

Urban Transportation Planning
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Lecture No. #25
Trip Distribution Analysis Contd.

This is lecture 25 on urban transportation planning. The discussion of the previous lecture will be continued in this lecture 2. You may recall that we finally discussed about the results of the calibration of the gravity models both doubly constraint as well as singly constraint for a real life situation pertaining to the city of Tiruchirapalli.

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Statistical Performance of Various Models					
Trip stratification (1)	Model number (2)	Value of parameter (3)	Measure of statistics		
			Normalized MABSERR (4)	Normalized $d\phi$ (5)	R^2 (6)
Work trips	I.a	-1.75	0.52	0.54	0.83
	I.b	-1.25	0.54	0.66	0.79
	I.c	-0.50	0.52	0.56	0.82
Educational trips	I.a	-3.00	0.43	0.52	0.94
	I.b	-2.50	0.51	0.68	0.90
	I.C	-1.25	0.48	0.56	0.92
Other trips	I.a	-1.85	0.60	0.62	0.77
	I.b	-1.25	0.66	0.74	0.75
	I.c	-0.50	0.61	0.65	0.77

Since travel deterrence function becomes discontinuous by virtue of the calibration procedure adopted for model II, the parameter value has not been indicated

To recollect the result of recalibration that we saw in the previous class, you can just look at the table, and recollect the numbers pertaining to the different parameters. The results shown here pertain to recalibration of only doubly constraint gravity model. Since single constrained gravity model involves number of parameters due to discontinuity in the function, those results are not shown here **right**. And we discussed about the logical correctness of the parameter values taking the example of model 1.a, pertaining to work trips and educational trips; and found that the travel deterrence in respect of educational trips is relatively higher, when compared to work trips; that is how we were able to appreciate the logical correctness of recalibration result. On the same lines, we will be able to compare and derive results or derive inference related to models 1.b and 1.c.

And the measures are statistics in the form of normalized MABSERR, normalized phi value and coefficient of determination also shown in the table, which indicate that all the models are statistically significant. And as I indicated to you, since travel deterrence function becomes discontinuous by virtue of the calibration procedure adopted for model 2, the parameter values have not been indicated in this table.

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Model I.a:

$$T_{ij} = X_i P_i Y_j A_j d_{ij}^\alpha$$

Model I.b:

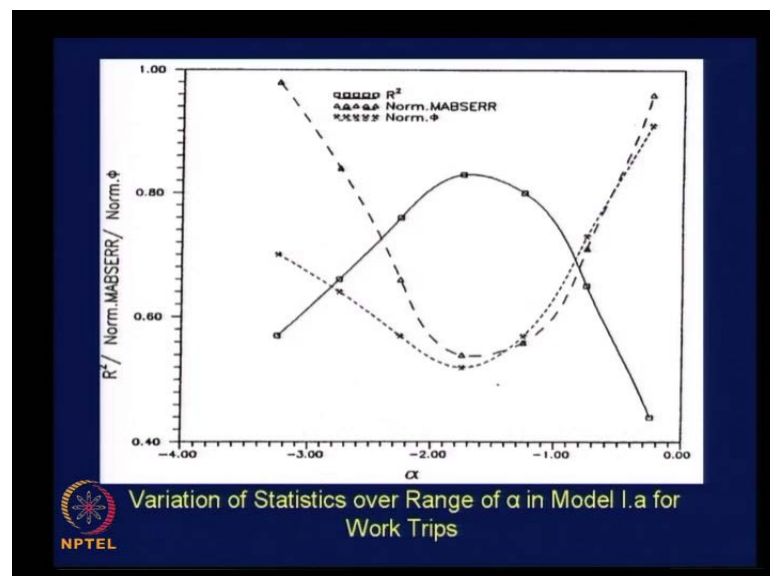
$$T_{ij} = X_i p_i Y_j A_j \exp(\beta d_{ij})$$

Model I.c:

$$T_{ij} = X_i P_i Y_j A_j \exp(\beta d_{ij}) d_{ij}^{-1}$$


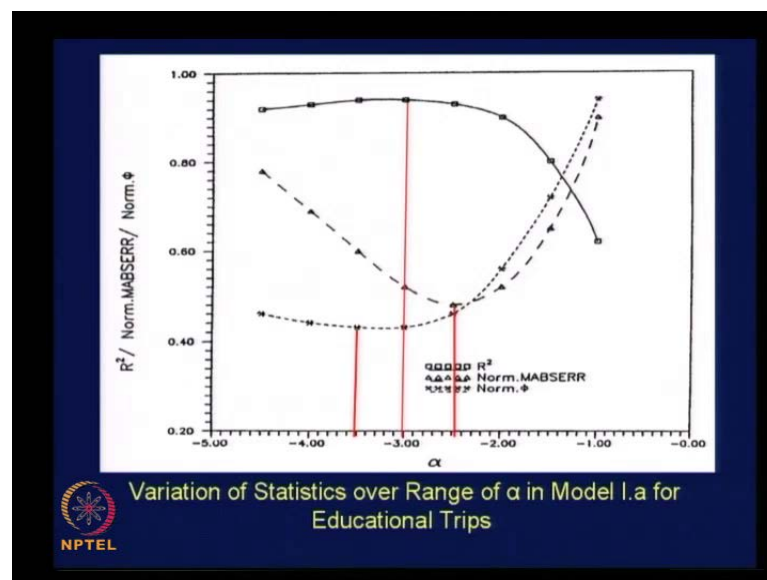
The models that we considered are these just to brush up your memory; I am showing the models again.

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And this is how we put the values of all the three statistics that we consider for checking the correctness of calibration. You can see the three curves pertaining to R squared, normalized MABSERR and normalized phi. And R squared is maximum at this point. You can see the phi value as well as normalized MABSERR values almost least at the same point; you can even draw a straight line connecting the corresponding points of all the three statistics. This is an ideal case, where all the three statistics are converging at a particular point. This need not necessarily with the case for all situation; there could be differences, which are sometimes minor; sometimes very significant may occur for work trips.

