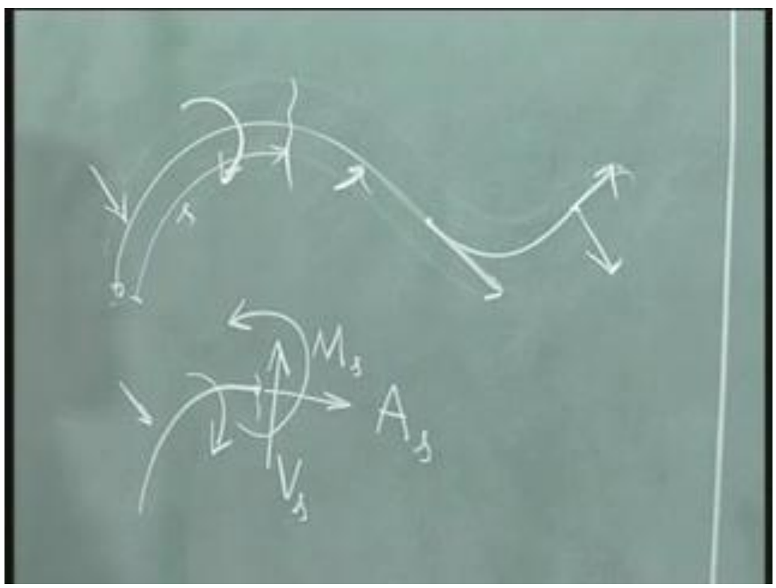
Engineering Mechanics
Prof. Siva Kumar
Department of Civil Engineering
Indian Institute of Technology, Madras
Statics – 2.8

In one dimensional structural systems, we looked at axial members. We also looked at a structural system that is made out of axial members. It's just a recap. If I have a one dimensional member, it is possible to draw it as a line diagram. Actually the member could be something like this and I just need to draw as a line diagram because it is treated as a one dimensional member. This may be subjected to different forces and moments something like this. Now we also looked at what type of forces will be revealed when we section this particular member right. Again as a recap, if I have to section at a particular point lets say it is s away from one of the ends. Then there will be a force that is along the axis of the member. So we will call it as A along the axis. One that is perpendicular to the axis, we call it as shear or V and it will also resist bending which is essentially a resistance offered through a moment reaction.

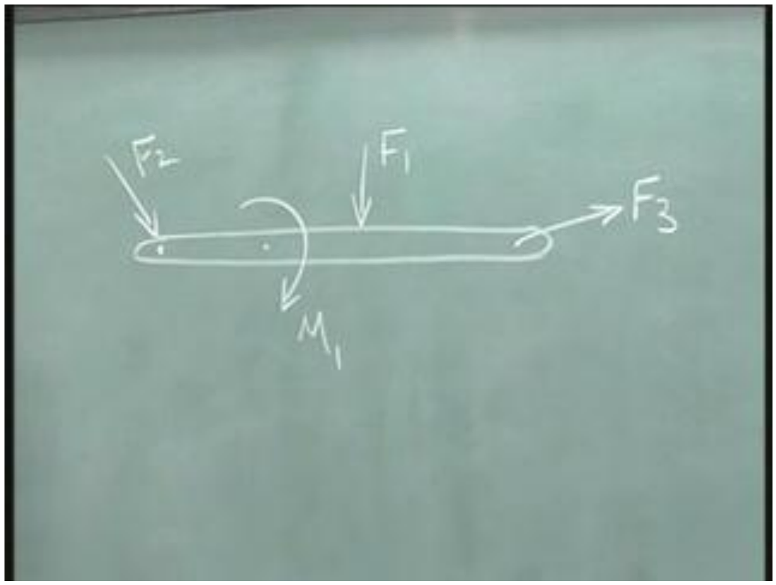
(Refer Slide Time 02:05)



If these are acting at a particular point, we will denote with that particular assignment s. s could probably vary from s equal to 0 to s equal to 1 as an example so which means I can cut at any particular section s away and then reveal these, depending on the forces that are there I can find out these particular internal forces. M_s, A_s and V_s are internal force resultants in the one dimensional member. In one of the cases that we studied, we looked at what is an axial member where only axial force is present and the other two were absent and we used that particular member in a truss system. Let's progress a little bit further and look at another system built out of a specialization of this particular one dimensional member.

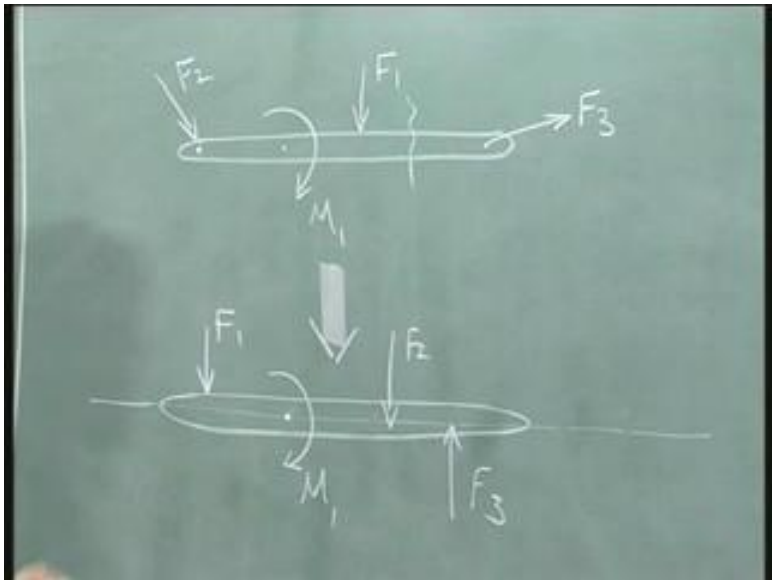
We are going to start with straight members unlike the axial members, this particular member could be subjected to moments and forces in between, apart from forces at the ends.

