Engineering Mechanics
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Statics – 2.4

Let's go back to this line diagram. This is a nice line diagram, I can use this for analysis. Next thing I have to do is to do analysis of this structure. Let's just for now have a force vertically acting here P. Before analysis I should tell you what are all the dimensions. Let us finish that. Whenever we give examples, we give nice simple examples and ask you for difficult problems, but we want to take representative examples. So here without loss of that representation, let's say I have equal lengths of BE and AB and let's say the height is such that this length is also L. What will this be? The height is root of 3 by 2 L. Find the required reactions from the supports? Where are the supports here? This is one support, this is another support. Support meaning supports this structure to a particular fixed frame.

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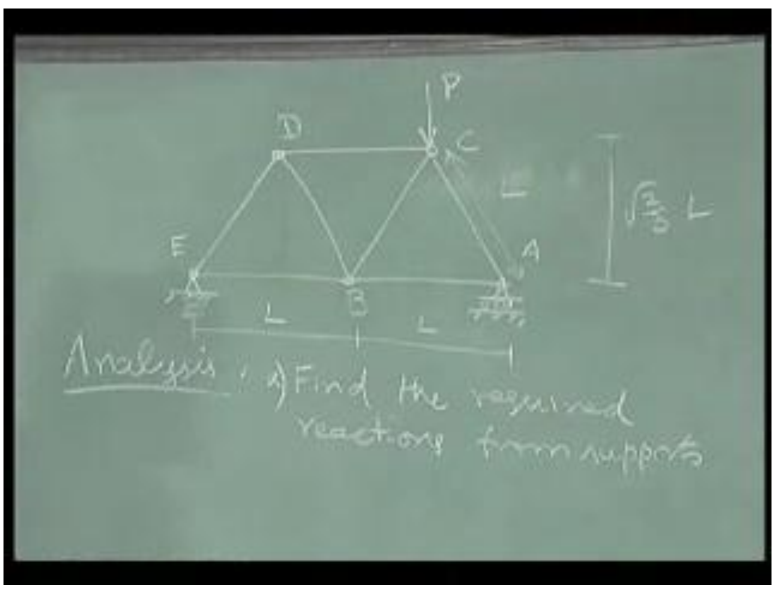

Find the required reactions from the supports, find the internal forces in members. It could be in all members so here I could have it as all or a set or a single member. Depending on my design requirement of having to build this, I would ask these questions and given these answers, I will be able to go back and say the structure is well built. It can take the load P that is acting on it. For example, if this is a piece of chalk and I wish to know whether this piece of chalk will hold a tension.

Now you notice I am going to increase, increase, increase, increase, increase, increase, increase, increase and then at a particular point of increased force, it breaks. If I wish to find out the force that is acting on each of these members is such that they are well within the breaking load then it is all right to have such a structure and safe at it. If I am going to use this as a piece of chalk, please remember I am applying a load at the center like this. Therefore, there will be a moment also coming to the picture. We will come to that particular type of structure at a later stage. If this is clear we will move on. For now let's assume that we have to find out all the reactions and all the member forces. How do I go about doing it?

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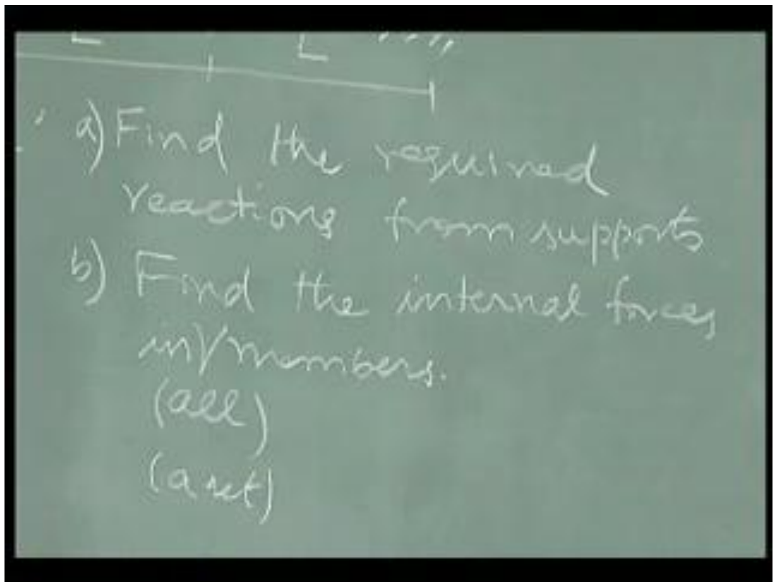

