Engineering Mechanics
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Statics - 4.2

I am going to continue discussion on virtual work method but before going further into virtual work principle, let's just have a brief look at the method itself. If you remember what we discussed earlier, we have what are called active forces. Let me just call them as active times the virtual displacements. Let's call them as delta i or let me put it as F i delta i. The summation of all these over i should be equal to zero is the virtual work method. Since this is a virtual displacement, we will call this as virtual. F active, it need not be said as F active. The reason why we have said F active is the forces that balance each other will have equal and opposite values that automatically cancel off. In a way I need not say F active here. One important thing to note is what is this virtual displacement? This virtual displacement is along the appropriate force, along F i. Along the direction F i and it has to have a positive notion, that's something you have to remember.

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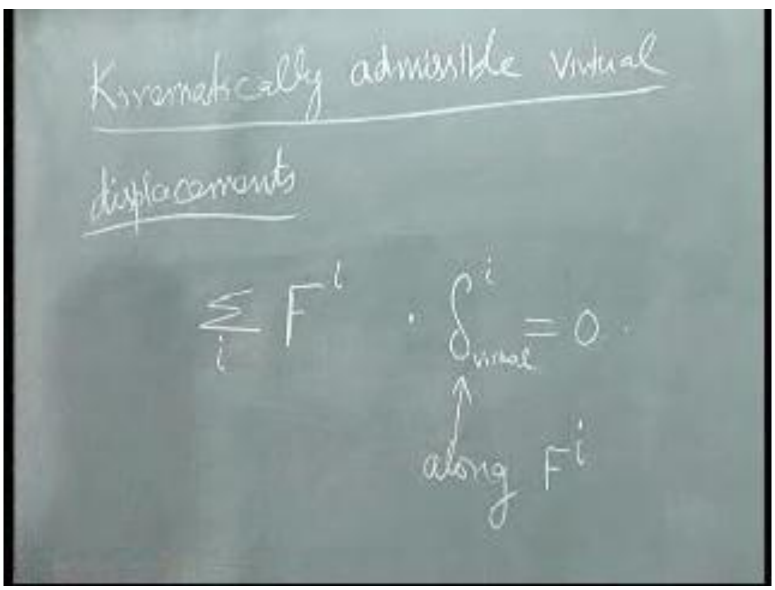

Another thing to remember is that this delta i virtual are as I have indicated here kinematically admissible displacements. They have to be kinematically admissible displacements. In a minute I will tell you why. We have force which are active external and otherwise, multiplied by the virtual displacements. The appropriate virtual displacements along F i are taken or another way to do it is take a dot product of this and the virtual displacement of that point, both are same in which case we can remove this. This is automatically satisfied and this delta i are kinematically admissible displacements. What do I mean by that? Let me ask this particular question.

Let's say I have a rigid body like this. Let us say I know some of the displacements. Let's just for a moment assume that the end displacement of this lets say A B is arrested. Now can I apply a virtual displacement like this? The answer is no because I have fixed this. This is a rigid body, there is no way I can pull it by a delta virtual and therefore I will call that as kinematically not possible. Kinematically in the sense I cannot move this guide like this and hope to get a virtual displacement. Another notion which is an offshoot of this, can I have for example a virtual displacement like this that I give. Again the answer is no, because when this rotates this point B will reach this point B prime. Since the length of this rigid bar remains unchanged and therefore this virtual displacement is an inadmissible displacement.

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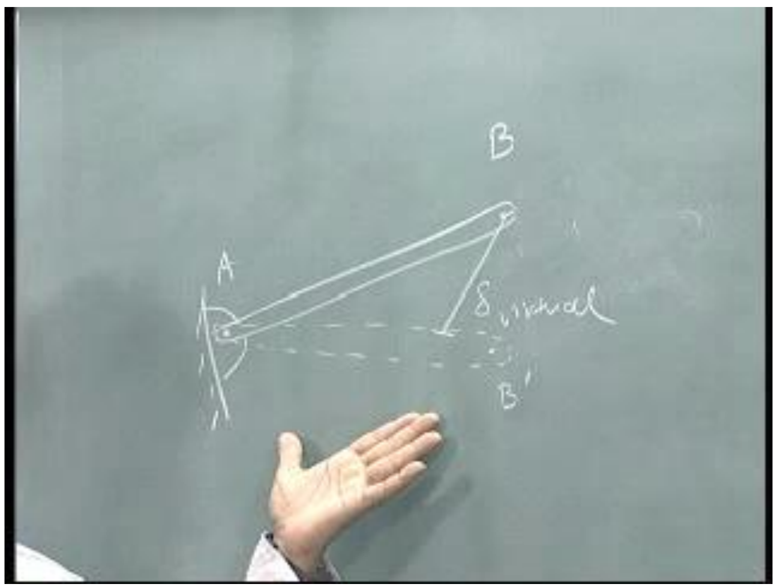

