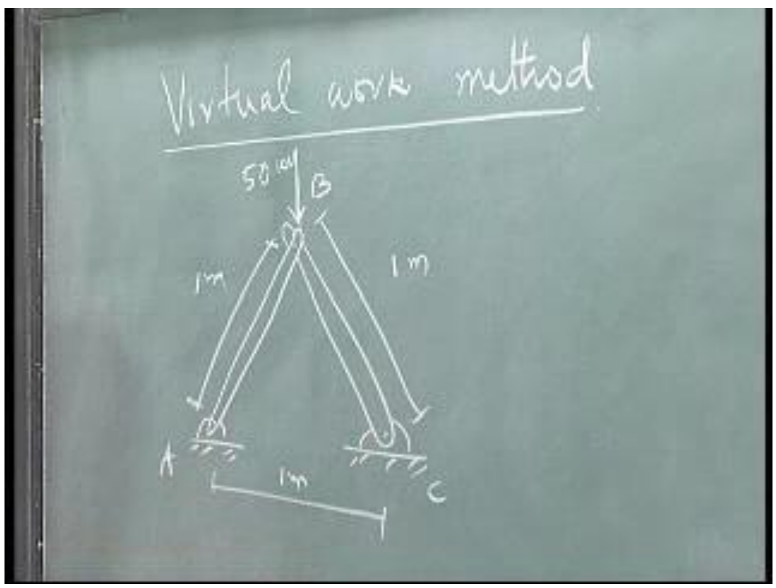
Engineering Mechanics
Prof. Siva Kumar

Department of Civil Engineering
Indian Institute of Technology, Madras

Statics - 4.1

Let us solve one of these problems using virtual work method. The problem is that of, a two bar truss pinned at both ends, let's call this as A B C and a force acting like this let's say 50 kilo Newton's. Each of these bars is 1 meter and distance between them is 1 meter. This is the problem that you can solve by using simple equations of equilibrium. Now is this a system with enough number of unknowns and number of equations. Let's examine if you remove this joint and this joint, these are two supports with respect to fixed frame of reference.

(Refer Slide Time 01:34)

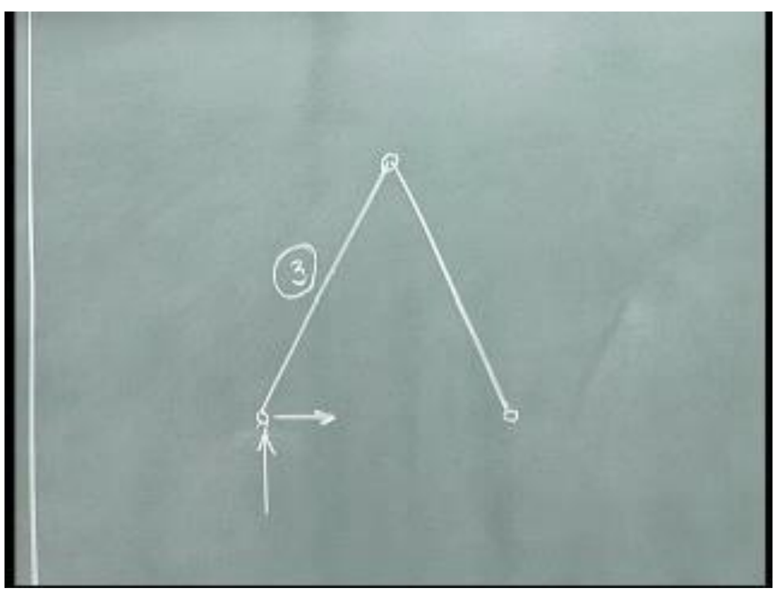


If we remove these two, we have 4 unknowns appearing. Two unknowns here and two unknowns here and we need to solve for those two unknowns. If you look at this particular system, this system has one degree of freedom. That's not very difficult to understand. I can fix one of these and then see whether the other bar is moving, the other bar can rotate about this particular path which means there is one degree of freedom. If there is one degree of freedom internally here, I can generate three for the rigid body plus one using the one degree of freedom which means 4 equations can be formed and we have 4 unknowns, we can solve for that. In that sense this is a determinate structure. Meaning, I can always find out the unknowns using just purely equations of equilibrium.

