

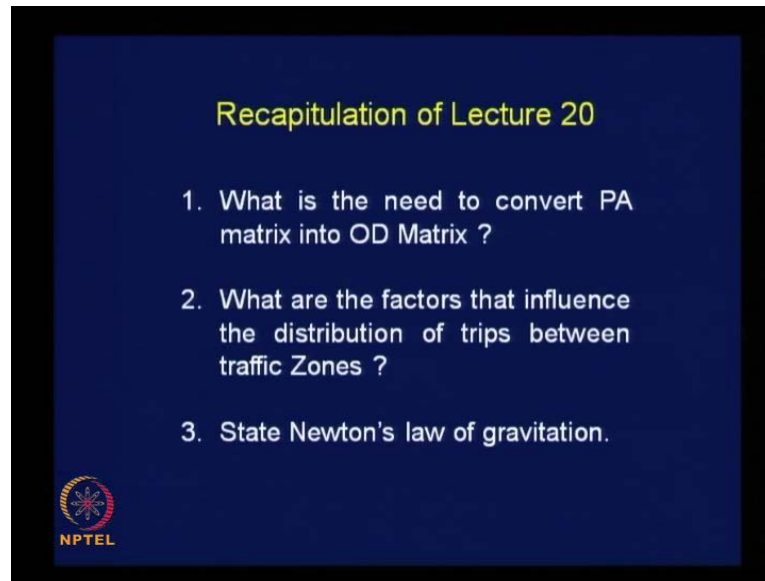
Urban Transportation Planning
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Module No. # 05
Lecture No. # 21
Trips Distribution Analysis Contd.

This is lecture 21 on Urban Transportation Planning. We will continue our discussion on Trip Distribution Analysis in this lecture. You may recall in the previous lecture, we started our discussion on trip distribution analysis with a understanding, that trip distribution is nothing but, the process of synthesis of trip interchanges between traffic zones. We also found that, it is convenient to present trip distribution data in the form of a matrix and we discussed about two types of matrices PA matrix giving information about zonal trip production and zonal trip attractions as well as OD matrix getting information about trip origins and trip destinations, then we understood the procedure of converting PA matrix into OD matrix.

And then we tried to identify, the factors that influence distribution of trips between traffic zones and based on the identify factors we found that Newton's law of gravitation can be consider as bases for framing an analytical formulation in the form of a model to explain trip distribution **right** and with this understanding to check our effectiveness of recapitulation of the points in the previous lecture, let us pose a few questions to us and try to answer.

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The first question is this, what is the need to convert PA matrix into OD matrix? Any responds, why should will convert PA matrix into OD matrix?

Represent means, it is the next step of fourth step of the fourth non ring process is.

Yes.

Trips in mind in that we need only OD matrix PA matrix only helpful.

That is a point, so we need to know the direction of movement between traffic zones while will do traffic assignment or route assignment in the fourth and last step of the planning process, so that is the reason why, we need to convert PA matrix into OD matrix and we should be conversant with the procedure.

The next question is this, what are the **factors that influence the distribution of trips between traffic zones**, factors influencing distribution of trips between traffic zones, anybody?

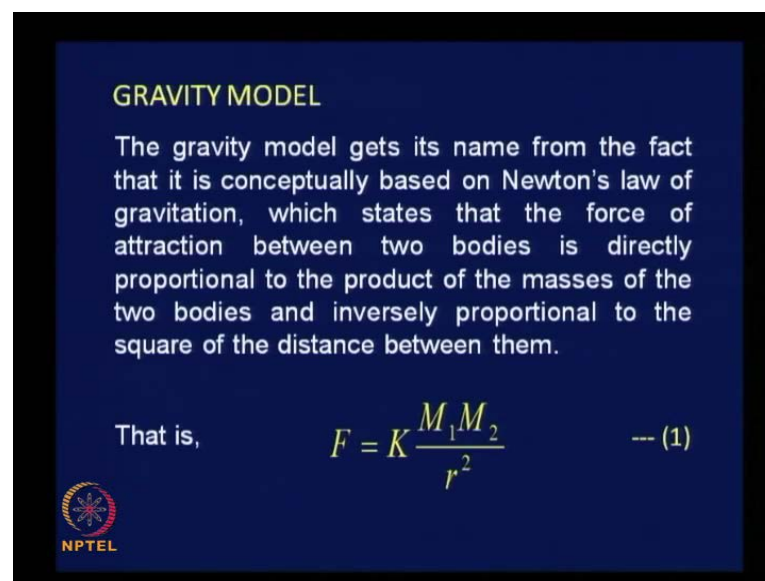
Travel time and generality cost, generality cost includes the aggregate money value that is including the travel time out of the ticket cost and level of comfort.

Yes, what you mean is the generalized the cost of transportation, it is major factor influencing trip distribution, that is one of the factors there are other factors that influence distribution of trips between traffic zones. We discussed about the relative

attractiveness of traffic zones. The amount of trip produces and attracted by different traffic zones, when the amount of trip attractive by a zone is large, obviously such a zone is slightly attracting more trips from various other zones. Similarly, if a zone is producing large number of trips the distribution of trip from that zone is slightly to extend over vast area covering more number of traffic zones is it not? So, that way we need to understand both the amount of trip produced, amount of trips attracted as well as some factor representing spatial separation between zones of the major factors.