(Refer Slide Time 03:49)



Now in this particular case if I cut any particular section, it will reveal all the three of them. As you can see here if I cut here, there is a horizontal force which will result in an axial force. There will be a moment because of the moments of these forces and there will also be a shear because there are forces in the vertical. In order to specialize this, let’s look at a particular case of straight members where there are forces only that are transverse to the axis of the member. What is the axis of the member? This is the axis of the member and the forces are either transverse to the members or there could be a moment acting on it. These are the only possibilities that could be there for example F_2. We have to avoid applying forces that are not perpendicular to the axis. In such a particular member, we can easily find that there is no axial force acting on it.

(Refer Slide Time 05:17)

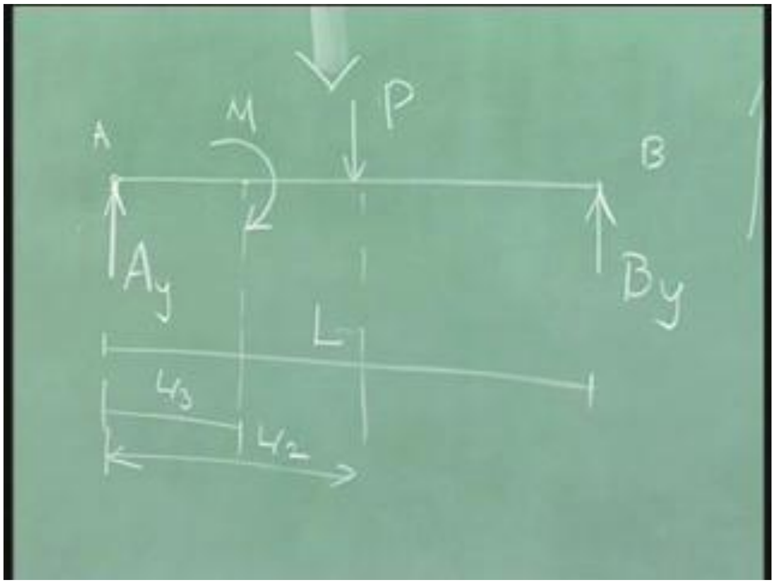


If I cut any particular section and find out the axial force, axial force is equal to zero. Such a member is called a beam member. We will look at this particular specialization and similar to what we did earlier find out the internal forces in these members. Like in the previous case, we can use a line diagram to draw this member which is a straight member A B. How many supports are necessary for this? There are no horizontal forces so I don't have to bother about it. There are vertical forces and moments, let us assume that this body A B is a rigid body. That's a good approximation to start with.

In general it can have one hinge and the roller support so that I have one unknown here, another unknown here and there are two force resultant equilibriums that I will have in order to solve for the problem. One is sigma F_y is equal to 0, if this direction is y and sigma M is equal to 0. M could be at a particular point that I choose. Remember any particular point I take, sigma M_0 is equal to 0. It is possible to get two equations and therefore if there are supports that have two unknowns, it is possible to completely solve for internal forces in this kind of members. Proceeding further let's just remove these supports.

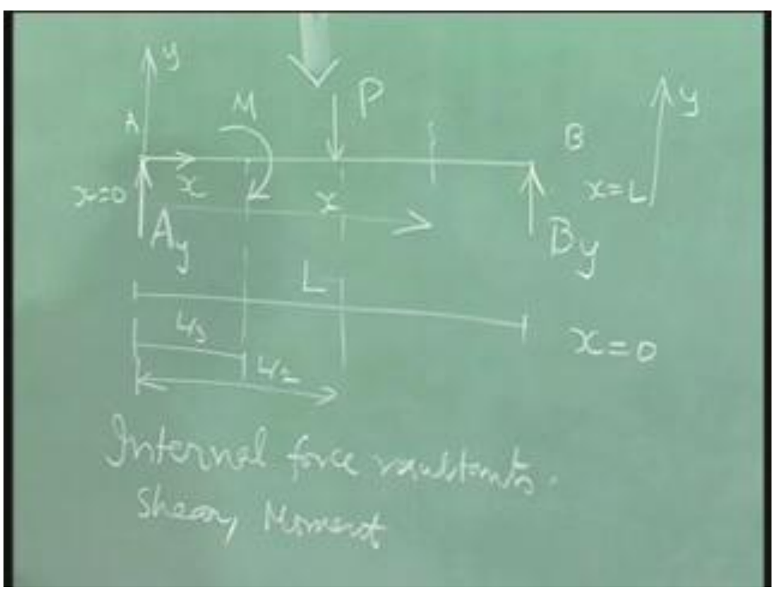
Since we already know that the reactions from these supports will be vertical, we will just draw something like this. This reaction is A_y and this reaction is B_y. Let's say there is a moment acting here M, for an example lets take a beam member of length L supported like this. If you have a support which is a hinged and a roller, we usually call that as a simply supported beam. Let's say I have one third away from A, there is a moment acting and let's say at the centre there is a force acting. Just to make sure you don't get confused, let me just use a notation P.

