Engineering Mechanics
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Statics – 2.9

I have a pair of scissors and small sheet, just to illustrate to you how I can find out bending moment and shear force for a particular situation. For example this is a sheet of paper and I am so used to doing this operation from my childhood. Cutting a sheet of paper using a pair of scissors. Can I analyze what kind of force is going on to this paper, so that I cut this particular sheet of paper? Let’s do and see. Let’s say this is the sheet of paper. Let me just draw it nicely. What is happening here? There is one leg of scissors, pair of scissors. This one leg I have drawn here, so that’s cutting it. It has to be this way because it is cutting along this. There is a other one which is just near like this so that when I cut I am applying force equal and opposite force at these two points. That’s not very difficult to understand.

If I apply a load over here and the load over here, the two loads that appear on the paper are exactly one and the same. So essentially I am applying a force P here and a force P here and I wish to use this set of forces in order to cut the sheet of paper. What is the distance between these two? Very small some delta. Let’s just blow this up and see what kind of forces appear. I have a P over here and a P over here, there is a small delta that rotates it. I am not showing that particular side of it by holding it tight and applying, there is a slight movement that takes place and that resistance is taken by my hand.

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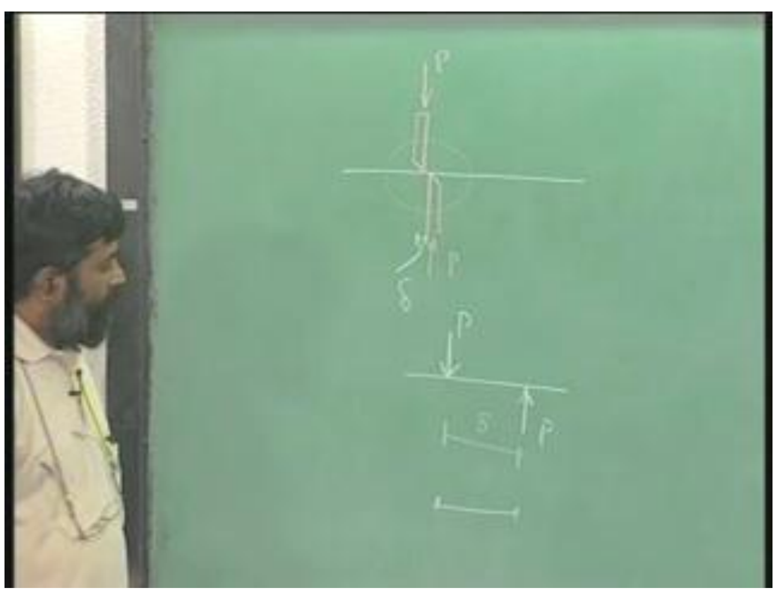

If I draw the shear force for this, pretty simple there is no force in between. There is a shear force equal to P here, constant over this delta region and goes to zero at this region. How about the bending moment diagram? Let me make it small so that I can draw the bending moment diagram. Bending moment diagram, if I take any particular point over here, I ask the question is it in equilibrium. The answer is no.

I should have applied for example I should had a cantilever or something like that which is nothing but the hand that is holding the sheet of paper. That is a moment that appears over here but then if I look at only between these two and look at this particular side, I cut it off like this. The moment is P times, if I take the distance from this as x it is P times x. It is some kind of moment like this and at this particular point it is P times delta. What do I know about delta? Delta is very small, very small compared to the total length of the paper and all this. Therefore P delta will be very small and therefore I can say this is roughly equal to 0.

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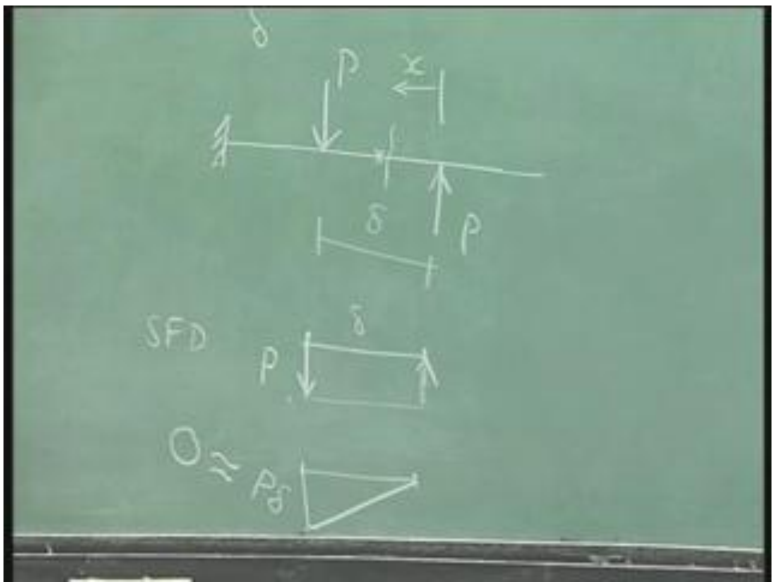

In this particular case of applying force using a pair of scissors, all I am doing is applying a shear force equal to the force that I have applied through my hand. Therefore I have a situation of nearly pure shear force. I am applying a pure shear force on this and that's enough to cut the paper. As delta becomes smaller and smaller, the efficiency of this scissors will become better and better. Now there could also be a situation where you can have pure bending moment for whatever reason. I will give you one example of that. This is usually called either two point bending or four point bending. It's a situation like this. This is a force P, this is a force P acting at L by 3 away as shown here.

What will be the reaction? Reaction is P and P over here. If I take the bending moment variation in this region, let me call this as A B C D. Let's first draw shear force diagram. I am going to focus only on this zone B C. If you notice there is a P over here like this and then this is P and I have to bring it down by P. This is negative, it's going to be 0 over here and I have a positive P shear and it ends like this. I have a negative shear and a positive shear but if you notice in between, you have a 0 shear situation.