We represent the spatial separations using the generalized cost of transport, which involves out occupied expenditure travel time, travel cost, comfort, connivance and so on, all transform into money value is it not? So that is how we need to understand the factors influencing trip distribution, basically there are three factors, trip production, trip attraction and generalized cost of transportation.


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GRAVITY MODEL

The gravity model gets its name from the fact that it is conceptually based on Newton's law of gravitation, which states that the force of attraction between two bodies is directly proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them.

That is,
$$F = K \frac{M_1 M_2}{r^2} \quad \text{--- (1)}$$

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Then, can anyone state the Newton's law gravitation, put up your hands if you are ready Newton's law of gravitation? Newton's law of gravitation states that, the force of attraction between two bodies is directly proportional to the product of masses of the two bodies and inversely proportional to the square of the distance between the two bodies is it not, that is the statement of Newton's law of gravitation. And mathematically we can give Newton's law of gravitation, the form of equation as given in equation 1, force of attraction F is equal to K times M 1 and M 2 divided by r square, r is the distance

between the two bodies, square of r is given in the denominator and K is the proportionality constant, because we represent the Newton's law in form of an equation here **right**. The same principal can be applied, to develop a model to explain trip distribution.


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The application of this concept to trip distribution takes the form,

$$T_{ij} = K \frac{P_i A_j}{W_{ij}^c} \quad \text{-----} \quad (2)$$

Trip productions and attractions of the zones replacing the masses of the bodies and the travel impedance between zones taking the place of r .

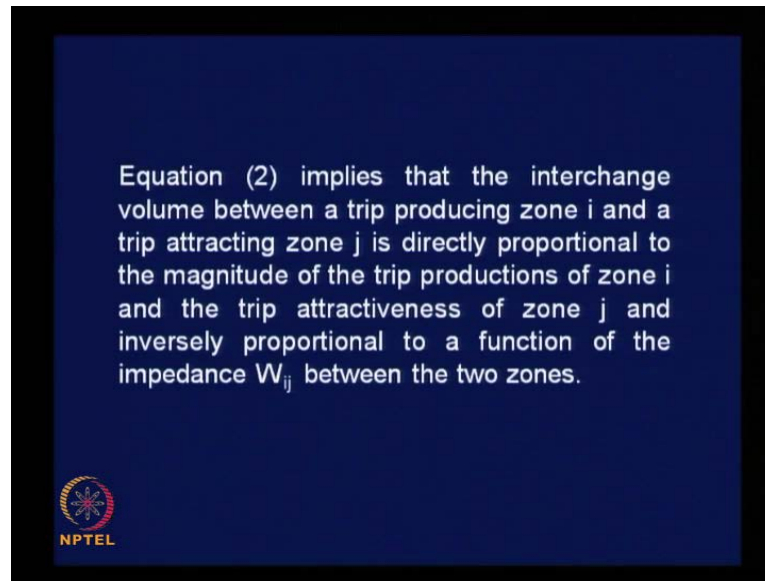
The exponent of the impedance term in the denominator, however, does not need to be exactly equal to two but may be replaced by a model parameter c .

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The application of this concept to trip distribution takes a form T_{ij} is equal to K times $P_i A_j$ divided by W_{ij} raised to power c , trip productions and attractions of the zones replacing the masses M_1 and M_2 is it not, of the bodies and the traveling impedance, which is proxy for generalized cost of transportation all the same travel impendent or generalized cost of transportation between zone taking the place of r .

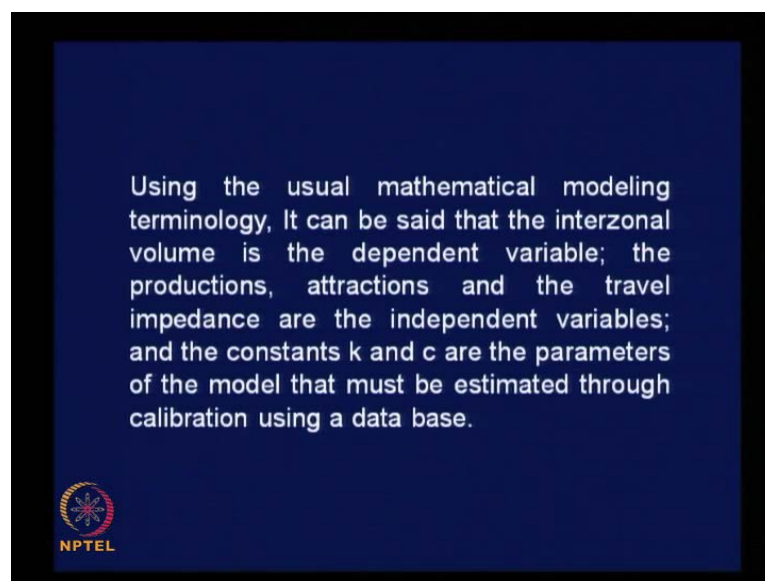
Otherwise, structurally this equation is similar to equation 1. The exponent of the impedance term the denominator, however that is not need to be exactly equal to 2 as in equation 1, you can take any value depending upon the actual field conditions, we have to really calibrate and find out the exact value of c , value of the exponent. May be replaced by a model parameter as shown here c , because it is going to be variable and it should be actually derived based on calibration.