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Look at this case, which is model 1.a for educational trips. The plots pertaining to the three statistics are shown here. And if you try to identify the least value pertaining to normalized phi, it lies between minus 3 and minus 4; the maximum value in respect of R squared is around minus 3; and the least value in respect of normalized mean absolute error is lying between minus 2 and minus 3. There is no concurrence between these three statistical measures in this particular case.

So, it is spread over some range. To manage this kind of situation, what is being normally done is that you need to take one statistic as the decision making criterion statistic, and use the other two statistical measures to counter check the correctness of your decision **right**. So, in this case, if you take say for example, normalized mean

absolute error as your decision or criterion statistic, you can check whether the other two statistical measures are fairly close to your decision criterion statistic. So, this kind of problems are likely in practice; we should be aware of it; it is not that always we will get a single value, which will be acceptable based on all the re-statistical measures. Clear?

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Trip stratification (1)	Model (2)	Value of parameter (3)	Measure of statistics		
			Normalized MABSERR (4)	Normalized ϕ (5)	R2 (6)
Walk trips	l.a	-3.25	0.44	0.58	0.92
	l.b	-2.75	0.45	0.58	0.91
	l.c	-1.50	0.43	0.50	0.92
Bicycle trips	l.a	-1.25	0.62	0.59	0.78
	l.b	-0.75	0.57	0.59	0.78
	l.c	0.25	0.59	0.60	0.79
Public Transport trips	l.a	0.00	0.69	0.71	0.74
	l.b	0.00	0.69	0.71	0.74
	l.c	0.00	0.72	0.75	0.70
Private Transport trips	l.a	-1.75	0.90	0.98	0.77
	l.b	-1.25	0.86	1.00	0.79
	l.c	-0.50	0.88	1.00	0.79
IPT trips	l.a	-1.75	0.78	0.79	0.68
	l.b	-1.25	0.75	0.76	0.70
	l.c	-0.50	0.76	0.76	0.70

This is a result of trip-distribution based on mode used for travel. We will just check the logical correctness of the parameter values, because the procedure is similar to what we have seen earlier in respect of distribution based on trip purpose nothing different. And let us compare the parameter values pertaining to model 1.a, in respect of walk trips and bicycle trips; for walk, it is minus 3.25; for bicycle trips, it is minus 1.25. This implies that travel deterrence in respect of walk trips is much higher compare to bicycle trips, because of sheer physical constraint, deterrence is more **right**. People do not travel long distance by foot. So, that is implied here through the result of calibration.

And if you compare the values for bicycle and private transport modes like motor cycle **right**. You get a value of minus 1.25 for bicycle, minus 1.75 for motor cycle; this implies travel deterrence is more by motor cycle and less by bicycle. Is it correct? If you perceive travel deterrence in terms of the energy used for transportation or human muscular power, then deterrence, when you use a motor cycle should be much less compared to usage a bicycle. Is it not?

So, here travel deterrence is a bundle of all the related issues as generalized cost of transportation. Travel time, travel cost, comfort, convenience all together is reflected. Can anyone ask why deterrence is more in respect of motor cycle compare to bicycle in this particular case? What could be the reason? It is obviously, because of the high cost of travel using motor cycle; they have to spend money for fuel, whereas bicycle is almost available free of cost, trip wise cost for bicycle is almost can be taken as 0. Is it not? That is why travel deterrence here is more. Clear? So, that is how we need to understand the effect of the bundle of the factors that influence trip-distribution; it is not simply only the distance or travel time or cost all together is taken into account, while deriving this parameters.

Then interestingly you can see that in respect to public transport, which is bus in this particular city, travel deterrence is shown as 0, no deterrence at all. Could it happen? For any mode, there should be some deterrence. Is it not? This implies that people do not mind travelling by bus, any distance in the city. This is mainly because of the reason that at the time of the study, the bus fare was highly subsidized; even for economically weaker section, it was throw away money, for travel by bus; it is quite cheap in that particular city. So, people did not mind using bus for any trip, any distance.

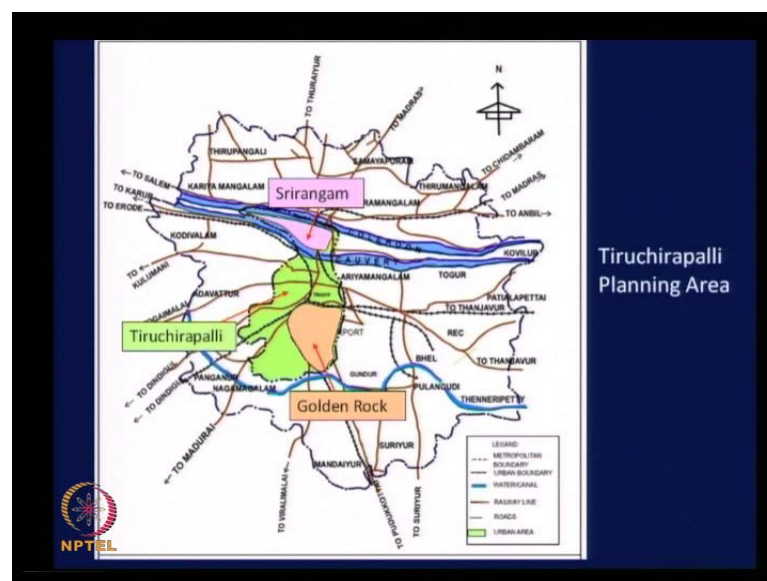
And when you do the analysis, when it is... When the situation is like that, you end up with values like 0. Even though if you go up to say 10 decimal places, you may get some numbers; when it round it off to three decimal or two decimal places we end up with only 0right. So, that is how we are getting zero for public transit in this particular case. Of course, it is not a metropolitan cities spread over say 50 or 100 kilometers, it is only a compact city that should be also be taken into consideration, when we discuss about the implications of travel cost, travel distance and so on. Clear? So, this is how we need to understand the correctness, particularly logical correctness of the results of calibration. And other statistical measures, you can always appreciate; based on the discussion, we had for the trips made for different purposes.