If I have to find out the reactions from the supports, one of the important things that I have to know is I have to remove these supports and draw a free body that is devoid of the supports. Let's do that exercise as a first step. Let me redraw now. I have removed at E the support, I have removed the support at A. I am not bothered about the others so I am just going to just take it as a particular body. At point E or at joint E, if you notice carefully this is a pin joint or hinged support which will introduce two reactions E along y direction and E along x direction. At A going back to this, we have a roller support which means the reaction from the fixed frame will be only in the vertical direction, I will have A_y. This is something that I have done when we dealt with simple rigid bodies that's basically what I have done here.

Now I will insert the external force on this. This is the external force. I don't bother about anything else inside. There are no other forces acting and this is a free body. I can always apply the equations of equilibrium. In this particular case it is stationary or it is a static equilibrium. I can employ the three equations sigma, let's say this is the x direction, this is y direction. I can employ these three equations. This moment can be found out with respect to any of them. Let me just take it as moment about E. These are the three equations or equivalent equations that I need to use. I will get three sets of equations. How many unknowns we have found out? This is P, this is a force that is applied. These are the three unknowns A_y, E_y and E_x to be found out. Three equations I can as well solve for it. Is it clear? This is something I have already done.

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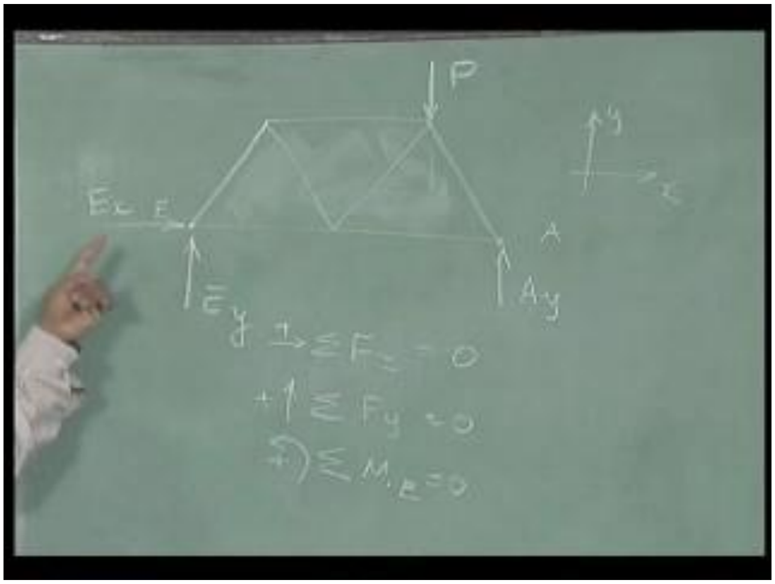

So let's just jump to the answers sigma F_x equals zero immediately tells us this is equal to zero. Instead of taking vertical equilibrium and moment equilibrium, I will take moment about this particular point which will involve A_y and P. This distance is L plus 1 by 2 which is 3 by 2 L and this distance is L and therefore if I take moment about this A_y gives a positive sense, P gives a negative sense. This implies A_y times 2L minus p times 3 by 2 L and this is equal to 0, I can cut off L. Net result will be A_y equal to P times 3 by 4 so 3 by 4 P. Since A_y plus E_y should be equal to P, E_y will automatically be 1 by 4 P.