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From the equation 2, previous equation T_{ij} to be equal to K times $P_i A_j$ divided by W_{ij} raised to power c , implies that the interchange volume between the trip producing zone i and a trip attracting zone j is directly proportional to the magnitude of the trip production, that is very important. The actual magnitude of the trip production of zone i and the trip attractiveness or again magnitude of the trips **attracted** attractiveness of zone j and inversely proportional to a function of the impedance, W_{ij} between the two zone in any zonal **(()) clear** that is a meaning of the previous equation.

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And the same equation or the same statement, that we made just now can be understood in a mathematical format like this, it can be said that the interzonal volume T_{ij} , interzonal volume is nothing but, the T_{ij} in the equation is the dependent variable. The productions P_i , attraction A_j and the travel impedance W_{ij} raised to power c or the independent variables, is it not and the constants K and c on the parameters of the model, that must be estimated through calibration using a data base.

Let us say, we are trying to calibrate a trip distribution model for base year condition. In the base year, you will have information about the actual trip distribution between zonal (i, j) that can be collected and it is possible to get T_{ij} value for your study area and P_i A_j values also will be known to you, so you have values for T_{ij} P_i and A_j using this values it is possible to find the values for the term K and c . And this process is what we call as calibration on trip distribution model.

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The parameter k can be eliminated from equation (2) by applying trip production-balance constraint, which states that the sum over all trip attracting zones j of the interchange volumes that share 'i' as the trip producing zone, must equal the total trip productions of zone i or

$$P_i = \sum_{j=1}^n T_{ij} \quad \text{--- (3)}$$

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Of course, the parameter k can be eliminated from equation 2 by applying trip production balance constrained **trip production balance constrained**, which states that the sum over all trip attracting zones j **right** of the interchange volumes, that share i trip producing zone i as a trip producing zone must equal the total trip production of zone i , you may recall when we discussed the convenient format of presenting trip distribution data, we discussed about PA matrix. We also know that if you had the cell elements along a row you will be getting the total trip production pertaining to that particular zone, we will be

adding $t_{11}, t_{12}, t_{13}, t_{14}$ and so on up to t_{1n} , then sum of all these cell values will give the total trip production for zone 1, P_1 will be obtained **right**, so that is information we have and use this formulation as a constraint in equation 2; and let say, how we can modify equation 2 using this constraint. This is the statement when we say that, sum of all the cell values along a row is **(())** is nothing but, P_i is equal to $\sum_{j=1}^n T_{ij}$, T_{ij} is nothing but, a cell value **right**, when you add cell values from 1 to n, you get P_i value total trip production.

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Substituting the value of T_{ij} from Eqn. (2), in Eqn. (3), we get

$$P_i = \sum_{j=1}^n \frac{K P_i A_j}{W_{ij}^c} \quad \text{--- (4)}$$

$$P_i = K P_i \sum_{j=1}^n \frac{A_j}{W_{ij}^c}$$

$$1 = K \sum_{j=1}^n \frac{A_j}{W_{ij}^c}$$

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Now, substitute the value of T_{ij} from the previous equation in equation 3, we get P_i can be written as $\sum_{j=1}^n K P_i A_j / W_{ij}^c$. Agree, we are substituting the value of P_i , just let us go back and see, this is what we do and we know the value of T_{ij} from equation 2. As per equation 2 T_{ij} is what? It is equal to $\sum_{k=1}^n K P_i A_j / W_{ij}^c$, that is what we do, we simply substitute that value instead of T_{ij} in this equation and you get this formula. P_i becomes $\sum_{j=1}^n K P_i A_j / W_{ij}^c$.

Let us call this equation as equation 4 and we can simplify this, \sum is over j only is it not, and K and P_i will not have any effect as far as the summation is concerned, take those two quantities outside the \sum sign and you can write P_i to be equal to $K P_i / \sum_{j=1}^n A_j / W_{ij}^c$ and P_i , P_i gets cancelled on both sides and we get simple 1 to be equal to $K \sum_{j=1}^n A_j / W_{ij}^c$


W_{ij} raised to power c . From this K can be written as 1 by $\sum_{j=1}^n$ is equal to 1 to n right, A_j by W_{ij} raised to power c or you can give the same information in different form instead of writing 1 by you can say raised to power minus 1 .

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From equation (4), K can be written as

$$K = \left[\sum_{j=1}^n \frac{A_j}{W_{ij}^c} \right]^{-1} \quad \text{--- (5)}$$

The expression for K ensures that the trip balance equation is satisfied.