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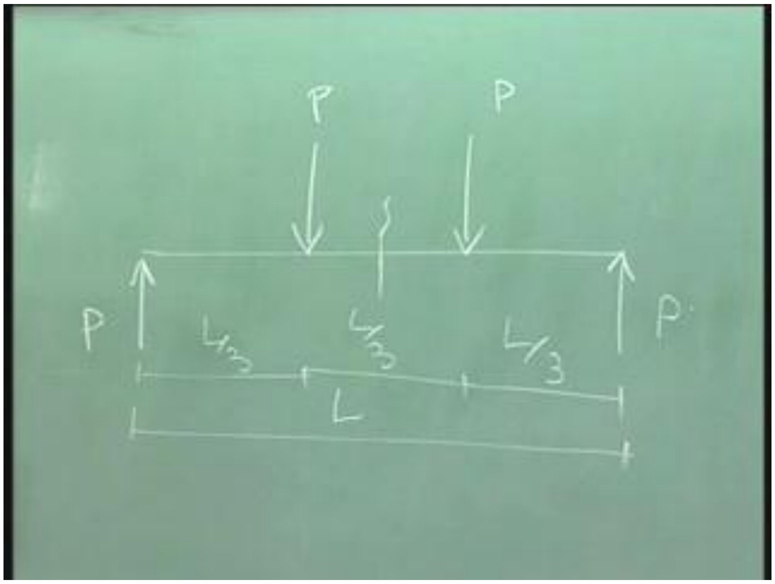

How about bending moment diagram? You will find out that the bending moment diagram can be written like, I have just copied over here. If I take a section in between, this P and this P, both forms a couple equal to P times L by 3.

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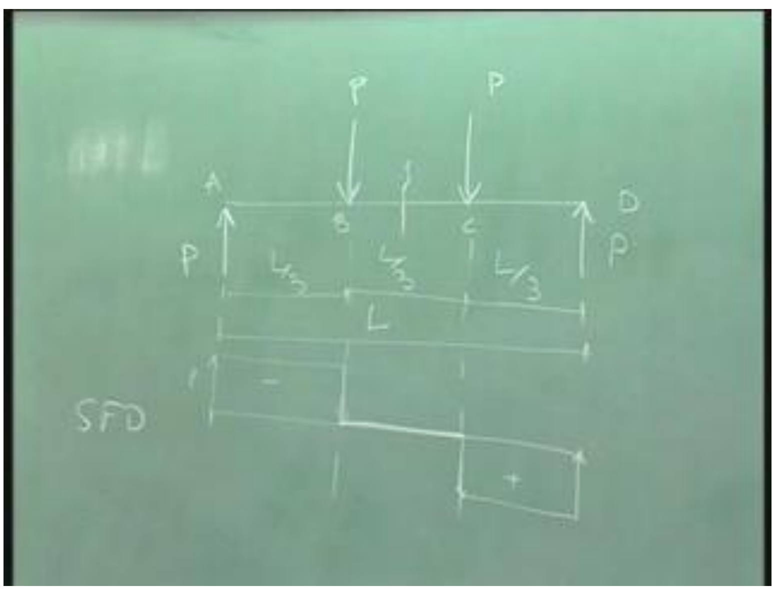

I can just replace this by a couple equal to P times L by 3. Similarly these two form a couple which is P times L by 3. There is no other force acting. Therefore in this particular zone, it is a pure bending moment acting. Just to make it clear here you have like this. I am cutting close to B and C and it reveals this pure bending moments equal to P L by 3 and P L by 3.

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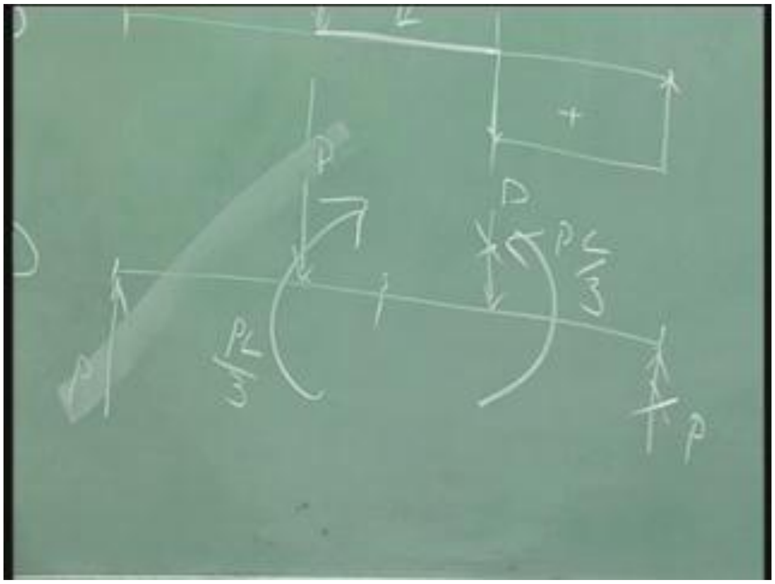

Therefore any other section here will be equal to P L by three. Therefore we have a constant moment within this zone B C which is a positive equal to P L by 3. Thus we can generate a pure bending situation, this is called the pure bending situation. Why pure bending? There is no shear, there is no axial force. The only force resultant that is present is pure bending. Now question. If I move this particular set any where around, do you think I will generate constant bending moment over that B C. For example instead of this being L by 3, L by 3, L by 3 supposing I moved to something that is at distance 'a' from the left hand side. Retaining this as L by 3, do you think I will have a constant bending moment over here? Please find out by yourself.