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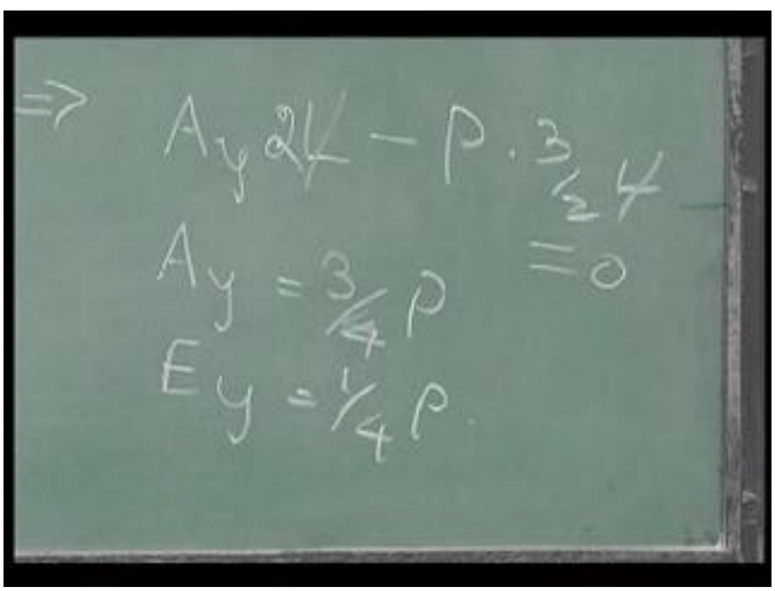

This is pretty clear. We have accomplished the first task as finding out what are the reactions. The next task that we have in hand is to find out the internal forces in let us say for now all members. As a first step in finding out internal forces, we know very well that we have to cut any one of those members for which we want to find out the internal force. Supposing I want to find out the internal force in BE, I need to cut that particular number. Let's seek as a first step to solve for internal forces. We need to cut and let's cut and see if we can find out how to solve for internal forces. If I cut a single member, please remember it is a part of a larger system. I cannot just draw free bodies and that's not going to give me anything that is going to be used. For example if I had cut this like this, let's say this is the cut.

It is not going to give me much of it. I will have equal and opposite forces acting on this like this. Since they are equal and opposite in this structure, I will not be able to solve for this internal force. This is an important thing to understand. If I can extract the free body out of this entire body, then it is possible to write down separate equations from which we can solve for internal forces. Given this, how do we go about picking out or removing certain free bodies from which we can get some results. If you look at it, these are the pins and if I draw a particular domain that engulfs each of these joints and then look at this particular zone alone. I can pick out each one of these as rigid bodies and draw the forces acting on that free body. Let's take E for example.

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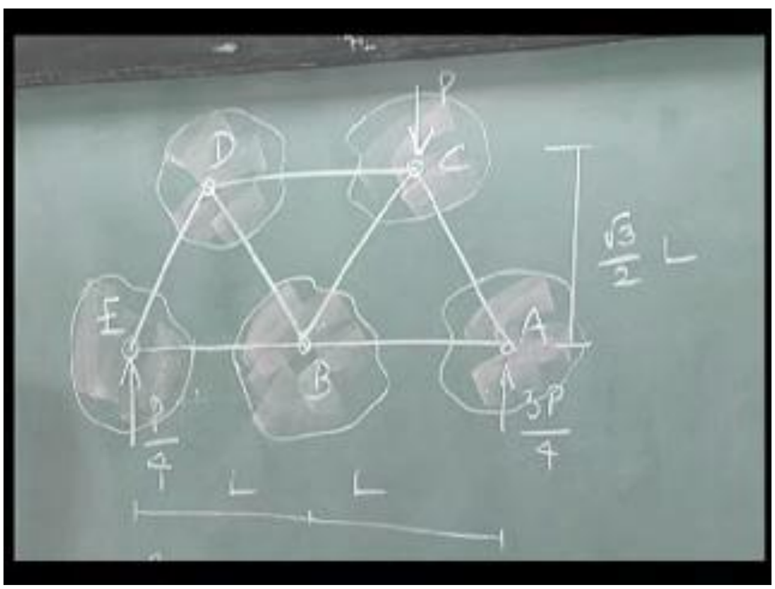

We have cut here this is E, we have B over here, we have D over here that these two members have been cut. There is an external force acting on it equal to p by 4. Since we have cut here, we will be exposing an internal force in each of these members that act along the axis of these members. I will tell you in a moment as to what direction force I have to insert over here. For now I will just put the internal forces to be like this. Since this is an internal force for the member BE, I am going to call this as F_{BE} or simply BE. Some people follow F_{BE}, some people follow just BE.