Now one of the important things to note here is, if I ask you how many degrees of freedom does this system have with the supports which means that if I have one degree of freedom here, have I arrested enough degrees of freedom for it to be stable or in other words cannot move at all. The answer to that question in this particular case is yes. If I ask you the system degrees of freedom, this is with the fixed points, the number of degrees of freedom is 0. Supposing I ask you the same question by removing this, how do you go about finding out the number of degrees of freedom for a system? This is the first and foremost important thing that you have to do in the virtual work method. Let's take this particular example because you can find out the degrees of freedom. There is no support here, there is one support here.

How do we go about doing this? We will resort to how we did in the earlier sections. We start with the fixed frame of reference. This is completely fixed and we know that there is a pin joint over here. When we draw it separately, I have these two members. There is a pin joint here, there is nothing over here. If I remove the fixed frame of reference and look at this particular free body, we have two unknowns that coming to this or in other words by releasing this there are two degrees of freedom that you release. The two degrees of freedom that you release are this direction and this direction movement. To put it the other way, the fixed frame basically freezes this point from moving in the two directions. The two directions can be x, y or x prime, y prime and so on. We have already done that. The body lost two degrees of freedom by fixing it to fixed frame of reference. Individually this particular rigid body has three degrees of freedom. Meaning, it can move without bothering about any of the other constraints. This can move with three degrees of freedom.

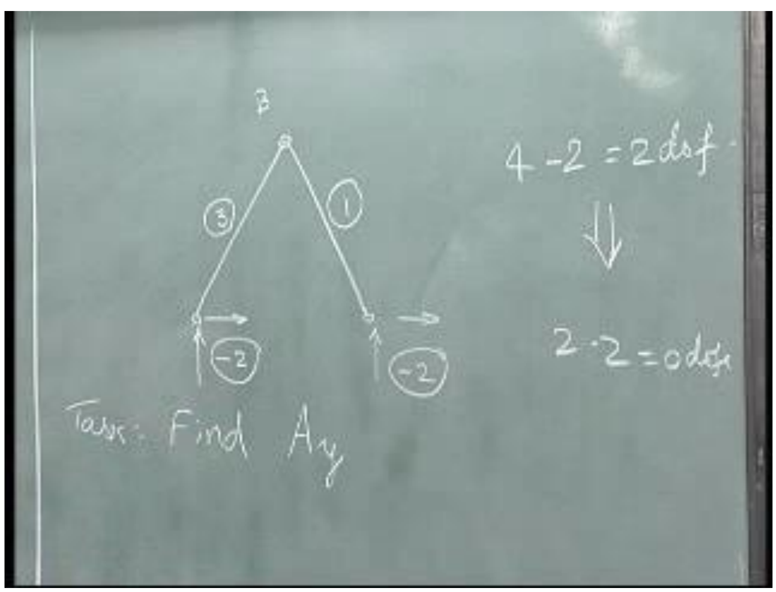
(Refer Slide Time 05:12)



This is something that we know and I am just talking about the planar rigid body here. I have minus 2 over here because there are two that are lost by fixing. There are 3 degrees of freedom for this and if I fix this rigid body also to the fixed frame, I know this can rotate about the point B which means this has one degree of freedom with respect to this. So I am going to write one here. I will repeat this. I look at this rigid body, it has 3 degrees of freedom. If I fix this rigid body and look at what is happening to the other rigid body, the other rigid body can have one degree of freedom which is rotation. So I put one here. Apart from that this particular rigid body has lost two degrees of freedom by fixing it to fixed frame of reference. Therefore the total degrees of freedom that this body can have is two. In this earlier problem we knew that there are two degrees of freedom that we arrested here. When we did that, this comes down to two minus two equal to zero degrees of freedom and this is an essential part in this method of virtual work. You need to find out the degrees of freedom of the particular system.