Now let's look at different scenarios of loading that can appear on a beam. Supposing I ask this question. Let's say this is a simply supported beam. Immediately I can draw like this. Let's say it has its own weight to be taken. Its own weight let's say it is weight per unit length given. This is nothing but mass times the area of the cross section per unit length. Now how do I represent it? People represent it in different ways. One of the simplest ways in which you can represent is draw something like this and say this is equal to w per unit length. They also represented using a diagram like this.

We call this as uniform because it is let's say, if it is a uniform cross section it is a uniformly distributed load. What's the direction of these loads? Transverse to the axis of the beam. Mind you it will always be transverse to the axis of the beam, if I am taking this particular structural member system to be a beam.

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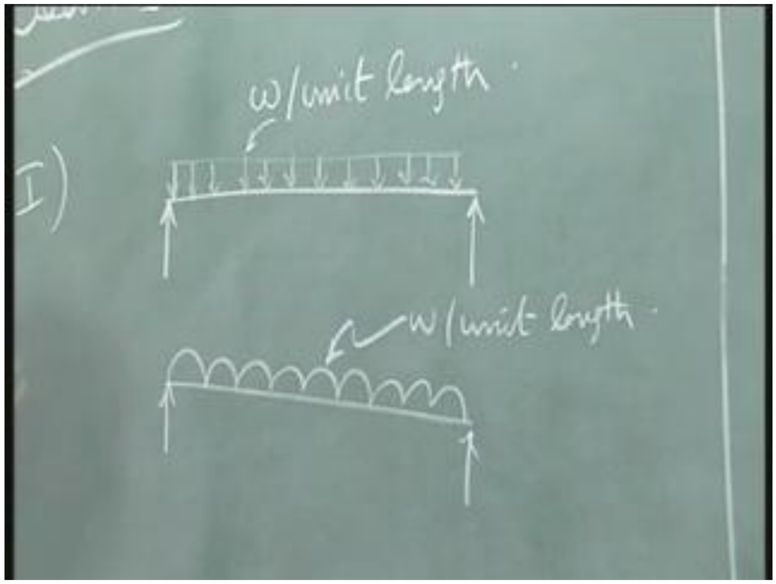

Therefore the support reactions will also be perpendicular to the axis of the beam. Another situation is for example I have a beam like this. Let's say it could be a wall and it is basically supporting a column of water. If I have to find out the distribution of force on this particular, let's say beam it could be a wall. Basically it can be idealized to be a beam. What will be the force on it?

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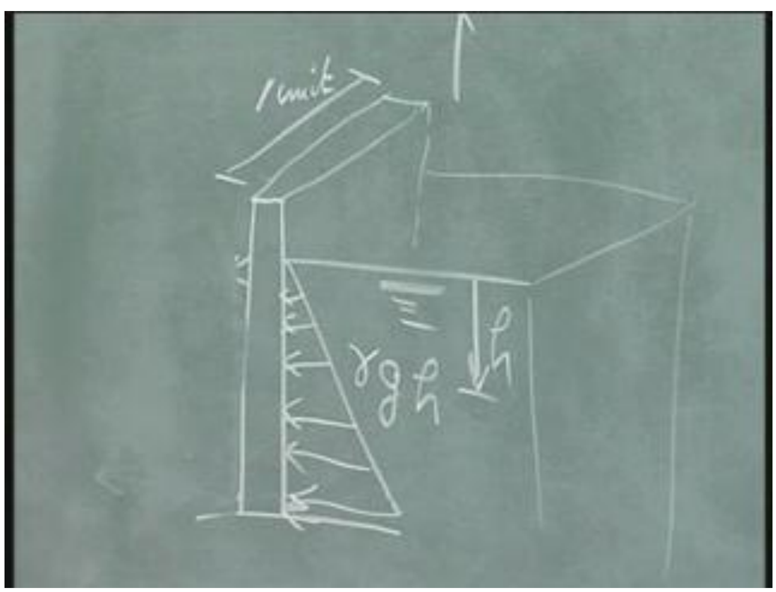

Supposing I take this to be unit length then at the top I have no pressure. As I go below, it will be gamma times g h where gamma is the specific mass of this particular. In this wall, since the pressure is acting this side, the load should be distributed like this equal to gamma g h. The gamma g is the specific weight times the height that will give you the pressure at any particular point and that's acting. Remember this h is varying with from this and therefore it's a linear variation over here. You can also imagine if you have a cross section that is varying, that will introduce variations in the load.

Simplest example could be, supposing I have something like this and a beam with a cross section. I know the weight per unit length is varying from one end to the other and this will introduce for example if the cross sectional area A is varying linearly from here to here. Then the load also will be varying linearly. This w per unit length is now a varying load, so I should write this as w of x per unit length and then define what is w of x. The third type is the type that you already know. For example if this is a platform and a man is standing on top. The weight of t he man can be replaced by a load equal to its weight and this is usually called the concentrated load. The width over which this load is applied is very small compared to the length of the beam that you can take this to be a load which is a pointed load or a concentrated load. This is also called point load for the very fact that it is acting at a point on the beam.

Another case could be think of something like this. I have something like a hook here and a weight is hanging over here or let's say there is a load. Let's take this particular example. If I have to treat this particular length as just a beam, what I can do is I can cut it off here and treat this as a separate body. So If I have to cut it off, I am revealing the internal forces over here. There is a force over here and that will create equal and opposite force. This is very simple to solve. This will be equal to w and if I take moment of this force about this particular point, let's say this distance is what shall we call. Say c in which case there is a bending moment acting in this direction. Therefore I will have something like this which is equal to W times c. So I can cut this off and replace with these two, a weight and a bending moment equal to W times c.