We are going to use just BE as the force and use that notation all through and therefore this is DE. There are two forces BE and DE which are yet to be found out that are acting on this particular free body. Is this clear? Apart from the external force that is already known, these are the two forces that act on it. If we examine this free body and ask the question how many equations can I write. Since these three forces are coincident at E, you can see that from E, BE is emanating, DE is emanating. The force at E, P by 4 is emerging which means these three forces are coincident forces and therefore I cannot write any moment equation separately. I will only have two equations for vertical and horizontal equilibrium.

If I take x as horizontal and y as vertical this way, I can now write the total force in y direction is equal to zero, the total force in x direction equal to zero. This lets say if this angle is theta which can be found out from geometry here. In this particular case we already know that this angle is equal to 60 degrees. Now which one would I choose, horizontal equilibrium and then vertical equilibrium or other way it depends on your convenience.

Supposing I take the horizontal equilibrium, what will I have? I will have BE acting along the positive direction. I am going to write this as positive and a component of DE along x direction which is again positive notion plus DE cos 60 degrees is equal to, there is no external force acting on it, is equal to 0. This is one equation that I can get. The other equation concerns vertical equilibrium. Upward is positive which means P by 4 plus DE sine 60 equals zero for static equilibrium. From these two equations, I can solve for BE and DE. Now which one will I use first to solve? Obviously BE and DE take part in this equation whereas in this equation only DE takes part and therefore I will solve this first and then go for this. That is one of the reasons why I started with sigma F_y equal to 0 and then said sigma F_x equal to 0.

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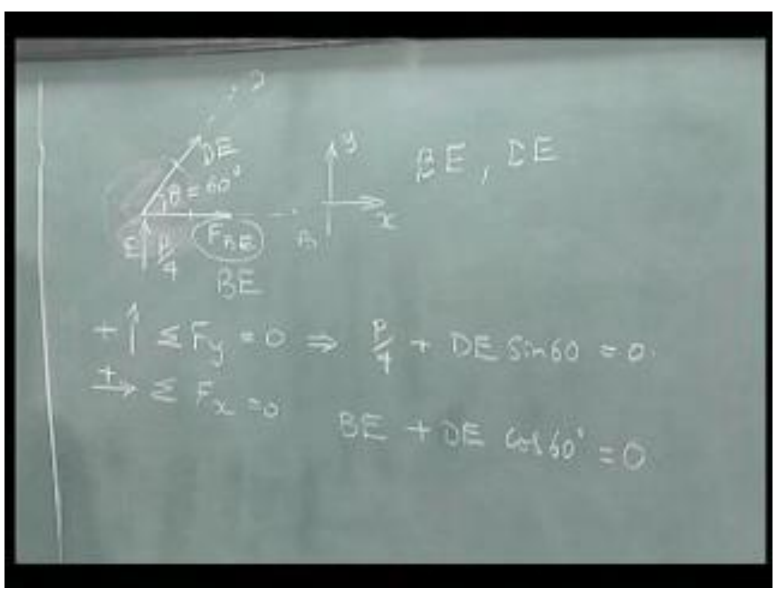

It’s a matter of choice as to which order you will go, so that you can solve in a simple way. If I had started from this, I will only get relationship between these two and then I have to use this equation in order to find out what each one is. Going back to this, we looked at this free body and from this free body it was possible to find out what is the force acting on this particular member BE and the member DE. Similarly I can do it for each one of these free bodies. Free bodies of each of the joints I am just going to write… What I have essentially done is I have engulfed so that I have each of these joints separated. I can have free bodies associated with each of the joints. So I am just going to call those as free body of the joints are used to solve for member forces.