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That is, what I have done from equation 4 K can be written as K equal to $\sum_{j=1}^n$ is equal to 1 to n A_j by W_{ij} raised to power c the whole thing raised to power minus 1 . Let us call this equation as equation 5. Now, the expression for K ensures that, the trip balance equation is satisfied, do you agree with this statement? The expression for K ensures that the trip balance equation is satisfied, what do you understand by trip balance equation? It is it nothing but, the fact that sum of all the cell values in the matrix along a row is equal to the total trip production for a particular traffic zone.

And that was given as input, that information was given as input and finally, we have just got the value of K to be this much, that is the implication. So we have got the value of K using that particular constrained in the form of A_j or involving A_j and W_{ij} raised to power c right, try to put this value of K in the original equation that, we got for T_{ij} equation 2, T_{ij} is equal to $\sum_{j=1}^n k$ times $P_i A_j$ divided by W_{ij} raised to power c , so that K within \sum is going to be replaced by this quantity, if we do so we will get this equation right.

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Substitution of the value of K, from equation (5), in equation (2) leads to the classical form of the gravity model.

$$T_{ij} = \frac{\left(\frac{1}{\sum_{j=1}^n \frac{A_j}{W_{ij}^c}} \right) P_i A_j}{W_{ij}^c} \quad \text{--- (6)}$$



So, we can write T_{ij} to be equal to $\frac{1}{\sum_{j=1}^n \frac{A_j}{W_{ij}^c}}$ multiplied by $P_i A_j$ divided by W_{ij}^c is it not, I am simply substituting value of K from equation 5, the previous equation in equation 2 and this leads to the classical form of gravity model clear and if you can modify and write the same equation in a compact form you can take F_{ij} to be equal to $\frac{1}{W_{ij}^c}$, this is just for convinces.

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Writing $F_{ij} = \frac{1}{W_{ij}^c}$

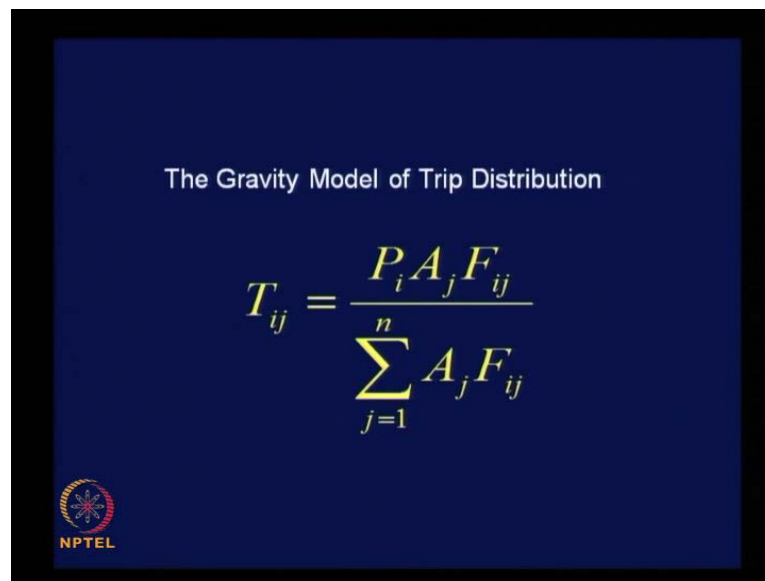
NOTE: The calibration constant c is now implicit in the friction factor, F_{ij}

Equation (6) can be written as,

$$T_{ij} = \frac{P_i A_j F_{ij}}{\sum_{j=1}^n A_j F_{ij}} \quad \text{--- (7)}$$


We change or take 1 by W_{ij} raised to power c as F_{ij} and the calibration constant c is now implicit in the frictions factor F_{ij} that is a point, because we take F_{ij} to be W_{ij}^{-c} that means when you estimate the value of F_{ij} you actually estimate value of c , that is the implication then using F_{ij} instead of one by W_{ij} raised to power c we can write the previous equation in compact form like this T_{ij} can be written as $P_i A_j F_{ij}$ divided by $\sum_{j=1}^n A_j F_{ij}$ right.

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The Gravity Model of Trip Distribution

$$T_{ij} = \frac{P_i A_j F_{ij}}{\sum_{j=1}^n A_j F_{ij}}$$


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And so the gravity model of trip distribution can be remembered in this form T_{ij} trip interchange between zones i and j a number of trips interchange between zones i and j is given by P_i trip production in zone i A_j trips attracted to zone j into a friction factor F_{ij} whole divided by sum over j from 1 to n of the product of A_j and F_{ij} ; please remember friction factor is zonal pair specific F_{ij} it should be like that, is init. Travel impedance cannot be same for our zonal pairs it should be specific to a zonal pair under consideration that is why it is F_{ij} clear.

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Finally, a set of interzonal socio economic adjustment factors k_{ij} are introduced during calibration to incorporate the effects that are not captured by the limited number of independent variables included in the model.