Now we have to make sure that this criteria is satisfied apart from the equilibrium equations. It should satisfy number one equilibrium equations. Any rigid body that we are looking at, the other is what we call as kinematically admissible displacements.

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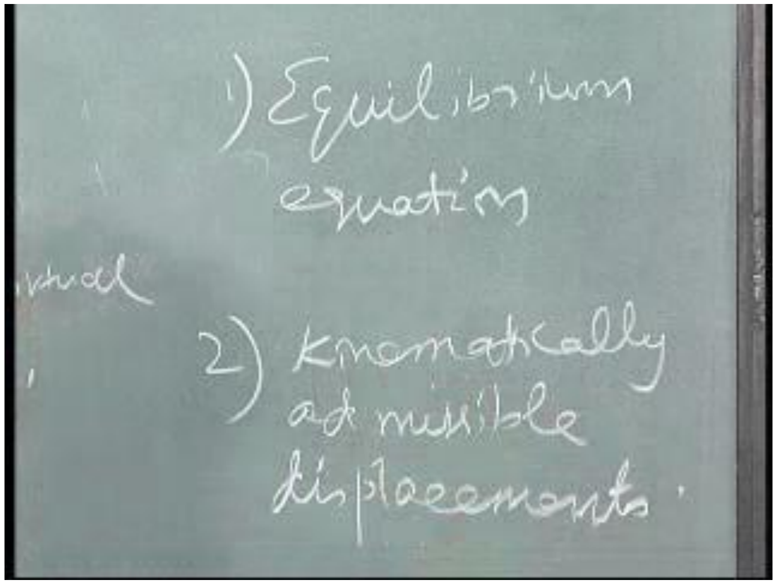

In a virtual work principle what we do is we will assume one of them and look for the other. If I apply let me call this as KAD kinematically admissible displacements, virtual KAD. I can seek to apply the virtual work method to satisfy equilibrium equation. Is this clear? Supposing I start with kinematically admissible virtual displacements for a particular system, then I can seek to apply virtual work method which is nothing but satisfying equilibrium equations. If I satisfy virtual work equation which is this, this also means the forces that I get here, I use here are equilibrated.

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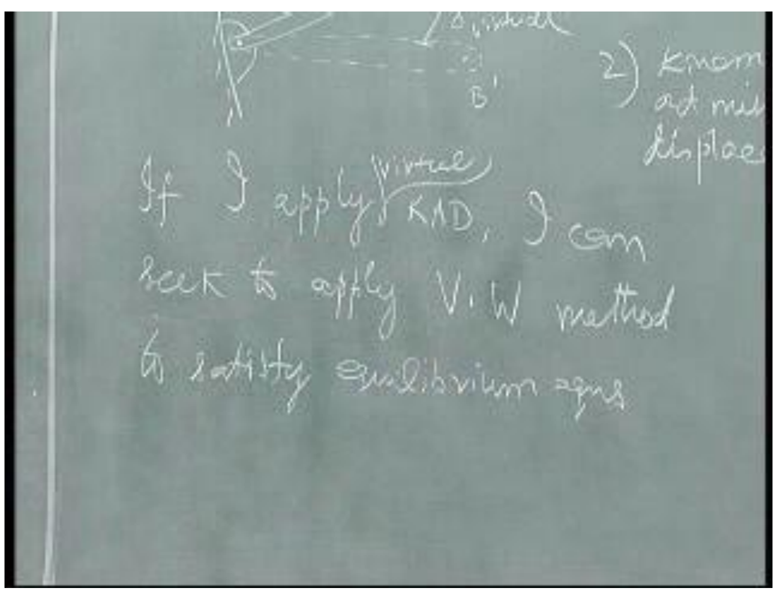