Now, we need to validate the model. Is it not? Otherwise your model cannot be considered to be valid model for application, for horizon-year condition. How do you validate a model? Calibration we can do, because data is available, you calibrate. Suppose you are asked to validate a model; how do you validate? One possibility is use this model to explain trip-distribution in another situation right, where trip-distribution is

known. Then you compare the actual field observed and model simulated distribution values. And if both are matching reasonably, you can say that model can be applied to a different situation.

In this particular case, model validation was done using hold-out sample. As a name implies, a certain portion of the sample data was held out from the pool of the data for the purpose of model validation. You may remember we talked about 49 traffic zones in the urban area that for model calibration purpose, data pertaining to how many zones were used. We used data pertaining to only 35 traffic zones; only 35 traffic zones. We held out data pertaining to 14 traffic zones, for the purpose of model validation. When you hold out data, you must also see that it pertains to one region or segment of the urban area. So, that it almost behaves like a separate entity, while you check your model for protecting the trip-distribution in that particular area.

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And this particular case, it was quite possible that way. As you can see the green colored one is urban area; that is our study area as far as trip-distribution is concerned, because we are discussing about distribution of trips within this green colored portion. This green colored portion actually consisted of three municipal towns; it is not a single municipality, three municipal towns grown to merge with one another. The boundaries of three municipalities almost merged with one another; the urban boundary was configured

combining all the three municipal towns; that is how initially this shape was obtained right.

The municipal towns involved were this. This is a municipal town named Srirangam; it is in between the two rivers island portions; and this is one corner of green colored portion named golden rock, municipal town, where there is a big railway workshop located; and rest of the area is Tiruchirapalli municipality; Srirangam municipal town, golden rock municipal town and Tiruchirapalli municipal town. The data that we used pertain to Srirangam and Tiruchirapalli, comprising 35 traffic zones; and golden rock area consisted 14 traffic zones, and this data was held out for validation.

You can say this is a totally a separate portion right. And this data was used to predict distribution (()) and distribution matrix was developed. So, we had two matrices; one the field observed, other one the model simulated matrix. And both the matrices were compact and the statistical measures where used for comparison namely normalized MABSERR and normalized phi value as well as the coefficient of determination. I will show you the result of the comparison directly.

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Results of Validation by comparing the model predicted and observed values of trip distribution in the hold out sample

Trip Stratification (1)	Measure of Statistic		
	Normalized MABSERR (2)	Normalized ϕ (3)	R ² (4)
Work trips	0.43	0.36	0.97
Educational trips	0.34	0.42	0.95
Other trips	0.46	0.34	0.94
Walk trips	0.31	0.37	0.95
Bicycle trips	0.40	0.36	0.97
Public transport trips	0.71	0.72	0.79
Private transport trips	0.78	0.82	0.60
Intermediate public transport trips	0.93	0.74	0.68

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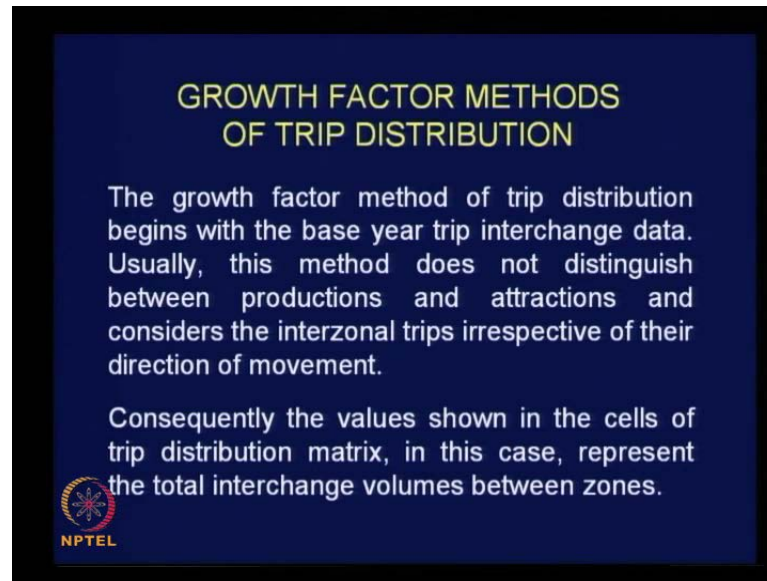
Results of validation by comparing the model predicted and the observed values of trip-distribution in the hold out sample: So, we considered all trip types; work trips, educational trips, other trips. Also trips based on mode used; walk trips, bicycle trips, public transport trips, private transport trips and intermediate public transport trips. So,

these were the model statistics; comparing the actual data and your model simulated distribution value. You can see in most cases, it is acceptable; R square value is very high. In one case, R square value is very low, 0.60 the same thing is reflected in respect of 5 and MABSERR values also; you can little higher value. Here also 0.68, again 0.74, 0.93, these are mainly because of the variation in the extent of usage of this two modes compared to Srirangam and Tiruchirapalli and golden rock. Variation in the characteristics of transport system creates this kind of problem.

So, this implies that the model is generally satisfactory; and there are certain issues related to the two modes. So, this has to be taken care of. What you can do is, having checked that it is reasonably okay; you can put this data also together along with the 35 zones, and develop a fresh a model, which can be more representative for the whole of the urban area. So, that is how normally we do as plan as at the end of the exercise. This is only to make sure that your model is reasonably okay; you gain confidence, then put the whole data set together, and recalibrate, and get your final model. Clear?

So, this is how we gravity models were applied for a particular case. And the models that we discussed so far are gravity models based on Newton law of gravitation. The input data to these models, we can recollect carefully are trip production and trip attraction values P_i and A_j right. And then these are the data corresponding to travel distance or travel time. These are the inputs to get T_{ij} value. And at early stages of planning process or I would say in late 50s and 60s, there were distribution models used, which are mainly based on the base year distribution matrix. And these models were known as growth factor methods of trip-distribution, growth factor methods.