Now if you notice, since this is a one dimensional member, I can draw a line over here, represent the bending moment, a moment applied at this particular point and the force applied at this particular point as W and W c. In addition to a point load here, you also have a point moment acting on this. The other situation is this is the example that I showed you. Supposing you have a pair of scissors that's blunt. The centre of the forces will be such that will be a distance and that will apply a moment. If there is sufficiently having a gap then you are basically applying a moment. That's also can be replaced like this. These are the some of the loads that may appear in a beam.

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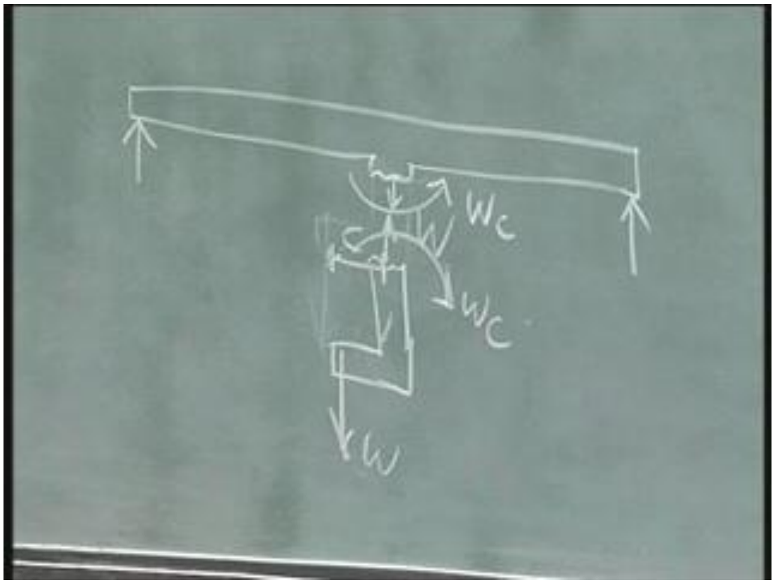

One last scenario which you may not expect in most of the situations is something like this. I uniformly distributed moment lets say C per unit length. Let me call this as m, small m per unit length acting over the entire length of the beam. Given this, there will be a combination of these forces that will come into picture while analyzing for various structures that use beams. One of the tricky examples is an example of the sort. I have a simply supported beam. Let's say at centre I have a moment acting M, let's say this length is L. Therefore this is L by 2, L by 2 and it would find out the shear force and bending moments.

What would be the reactions? That's not very difficult to find out. If I have to find out the left hand side reaction, I have to take moment about this. I have anticlockwise moment here M due to this and this is a clockwise. This force applies a clockwise about this and lets say this reaction is A_y, this is A and this is let's say B_y and this is C. A_y times the length L, so this minus A_y times L equals 0. This implies A_y equals M by L and therefore I can replace this by M by L. How about this side? There is no other force acting on this free body. If this is M by L, this will be minus M by L or in another words I can change this direction and write it as M by L. This is important to note. I have an equal and opposite forces here. In fact they form a couple equal to M. That's a way of understanding.

How do I draw the shear force diagram? In a shear force diagram I don't have to care about the moment that is acting on this. It's pretty simple. I have M by L acting here and therefore there will be a shear.

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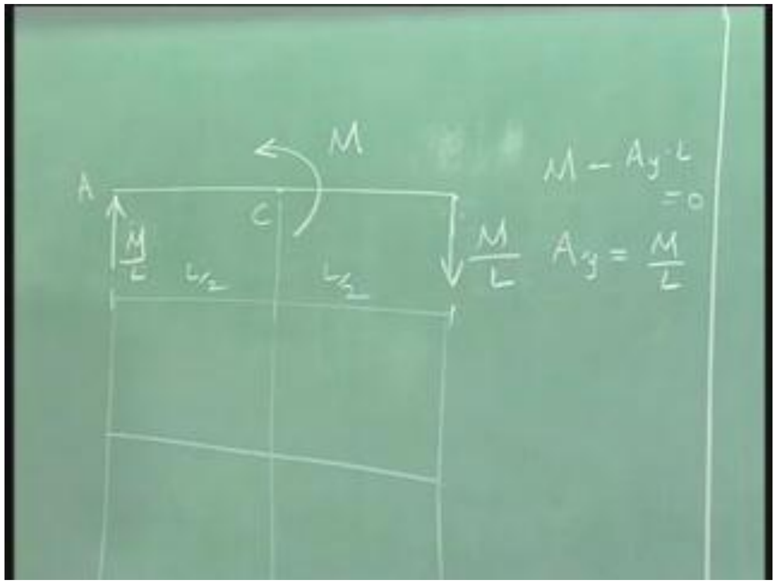

There is no transverse force acting on this, also on this and therefore it will continue to be like this and at this particular point, it will dip down by M by L and this is the shear force diagram.

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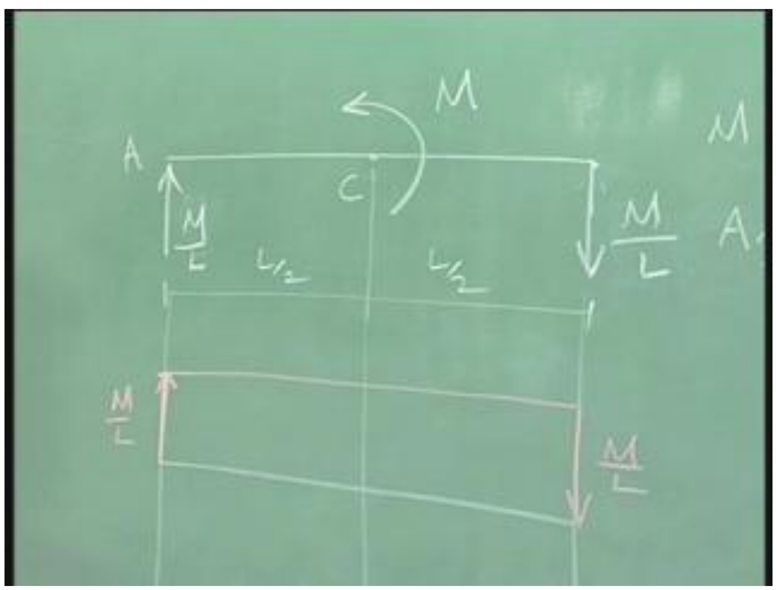