(Refer Slide Time 08:54)



What do I want to find out? I want to find out internal forces where I have to choose a particular point from which I consider. In this particular case let me consider A as the point from which I look at internal forces. I am going to take the coordinate system from left. What do I have here? We have a beam which is from x equal to 0 to x equal to L and if I have to find out the internal force resultants, there are two types of force resultants that we can find. One is the shear and the other is the moment. Let's look at an example let us say I am going to take a section x away. Let's say at this particular zone.

(Refer Slide Time 10:16)

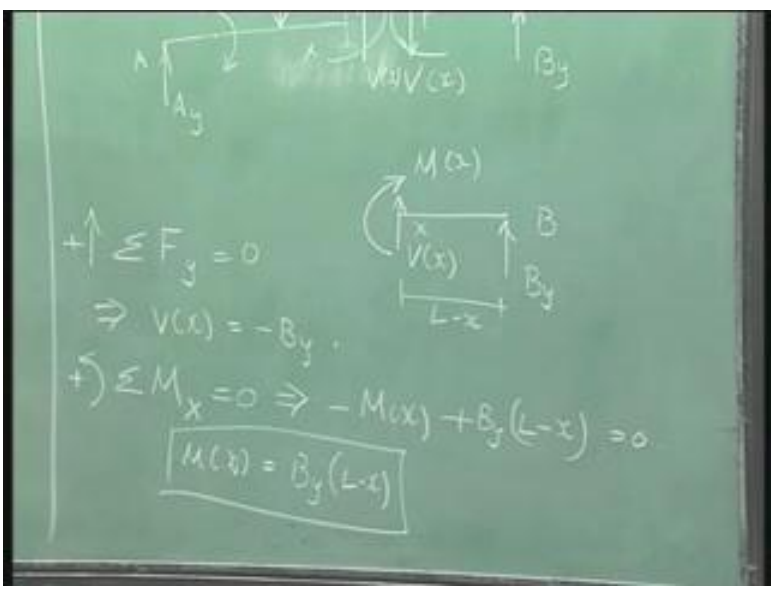


I want to find out the shear and moment at this particular point how do I find out? Again whenever I need to find out the internal forces, what should I do? I need to section it. There are two ways I can section, one is like this. The other is the left side of it. Let's call this as x, so I have a moment over here, the external forces are acting. Now this will reveal a shear force and this will also reveal a moment. Now equal and opposite forces I have to draw over here and what I need to find out are these V and M. To make it very clear, we will write it as V at x and M at x.

Is this a free body? The answer is yes. Is this a free body? The answer is yes. This case is very simple. If I need to find out these force resultants, one is like this and the other is like this. How will I find out the shear force V (x). That's pretty simple, I just have to take vertical equilibrium. Let us say I have found out the reactions and this is A_y and B_y to the fixed support. If I take sigma F along the y direction, let's say upward is positive equals zero or static equilibrium, immediately this will give me or this particular body V of x is equal to minus B_y. How do I find out this moment?

Now I can either take point B or point x. If you take point x, I can do away with this particular shear force. If I do that sigma M at x, total x is equal to 0. Let's say this is counter clockwise sense is positive. If this is equal to 0 this will imply, I have a moment which is clockwise sense which means minus M of x and I have B_y which is in the counter clockwise sense which means plus B_y. This length is if it is x away from here, this length is L minus x. So this is L minus x and therefore M of x is equal to B_y times L minus x. Since this is equal to zero we get this.

(Refer Slide Time 14:00)

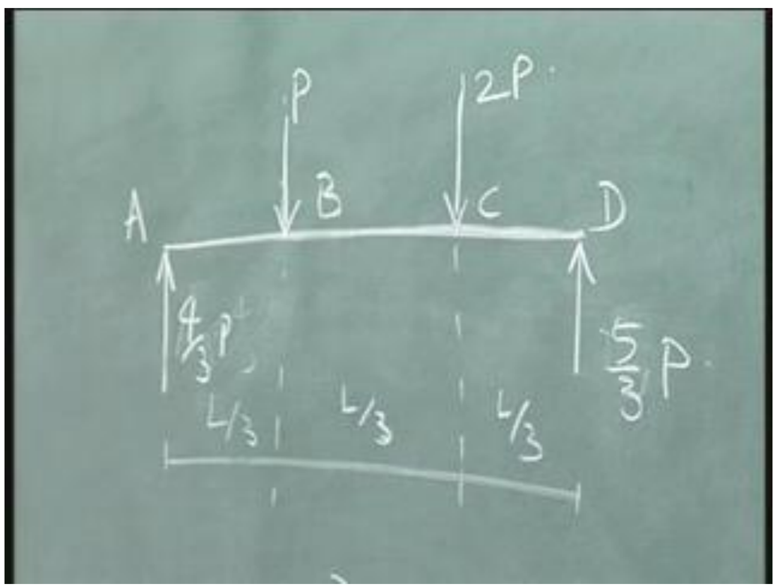


Is this clear? Now please remember this M of x and this M of x are exactly the same. This V of x and this V of x are one and the same. One of the confusions that will happen is which direction should I take? Vertically this way or this way? We will come to it when we talk about sign conventions for internal forces. Is this clear? Now please remember if

I had taken x, if I had moved x in this direction, this x will change and therefore M of x will change. The expression for V of x seems to be independent of x which means in this zone away from this force, any point that I take the shear force is equal to minus B_y. This particular exercise of finding out V of x M of x, I have to do throughout for x equal to 0 to x equal to L. Now how do I represent such a variation? Let's say this is the problem that I want to solve. Let me just denote the points at which the forces are acting as B A, B C. Remember this is a continuous member unlike the truss. Force P is acting here, force 2 P is acting here. L by 3, L by 3, L by 3 away from each other. What's the first task? I need to find out what is A_y and what is D_y that's pretty simple.