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**GROWTH FACTOR METHODS
OF TRIP DISTRIBUTION**

The growth factor method of trip distribution begins with the base year trip interchange data. Usually, this method does not distinguish between productions and attractions and considers the interzonal trips irrespective of their direction of movement.

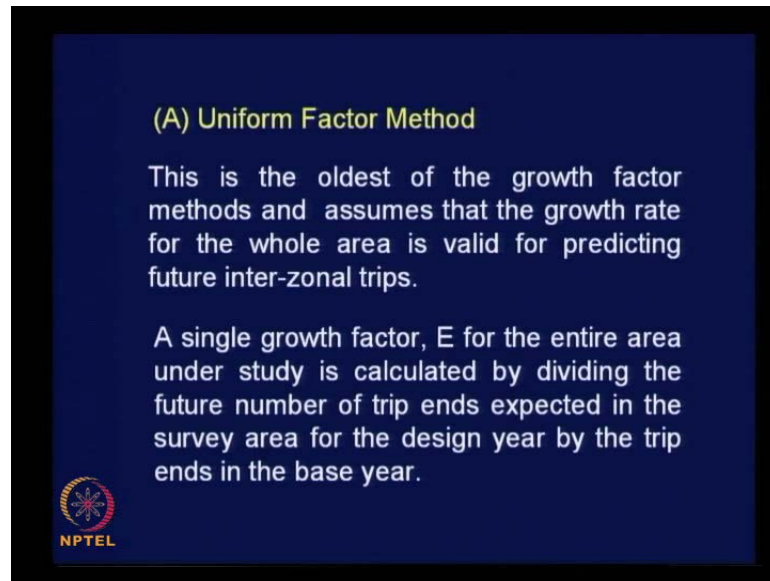
Consequently the values shown in the cells of trip distribution matrix, in this case, represent the total interchange volumes between zones.



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The growth factor method of trip-distribution begins with the base year trip interchange data; that is very important; base year trip interchange data is the starting point; you must have the complete matrix available for base year condition. Usually this method does not distinguish between productions and attractions; and considers the interzonal trips irrespective of their direction of movement; it is simply a p a matrix, which has got no directional meaning **right**. We use p a matrix pertaining to base year as database for distributing trips based on this particular method. Consequently the values shown in the cells of the trip-distribution matrix, in this case, represent the total interchange volumes between zones; total trip interchanges irrespective of the direction of movement **right**. That is what is used for subsequent analysis by this method.


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(A) Uniform Factor Method

This is the oldest of the growth factor methods and assumes that the growth rate for the whole area is valid for predicting future inter-zonal trips.

A single growth factor, E for the entire area under study is calculated by dividing the future number of trip ends expected in the survey area for the design year by the trip ends in the base year.



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There were different approaches available. The simplest of all the methods is called uniform factor method. This is the oldest of the growth factor methods; and assumes that the growth rate for the whole area is valid for predicting future interzonal trips. What does it mean? So, a city grows over time; the rate of growth in a particular city may be more on one side, less on other side may be moderate in another region. That is happening, if you take Chennai city, southern part of Chennai city is growing at a faster rate compared to northern part due to various reasons, geographic as well as socio economic factors, the growth rate towards south is much faster compared to the growth rate on the northern part.

And here the assumption is that the urban area grows uniformly over the entire urban space; that is assumption; that is why it is called as uniform factor method. So, there is a constrained; so this is the basic assumption. Once you make this assumption, then you can use single growth factor; when the growth rate is assumed to be constant, a single growth factor E, E stands for expansion. E for the entire area under study is calculated by dividing the future number of trip ends the expected in the survey area for the design year or for horizon-year by the trip ends in the base year. Clear? Number of trip ends expected in the survey area for the design year; that is your horizon-year by the trip ends in the base year; just we divide 1 by the other.

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The future trips between zone i and j, namely, T_{ij} are then calculated by applying the uniform factor E to the base year trips, t_{ij} between zones i & j.

Thus,

$$T_{ij} = t_{ij} \times E$$



The future trips between zones i and j for example, namely T_{ij} , when calculated by applying the uniform factor E to the base year trips say given by small t_{ij} , lower case T_{ij} , because we are using both simultaneously, it is better to distinguish between horizon-year and base year values, between zones i and j. Thus we simply express capital T_{ij} to be equal to small t_{ij} into the expansion factor or growth factor. What is small t_{ij} ? It is nothing but the cell value in the base year matrix, base year trip-distribution matrix; we simply multiply the cell values by one factor.

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Example:

i \ j	1	2	3	q_i	Q_i
1	60	100	200	360	360
2	100	20	300	420	1260
3	200	300	20	520	3120
q_j	360	420	520	1300	-
Q_j	360	1260	3120	-	4740

q_i, q_j = base year trip interchanges
 Q_i, Q_j = horizon year trip interchanges.



Now, how do we get that factor? That is considered as numerical example. There are three traffic zones, and production attraction values are given. And their total of the production values for zones 1, 2 and 3 are 360, 1260 and 3120 **right**; and the total of attraction values are again 360, 420, and 520. Let me repeat; total of the production values for base year condition, **base year condition** given as small q_i are 360, 420 and 520 **right**; similarly, trip attractions zone wise for base year small q_j are 360, 420 and 520 **right**. And we have predicted the trip attraction values for the horizon-year as given here. We use trip production and trip attraction models, and then predict the horizon-year trip production as well as trip attraction, and those values are taken for distribution purpose. Is it not? That is how you must understand these numbers. These are horizon-year values 360, 1260 and 3120 **right**. And similarly horizon-year values of attraction are 360, 1260, 3120.

Interestingly, we find that there is no increase in trip production at all in zone 1, it remains static **right**; both base year and horizon-year values are same; it may happen in certain cases. Similarly, in respect of trip attraction also we find 360 and 360. Whereas 420 become 1260 in the horizon-year, and 520 become 3120 in the horizon here. Similarly there a tremendous growth in respect of attraction also for zones 2 and 3. So, instead of total of 1300 trips, it becomes 4740 trips in the horizon-year. Clear?


