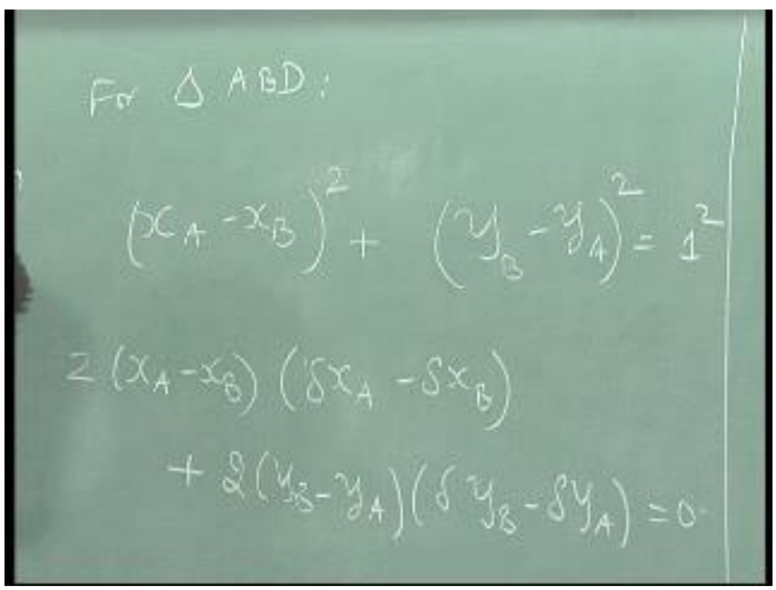

We have achieved a relationship given x_A minus x_B and y_B minus y_A, according to the geometry we have achieved a relationship between these displacements. What is x_A minus x_B? It is half, half times… so let me just strike off two here, this is half so 1 by 2 delta x_A minus delta x_B plus y_B minus y_A is a positive quantity which is root 3 by 2 times delta y_B minus delta y_A.

Again I will reiterate, this can be used for any configuration whereas this can be used only for this right angle configuration. You have to make that distinction when you do this. I already know delta x_A is equal to 0, so I can erase it. Now this relates delta y_A to delta x_B and delta y_B. The net result will be delta y_B equals, I will get something in terms of x_B and y_B.

I already have what is delta x_B, if I substitute here I will get in terms of delta y_A. What is this? You are getting this half and this two goes off, I get it as half of delta y_A. Let's see if it is reasonable. Let's go back to this. If I move this a little bit, what should be the movement here? Will it be the same as this? The answer is no. This will be less than this. How much less? Perhaps as I move towards C, the total y displacement is equal to 0. So half way I should get half the displacement that I have got. Roughly I have a mental picture of okay, delta y_B is half of delta y_A, it seems all right.

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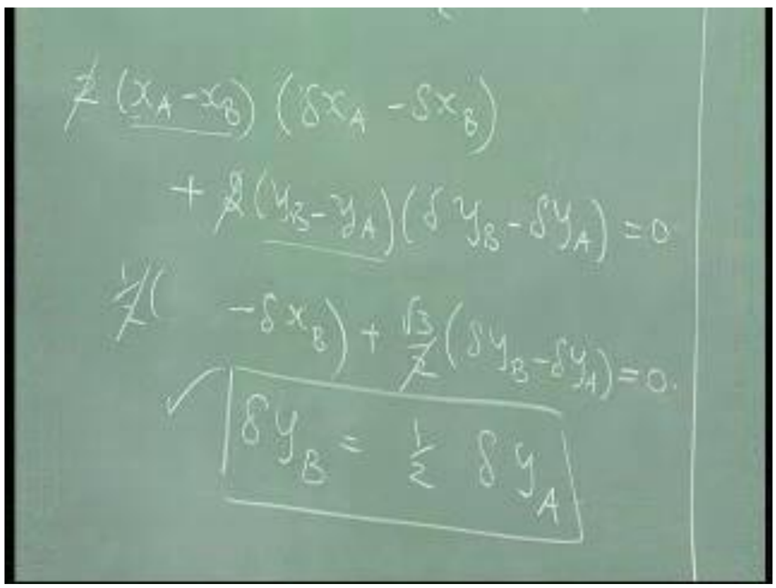

Now going back to this particular problem, do I need delta x_B? The answer is no, because I need to get only the values of delta y_P and delta y_Q. For that I don't need x.