What sense is this? Negative sense. Remember upward is negative and downward is positive is what we used as sign convention. How about bending moment diagram and that's a question, that most of us have problems answering. But if you think rationally, it becomes very simple. Like before if I cut anywhere, this is the one that is going to take part. This M by L is going to take part in the bending moment and it is going to have a positive sense and therefore automatically I will have like this, positive. There is no transverse force on it and therefore the shear force is constant. If the shear force is constant, bending moment has to be linear starting from a 0 over here to a value equal to M by L times L by 2. So M by L times L by 2 is M by 2 for this particular point.

What is the moment? What is the sign of this? It is equal to positive. You can use the trick that I told you earlier. In order to find out for the right hand side, instead of taking a section that is this, if you take the section that is to the right, I have a force acting downward. So If I have a force acting downward like this and I wish to find out the bending moment. What is the positive sense of this bending moment? Is it this way? The answer is yes, if clockwise is positive because negative direction is… this is minus x direction. If I need to find out this M at x, it is equal to M by L times L minus x. Therefore if I substitute so M of x will turn out to be. Is it positive or negative? M of x is in the clockwise direction. M by L times L minus x is also in the clockwise direction and therefore i will have a minus M by L times L minus x.

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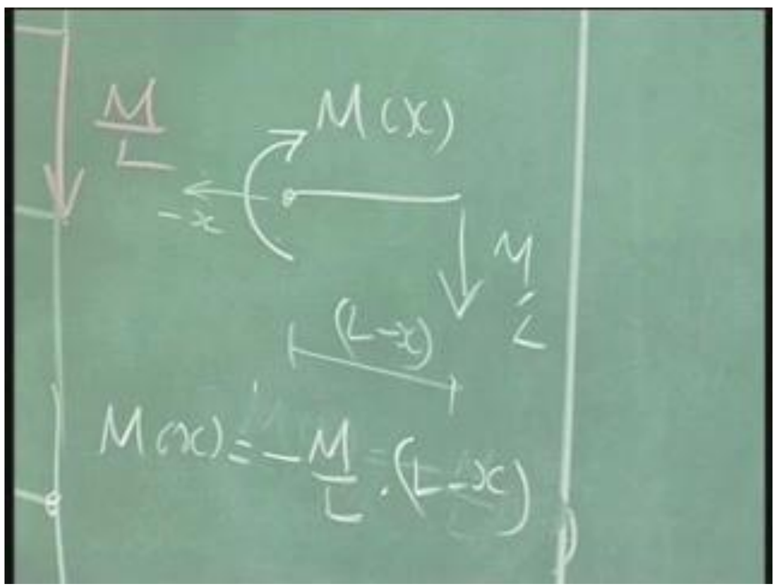

Let me make it clear here. M of x is equal to minus M by L times L minus x. Sometimes I may go from Goa to Chennai, what’s the value of this at x equal to L? Naturally it is equal to 0. Is it going to linearly vary? The answer is yes. Is L minus x always positive? The answer is yes. There is a negative sign here which means the bending moment will always be negative, it is linearly varying. At this particular point, it is equal to 0 and what about at x equal to L by 2. It is L minus L by 2 which is L by 2, M by L times L by 2 is minus M by 2 and therefore I will have something like this.

This is M by 2. This is the bending moment diagram. Most often you will have a confusion over here because you have minus here plus here. It doesn’t make sense, why I should always have a continuous scope for M and that creates a problem here. The work we have drawn here is correct. Please always remember this particular notion. If I am drawing the shear force and if there is a concentrated load for example in this case I have a concentrated load that pushes the value of shear force by that value, abruptly at that particular point. I go on at this particular point, I have a force therefore it pushes it down.

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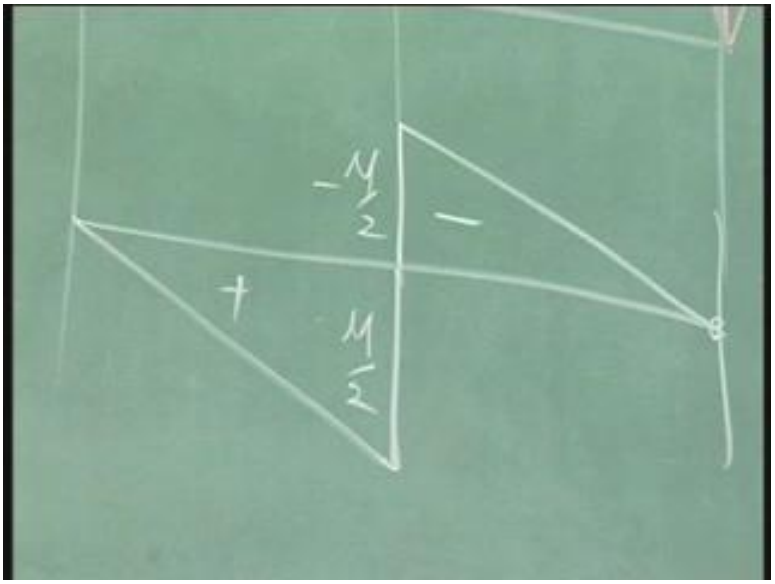

Similarly if I have a bending moment diagram, it will be continuous until I see a concentrated moment like this. That will create a jump, a sudden jump here and that's the jump that you see here equal to total M, M by 2 and M by 2 total together will be equal to this M that you have applied. Now the question. Supposing I move this M, away from the centre to some other point over here. Let me draw it and ask you the question. Let's say I have moved it to L by 3 away. What happens to the reactions? Most often if I ask you to answer quickly, you will say that this force will increase compared to M by L but if you do the calculation, it will remain M by L here and minus M by L here. There is often a confusion that you will have. I can draw the shear force diagram. Shear force diagram will not change at all irrespective of my moving the moment over here, the shear force diagram will be essentially the same.