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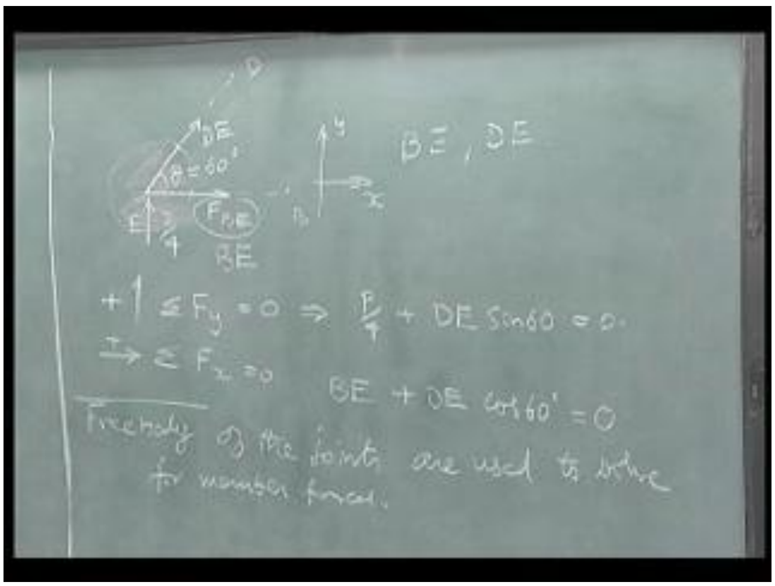

Now what’s the guarantee that this particular free body is a stable body in or in other words, it’s a stable and stationary in this particular set of forces that are acting here. We will postpone that and look at it separately in a clipping. Such a method where I used the free body of the joints or free body associated with joints in finding out the member forces, I am going to introduce here (Refer Slide Time: 18:23). Member internal forces or simply member forces is called method of joints. Now in this particular case, I can draw free body of A B C D and E so that I can solve for all the member forces. But I am not clear which one to start with. Shall I start with B, shall I start with D, C. That’s a question that will arise. What do you think, what criteria will you adopt in order to choose? This is one of the problems that many students will face. The best thing that you can do is examine.

If you look at this particular joint E, I have BE and DE as unknown member forces acting. Are there any other unknowns? No, and for every joint how many equations can I write? If I go back to this, I can write two equations for every joint and therefore if I am able to generate two equations for two unknowns, I can directly solve for those unknowns. Going back to this and therefore the first attempt that i will make is to find out, if there are joints with only two member forces. What are they? E, A are the two joints which have only two member forces. From these what will I find out? I will be able to find out BE, I will be able to find out DE, I will be able to find out AC and I will be able to find out AB.

Now if I ask the question, what next? Then I will go to any one of these and ask the question, how many are still unknown here. Since this is already known, there are two of them that are unknown here. Since this is already known, there are two of them that are unknown. Since these two are already known, these two are unknown and therefore they actually have equal importance with each other but if I take this particular joint, there are no external forces and therefore this will be a simpler one to do compared to C or B.

The next choice of taking equilibrium will be D and naturally the other one would be P or B. The choice may be P, if I am taking vertical equilibrium where P will directly come in. Here I have to take the components. Mind you if I do D, I would have already found out these two. The only one that is left is here and if I take C, I would have found out this. The equilibrium of B is a redundant one. That's how I will solve for all the member forces. Is this clear? Just to recap, I draw a line diagram of this particular truss member, find out the reaction forces so that I can draw a free body diagram of this truss.

Assuming that I already know that this entire structural system forms a rigid system. I can now start to examine and find out the internal forces by drawing the free body of each of these domains that encircle as single joint or in other words free bodies associated with each of these joints. After doing that I will pick the appropriate order of these joints for writing equilibrium equations to solve for the internal forces in these members. Mind you this is a simpler problem because over the entire length of the members, the axial force is the same and therefore it's a single value that I need to find out. This is a simpler structure that I can think of. In a more complex structure, I may need to find out the internal forces over the entire length of the member. That is a simplicity that this system, called the truss system offers. Thank you.
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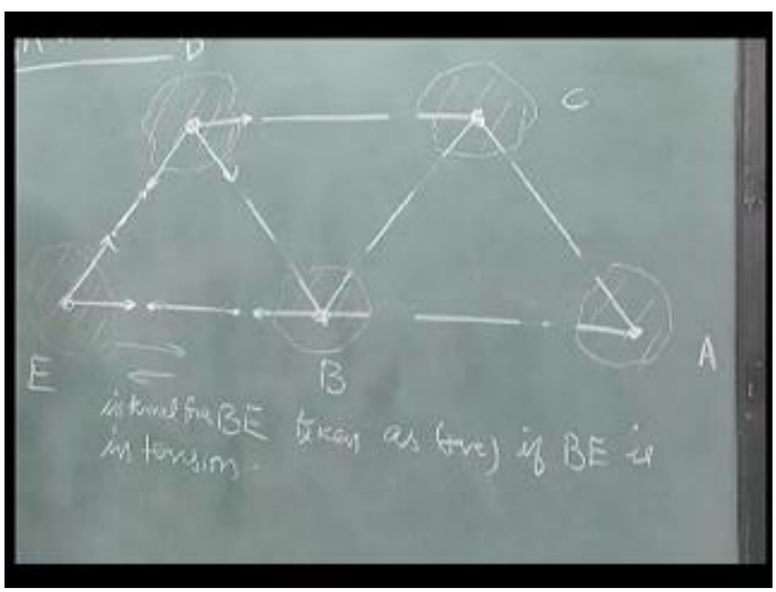