In order to find out D_y, I will take moment about this. Let's just mentally find it out. We have a P times L by 3 and 2 P times 2 L by 3, so D_y times L which is in the anticlockwise direction is equal to these two particular forces, these two moments which is P times L by 3, due to this and 2 P times 2 L by 3 plus 2 P times 2 L by 3. That will give me D_y is equal to 2 P, 2 into 2 is 4, 4 by 3 plus 1 by 3 is 5 by 3. 5 by 3 P, this is D_y. Total is 3, I have a 5 by 3 here which means this is 4 by 3. So this is 4 by 3 P, so let me just replace it and write it as 4 by 3 P. This is 5 by 3 P.

(Refer Slide Time 17:06)

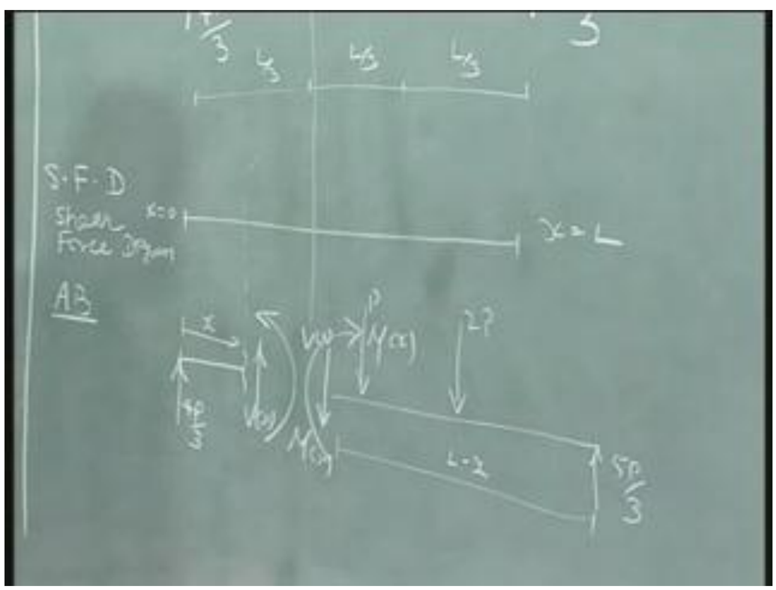


So having found out the reactions, now we wish to find out the internal forces of the members. What are the internal forces? The shear and the bending moment, at every point on this member. Are they the same? Am I going to have the shear force same all through. The answer is no. Am I going to have moment all through the same? No, may turn out be the same in certain cases but in most cases I need to find out at every x from A to D.

So let's see how we will proceed. Let me just draw this again. We have a 4 P by 3 and a 5 P by 3 like this. If I cut at any particular x from this, I will reveal transverse shear force and the bending moment. So first thing I have to do is to represent x axis. In this case only one axis is enough. I have just represented y, so that when I take the vertical equilibrium it makes sense. Now one thing that I know is the shear force cut at any particular section need not be the same and therefore I should probably have the variation of the shear force represented here. I am going to call this as shear force diagram that's going to show me from x equal to 0 to x equal to L. What is the variation of the shear force?

Let's look at this particular side. Let's focus on this region between this reaction 4 P by 3 and P. Let's draw that, so at a section x away from A, for the section A B I have a force acting like this which is 4 P by 3. There is no other force. Since I have cut like this, I will be revealing a shear force and a bending moment. Now the other side of it will be something like this. This is x away and therefore this is L minus x equal and opposite shear and equal and opposite moment at x. There will be the other forces that will appear on this. There will be the one force here P, another force 2 P, another force 5 P by 3.

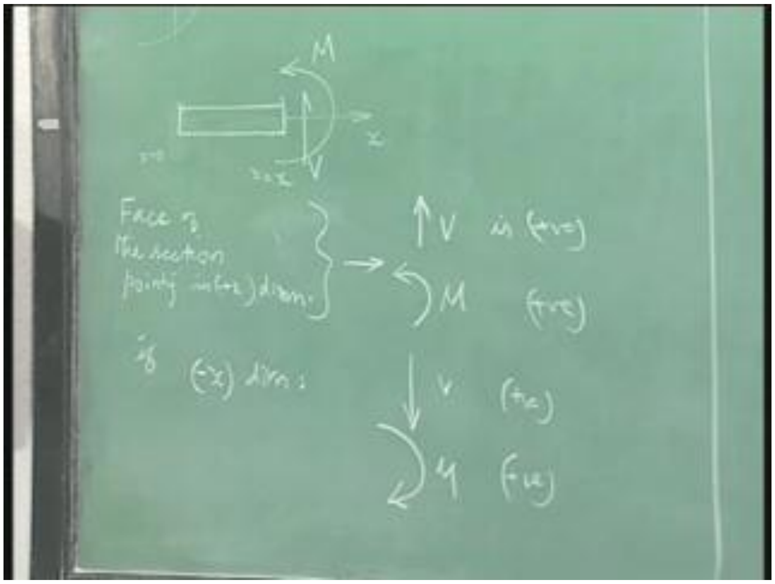
(Refer Slide Time 21:04)



