Engineering Mechanics
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Statics – 2.10

Now let's look at a general scenario. Let's say we have a beam which is extending so I am just going to cut off a small portion here and look at what forces exist within that. Since I am sectioning at this two points, we will have shear forces acting like this. I am just drawing the positive sense always. There is a moment acting here as shear. What is the positive sense? Here it is like this and this could be x_1 and this could be x_2. So M at x_1, V at x_1, M at x_2, V at x_2 and there could be some load acting in between so let me call that as W of x per unit length.

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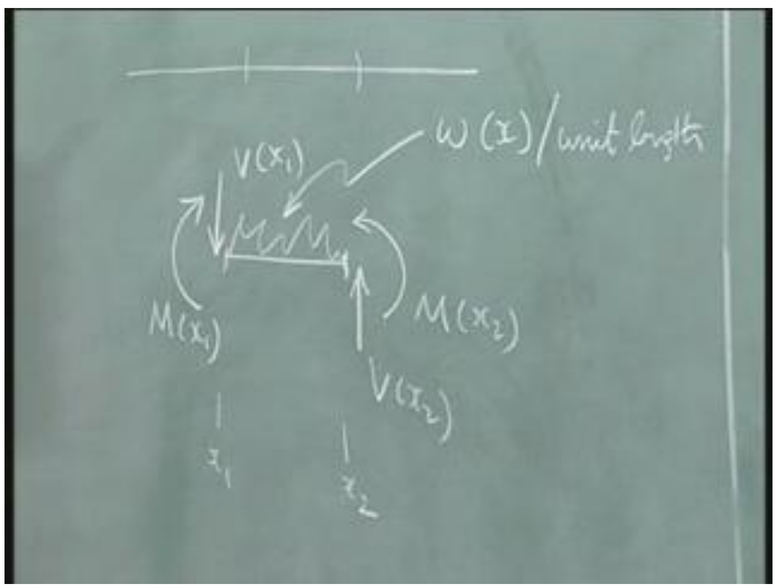

There could also be moments, for now let's assume that we have no concentrated or distributed moments directly acting on this beam. We need to find a relationship between all these quantities that we have M, V and W. How do we go about connecting these? We can use equations of equilibrium here in order to make a connection. Let's do that exercise a little bit more rigorously. I am going to take lets say this is the reference point. There are some forces acting lets say I am bothered about a section at x and I would like to know how the shear forces V of x and M of x is varying. Now one way to do that is I can section a very small segment at x, let's say dx and examine that particular small element. Let's do that exercise and see what we have. This is a length which is just dx in length, this is x away from one of those reference points. There will be a load acting on this, with no loss of generality within this small length dx, the variation can be neglected and we have W over here, W at x per unit length.

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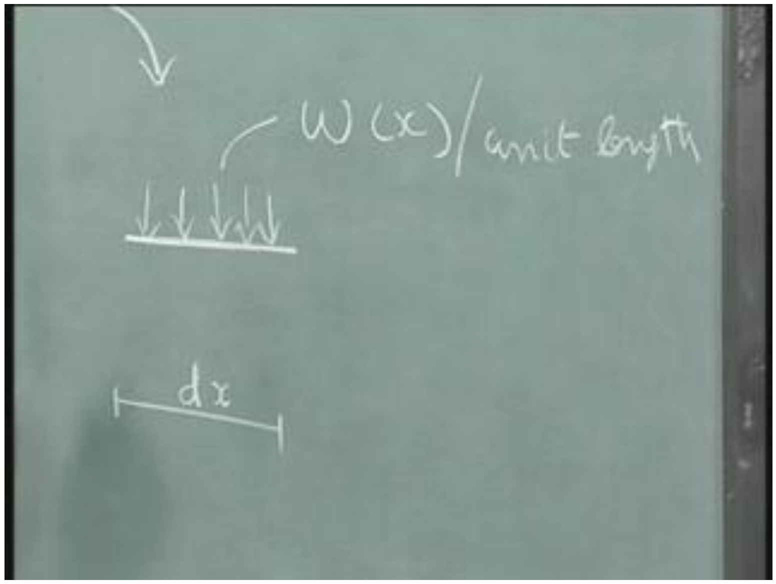