Then, the resulting gravity model formula becomes,

$$T_{ij} = \frac{P_i A_j F_{ij} K_{ij}}{\sum_{j=1}^n A_j F_{ij} K_{ij}} \quad \text{--- (8)}$$


And finally, a set of interzonal socio economic adjustment factor given by notation small K_{ij} lower case K_{ij} are introduced during calibration to incorporate the effects that are not captured by the limited number of independent variable included in the model. We have just only three independent variables in this model T_{ij} is the dependent variable, P_i , A_j and F_{ij} with indicated the travel impedance or the independent variable, sometimes it may not be possible to explain the trip interchanges T_{ij} with these three independent variables also only **right**.

In that case, there may be a need introduce the factor given the notation K_{ij} to take care of the unexplained part of the distribution of trips between zones by the three independent variables, will it happen like this in practice. You consider the case of two traffic zones where in one zone there is a huge employment attraction center, let say public sector manufacturing plant like BHEL factory employing more than 10000 employees.

And normally it is see practice in India that large scale public sector undertakings develop residential quarters also for the employees to accommodate as much of the employees as possible, normally more than 80 percent of the employees a public sector undertakings or residing in the quarters of that particular company and when they develop such residential quarter, they will see that, they were as close to the employment place as possible so that commuting time can be reduced, while occurring land for the

factor itself they will keep this residential requirement in mind and they will acquire contiguities land spaces.

Both for the employment location as well as for residential location, so when you come across such a situation in a urban area, then you will find that a large number of work trips or attracted between this two zones employment center in one zone and large reason as residential colony in another zone, there will be a large number of trip attraction or trip interchanges between this two zones compare to all other zonal pairs. So this is the special case your model developed based on trip interchanges over the entire urban area may not be able to explain this particular situation, so in such case you must check to what extent it is higher than the normal average trip interchange.

And then to compensate for that introduced a coefficients indicate by K_{ij} for this particular zonal pair, if trip interchanges among majority of other zonal pair or matching with average that you get using the model, then you can take K_{ij} to be simple 1, it will not make any difference may be in that particular case you may have to take a K_{ij} value much higher than 1 may be 2 or 3 depending upon the situation and other possible examples are there may be a particular religious or ethnic group of people living in one traffic zone their place of worship might be located in a different traffic zone, then you will find that there is large number of trip interchanges between these two zones compare to the average trip interchange in zonal pairs in urban area **right**.

So, these are all the situations which may not be effectively explained by with general gravity model, so that is the purpose of introducing this particular factor indicated by small K_{ij} zone to zone adjustment factor to take into account the effect of socio economic connectivity between zonal pairs, then the resulting gravity model formula becomes as given equation 8. T_{ij} as to be written as $P_i A_j F_{ij} K_{ij}$ zone to zone adjustment factor divided by $\sum_{j=1}^n A_j F_{ij} K_{ij}$ we are introducing K_{ij} both in the numerator as well as in the denominator.

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The General Formulation of the Gravity Model:


$$T_{ij}^m = \left[\frac{P_i A_{jk} F_{ij}^m K_{ij}}{\sum_{j=1}^n A_{jk} F_{ij}^m K_{ij}} \right]_p$$

where,

T_{ij}^m = trips produced in zone i and attracted to zone j by mode m in kth iteration

F_{ij}^m = empirically derived travel time factor for mode m which expresses the average area-wide effect of spatial separation on trip interchange between zone i and zone j.

k = iteration number, p = trip purpose



The general formulation of the gravity model very general formulation we are trying to generalize it further can be written like this, please note we have written T_{ij}^m there is one more subscript T_{ij}^m and we have introduced superscript for T it is T_{ij}^m it is T_{ij}^m is equal to $P_i A_{jk} F_{ij}^m K_{ij}$ for trip attraction also we are introducing the subscript k $A_{jk} F_{ij}^m$ or F_{ij}^m we can say into K_{ij} whole divided by $\sum_{j=1}^n A_{jk} F_{ij}^m K_{ij}$ whole to within brackets and for the bracketed quantity we have given a subscript p , so this is a very generalized formulation of gravity model.

That is a reason for introducing all this subscripts and superscripts T_{ij}^m is this trips produced in zone i and attracted to zone j by mode m in kth iteration F_{ij}^m is empirically derived travel time factor or friction factor for mode m for a specific mode for mode m which expresses the average area wide effect of spatial separation on trip interchange between zone i and zone j, k is iteration number and p trip purpose this is a very general form of in gravity model.

Now we should be very clear to answer the following questions, should we have to do trip distribution based on the mode used for trip making is it necessary, any responds why not put the trips made by all the modes together and distribute the trips as simple person trips is it necessary to distribute the trips based on mode used for travel necessary or not of course, we equation implies that it is necessary you consider trips made by foot walk trips what would be the extent of distribution of trips from one traffic zone may be

it may extent over the adjoining traffic zones, the distance involved in not more than 1 and half hour 2 kilometers maximum. So distribution zone is becoming small and the zonal paires involve.

Or going to be adjustment zonal pairs we consider trips made by bicycle it may extent little more covering few more zones but, still the distance covered am not be normally more than 5 6 kilometers **right** that will be regional distribution of trips by that particular mode of bicycle. If you consider public transport trips it may extend widely covering the urban area which is cover by the route of operation by the transit service there mean may be areas where you do not have good public transit service and if you take the trip interchange using personal motorized vehicles like motor cycle, car a trip interchange distance involved and the pattern will be totally different.