If I satisfy this condition, I am satisfying equilibrium or in another words these are equivalent to equilibrium equations. This must be understood clearly. One may ask this question, supposing I start with forces that are in equilibrium. Can I seek to find out kinematically admissible displacements? The answer is yes. Let me write it down on a separate board. If I have virtual, remember I am using this virtual forces that are in equilibrium applied to a system, what does the virtual work equation give? The virtual work equation renders kinematically admissible displacements. I can get kinematically admissible displacements. We will focus on this method first, I will try to give one example in this method, just to give you a completion.

Now when I say virtually kinematic admissible displacements or kinematically admissible virtual displacements, what should I satisfy? It should be admissible to the assumption that I have made which is the bodies that are linked together are rigid. Let me give you a couple of more examples of what are kinematically admissible displacements so that it is clear to you. Let’s take the system that we were looking at. This is the system like this and we have hinges on both sides. Now it is our choice to release one of them, so let me just do that exercise of releasing this side, though the problem that we solved is pertaining to something else. Let me release only the vertical displacement here, so I am just going to substitute with releasing only the vertical displacement. I apply a virtual displacement here to look at the equilibrium. Of course when I release, mind you I have to apply the appropriate unknown reaction. In this particular case let’s say this is, I think A B C was what was used. This should be C along the y direction.

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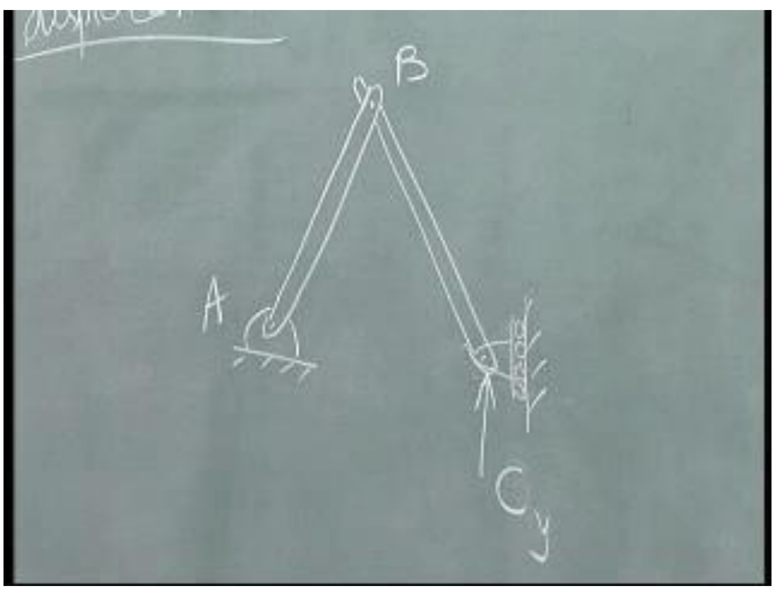

Now let’s get a feel for what should happen. Supposing I push this guy up let’s say I have taken it all the way here. The point C has now moved to C prime, just to make it very clear I am going to make this entire diagram as a line diagram. That’s make it simple for me to draw also. Let me just erase it, draw the line diagram this is what we have.

Mind you this is a hinge to the roller support on to the fixed frame of reference. This is B, this is A, when I move this point C to C prime what should I get? I should get, remember the length B C cannot change. It's a rigid body. Length A B cannot change, it has to be a rigid body. When I draw the geometry that is related to this movement, I have to make sure that A B and B C lengths will be maintained. If I don't do that then what I am primarily doing is I am violating the condition that they are rigid bodies. The reason why we have introduced rigid bodies is something we will not focus on right now. If they are flexible bodies it is a different scenario, we will discuss that when it comes to the topic of strength of materials or mechanics of materials.