This is the external force that is acting transverse in direction. Now let's draw the bending moment, this as a negative sense like this and therefore we will have this is the positive sense of the shear force V at x and bending moment will be like this which is M at x. Since we have cut this side also, we have moment and shear force. This is M at x plus dx because I am taking dx away from this in the positive direction and this shear is V, at x plus dx.

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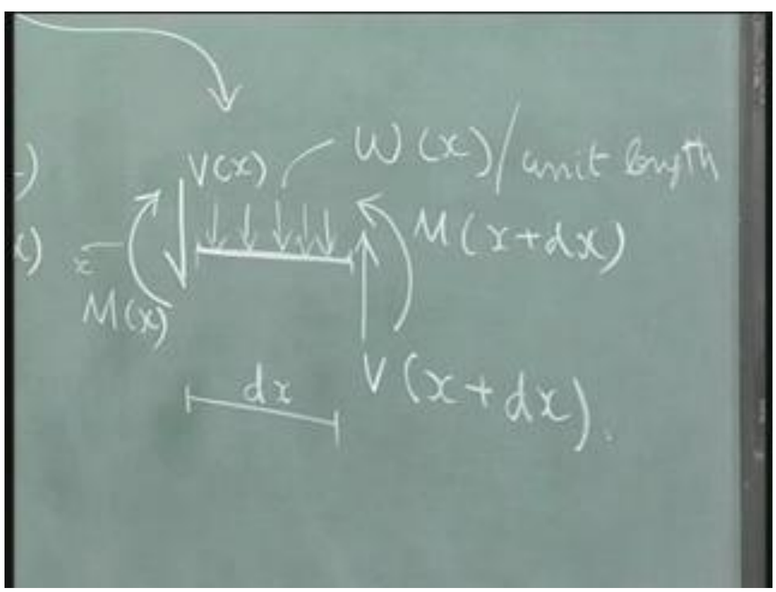

If V is varying from x to dx, this will show up as a different value compared to this. There is a small difference in the value and if I can connect these M of x, M of x plus dx, V of x plus dx to V (x) and see what we get, this may probably give a sense of the relationship between shear force and bending moment. Let's do that exercise. What all equations can we write from this? Definitely one equation that I can write is vertical equilibrium, sigma F along y, lets say upward like this is positive equals zero, if it is in static equilibrium. The other is bending moment about a particular point. I can choose a centre or one of the edges, draw and find out the equilibrium of it.

Let's say M at a particular point lets say in this particular case shall we choose this centre M at o equals zero. We have already learned that we can take any one of those points in order to write down these moments. Let's see what we get out of these. Let's first start with sigma F y equals 0. What all will take part in it? This shear force V (x), this shear force V of x plus dx and W. So let's write one by one. This is negative in direction so minus V (x). This is positive so plus V of x plus dx and there is a load over here, distributed over this length which is dx. So W times dx is the total force and it is acting downward so minus W times dx. The total should be equal to 0 for static equilibrium. This immediately reveals I am going to write this in a way that you already know is equal to W times dx. I am going to take dx down like this. Am I correct in the sign? Yes, I have taken W over to the other side. Remember this is W at x. What is this? Limit, let me just correct myself and write it as delta x here small length. I am going to have delta x here, delta x here. Limit as delta x tends to 0, what is this? It is nothing but dV by dx and that is equal to W of x.