How about the bending moment? Having given this understanding that this concentrated bending moment will give a sense of sudden shift. I know there is no transverse force acting here which means its derivative as I have already pointed out here, d square M by dx square is equal to minus w of x. The w in this particular zone is 0 means this is 0, d square M by dx square is equal to 0 means dM by dx is constant. If dM by dx is constant, M is linear in x and therefore this M by L will create, let me draw till this a positive sense like this. What is this? It is M by L times L by 3 which is M by 3. This is positive moment, you can do it yourself and see.
I can use the same principle that I used over here for this concentrated moment. What is this concentrated moment going to do? It is going to up because it is counter clockwise, it is going to up by a value equal to M. I already have M by 3 here which means this is 2 M by 3. I know it is linear between this and this because there is no transverse force acting

and I just connect it at this end, it has to be equal to 0. I finish it within no time. Please practice a lot of these, if you have to immediately solve some problems. By practice and understanding how these intricacies are useful, you won't be able to solve the problems faster and correctly. Thank you.

Let's take another simple example this time using a distributed load. This is also a simply supported beam with self-weight acting on this. Let's assume that the cross section is uniform all through, so we have a uniformly distributed load. So that I can draw something like this. How do I draw the shear force and bending moment diagram for this? Now if you look at it, we have force every where so unlike the other case where we had two concentrated loads and we were able to separate them. Here there is no need to separate, there is no distinct change that is occurring from one point or one zone to the other zone. Let's now start to draw shear force diagram. Let's adopt the method that was given earlier. I take an x from the left hand side. I need to take a section. What I have basically is the self weight acting on it, cut at a distance x. There is a reaction over here. Let's say this self-weight is w per unit length. Let's say the total length of the beam is L and therefore the reactions will be W L by 2, W L by 2.

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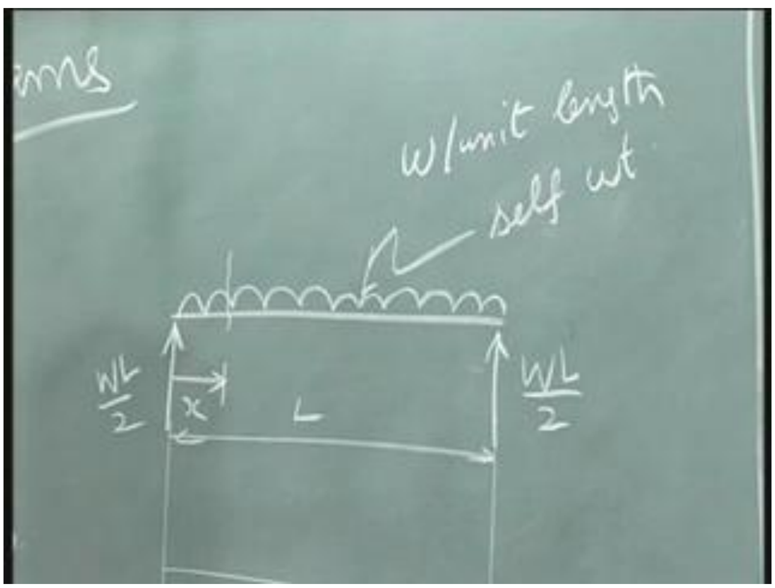

So W L by two. Is it W L by 2? The answer is yes because W is per unit length and this is acting over the entire length L so the total weight is equal to W times L. Since it is symmetric immediately I can distribute half and half here. Therefore it is W L by 2 here and this is W per unit length. Upon sectioning I should be revealing the shear force. Again I am drawing the positive sense of these, the shear force and the bending moment at x. What's the shear force on this?

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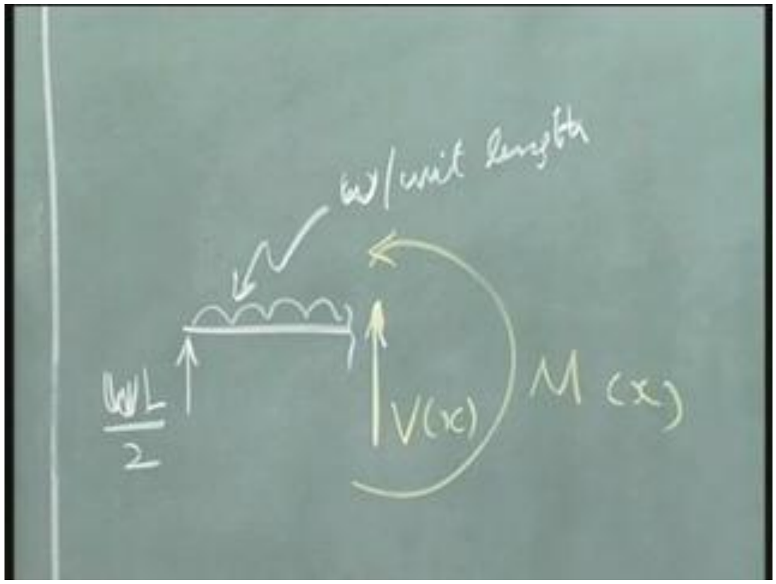

It is if I take this to be positive sigma V along the y direction. If this is y, this is equal to 0 implies W L by 2 minus W times, this length is x and therefore W times x.