So, we should be convinced that trip distribution should be done after segmenting data based on mode used for travel, that is why we discussed mode choice analysis first before getting into trip distribution analysis **right** it will be more accurate and appropriated if we distribute trips based on mode used for travel. Additionally it will be more appropriate if segment data further based on the purpose of trip; trip purpose if you take a trips made for work, the employees are no control of the location of the work place.

Some employees may have to travel very long distance, commute very long distance for work some may have to travel medium distance and short distance and so on, these are mandatory trips and the distribution pattern of trips for work will be totally different from the distribution pattern of trips for shopping for example, from a particular zone shopping we have a choice of going to this place or that place depending upon our need and weight of the purse, you cannot say that shopping trip distribution will be similar to work trip distribution. Similarly, trips made for educational will have different pattern may not be all as long as the work trips may be slightly less than the length of the work trips, because you have some choice limited choice for fixing your place of education for rewards.

So it is appropriate to segments a data based on trip purpose also, if you segregate data based on more use for travel and within that a trip purpose you will be getting highly homogenous data base for calibrating for trip distribution model which will be very

effect in predicting the future trip distribution. So that is the idea of introducing m and P then, why K iteration number why should we do iteration and getting T_{ij} value, please remember we have introduced one constrained already in this model formulation with regard to trip production, the sum of the cell values along the row should be equal to total trip production that constrained was taken into the modeling process itself we derived K value based on the constrain.

So, this formula will give a result which will give this are the total of cell value is to be total trip production will not be any change but, there is a another constrained related to the trip distribution matrix the column totals cell **cell** values along a column should give you the total trip attraction by each of the traffic zones, that this constrains was not brought into your model formulation **right**. So, when you used this gravity model or when you tried to theoretically explained a trip distribution, we may get some trip distribution pattern or some cell value for each other matrixes, that will added to the condition that along a row the total of the cell value gives you the total trip production.

But need not added to the another constrain, namely the cell values along column should be equal to the trip attraction for the traffic zones, that will not be added to, because that constrained as not be brought into our model formulation **right**. We are trying to explain theoretically it is attend trip distribution process and try to get the cell values in the matrix finally, and since we have introduced one constrained so trip production values will not get affected in the result, trip attraction values will change there will be some changes and we must try to bring those attraction values also to match with the actual observed attraction values by some iterated process **right**.

That is why our to imply that, we have introduced this subscribe K here and K is introduced in respect of T_{ij} and A_j only, you can look at because it is related to attraction. Iterated process will influence A_j value which will result in a change in T_{ij} value, that is why iteration is necessary clear and with this understanding, let us take very simple numerically example and see how gravity model can be applied to this particular case, the number of work trips produced in attracted to three zones 1 2 and 3 by public transit or as under clear.


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Example:

The number of work trips produced in and attracted to three zones 1, 2 and 3 by public transit are as under:

| Zone | 1 | 2 | 3 | Total |
|---------------------------|----|----|----|-------|
| Trips produced (P_i) | 14 | 33 | 28 | 75 |
| Trips attracted (A_j) | 33 | 28 | 14 | 75 |

The friction-factor values between the various zones, obtained as a result of calibration, can be taken from the following matrix:




Now, this segmentation information is already given to you this pertness to work trips data is already segregated based on trip purpose and mode is public transit. So, what you get here is segmented data are two levels one based on mode used other one based on trip purpose already data given to you is segmented clear. Zone number trip produced P_i and trips attracted A_j zone 1 produces 14 trips and attracts 33 trips, zone 2 produces 33 trips attracts 28 trips and zone 3 produces 28 trip ends infect and attracts 14 trip ends and total of the trip ends produce and attracted or 75 each.

This information given to us these trip ends paten to trip made for work by public transit the friction factor F_{ij} values between the various zones obtained as a result of calibration can be taken from the following matrix, the calibration result is also given to you because we are trying to understand the application of gravity model in this case.

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| P \ A | 1 | 2 | 3 |
|-------|----|----|----|
| 1 | 13 | 82 | 41 |
| 2 | 50 | 26 | 39 |
| 3 | 50 | 20 | 41 |

Distribute the trips between the zones taking the zone to zone adjustment factor $K_{ij} = 1$




So, you have been given the result of calibration F_{ij} values for the different zonal pairs are given to us, these are the F_{ij} values. As I told you friction factor is specific to zonal pairs, so obviously it will be in the form of a matrix.

And this is what we need to do distribute the trips between the zones taking the zone to zone adjustment factor K_{ij} to be equal to 1, further simplification since we have only three zones it is found there are no special socio economic factors, that deviate or gives you significantly different value compare to what you will be getting by using the gravity model. So, K_{ij} can be taken to be 1 for all the three zonal pairs clear, so this is the information given so it is going to be direct application of the given values in the equation for trip distribution.