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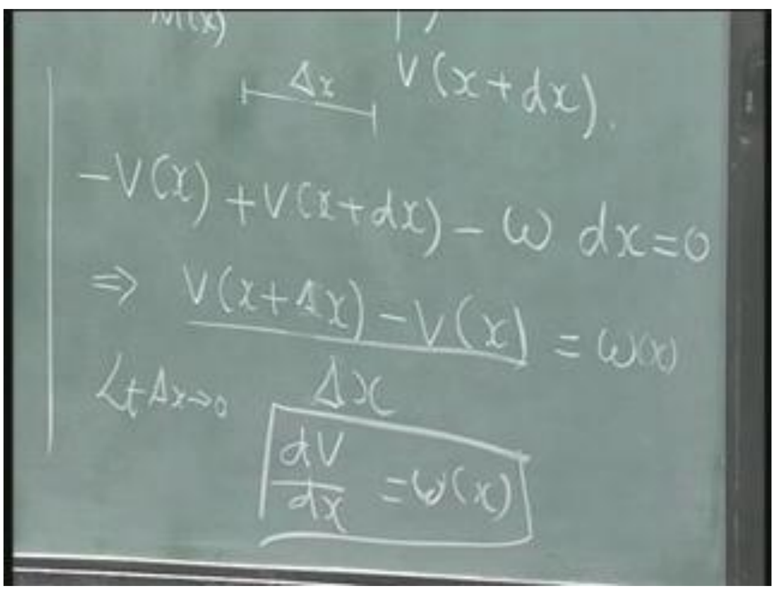

Therefore we have a relationship. It essentially tells me that the variation of shear force along x gives me W of x which is the transverse load applied. One result that we get is dV by dx equals W of x. We will get one more result from this. Let's do that separately. To be specific I will write it as W of x. Let me denote all the forces, this is V at x, this is M at x, M at x plus dx. Notice that I am using positive sense of all of them and this is V of x plus dx. I have represented all the forces and this length, let me again make this as delta x so that we are clear about it. Now we seek to find the moment equilibrium at a particular point. Which point is better to choose here? We will choose one of those points. Shall we choose a point here or a point here or a point here?

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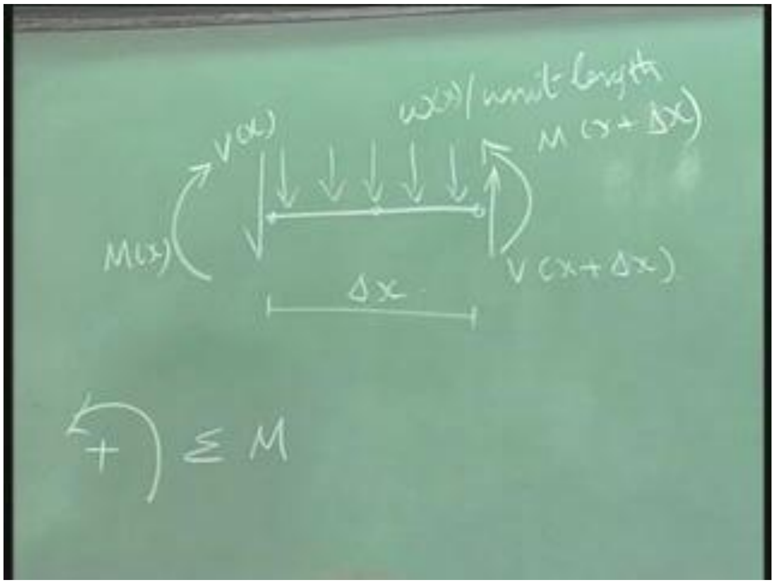

If I choose a point over here, I will not have this shear force coming into picture. We choose a point over here, remember this particular load is equally distributed and therefore that will not contribute to the moment over here. If I take this V of x will not contribute to it, either way it does not matter. Let's choose at the center. So o, please remember this is at a distance delta x by 2, delta x by 2 away. This is equal to 0 will imply, let's take one after the other. This M of x is in the clockwise sense, so minus M at x, as shear force that is acting downward like this will produce a counter clockwise moment. Therefore it is plus V at x times delta x by 2. We have accounted for this and this.