Now the confusion arises as to; is the vertical force positive for shear force or vertically upward or vertically downward force positive? Which moment is positive? These are the doubts that will appear. Let me just spell that particular doubt now. What is my direction? This is my x direction. This is y counter clockwise is like this. If I draw a member starting from the left, this is x equal to 0 to x equal to x and if I reveal the shear and if that shear is vertically upward, I will call that as positive. Similarly for this particular section, let me draw it a little better to give you an understanding. Let's say this is the section that I have made. The face of the section is directed as normal x in the positive x direction. I am going to choose that particular face.

So face of the section pointing in positive x direction. For this vertically upward V is positive. As you can see this is a positive because the face has its normal directed along positive x direction. On this particular member if I have a counter clockwise moment M then that is also in the positive sense.

(Refer Slide Time 23:57)



But if I have a section in the negative direction, I will just take the opposite of this. This direction is positive and this clockwise moment is positive or in another words if it is upward, it is negative. If it is counter clockwise it is negative on a face whose direction is negative x direction, simple. If I follow this sign convention there is no problem, I can easily figure out which is positive and which is negative. This will be a confusion if I just leave it as it is. In this particular case this face has a positive x direction like this which means upward shear, this is a positive shear, this is a positive bending moment. If I get values which are negative that means it points the other way. Having done this, it is now simple to draw the variation of shear and bending moment.

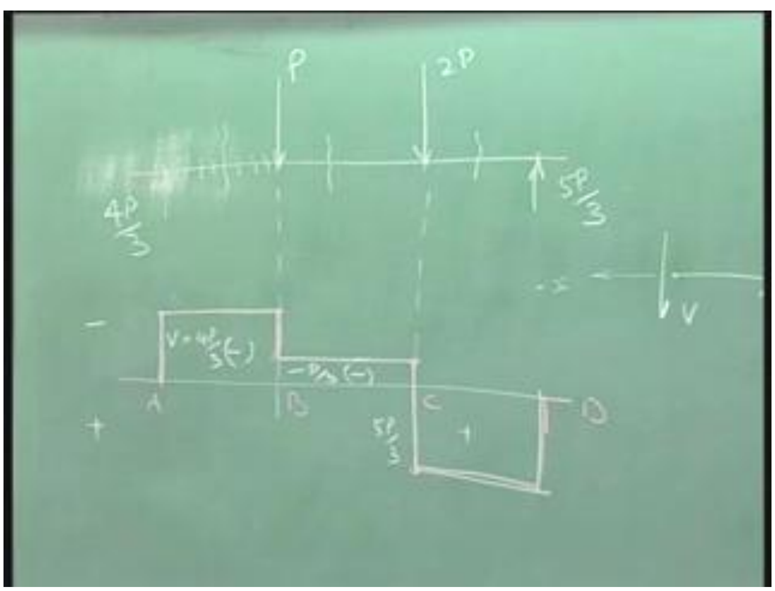
Let’s look at the problem. We have like this, if I cut like this and separate this particular section I know, if I take section here or here or here or here or here or here there is only one force acting and the shear force that I will get will be the same all through. Let’s find out what will be that shear force. We know for this particular section along the x direction positive, vertically upward is a positive shear. I have a 4 P by 3 here which means this has to be negative 4 P by 3 and therefore at this particular point, I will just represent it. For now I am going to take top portion as negative and bottom portion as positive. I will tell you in a moment why I am doing this. If I do it this way starting from this particular point, the value of the shear is 4 P by 3, till I reach this particular point. Let’s examine what will happen here.

If I cut any section here, there is a 4 P by 3 acting upward and a P acting downward. The net force is P by 3 acting upward. The shear has to act in the downward sense in order to counter it, so it is still negative minus P by 3. When I come to this section, I can probably look at the other side of the section. Now what is the positive sense of shear here? This is negative x direction and therefore downward is a positive notion. So is this positive? The answer is yes, V will be equal to 5 P by 3 and therefore I will have to take it as this is negative, this is negative and this is 5 P by 3 positive all the way through.

Now I have represented the variation of the shear force. I am just joining these. These are points of discontinuity because forces are acting at those points. The value of the shear goes from minus 4 P by 3 to minus P by 3 here, from minus P by 3 it goes to 5 P by 3. If I go a little further away the shear force is equal to 0. This is just to make sure that I represent 0 here, 0 here, I can extend a little bit further. This is A, B, C, D. is this clear? Look at how I have done it this way. It is possible to take the left side also in which case I should have taken all these three forces into account.

A simpler way is to take the section to the right, only one force is acting on it but I have to take the proper sign convention. Since the face is having a normal which is in the negative x direction, downward now is positive sense for you and therefore we get it like this. One might want to check whether this result is correct. There is a simple check that I can give you, that will be very useful in checking whether this is right or not. The first check is between this point and this point, there is no other force acting which means it will be horizontal variation from here to here or in another words, the shear force is constant from A to B, similarly from B to C, similarly from C to D.