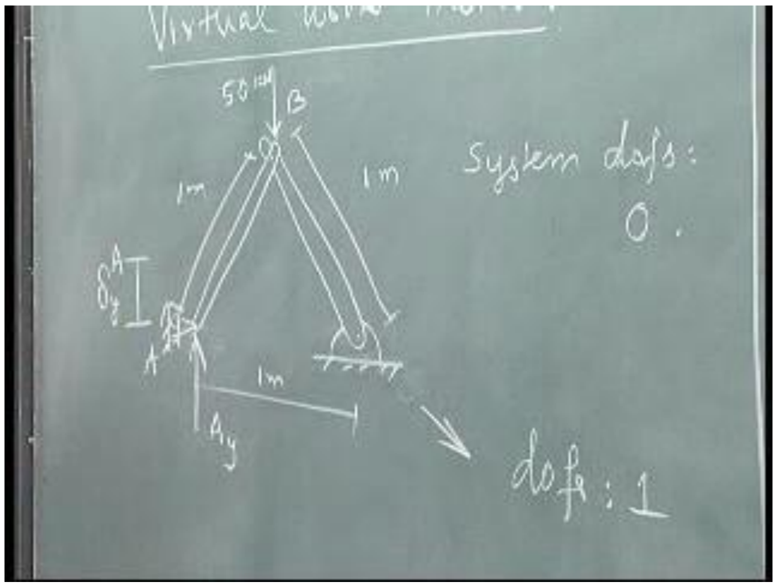
Supposing I want to solve this particular problem with two bars pinned together with a hinge support at both ends. You know the degrees of freedom is 0 which means I cannot solve this problem directly using virtual work method. I need to release at least one degree of freedom and that one degree of freedom can be associated with the kinematics or the movement of the structure. Please understand that in virtual work method there has to be movement possible in which case, supposing lets ask this question what do I want to find out? Supposing I say which one do you want? Tell me something that you want to find out. Shall we say vertical reaction here? The task is to find out vertical reaction at A, let's say. Now please remember the fixed frame of reference will react because it is constraining the vertical movement and therefore there will be a vertical reaction coming up. Supposing I release that and apply A_y which is the reaction, it is one in the same at equilibrium.

(Refer Slide Time 08:27)



Please remember the two are different in terms of movement. I will repeat this once more. Supposing I release this and let it have a vertical degree of freedom. I apply a force A_y. Is this body in equilibrium? The answer is yes, if I have found out A_y properly. But is it kinematically stable? Not really because it can move, even a small disturbance is enough for it to move. Whether it is a stable equilibrium or unstable equilibrium is another story to find out. But in this particular case if we seek to find out lets say A_y, we need to release that particular degree of freedom or that displacement. Once we do this exercise, we know the number of degrees of freedom for this body is equal to one. What is this degree of freedom? Vertical movement, so let's say the vertical movement I am going to call as delta y at A.

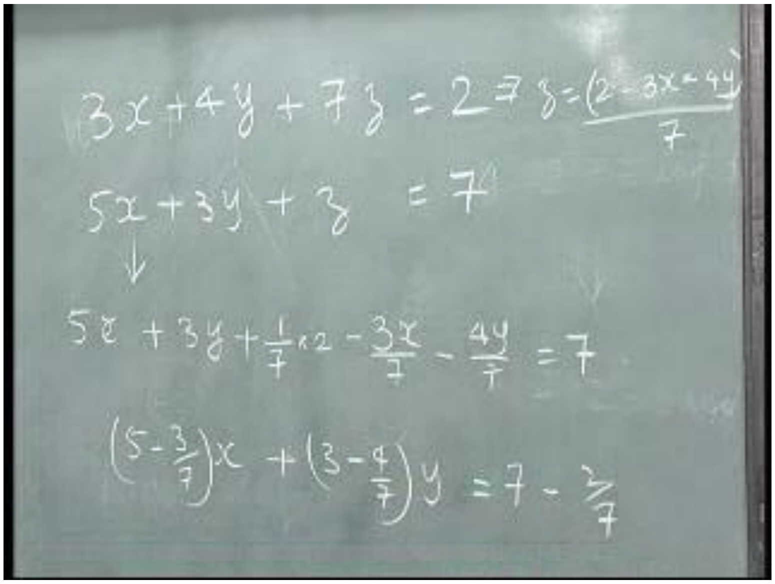
(Refer Slide Time 09:55)