This force will have a resultant over here and another resultant over here, those two will cancel each other. So I am not going to bother about moment due to W of x. To the right hand side about o, M of x plus delta x is in the anticlockwise sense so it is positive M at x plus delta x and the other bending moment contribution is from V of x. That is also counter clockwise and that is V of x plus delta x times delta x by 2. Have I left out anything? Nothing else, this is equal to 0.

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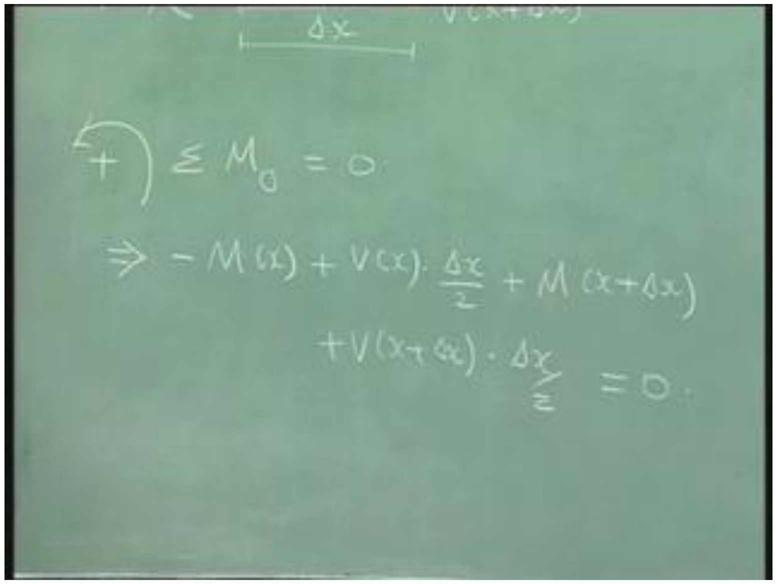

Notice that we have M of x plus delta x minus M of x. Let's write it down, M of x plus delta x minus M of x plus (V of x, V of x plus delta x) V of x plus V of x plus delta x both multiplied by delta x by 2 is equal to 0. Dividing throughout by delta x, we will have M of x plus delta x minus M of x by delta x by 2. I am going to divide by delta x by 2 is equal to, I am going to take to the right hand side minus V of x minus V of x plus delta x.

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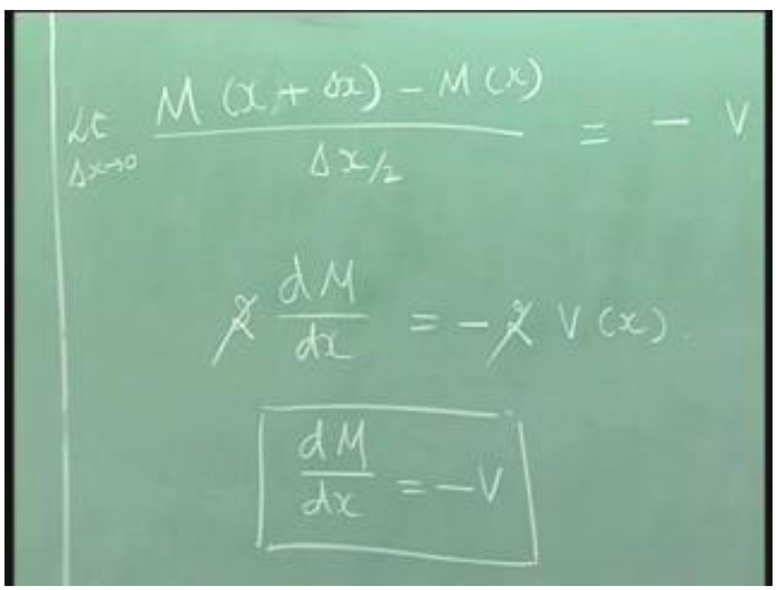