Given that what is the possibility? Now lets look at this A B. Please remember it is hinged over here which means, the only way A B can move is with A pivoted at this fixed support or fixed hinge support, it can only rotate. Let me just draw that particular thing. It may be worthwhile drawing using different color. I am just going to draw, just to indicate what is possibility of this movement. What should I do? It's very simple, geometry helps me here. Now I take the length B C and find out where it cuts. I will take a compass, I will make an arc and this length is that of B C. Let's write it as l of B C, it maintains that. Where it cuts is the only point possible, if I move this point C. Are you with me? A very simple geometrical explanation and therefore now I just need to connect, let me just do the connection. I will take a straight edge, all I have to do is now connect it. Don't mind me using some simple ruler type here. Let me use another color just to represent how it would have moved. What do I know about this? If this were the virtual displacement of C, this is the virtual displacement of B C, so let me just use the direction also. From C it has gone to C prime, this is B prime. How about A? A is a pinned support, so it remains here A prime. Is this clear?

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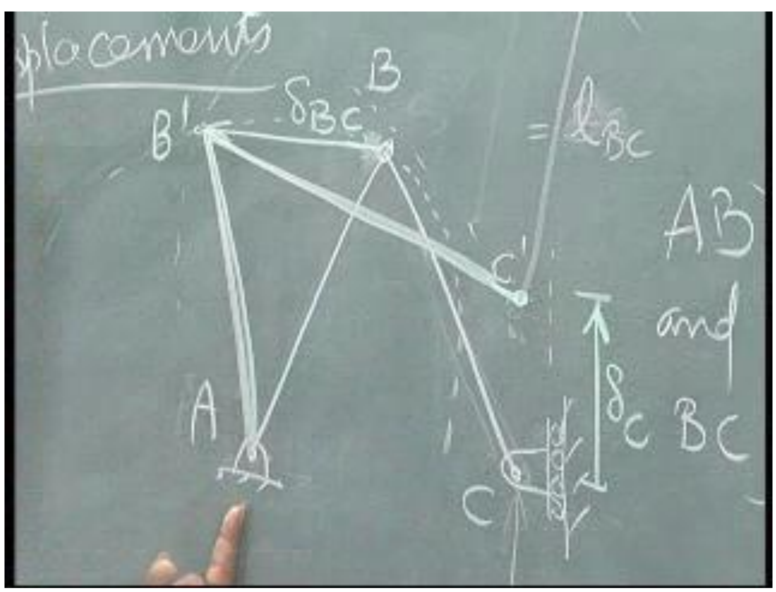

If I do this what I get as delta C and this should be delta B I am sorry (Refer Slide Time: 15:26) and delta B are kinematically admissible displacements. Is this clear?

Let’s do one more, just to get the hang of it. I am going to use the same thing but release a different displacement. I have like this, this is A, B and C. Let me retain this hinge support here, let’s say I want to find out the horizontal reaction now. What would I do? This is a pinned support, I would want to release the horizontal displacement and apply a force. In this particular case let’s say I am applying the reaction lets say this is C_x. Mind you this is a choice that I have made. If the reaction is not this way, it has to be this way. I will automatically get a negative value for this.

Now I have to free the horizontal displacement, so now I am consistent with what it should be. Whenever I release a displacement, I have to apply the appropriate reaction for it. I have done that job. How do I go about solving this? Of course one of the important things that you have to do all the time is write the references and the references have to be with respect to the fixed frame of reference. Please don’t make a mistake on that, so the best way is to use this as a reference. This is x, this is y so that I can write this as y C, x C, x B y B. x A, y A is zero all the time.