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Solution:


The general form of the gravity model for this case is as follows:

$$T_{ij_k} = \left[\frac{P_i A_{jk} F_{ij} K_{ij}}{\sum_{j=1}^n A_{jk} F_{ij} K_{ij}} \right]$$


The general form of the gravity model for this case can be written as follows, I am not given the super scribe m because mode is already given to this I also removed this subscript p at the bottom of the bracket, because the purpose is also given to us. We distribute trip made for work and within work trips **trips** made by public transit clear, so m and p are gone, you are not introducing those subscript as well as superscripts. So, we can write T_{ij_k} to be equal to $P_i A_{jk} F_{ij} K_{ij}$ whole divided by $\sum_{j=1}^n A_{jk} F_{ij} K_{ij}$ and what is the value of K_{ij} in this case, K_{ij} has been given as 1 the value of zone to zone adjustment factor is 1 and value of n is three we have only 3 zones.

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Since there are only three traffic zones and since $K_{ij} = 1$ for all zonal pairs for the given problem, the equation becomes,

$$T_{ij_k} = \left[\frac{P_i A_{jk} F_{ij}}{\sum_{j=1}^3 A_{jk} F_{ij}} \right]$$


So we can further simplified this equation based on the fact, that K_{ij} is equal to 1 and number of traffic zones are only 3, we can write T_{ijk} to be simply equal to $P_i A_j k F_{ij}$ divided by j is equal to 1 to 3 $A_j k F_{ij}$; now we have been given the values $P_i A_j$ as well as F_{ij} friction factor values are also given **right** and we need to work out only T_{ij} values clear.

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First iteration ($k = 1$)

$$T_{11} = \frac{P_1 A_1 F_{11}}{A_1 F_{11} + A_2 F_{12} + A_3 F_{13}}$$

$$= \frac{14 * 33 * 13}{33 * 13 + 28 * 82 + 14 * 41} = \frac{6006}{3299} = 1.82$$

$$T_{12} = \frac{14 * 28 * 82}{3299} = 9.74 \quad T_{13} = \frac{14 * 14 * 41}{3299} = 2.43$$

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Simple substitute the corresponding values in this equation, accordingly we can write T_{11} interzonal trip distribution within traffic zone 1 to be $P_1 A_1 F_{11}$ divided by $A_1 F_{11} + A_2 F_{12} + A_3 F_{13}$ and substitute the values 14 into 33 into 13, 14 is P_i value 33 A_j and 13 F_{ij} whole divided by 33 into 13 plus 28 into 82, 82 is again F_{ij} value plus 14 into 41 that is equal to 1.82 it is a fraction **fraction** we will have the fraction as it is at this stage and finally, we have to round it off to the nearest whole number because T_{ij} is nothing but, the number of trips distributed between traffic zones it cannot be in fraction.

Since, we are the middle of we analysis we need not have to worry about rounding the fractional part to the whole number at this stage; similarly, T_{12} since denominator is same and just putting the final value of the denominator numerator only will change 12 is 14 into 28 into 82 whole divided by 3299 that is equal to 9.74 **right** and then T_{13} between 1 and 3, 14 into 14 into 41 whole divided by 3299.

Same denominator which gives you the T_{ij} value of 2.43, so T_{11} 1.82, T_{12} 9.74, T_{13} 2.43 that is our interest, this is distribution of trips from one to other zones.

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$$T_{21} = \frac{33 * 33 * 50}{33 * 50 + 28 * 26 + 14 * 39} = \frac{54450}{2924} = 18.62$$


$$T_{22} = \frac{33 * 28 * 26}{2924} = 8.22$$

$$T_{23} = \frac{33 * 14 * 39}{2924} = 6.16$$

And similarly, let us do the calculation for zone 2 to 1, T_{21} , 33 into 33 into 50 ; 33 stands for what? T_{21} is nothing but, T_{ij} , obviously 33 the first 33 stands for production in zone 2 and the next 33 stands for attraction in zone 1 and 50 is F_{12} , the friction factor for travel between 1 and 2 is T_{21} from 1 to 2 right.

whole divided by 33 into 50 plus 28 in 26 plus 14 into 39 that gives you the value of 18.62 trip interchanges between 2 and 1, T_{22} interzonal trip interchanges in zone 2 33 into 28 into 26 whole divided by the same denominator 2924 is the value of 8.22 , then T_{23} 33 into 14 into 39 whole divided by 2924 6.16 ; so from zone 2 we have trip interchanges of 18.62 8.22 and 6.16 .

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$$T_{31} = \frac{28 * 33 * 50}{33 * 50 + 28 * 20 + 14 * 41} = \frac{46200}{2784} = 16.59$$
$$T_{32} = \frac{28 * 28 * 20}{2784} = 5.63$$
$$T_{33} = \frac{28 * 14 * 41}{2784} = 5.77$$


Similarly, from zone 3, T 3 1 is 28 into 33 into 50 whole divided by 33 into 50 plus 28 in 20 plus 14 into 41 that is equal to 16.59, trip interchanges between 3 and 1, T 3 2 28 into 28 into 20 whole divided by 2784 that is equal to 5.63, T 3 3 28 into 14 into 41 whole divided by 2784 5.77 right.