Now if I take the limit as delta x tends to 0, I know M of x plus delta x minus M of x by delta x, delta x tending to 0 will give me dM by dx. I have a by 2 which upon going up, will give me 2 times dM by dx is equal to minus of… When delta x tends to 0 this will be

V of x. This is already V of x so we will get minus 2 V of x. Is that all right? Canceling these two's here, we get dM by dx equals minus V. Therefore in this relationship, we got one more which is dM by dx is equal to minus V. We have two relationships that we could find out from equilibrium of a small infinitesimal element for a beam which has transverse force acting on it. Remember in this particular case I don't have to bother about what support reactions and all those. That will come into play in an automatic sense. One more thing to note here is I have dV by dx is equal to W of x but V is equal to minus dM by dx.

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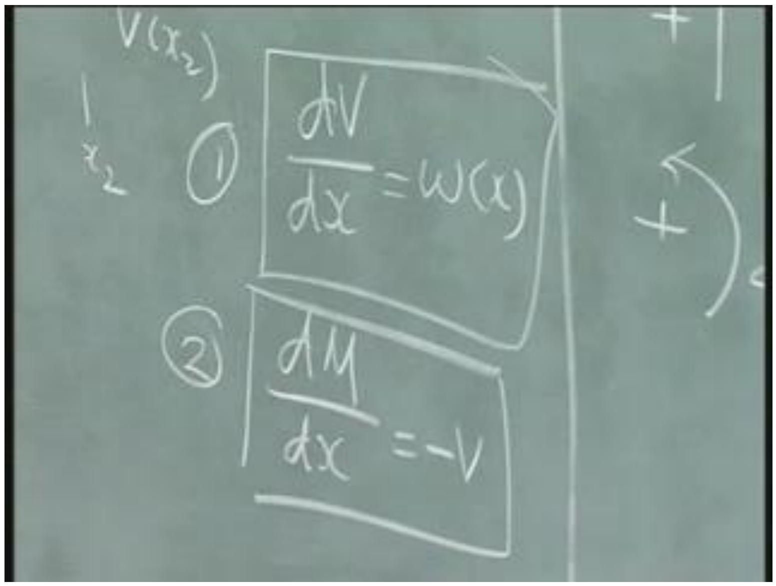

Therefore I can now write d of minus d m by d x by d x is equal to W of x. What is this? This is nothing but minus d square m by d x square. This is equal to W of x. I am going to take this minus over to the other side which is minus W. If I know how the force W is varying, I can find out how shear force is going to vary. If this were a constant force, what do you conclude for as far as variation of v is concerned? dV by dx is constant which means V is linear. If V is linear, d M by dx is one order, M has to be one order higher than V so that M has to be quadratic. For a constant W, I have a linearly varying V. For a linearly varying V, I have a constant here. These are the results that we got.

Let's observe and find out what all we understand from this. For example if I have a beam like this, lets say we will take those that particular example where we had a P acting, 2 P acting at l by 3, l by 3. What do you see in this zone? In this zone W of x is equal to 0. There is no force acting in between and therefore what should I notice as far as dV by dx is concerned? dV by dx is equal to 0 implies V is a constant. In this particular region when we drew shear force diagram, we found that the shear was not varying between this point and this point. If shear force were constant and I used this fact over here, we have a dM by dx is equal to constant because in this region V is a constant which means this is a constant, negative constant means m is now linearly varying. This implies that M is linear.