Now, we have the base year trip-distribution matrix with us; and base year trip production and trip attraction values for all the zones is known; horizon-year trip production and trip attraction values are known. So, with this information, we have to get the horizon-year trip-distribution matrix. Assuming a uniform growth factor that is it; of course, this is what I said q_i q_j base year trip interchanges, and capital Q_i capital Q_j horizon-year trip interchanges.

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Solution: $E = \frac{4740}{1300} = 3.646$

By multiplying the cells values of the base year matrix by the uniform factor of 3.646, the following matrix results.

i \ j	1	2	3	$Q_{i(cal)}$	$Q_{i(given)}$
1	218	365	729	1312	360
2	365	73	1094	1532	1260
3	729	1094	73	1896	3120
$Q_{j(cal)}$	1312	1532	1896		
$Q_{j(given)}$	360	1260	3120		

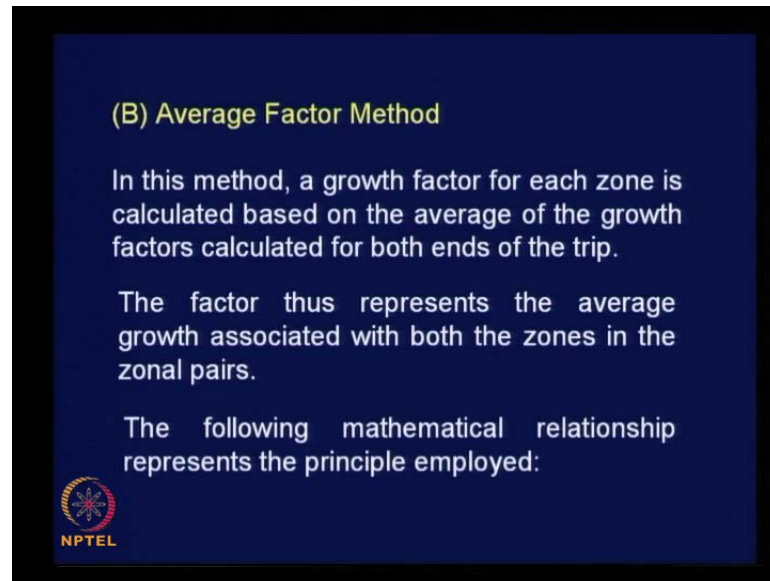


This is the expansion factor, growth factor. Is it not? You can now look at these numbers; this is the total of productions and attractions for horizon-year; this is the total for base year. When we assume uniform growth, simply divide the totals. E is equal to 4740 divided by 1300. So, the expansion factor, a growth factor is 3.646. And then we get the horizon-year matrix by multiplying the cells of the base year matrix by the uniform factor of 3.646, the following matrix results. So, this is the horizon-year trip-distribution matrix. And this is horizon-year value as per calculation; after multiplying the cell values by the growth factor, we are getting a set of values of trip production p a values **right** after distribution; and these are trip attraction values, but the actual predicted values are these.

Here, we find there is a problem. After distribution, we find the total of the cell values are not matching with the predicted values of both production as well as attraction **right**. So, this is the major drawback of this procedure, when **when** can you use this kind of distribution procedure? Think about this; we will answer this question a bit later; still this procedure may be applicable for specific situation. Here definitely we cannot accept this kind of variation; Is it not? When we distribute, there is a total distortion of

P_i and A_j values, but still this was adopted initially, because it was found to be acceptable for various reasons about which will discuss bit later.

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(B) Average Factor Method

In this method, a growth factor for each zone is calculated based on the average of the growth factors calculated for both ends of the trip.

The factor thus represents the average growth associated with both the zones in the zonal pairs.

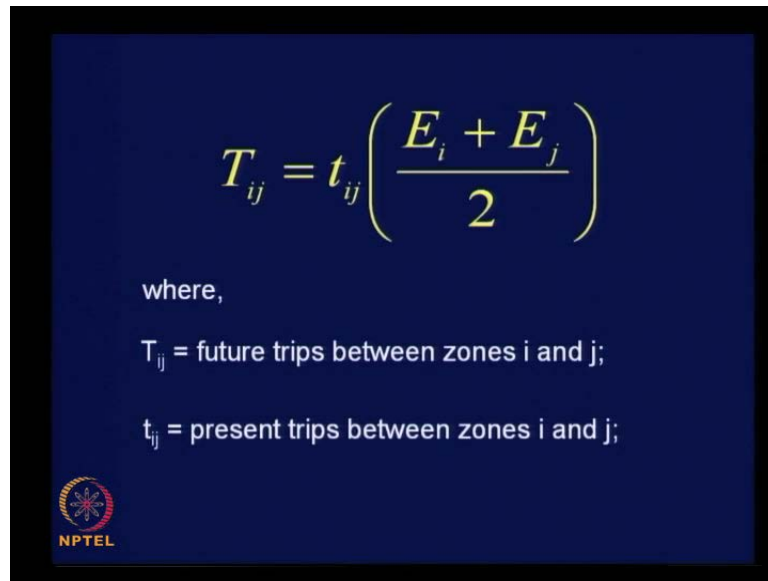
The following mathematical relationship represents the principle employed:



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Then there is another method; try it out; which is named as average factor method. In this method a growth factor for each zone is calculated based on the average of the growth factors calculated for both ends of the trip; that is why it is called as average growth factor or average factor method. So, trips are interchanged between zonal pairs. So, there are two zones involved in respect of any trip. You look at the growth rate of both the zones; take those two growth factors, and take the average; which will be more logical compare to the previous case where we just assumed uniform growth. The factor thus represents the average growth associated with both the zones in this zonal pairs **right**. And the following mathematical relationship represents the principle employed.