Now one of the most important concept that we have to understand here is about sign convention. I skipped it when I was solving for the problem. Let's go back to that and ask the question, how do I make a sense of positive, negative and that becomes a big confusion here. I will just tell you in a moment, how it becomes a confusion.

We started with this problem. I have just redrawn in a bigger way here. These are the free bodies that I have taken for just an understanding and to just indicate the forces over here. Assume we have cut like this. We have cut each of these members and therefore there is an exposure of forces. Let me just draw this small one here also. This will have equal and opposite forces. Now at this stage I will have a doubt which direction should I take it as positive. This direction or this direction? Is this positive or is this positive for this particular internal force? Now this force, this force, this force, this force, all these forces indicate internal force in this particular member and therefore this confusion is a very common confusion. In order to understand this, the first thing that you have to remember is the entire member BE is having a single internal force. Let's say BE in this particular case. This internal force BE is taken as positive if BE is in tension.

If you remember we started with this particular notion. I showed this particular member, asked my friend venketesh rao to pull it. When you pull it, it is in tension which means I have a positive internal force and when I push it, I have a negative internal force. Now examine these to get an idea of what kind of force I have drawn. Let me just encircle them to get you an idea. Let's take this member B E. I will just cut both sides, expose the force. Remember this is equal and opposite, this is equal and opposite. What kind of internal force is acting here? It is a tension force. How about here? This is member DE so this is DE, again I have indicated forces in such a way that this particular member is being pulled both sides.

This is also under tension. If you notice all the other members, I have drawn forces in such a way that each of these members are assumed to be in tension or in other words, this is a positive value, if they are in tension. I will have a positive value here, if this is what it is. In the result of the forces, supposing this value that I get is a negative value, it means that it is pushing this member into compression. What I am going to do is I am going to start with a value which is positive all through. So I will take the positive notion for each one of these. If I draw this, I know that this member is in tension. Equal and opposite is what you have here, so I will indicate it to make sure that you understand. Look at how the direction is here. The same force as a direction which is opposite are the other free body. Why? because equal and opposite have to be exposed.

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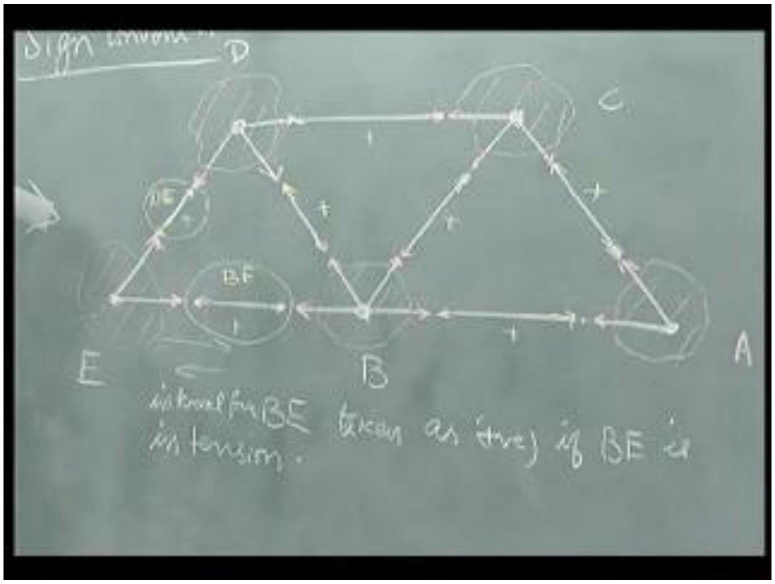