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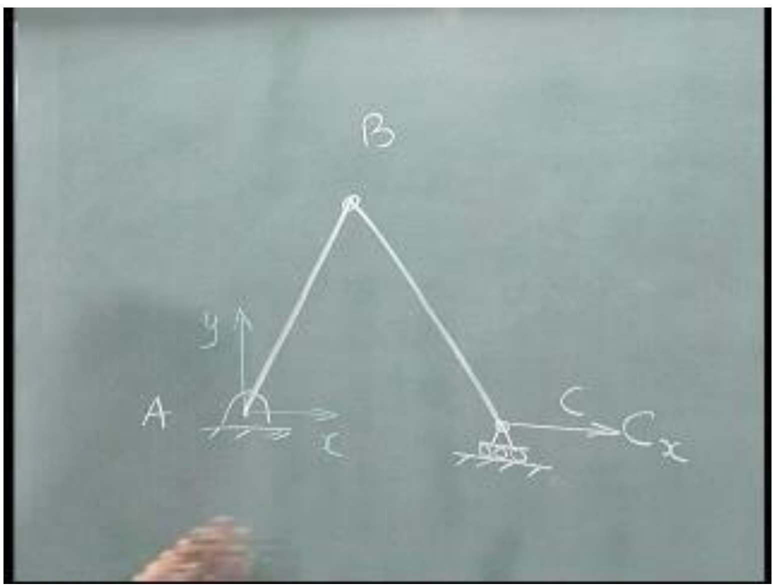

If I have to find out C_x, automatically I have to give a displacement to it. Let me use another color here, so let’s say I have moved it to this. Along with this support, I am just moving it like this. What is this? I am putting a direction over here, please remember direction is also important because when I take the dot product, I can get negative or positive and those signs are very important. If you miss it, you miss the entire system. What is this? I am applying delta C and I seek to find out where ever the active forces are present, the virtual displacements. If possible on all the points but I don’t need for all the points. I need points where active forces are there. I have not represented any of those active forces over here but our aim here is to find out kinematically admissible displacements. How do I go about? There is nothing different it’s the same. What do I know? I know that lengths A B and B C do not change consistent to the assumption of rigid body systems.

Again I will do the same thing. I have to show make sure that the length, lets say this is C prime, this is C, length C prime B prime is the same. I will do the same exercise. I can either start from here or I can start from the fixed frame of reference. I know A B again can only rotate, so that remains the same. I will just draw the possible path of point B. What is the possible path of B C? I just have to find out. I know that B can only rotate like this. If I take in a similar way and draw a radius equal to B C, we will get something like this. Where it cuts is B prime and therefore this is delta B, simple and clear. What do I know? delta B and delta C are kinematically admissible displacements. If I apply a delta C, I should get the delta B as denoted here. It's better to denote them as vectors, so that you are clear about the direction and if I have a force acting on it, I will take the appropriate dot product of it.

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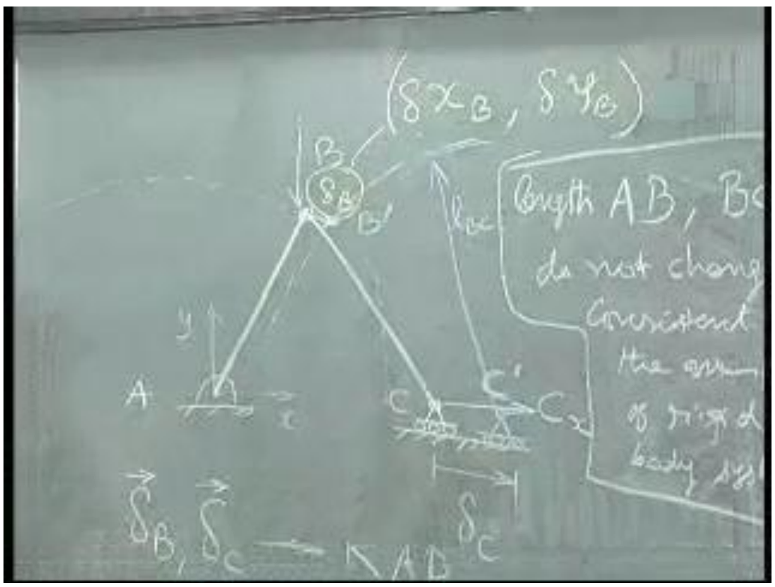

After all work is nothing but force dotted with displacement. Is this clear? That's how I go about finding displacements that are consistent with the rigid body assumption. As long as I maintain this, absolutely no problem. I will get equations of equilibrium that are correct. Again I will repeat this because this is very important. Please make sure the directions or the virtual displacements are properly found out. In this case the direction of delta C is indicated here. Similarly you will find the direction to be this. If you are correct, you will find the displacement to be this. Basically this sometimes is written as delta change in the position of B, delta x_B and delta y_B, the coordinates of this can be written like this.