This vertical movement is such that the body is in equilibrium. Slight virtual displacement of this will give us an idea of equilibrium of this body. That's the idea of virtual work principle. Let's move on and find out what we should do next. One more important point before moving. What do I mean by this degree of freedom? It's not very clear to me, so I am going to make it clear by giving a simple example. Let's say I have an equation ax, let me just give numbers, 3 x plus 4 y equals 7. Let me add one more plus 7 z is equal to 2, 5x plus 3y plus z equals 7. Let's say I have these two sets of equations. Let's say unknowns are x y and z. If I ask you the question how many degrees of freedom this set of equations have? Is the question very clear? But if I ask you, can you reduce this in such a way that you can write one equation in terms of one of the variables. Let's see if we can.

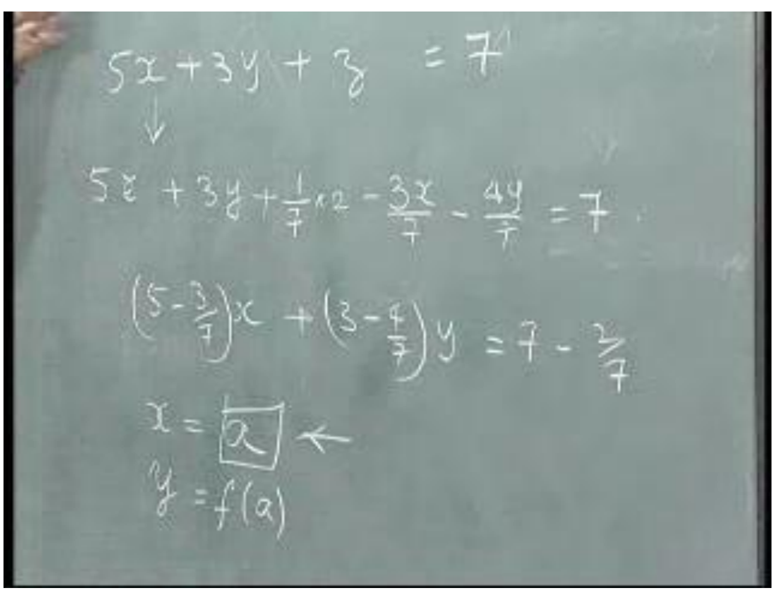
From the first equation I can write z equals 2 minus whatever in terms of x, so let me write it as 3x minus 4y divided by 7. What I have done here is I have written z in terms of x and y. If I substitute over here, z is eliminated and we have an equation that is in terms of x and y. Now we have reduced, supposing I am not using this equation. I have inserted that into this. We have reduced this equation into having only two unknowns. Is this clear? Let's do that exercise because that will be useful to us. Let me do that reduction, 5x plus 3y plus 1 by 7 times 2 minus 3x by 7 minus 4y by 7 equals 7. This is what we have. Let me just regroup fast, (5 minus 3 by 7) x plus (3 minus 4 by 7) y is equal to 7 minus 2 by 7. Is it okay?

(Refer Slide Time 13:04)



If you notice here, this is a equation in terms of x and y. Can I ask this question? Can I write why in terms of x? The answer is yes. In this particular case, even though I have two unknowns, I can write one of them in terms of the other. Let us say I am going to put x is equal to some arbitrary a, can I write y as a function of a? Again I will repeat this. If I say x is some parameter a and I ask you, can I write y in terms of a, can I write z in terms of a? The answer is yes. This parameter essentially is the backbone of these two sets of equations.