Similarly, in each one of these. So that I get an understanding of how to draw the positive values of forces for each of the free body. As you can see in this particular case, I don’t have to waste any time in order to draw this particular notion, having drawn each of these members separately as tensile forces on these members. To repeat, if I snap out a part of this particular member BE and assume that it is under tension, I will have these two forces. Since I have removed this part, the other part of it should have equal and opposite reaction and therefore these two conventions. If this is clear, I will tell you a simple way of drawing the correct positive directions of each of these internal forces. For that I am just going to erase these individual members that I have drawn. The question is how do I make sure that I draw forces that indicate positive values and if I get a negative value for this, it means that the member is in compression.

Let me just write down the member forces here. This is BE, this is DE and so on. Note that if I choose this particular joint E, each of these forces seems to be pulling this particular joint away from this particular joint. If this is the body, they are all away from the free body that I have drawn. If I take this, they are all emanating from this particular joint. Emanating like this, emanating like this, emanating like this, emanating like this, emanating like this.

Similarly, from point B emanating like this BC, emanating like this AB, emanating like this BE, and emanating like this BD. Very simple. When I draw these free bodies of every joint, I will take the directions which are emanating from that particular joint and all these are treated as positive internal forces. If I get a negative value for any one of these, it means that particular member that I have for example BE, if I get a negative value for BE it means that member is in compression, as simple as that. This is one of the most simplest one but yet people make lots of mistakes in inserting this particular sign. Thank you.

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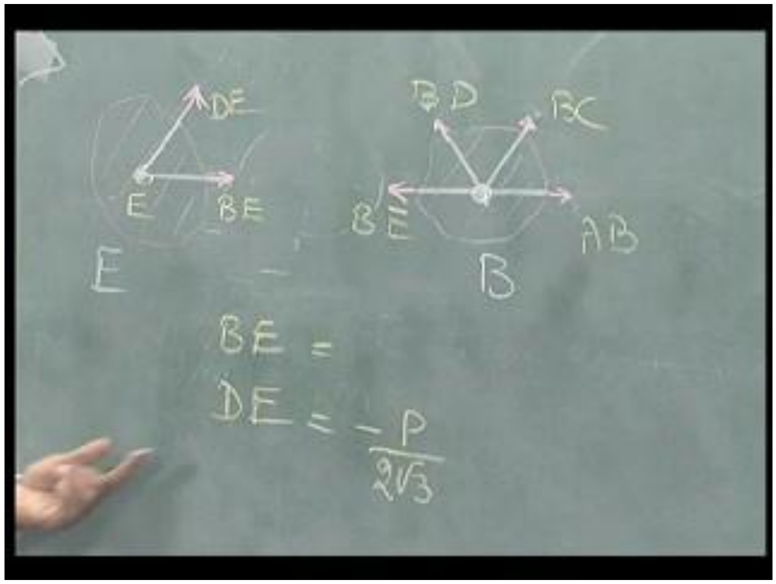

In this particular problem let's examine, I have a force P acting on this. We started with this particular joint E and found out what is BE and DE. BE happens to be… can you tell me what it is? DE is, we get minus P by 2 root 3. We happen to get from equilibrium at E in the vertical equilibrium, we get DE is equal to… Let me just insert support reaction here. We happen to get DE equals minus P by 2 root 3. What is this equal to? This is equal to minus, so what should happen is this direction happens to be like this. Mind you this direction also should be this way. This is an important thing to note. All you have to do is if I get a negative value, if I retain the negative value, I can have a direction like this and insert a negative or if I change the direction, it is equal to P by 2 root 3.

Other way of looking at it is I can retain the direction that I have which is like this and like this and insert the value minus P by 2 root 3, same here minus P by 2 root 3. One of the common mistakes done by the people is since this direction is minus, this should be plus. This is an internal force, if this I got as minus, this also should be equal to minus. When I solve at D, this particular direction is taken as it is and the magnitude is taken as minus P by 2 root 3. This is important to note. If this sign convention unless done properly, you will get into results that are not correct.