(Refer Slide Time 29:49)

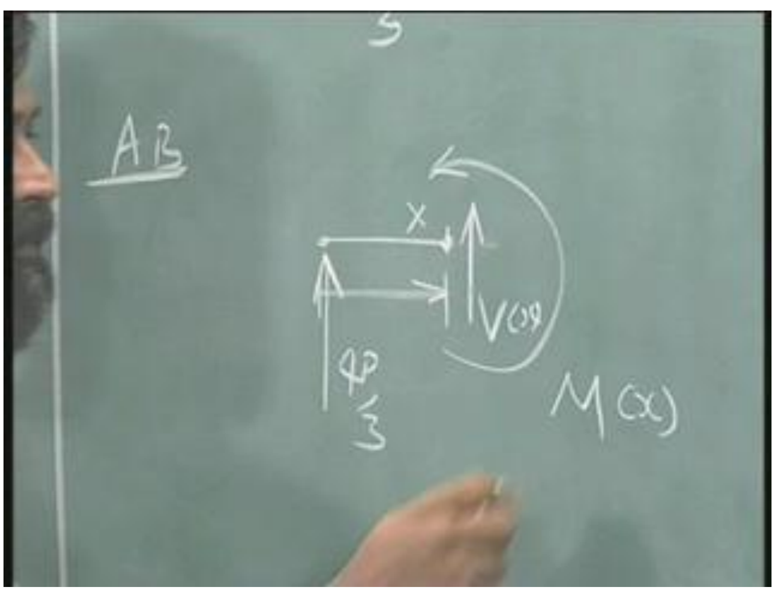


That is one check. The other check is if you look at this particular point to the left and right, the difference between the shears will be 4 P by 3. Therefore if you look at this particular point, push it up by 4 P by 3, you will reach this particular point. There is no force and therefore it is constant here. There is a downward P so I will put a downward P here. I will reach minus P by 3. There is no force in between, I can go straight. At this particular point I have a 2 P acting, I already have minus P by 3. I just have to add 6 P by 3 so that I get 5 P by 3 as a positive. I further go down to this, there is an upward force acting which is equal to 5 P by 3, this is downward and therefore I reach this particular point. This is a nice check where I start from the left to the right to close the gap completely. This is a nice check that you can make especially for such a problem. Thank you.

We drew shear force diagram. Just a recap of it. The simplest method that I talked to you about negative is on top, positive on the bottom. Drawing this way has an advantage, we start from the left most side, this is 4 P by 3 so I take it over by 4 P by 3. There is no other force over here, so it's a constant. at this there is a P downward, so it reduces this 4 P by 3 to P by 3 so that this is equal to P and this is a constant till we reach this force 2 P. 2 P acts at this particular point so this is negative. When it acts downward, it pulls this curve downward by 2 P so that we get 5 P by 3 over here and this total is 2 P, this is 5 P by 3.

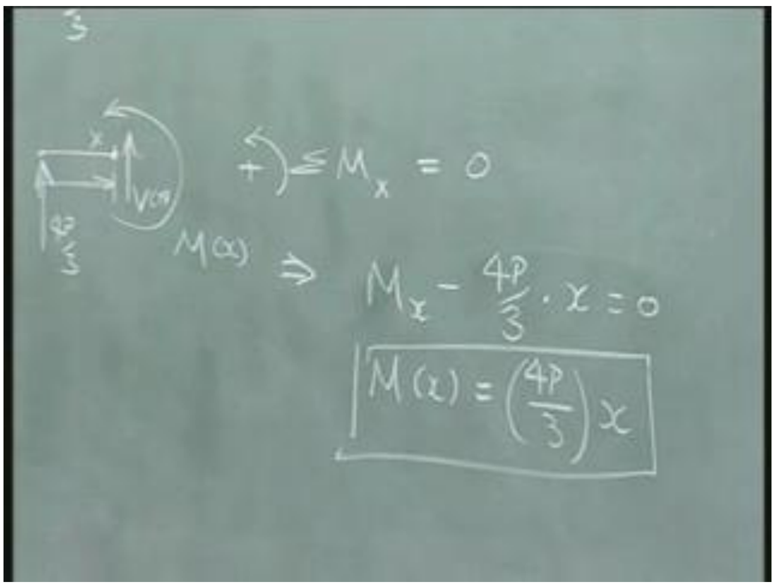
So nice direction will give you a better view of this. This is a 4 P by 3 acting here, P acting here, 2 P acting here and 5 P by 3 acting here. The simplest way that you can draw the shear force diagram. The other diagram that you need to draw a little inclined diagram here. In a similar way we need to find out the other internal force. The first internal force resultant that we found out was shear force. The other is bending moment therefore we call this as bending moment diagram. It basically has to convey the variation of the internal force resultant which is the bending moment over the length of the beam.