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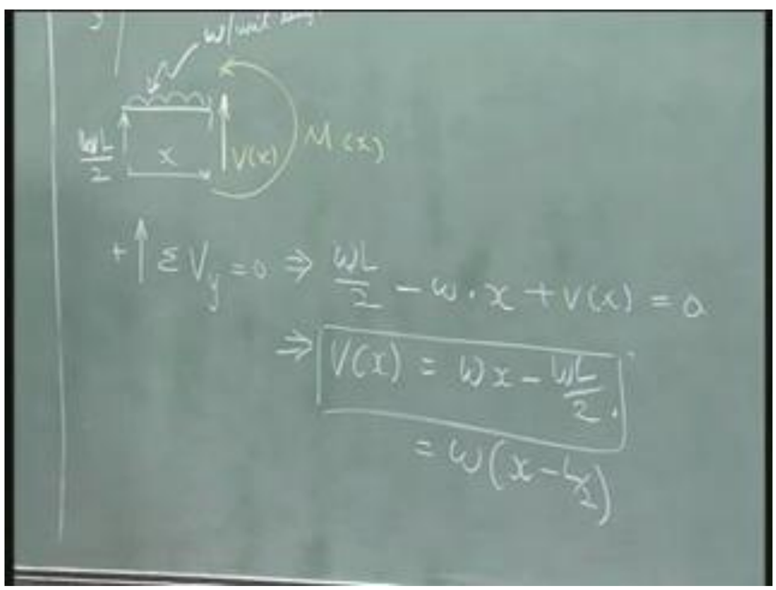

Let me use the same symbol so that there is no confusion, plus V of x is equal to 0. This immediately gives us V of x is equal to W x minus W L by 2. This can also be written as W into x minus L by 2. Notice here unless x goes to over L by 2, this value is negative. We have a negative shear till half the distance and the rest of the distance will have positive. Now do I need to write separately for this half? Not necessary, this will completely give the value, given x varying from 0 to L.

How do I draw this? If you look at this, this is a linear variation in x. At x equal to 0 it is W L by 2 and that's naturally true because I have a W L by 2 over here. So this is W L by 2 and then we have a uniformly decreasing value to this and at centre x equal to L by 2, V of x is equal to 0 from this and therefore it passes through this and it is linear. You have something like this. Started with W L by 2 and this is a negative, it becomes positive and finally it closes with W L by 2.

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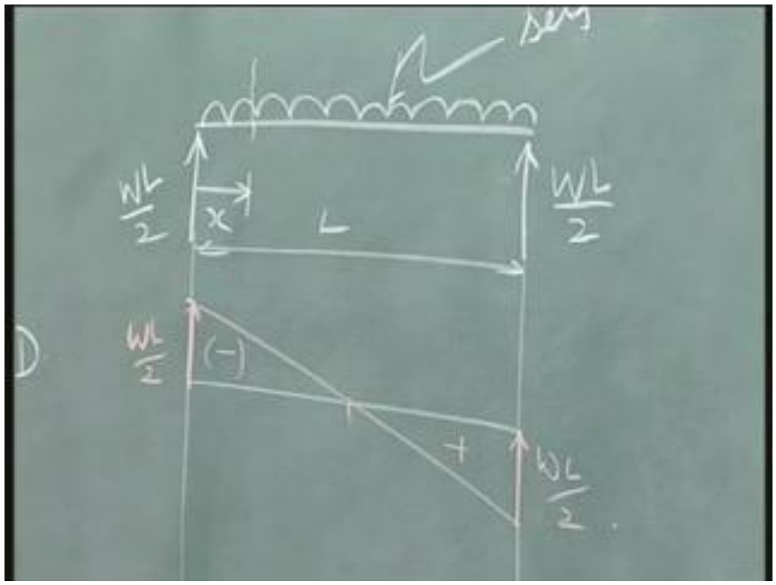

Bending moment diagram. How do I draw the bending moment diagram? Again the same principle. Let me use the same free body diagram over here. In order to find M of x, I should choose this point over which I find the equilibrium of moment. So M at x equal to zero, this will give me… V of x will not take part, W L by 2 will take part in the clockwise sense. Therefore I have minus W L by 2 times x. Many times people may forget this x and how about this. This is uniformly distributed self weight over this entire length. Now there are two ways of doing. You can integrate from one end to the other.

The other way of doing it is the total force can be replaced by this equal to W times x because that's the total force. That is acting at the centre of this x which is at a distance x by 2 from this x. So W x times x by 2 will be the moment created by that and that is in the anticlockwise sense. We have plus W x times x by 2 and this is a positive moment equal to 0. This will give us M of x is equal to W L by 2 x minus W x square by 2.