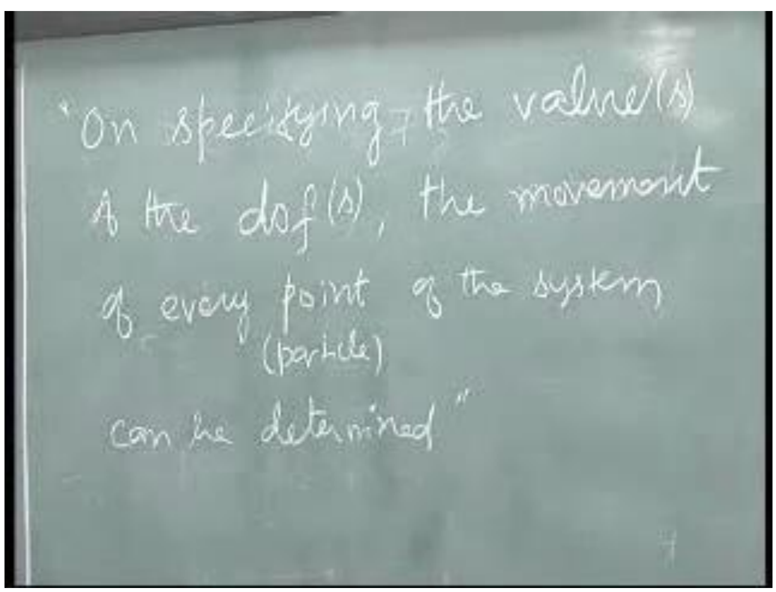
(Refer Slide Time 13:58)



Now in this particular case that parameter is nothing but x or in other words, I can write everything in terms of x, y as well as z can be written in terms of x. This two set of equations is a single parameter family and this we call as one degree of freedom, I can give you the value of a and you can find out x and y, if I give a different value of a, we will get different value. I have the freedom to give you the value and you can find out the other variables. Supposing I give one more equation which is independent of this. Then I know the number of degrees of freedom is 0 because everything has a single value. I could have solved for x y and z.

Supposing I had not given this at all, then I have this equation. Do I have this equation? Answer is no. I don’t have this equation, I have one single equation as you can see over here. Is z a function of x and y? Unless I give you the value of x, unless I give you the value of y also, I cannot find out z. This directly means that there are two degrees of freedom. If I give a system and you are able to write only one equation like this from equilibrium then I can say that I have a two degree of freedom system. In this particular case when I say one degree of freedom system it means that there is one movement that I do, the movement of every point of this body can be found out using this one degree of freedom. Let me write that down so that it’s easy. On specifying the value or values of the degrees of freedom, dof is the short form, the movement, (moment is different from movement) of every point of the system, I can also write this as particle of the system can be determined. This is very important.

(Refer Slide Time 17:20)



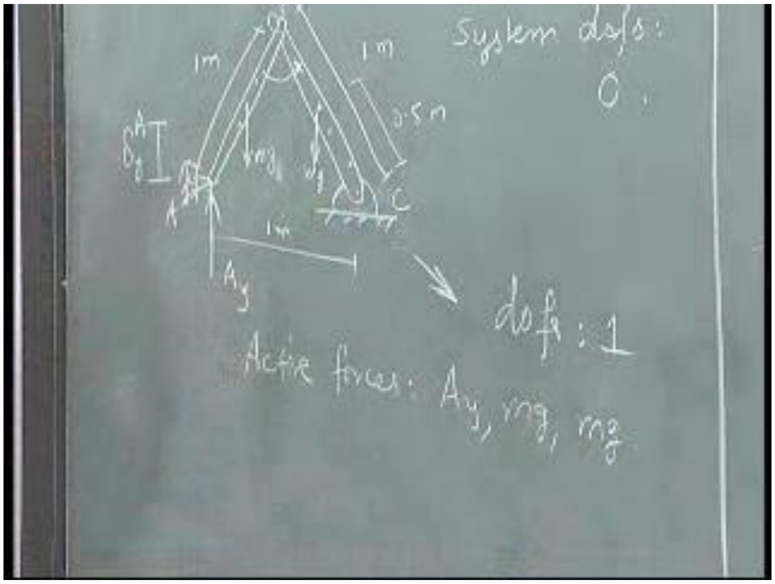
This is what we call as kinematically admissible displacement. For example in this if I move this guy, every point of this would have moved. I am going to show that in a simple example and therefore if I have to find out how much this has moved, how much this has moved, how much this has moved, how much this has moved, how much this has moved, all these will be in terms of delta y A. Is that clear? Of course this point we already know it is equal to zero, displacement of this is equal to zero.