(Refer Slide Time 35:17)



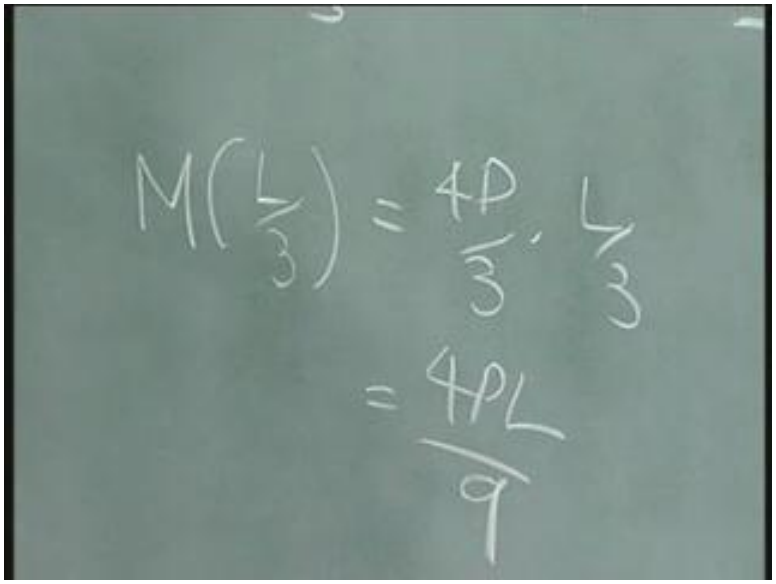
Now like what we did here, we will now find out the bending moment. Let's take the zone A B, lets take a distance x away from the left most side because this is the reference point. We have a force acting on this equal to 4 P by 3. Since we are cutting in between, only internal forces will be revealed. One is the shear force as you already know, as you have already calculated. The other is the bending moment. Look at how I have drawn. This is a positive sense of shear force and the positive sense of bending moment at x. Now can I calculate M of x without having to calculate V of x. The answer is yes. I just need to find out the bending moment about this point, let's call that as x. Since it is in static equilibrium it is equal to 0. This implies I take this particular point, this moment is always in the positive sense. The other force 4 P by 3 gives an anti clockwise sense. We have M x minus 4 P by 3 times the distance is x here is equal to 0 which means M of x is equal to 4 P by 3 x.

(Refer Slide Time 36:08)



Is it a positive bending moment? The answer is yes because P is positive, x is positive, 4 P by 3 x is positive. Therefore this is a line on this distribution which has a slope of 4 P by 3. We have an L by 3 length over which this is applicable, so at this particular point x is equal to L by 3 and therefore M at L by 3 is equal to 4 P by 3 times L by 3 and that will be 4 P L by 9.

(Refer Slide Time 36:50)

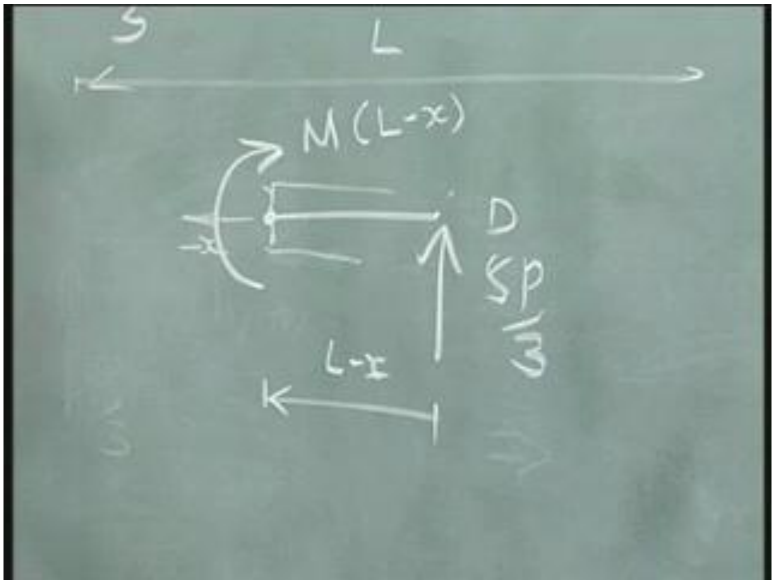


That’s the value you will have over here. Is it positive? The answer is yes. Sometimes the positive is brought downward. Let me use that particular convention. I have a positive over here, the value here is 4 P L by 9. I know it is a straight line or linear distribution and therefore this is the variation till L by 3. Now if I have to take, let me just erase this and go on to the other portion which is B C. I can do a similar thing, I can cut here, take the free body and draw. Since there is no force here, one thing that we found out is the variation of the bending moment is linear because only this force is active in it.

A wiser thing to do in this particular problem is to take the right hand side portion which is C D. Let’s do that and you will see for yourself that drawing this bending moment diagram becomes easy. Let’s take a section like this which is the best section to take this side or that side. The answer is of course this side. So let’s take that. If I take from the right hand side, this total length is L and therefore this with reference to this is L minus x. This distance is L minus x. Why do I need this? Because I need to find out the bending moment.