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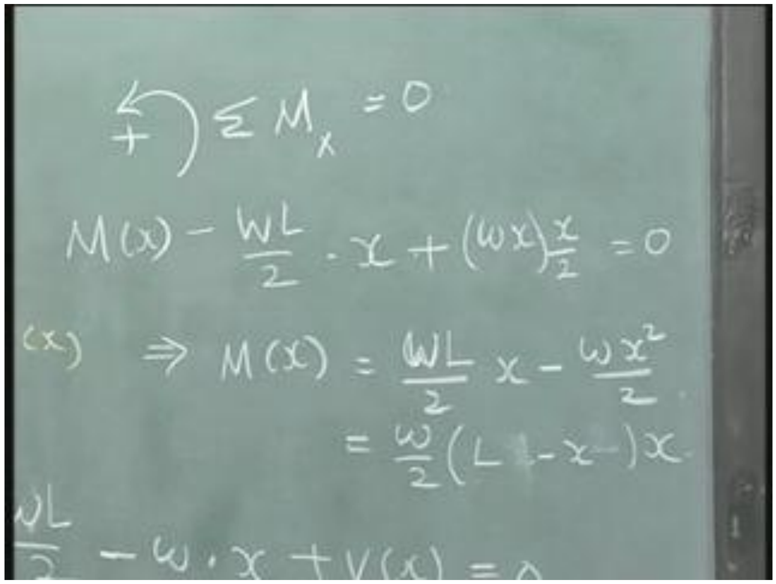

You can also write this as W by 2 into L x minus x square. You can take the x also outside and write it as L minus x times x. Look at the symmetry L minus x times x. If I go from the left hand side it is x, if I go from the right hand side it is L minus x. It is L minus x into x. If I put x is equal to 0, this entire expression is equal to 0 because x equals 0 here. If I put x equal to L this will go to 0 therefore the entire expression is 0 which is true because these two points are simply supported points. Now anywhere in between what is the sense of this particular value? X is a positive value, L minus x is also positive value as long as I am between 0 and L. Therefore this entire thing is a positive value. Positive we denote by down below.

What kind of variation is this? It is a quadratic variation. Therefore primarily I will get something like this. How about at the center? What is the value at the center? The value at the centre is L minus x is L minus L by 2 which is L by 2 times L by 2. L by 2 times L by 2 is L square by 4 times W by 2, you get W L square by 8. This is W L square by 8 parabolic variation. Zero at this point and this point and this is a positive value. There is one thing that you should note over here, at the middle the shear force is equal to 0 whereas bending moment is the maximum.

Shear force is linearly varying whereas the bending moment is quadratic. The previous examples that we solved, we had constant shear forces and the variation of bending moment was linear. For a constant we had linear, for a linear shear force diagram we have a quadratic. There seems to be a relationship between these two diagrams and that's what we are going to explore next. Before doing that I have one simple question and that is this. I have something like this as shown over here. This is uniformly distributed load, due to its self-weight W per unit length.

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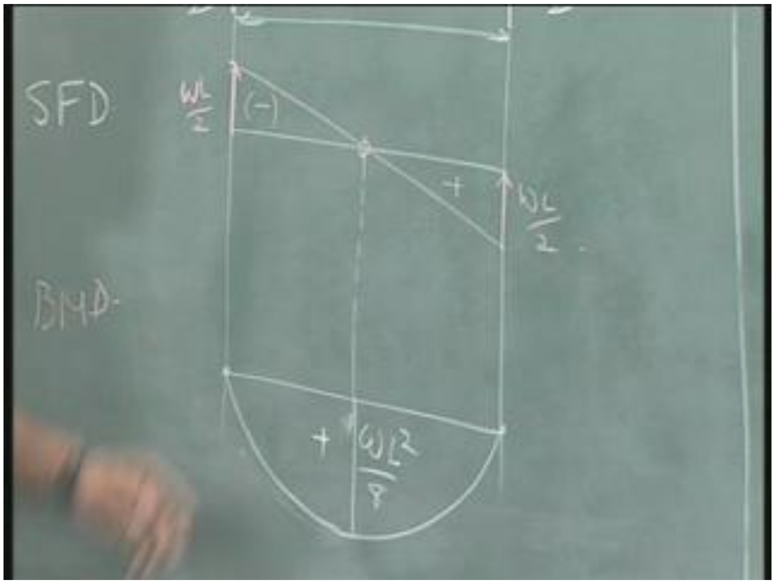

Can I find out the total resultant force on this? The answer is yes and if I replace this uniformly distributed load with that, it is nothing but the total weight acting at this particular point which is equal to m times g where m is the mass of the entire body. I will use capital M.

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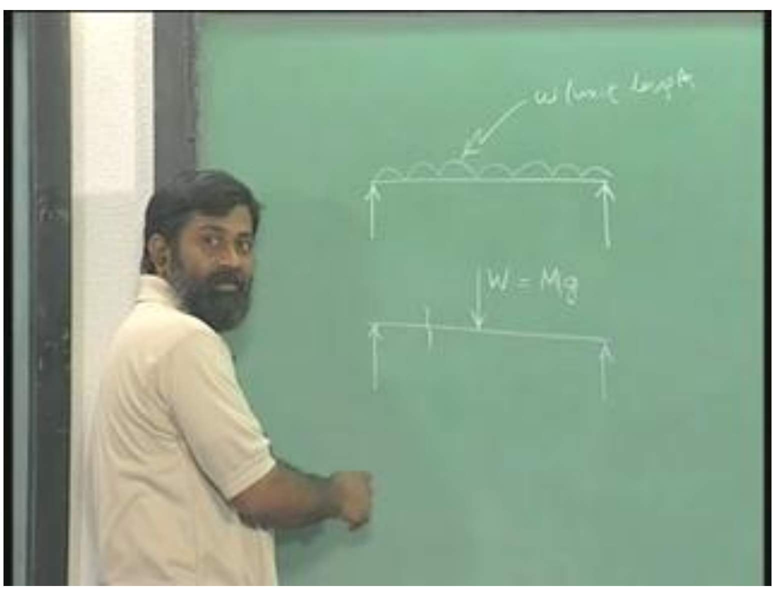

The question is can I use this diagram in order to draw bending moment and shear force? The answer is natural. The answer is no because if I cut anywhere here, what I have is a constant shear force and linear bending moment which is not the story over here. This is a common mistake done by many, so I just want to point this out.