Therefore since I can find out the displacement of all these with respect to delta y A, supplying a single value of delta y A, I can find out the displacement of the entire system. Therefore degrees of freedom is equal to one. Supposing it were two, I have to specify the values of two of those and then I can find it out. Is this clear? This is the first exercise. After finding out the degrees of freedom, you also have to tell which one is the degree of freedom. Here lies the trick. There are many many ways in which you can do this. Is it necessary that it has to be delta y A, that has to be the degree of freedom? Not necessary. Remember if I specify this, can I find out this? Let me ask you this question. I have written 2x plus y is equal to 7, let's say this is the degree of freedom. If I specify x, can I find out y? Supposing I give you y, is it possible to find out x or in other words if I give you displacement of any other point I should be able to determine which means every other point also is a candidate for degree of freedom. Please don't misunderstand what I am talking about.

I can choose any one of these displacements as the degree of freedom. All the other point, the movement of all the other points can be determined using that particular degree of freedom. There are several options for example I can take the angle between this, the angle is changing. If I specify this angle then every point of this can be found out. It is a matter of choice and the moment we have choice, we also have a problem as which one and that's basically is the crux of the problem. Many times we don't recognize, this is a degree of freedom that I can use. But any one of them that we have used, you will get the proper kinematically admissible displacement.

After having found out the degrees of freedom, you also need to find out next step of finding the active, mind my words active forces in the system. What do I mean by that? There is no cancellation that happens or there is no internal force that takes part in this virtual work method. For example here I have a 50 kilo Newton, let me just change it a little bit so that it's easy for you to comprehend. Let's say there is only the self-weight for which I want to find out. There is a self-weight here, I am just using the center of mass here. This is mg_1 mg_2. In this particular case since they are same length and lets say mass per unit length is the same I will write it as mg, mg acting at the center.

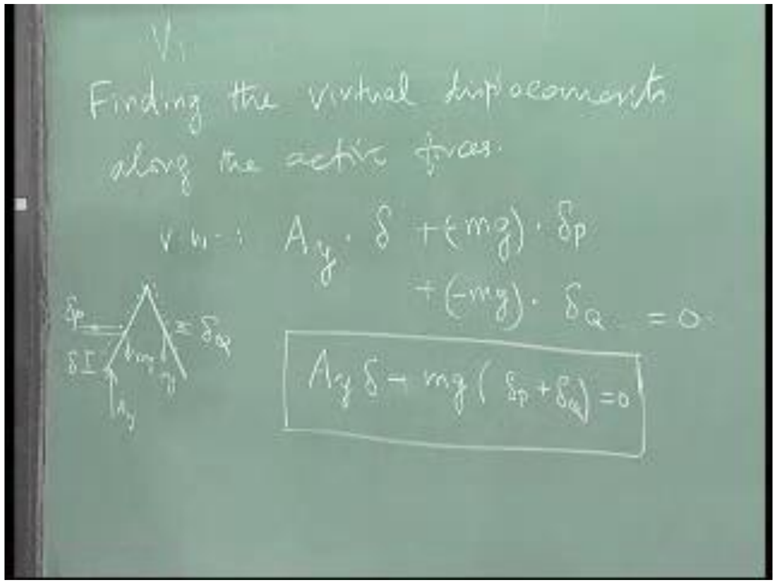
(Refer Slide Time 22:58)



Now it's very clear, the external force is A_y, mg mg will take part in the virtual work equation. Now how about the reaction at B? One of the confusions that happens in this virtual work method is which force should I take? Please remember only when I remove the pin that joins the two bars, I will reveal a force and they are equally opposite force that are internal to this particular joint B. The moment I have one force and another force opposite to each other and there is a displacement. The net force there is equal to zero or in other words it does not matter what reaction the bar A B offers on B C. I will not have to bother about it. This is the beauty of virtual work method.