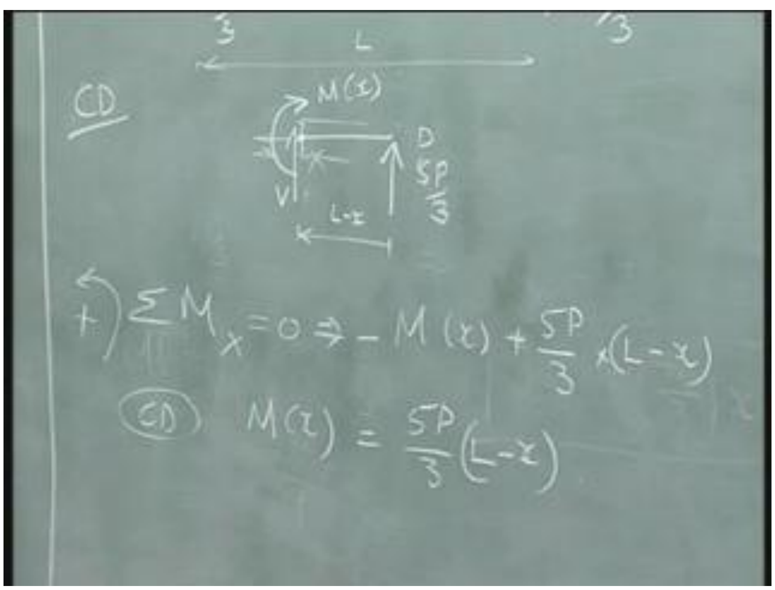
Now let’s draw the positive sense of the bending moment. This is the direction of the face that I have. For example if I have something like this which is the beam, the face has a negative direction minus x as the normal to that face which means which is positive according to the sign convention, it is the clockwise direction that is positive and therefore let me impose a positive M in this particular case at L minus x apart from the shear force.

(Refer Slide Time 39:46)



Therefore I will get M, taking ~~bending moment about this particular moment~~ total moment about this particular point, let's call this as x again. Anti-clockwise being positive is equal to zero, for this particular free body diagram. This implies I take this particular point, this is negative so minus M of x is equal to remember this is x, so I can write this as M of x. This may be a confusion, so let me not write it like this. Let me write it as M of x, so minus M of x. The other one is in anti-clockwise direction moment, so it is plus 5 P by 3 times, the length here is L minus x. This directly gives me M at x in the portion C D is equal to 5 P by 3 L minus x, simple.

(Refer Slide Time: 41:18)

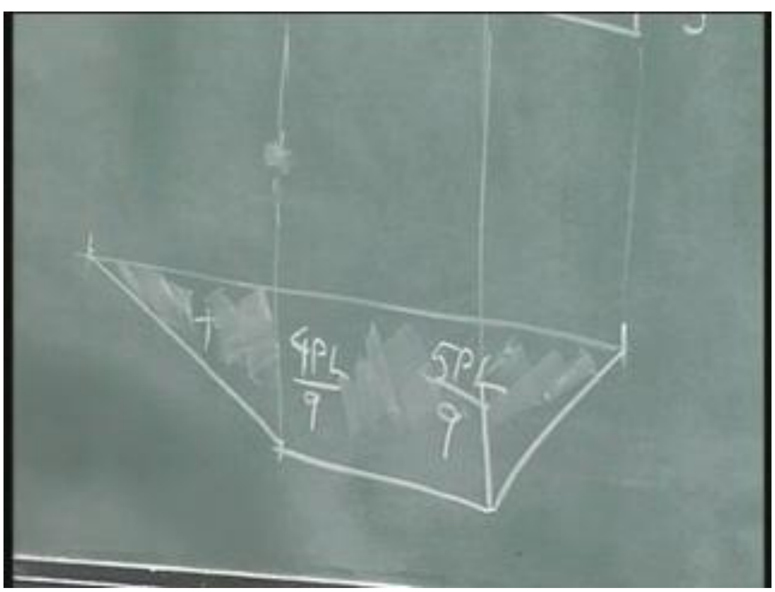


What should be the bending moment at this particular point? This particular point and this particular point are hinged supports, naturally I should get the bending moment to be zero. If I substitute x equals L, this is equal to 0. How do I find out the bending moment at this particular point? This particular point C is 2 L by 3 away from left hand side or in another words I just have to substitute x equals 2 L by 3. So M at C is equal to 5 P by 3 times L minus 2 L by 3 and that is equal to L by 3 which is 5 P L by 9. Is it positive sense or negative sense? Simple, it is actually positive sense and therefore this value is 5 P L by 9.

Is this a linear equation? The answer is yes, it is linear in x and that's what we find here. We have essentially moved from this point to this point. Now how about the zone in between? Is this also going to have a linear variation of moment? Let's see, supposing I cut a section over here. I should take moment of this and moment of this. Moment of this is this force times x which is linear in x. Moment of this is P times x minus L by 3, again it is linear in x and therefore the moment in between will also be linear in x. Or if I ask you the question, if I found out within the zone B C the bending moment, what will be the bending moment here? It will naturally be exactly the same as what we found out for the portion A B at B.

Similarly for portion C D at C, what ever is the value that we found out to the right of C will be the same as left of C and that automatically gives an idea that I just have to join these two ones. Therefore if you notice, I didn't have to directly find out within this particular sub and yet could draw the bending moment diagram. What's the use of this bending moment diagram? It tells me what this particular beam is undergoing under that particular load. Just to give you an idea, I am just using a ruler here. It is simply supported at the ends. There is a one force at L by 3 and another force is 2 L by 3, 2 P and P are acting here. Naturally it will start to sag like this. Now this sagging is due to the moment that appears on the beam.

(Refer Slide Time 44:45)



If I keep increasing this value of the force P, this value if I keep on increasing, this will take more and more and more moment. There will be a particular point at which it will break. Supposing I know what will be the bending moment for which this particular ruler will break. I can assess from this where that would occur. For example in this particular problem, I know I have to be careful about this particular point. If I strengthen this and it might probably take more load. So from design perspective all these are useful.