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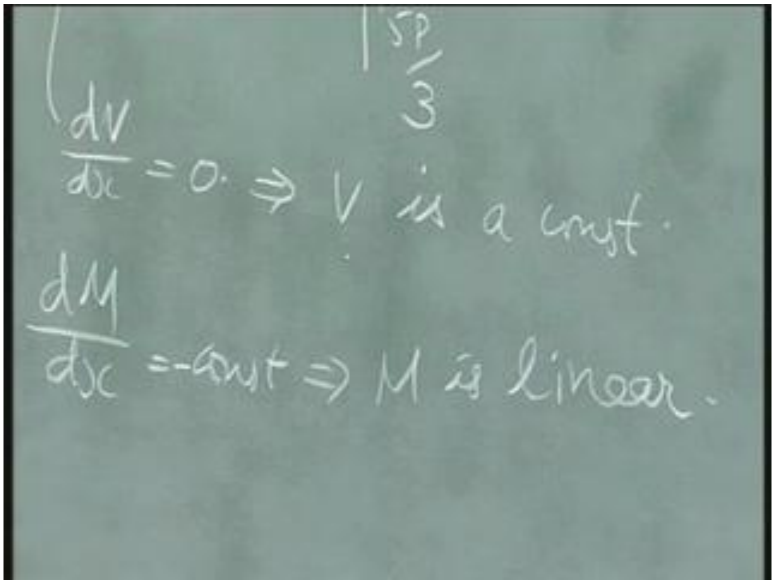

On the other hand, the example that we took for uniformly distributed road. For this case if you remember we had shear force diagram something like this W L by 2 and it was 0 at this point and it was varying linearly like this. Is this correct from this conclusion? W is a constant which means dV by dx is a constant means V is linear, I get a linear variation.

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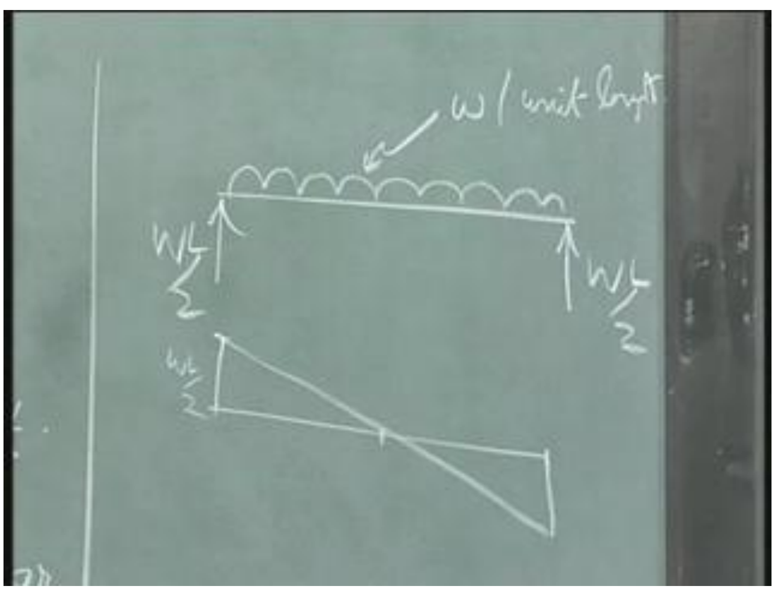

Similarly if I have to draw and this is shear force diagram. If I draw the bending moment diagram, dM by dx is equal to minus V of x. I can use this fact to draw that or from the basic free body diagram I can draw. I got it to be something like this, the positive value and the centre was equal to W L square by 8. If I remember correct and it was the maximum at the center. Is that correct? Let’s look at this particular point. This particular point V is equal to 0, if V were equal to 0, dM by dx is equal to 0. When do you have dM by dx equal to 0?

If there is a function M of x and I want to find out the maximum or the extremum value of m of x. I take the derivative and said it is equal to 0. That’s basically what I can do here and I get dM by dx to be 0 or in other words M is maximum. Therefore I have maximum value over here or in other words the slope of this distribution at this point is horizontal equal to 0. There are some conclusions that you can make for example in a beam where I want to find out where the maximum bending moment would occur. It is enough to draw the shear force diagram, find out where zeros are existing in shear force and that will immediately tell me where I should be finding out the bending moments. That’s the advantage of using these expressions.
Thank you.