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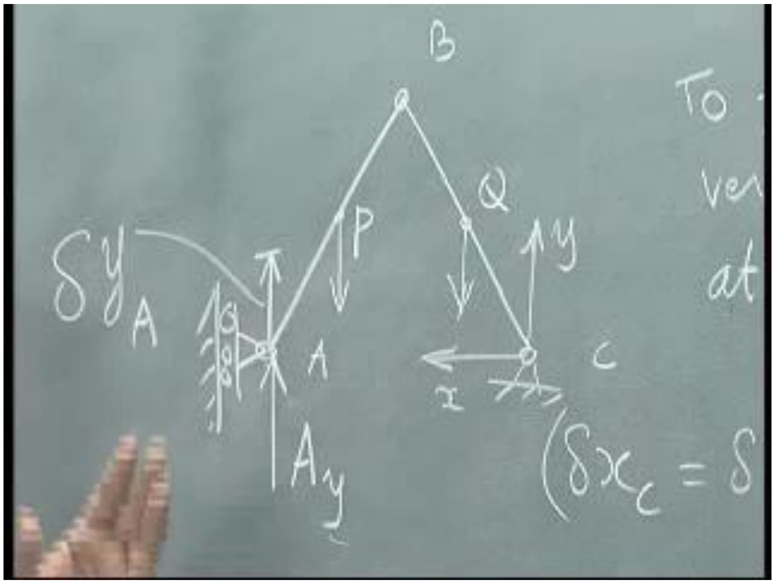

How do I find out delta y_P? Since it is at center, I average delta y_A and delta y_B, I will get for this. I will average delta y_B and delta y_C, delta y_C is 0 and therefore delta y_B by 2 will be automatically delta y_Q. That's what I am going to do next, delta y_P is equal to, since it is midway delta y_A plus delta y_B by 2 and that is equal to delta y_A plus half of delta y_A divided by 2 and that is 3 by 4 delta y_A.

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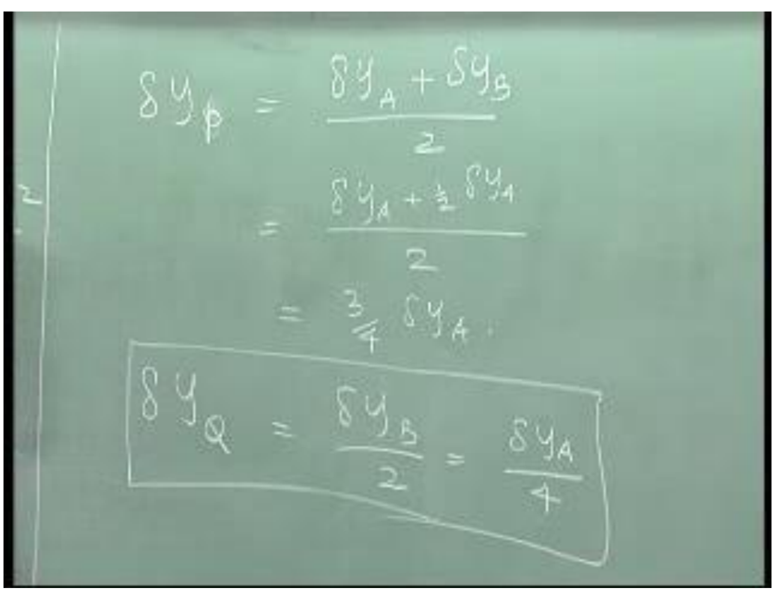

Similarly I can find out delta y_Q which is nothing but delta y_B by 2. What is delta y_B by 2? It is equal to delta y_A by 4. Now if you look at it, delta y_P is written in terms of delta y_A. delta y_Q is written in terms of delta y_A. When I now write down the entire virtual work equation, I will find that I would have written all of them in terms of delta y_A. Let's just finish that particular exercise. We will go back to that. Let me just write down clearly what we obtained. Applying delta y_A we get delta y_P equal to 3 by 4 delta y_A and delta y_Q equals 1 by 4 delta y_A.

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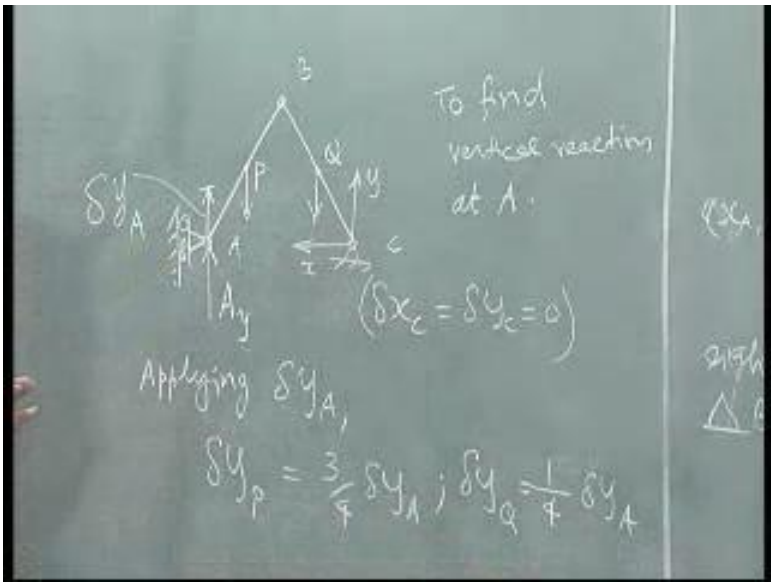

Why did we look at delta y_A, delta y_Q? Because those are the active forces for which we need to find out. What is the direction of these? This is upward direction, this is upward direction because they are positive notions.