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$$T_{ij} = t_{ij} \left(\frac{E_i + E_j}{2} \right)$$

where,

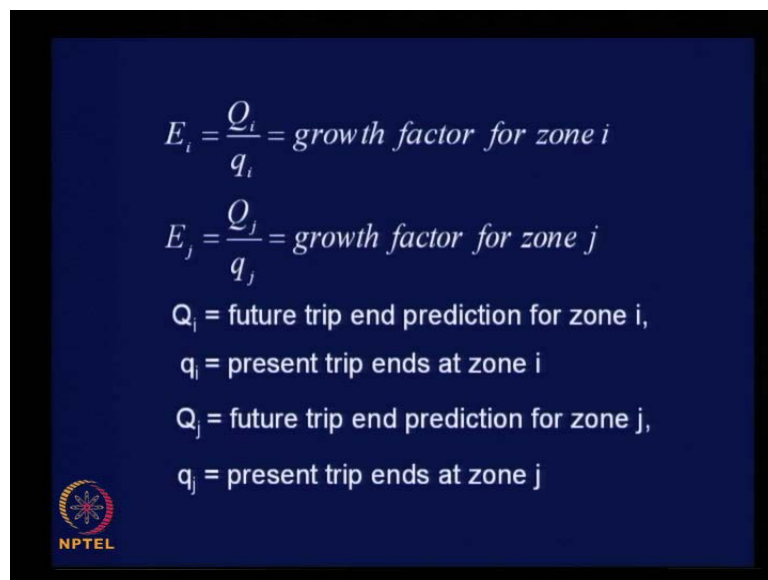
T_{ij} = future trips between zones i and j;

t_{ij} = present trips between zones i and j;




As you can easily guess T_{ij} was obtained as small t_{ij} multiplied by E_i plus E_j whole divided by 2, where capital T_{ij} is a future trips between zones i and j; small t_{ij} present trips between zones i and j.

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$$E_i = \frac{Q_i}{q_i} = \text{growth factor for zone } i$$
$$E_j = \frac{Q_j}{q_j} = \text{growth factor for zone } j$$

Q_i = future trip end prediction for zone i,
 q_i = present trip ends at zone i

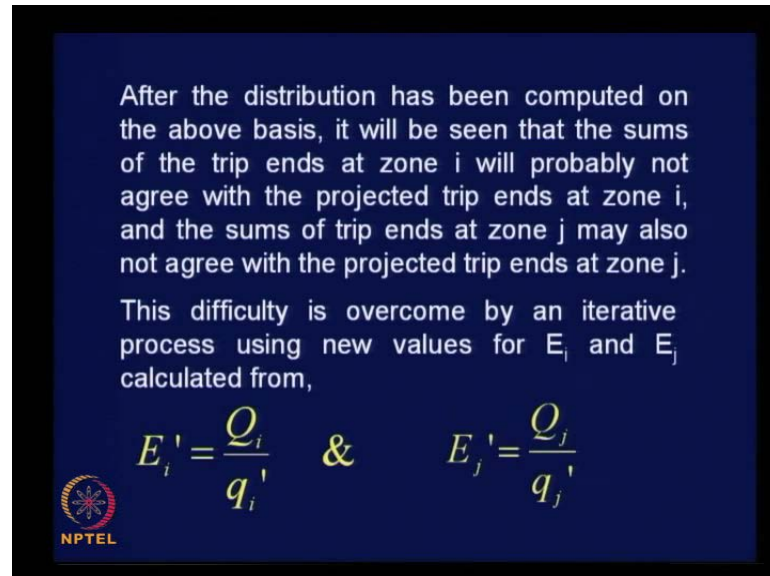
Q_j = future trip end prediction for zone j,
 q_j = present trip ends at zone j



And E_i is Q_i by q_i , capital Q_i by small q_i ; growth factor for zone i, production zone growth factor; and E_j attraction zone growth factor Q_j by small q_j growth factor for zone j; and Q_i obviously is future trip end production for zone i; and small q_i present


trips at zone i; and Q_j future trip end production for zone j; and q_j present trip ends at zone j. Nothing complicated; we simply calculate two growth factors and take average.

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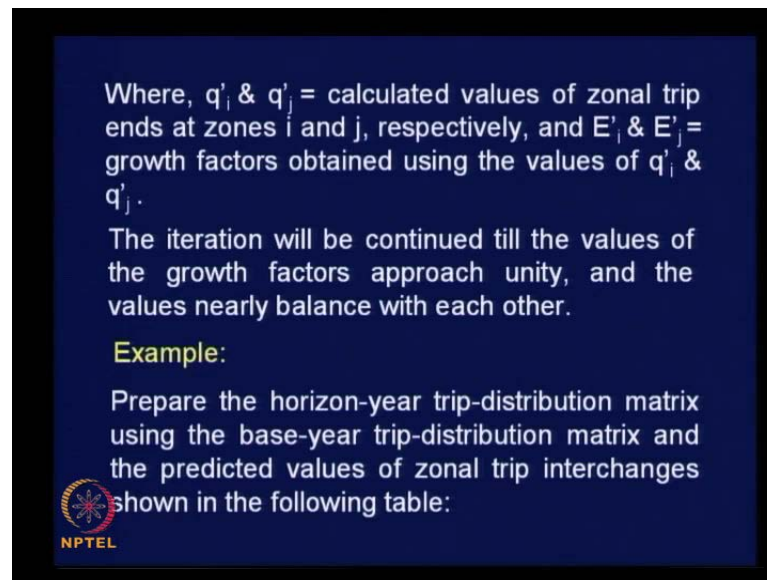
After the distribution has been computed on the above basis, it will be seen that the sums of the trip ends at zone i will probably not agree with the projected trip ends at zone i, and the sums of trip ends at zone j may also not agree with the projected trip ends at zone j. This difficulty is overcome by an iterative process using new values for E_i and E_j calculated from,

$$E_i' = \frac{Q_i}{q_i'} \quad \& \quad E_j' = \frac{Q_j}{q_j'}$$

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Now, after the distribution has been computed on the above basis, it will be seen at the sums of the trip ends at zone i will probably not agree with the projected trip ends at zone i, as we have seen in the previous case. And the sums of the trip ends at zone j may also not agree with the projected trip ends at zone j; both productions and attractions may not agree or tally with the predicted values for the horizon-year. In that case, this difficulty is overcome by an iterative process using new values for E_i and E_j calculated from a correction procedure. We calculate a new growth factor E_i' to be capital Q_i divided by q_i' ; and E_j' is calculated as Q_j divided by q_j' .