So, we have done the calculation involving all the three traffic zones, let us put this numbers in a matrix form and that take overview and check whether we have arrived at the final result or we need to do some more iterations right

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The matrix then becomes:

| P \ A | 1 | 2 | 3 | Total P _i |
|--------------------------|-------|-------|-------|----------------------|
| 1 | 1.82 | 9.74 | 2.44 | 14.00 |
| 2 | 18.62 | 8.22 | 6.16 | 33.00 |
| 3 | 16.59 | 5.63 | 5.77 | 28.00 |
| Total C _j (1) | 37.03 | 23.59 | 14.37 | |

It can be seen that the total trip productions, after the distribution, match with the corresponding predicted values, but the attractions do not equal the predicted attractions. Further iterations are, therefore, necessary.



This is the result given in matrix form $T_{11}, T_{12}, T_{13}, T_{21}, T_{22}, T_{23}$ and T_{31}, T_{32}, T_{33} is initial, that is what we calculated and if you look at this sum of the cell values along rows to get a number as 14, 33 and 28 trip production values for zones 1, 2, 3 or respectively 14, 33 and 28 these are the actual given values to us initially, when you started applying the gravity model there is no change in the PA values where as if you look at the total of the cell value along columns, we get values of 37.03 for zone 1 23.59 for zone 2 and 14.37 for zone 3, what were the actual values of attraction for zones 1, 2 and 3.

Now, what we do with this result can we say that, this distribution is acceptable it is not because, a distribution process should not distort the basic values of trip production and attraction. A trip attraction zonal trip attraction values are distorted, because of the distribution process the reason is we have assumed a set of independent variables to be influencing the trip distribution and use those independent variables to get to distribution and in the development of the model we introduced one constraint, to see that the total of the cell values along a row gives you trip production, that constraint was introduced in the model, that is how we are getting undisturbed values, unchanged values are of trip production for a zone.

The other values trip attraction values are changed, now we must restart to some iterated process to see that, the trip attraction values also brought back to the original values. Simultaneously holding trip production values unchanged, that will be held because it is model the framework itself and let see how to go about doing this of course, the statement is given here it can be seen that the total trip productions, after the distribution match with corresponding predicate values but, the attractions do not equal the predicated attractions further iteration as therefore, necessary now the question is, how to go about doing iteration?

Our main objective is to push this disturbed values close to the actual observed value for each of the zones that is our intention


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SECOND ITERATION:

The following formula can be used to adjust the trip attraction values for subsequent iterations.

$$A_{jk} = \frac{A_j}{C_{j(k-1)}} * A_{j(k-1)}$$

Where,
A_{jk} = adjusted attraction, iteration k
A_j = desired attraction
A_{j(k-1)} = adjusted attraction, iteration (k-1)
C_{j(k-1)} = calculated attraction, iteration (k-1)



So, this is how we do iteration the following formula can be used to adjust trip attraction values for substantial iterations, we say A_{jk} to be equal A_j divided by C_{jk} minus 1 into A_{jk} minus 1, where A_{jk} is equal to adjusted attraction partnering to iteration k **right**, A_j desired attraction which as originally to input for the model desired attraction, A_{jk} minus 1 adjusted attraction partnering to iteration k minus 1.

C_{jk} minus 1 is equal to calculated attraction as per iteration k minus 1 **right**, so we have some value of trip attraction as result of the calculation indicated here as C_{jk} minus 1 that is what we get as the total of the cell values along each column at the end of the calculation **right**, that is indicate here as C_{jk} minus 1, A_{jk} minus 1 is nothing but, the attraction values given initially for that particular iteration as trip attraction values A_j is the desired value, which was attained originally as trip attraction values for the traffic zones **right**.

Let say your calculate value is small compare to A_j value desired value, then you may have to increase the trip attraction values some by sum means **right** that is what we do see when we have a smaller value in the denominator corresponding to the observed value and multiplied that, by the trip attraction value that you have given as input for that particular iteration for the subsequent iteration you are going to get a value higher than the value that used for previous equation isin it. So that is how A_{jk} will be defiantly higher than C_{jk} minus 1 **right** instead if C_{jk} minus 1 is larger and A_j, then you will

get reducing effect, because a denominator is larger **right** so that is how we intimately adjust the trip attraction value and give the adjusted value as initial trip attraction value for subsequent iterations and this iterated process is continued until your calculated value of attraction and the desired value are matching to a significant extent. This is how we just adjust a trip attraction value for subsequent iterations, let us stop here and recollect what we did today.

We essentially understood this structure of the gravity model for trip distribution in this class and we also know, how the trip production constrained is introduced in the gravity model formulation, to make sure that while your distributing the trips trip production values for the zones are not affected and only trip attraction values are influenced by the distribution process and finally, we have discussed about a methodology to adjust the trip attraction value also to get the desired or actual observe values of trip attraction for each of the traffic zones so that, you actually distribute the trips based on the effective to casual factors without affecting the basic information related to trip production and trip attraction of a traffic zones, will continue our discussion the next class.