So very simple active forces involved in this particular case. Since I have released this A_y, mg, the other mg. Mind you here also I don't have to bother about the reaction. The reason is there will be equal and opposite reaction and the displacement will come into play. Here one more important point is this is the displacement is equal to 0. In the virtual work method, if either displacement of the force is equal to 0, the virtual work is equal to zero and therefore it does not take part in the equation. What does virtual work principle say? The total virtual work due to a small displacement given for the degree of freedom is equal to zero. That's the statement that we will be using effectively here. But before that mind you supposing I use this delta y A as the displacement given to me, I have virtually moved this point A vertically by delta y A. For now let me just remove this delta y A and all those because it's just one value. I have moved it by delta. What are the other things that I need to find out? I need to find out the kinematically admissible or in other words what happens when I move this delta at these two points? How much do they move vertically? If I know that information I have solved this problem because I just need to use those and write down the virtual work principle. So The next set of values that I need to find out are finding the virtual displacements along the active forces. Once I know that it's pretty simple.

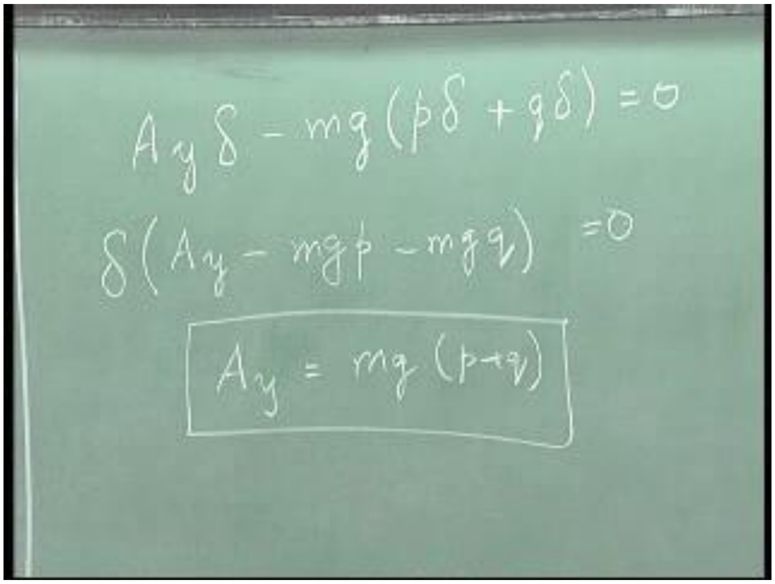
(Refer Slide Time 27:38)



Let me just number these, name these. Let's say this is P, this is Q. A B C P Q, so if I have to write the virtual work equation, the virtual work equation will be A_y times movement in the same direction as, movement in the same direction of this force will give me a positive virtual work. I need to be bothered about the directions of both the force and the displacement. If they are along the same direction, it is positive. If they are opposing each other, it is negative. If it is both same directions, it is positive. This is something we have. If I have to find out this and this, first let me write down the virtual work due to A_y then I have mg is acting downward. Let me draw this over here so that it's easy. This has moved over here, let's say this is the displacement, this is delta. There is A_y acting like this, there is an mg acting like this, there is an mg acting like this. Let us say this movement in the upward direction is delta P.

This one again in the upward direction is delta Q. Mind you movement upwards, force downwards so I will have minus mg times delta P. Similarly plus minus mg times delta Q and this should be equal to 0. This gives me A_y delta minus mg times delta P plus delta Q equals 0. This is the virtual work equation. If I find out delta P and delta Q in terms of delta, let us say let delta P equal to, let me just give some number p times delta. Similarly delta Q is equal to q times delta. This p and q I need to find out from kinematics. If it is so, I can insert these into this, I will get it to be A_y times delta minus mg times delta P is p times delta, delta Q is q times delta equals 0. You can see that delta is common throughout or in all the terms I get delta A_y minus mgp minus mgq equals 0. What do I know about delta? Delta is a non-zero displacement that I have given which means this has to be equal to 0. So what I get is A_y equals mg times p plus q.

(Refer Slide Time 29:05)



Is this clear? What is left out is how do I find out delta p, delta q? The moment I find those two out then I have solved this problem for A_y. Is this clear so far.