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Where, q'_i & q'_j = calculated values of zonal trip ends at zones i and j , respectively, and E'_i & E'_j = growth factors obtained using the values of q'_i & q'_j .

The iteration will be continued till the values of the growth factors approach unity, and the values nearly balance with each other.

Example:

Prepare the horizon-year trip-distribution matrix using the base-year trip-distribution matrix and the predicted values of zonal trip interchanges shown in the following table:




You may wonder; what is q_i and q_j ? They are the actual calculated values of zonal trip ends at zones i and j ; say after distribution you add up the values; that is the calculated value; they are termed as q_i and q_j respectively. And E_i and E_j are growth factors obtained using the values of q_i and q_j . And then the iteration will be continued till the values of the growth factors approach unity, and the values nearly balance with each other. Why should the growth factor become unity? How do we get the values of E_i and E_j ?

At every stage, we take the calculated values of q_i and q_j to get the value of the subsequent growth factor for the subsequent iteration **right**. So, when you do the iterations then you calculate values matching with the actual or your predicted value, the result is going to be 1; the growth factor will be 1. Is it not? So, when E is 1, your calculation matches with the actual predicted values of P_i and A_j **right**, that is what is meant here. We will take a small numerical example and see, how to apply this particular method. Prepare the horizon-year trip-distribution matrix; using the base-year trip-distribution matrix, and the predicted the values of zonal trip interchanges shown in the following table.

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i \ j	1	2	3	q_i	Q_i	$E_i = Q_i / q_i$
1	60	100	200	360	360	1
2	100	20	300	420	1260	3
3	200	300	20	520	3120	6
q_j	360	420	520		-	
Q_j	360	1260	3120	-	4740	
$E_j = Q_j / q_j$	1	3	6			




These are the values shown. We have three traffic zones; and the values are similar to what we have seen in the previous example **right**; the cell values are same. For the base year condition or the production values of zones 1, 2 and 3 are 360 420 and 520 as we have seen earlier. And for base-year condition, the attraction values of zones 1, 2 and 3 are 360 and 420 and 520; and the predicted values of productions for horizon-year condition are 360, 1260 and 3120; again the same as we have seen in the previous case. And predicted attraction values for horizon-year condition are 360, 1360 and 3120 **right**.

And to start our first iteration, we can straight away calculate the growth factors by dividing the base-year values and horizon-year values of productions as well as attractions; that is what we do 360 by 360 - 1, 1260 by 420 - 3 and 3120 by 520 - 6. Similarly, we get values 1, 3 and 6. And total of trip productions or trip attractions is 4740 for the horizon-year condition. Clear? So, we are going to use these growth factors 1, 3, 6 and 1, 3, 6 for subsequent calculations.

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Solution


$$T_{11} = \frac{1+1}{2} \times 60 = 60 \quad T_{23} = \frac{3+6}{2} \times 300 = 1350$$
$$T_{12} = \frac{1+3}{2} \times 100 = 200 \quad T_{31} = \frac{6+1}{2} \times 200 = 700$$
$$T_{13} = \frac{1+6}{2} \times 200 = 700 \quad T_{32} = \frac{6+3}{2} \times 300 = 1350$$
$$T_{21} = \frac{3+1}{2} \times 100 = 200 \quad T_{33} = \frac{6+6}{2} \times 20 = 120$$
$$T_{22} = \frac{3+3}{2} \times 20 = 60$$


T 11; what is the growth factor for zone 1? It is 1 both for production as well as attraction. So, 1 plus 1 by 2, and cell value was 60; so, it is 60. T 12 - 1 and 3; is it not? For zone 2, growth factor was 3 both for production as well as attraction. So, 1 plus 3 by 2 into cell value was 100, so we get 200. Are you able to appreciate? Then 13 for zone 3, growth factor was 6, zone 11, so 1 plus 6 by 2 and the cell value of 1 3 was 200, so we get 700; 2 1 - 3 plus 1 by 2 into the corresponding cell value of 100, so we get 200; 2 2 - 3 plus 3 by 2; is it not? It is 22 into 20, which is 60. T 23 - 3 plus 6 by 2 into 300 that is 1350; T 31 - 6 plus 1 by 2 into 200, which is 700; clear? T 32 - 6 plus 3 by 2 into 300 - 1350; T 33 is 6 plus 6 by 2 into the cell value of 20, which is 100 and 20. Clear?

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The calculated T_{ij} Values, given in the following Table, represents the horizon-year trip-distribution matrix.

i \ j	1	2	3	q_i'	Q_i	$E_i' = Q_i / q_i'$
1	60	200	700	960	360	0.375
2	200	60	1350	1610	1260	0.785
3	700	1350	120	2170	3120	1.438
q_j'	960	1610	2170			
Q_j	360	1260	3120			
$E_j' = Q_j / q_j'$	0.375	0.785	1.438			



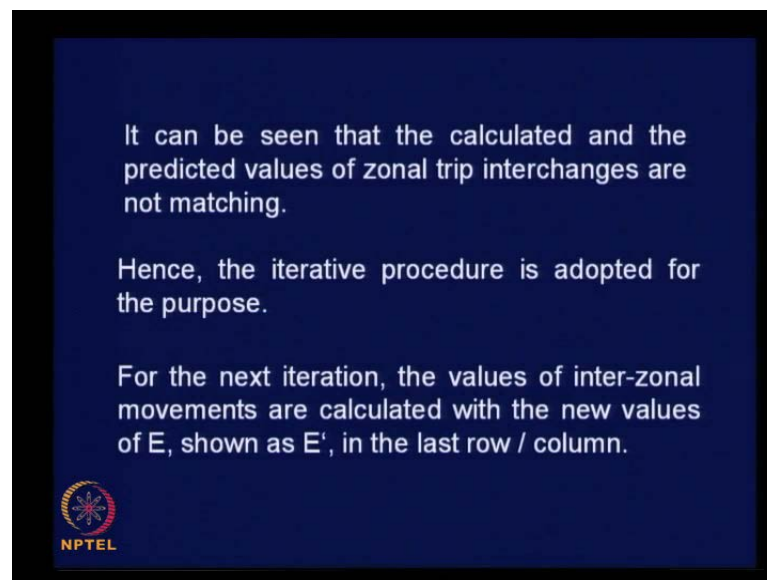
Now, we have the cell value; let us look at the result in the form of a matrix **right**. These cell values are the recently calculated cell values 60, 200, 700, 200, 60, 1350, 700, 1350 and 120 are the calculated cell values. When you total up, we get q_i' and q_j' values. We find the totals are 960, 1610 and 2170 for production; similar numbers for attraction values also 960, 1610 and 2170. But the predicted production values are given here as capital q_i ; these values are totally different from these values **right**. And here also we find that there is a significant difference; what we do then? If there is no difference between q_i' and q_i will be happy; is it not? It is not varying much; it is matching with the predicted total production for each of the zones **right**; or in other words if q_i' minus q_i or q_i minus q_i' is unity 1; it means that both the numbers are same and probably, we need not have to worry about the deviations. Is it not?

Let us check, what is happening here; let us divide Q_i by q_i' ; similarly here Q_j by q_j' ; and check whether they are closer to unity or not when we divide we find values are totally different from unity in all the three cases here also, they are different **right**. And by division, what do we do actually? We work out a new growth factor E_i' and E_j' ; and these growth factors will be used for subsequent iteration to distribute the trips. What will be effect of these things? Will it really get us the desired result? It will; for example, you see here. Our calculated value is much higher than the actual predicted trip attraction here; 960 against 360. When we just work out E_j'

value, our growth factor will have a decreasing effect in fact, **right**. So, that is what we want? We want a reduced value of q dash j in the subsequent iteration. Is it not? So, that is how, we automatically get the reduction effect by this growth factor **right**.

Contrarily, if you look at here for zone 3, the calculated value is 2170 **right**, and the predicted value, which you desire is 3120; we have to get a higher value. So, if you work out a growth factor, it automatically becomes more than 1, 1.438. So, use these growth factors for subsequent iteration **until** and continue the process until a growth factor value becomes nearly unity; that implies that your calculated and the predicted values of the productions and attractions are matching. Is that clear?

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This is what I said. It can be seen that the calculated and the predicted values of zonal trip interchanges are not matching. Here is the iterative procedure is adopted for the purpose. For the next iteration, the values of inter zonal movements are calculated with the new values of E shown as E dash in the last row or column, as we have seen in the matrix. I am not going to show you the subsequent iterations, and it is up to you to continue the process, and have a feel of the benefit of the iterative process. I will encourage each one of you to continue the iterative process, and see that you end up with growth factors of values nearly unity; it is possible; only thing is number of iterations depends upon the actual numbers we deal with. Clear?

Now, I have a question to you. We have discussed about two types of growth factor methods; uniform factor method and average factor method. What is the basic difference you see between the gravity model method of trip-distribution and growth factor method of trip-distribution? Are they differences? What is a difference in respect of input data? What is an input information given in respect of gravity model? Gravity model structure you should recollect T_{ij} is equal to $A_j P_i$ into $A_j F_{ij}$ divided by $\sum A_j F_{ij}$. Is it not? So, the basic input to get T_{ij} value is trip production, trip attraction and the parameter value pertaining to the friction factor. Whereas, what is a input information that you give for growth factor method of trip-distribution?

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Trip interchange itself; you should have complete matrix for base year condition. What you do is, you prepare the matrix of trip-distribution, and then manipulate with the matrix; manipulating the cell values in such a way that you are able to get the horizon-year predicted values of P_i and A_j that is what we do. We multiply the cell values by some factor **right**; in such a way that when you add up the cell values along a row, you must get the total production that you predict for the horizon-year condition for all the zones. Simultaneously you should be getting the production values that you predict for the zones **right**. So, the input data required for the growth factor method is the base year trip-distribution matrix itself. So, data requirement is elaborate in the case of growth factor method compare to gravity model. Is it not?

Any other difference you feel between these two methods? Think about it. Another related question is, can we really apply this growth factor method for any situation in practice; because we have come across number of problems in the use of growth factor method. Let say we come across situation, where a city has grown to a saturation level, and it is not going to grow any more in the next decades; there are cities of that kind in developed countries. Almost reach a saturation level, growth rate will be minimal **right**; if at all grows, it will grow bit by bit everywhere all over uniformly. Under such conditions probably your growth factor method, you may have the initial matrix after distribution for the base year condition, you can simply manipulate the values for getting the future trip-distribution scenarios. It is not that this method is totally useless; there are situations where this method can be advantageously made use off.

Now, to sum up what we have discussed in this lecture. We started our discussion on appreciating the result of calibrational gravity model for a case study. The city considered being Tiruchirapalli city. And we were able to understand the significance of the result of calibration in the form of the values of the parameter reflecting the logical correctness of the explanation that will be provided by the models for different purposes as would as trip-distribution by different modes. Then we discussed about model validation procedure by using hold out sample, which is the very important step; unless a model is validated, you cannot consider a model to be fit for application for future condition or for a different situation. So, in this case, we found how hold out sample can be used for model validation; and how to check for the correctness or validity of the model? **Right.**

Then I suggested that even the hold out sample can be combined with the original sample, and a new model can be developed as a representative model for the whole area, when you hold out sample from the same data set pertaining to same city. Then we discussed about the earlier attempts made for trip-distribution by growth factor method; under which we discussed about two basic methods; number one being uniform factor method, which assumes uniform growth rate for whole of the urban area, and then the average growth factor growth factor method or average factor method, which takes the average of the growth of the origin and destination zones, when we distribute trips between zonal pairs. With this, we will complete our discussion for today. We will continue on this topic in the next class.