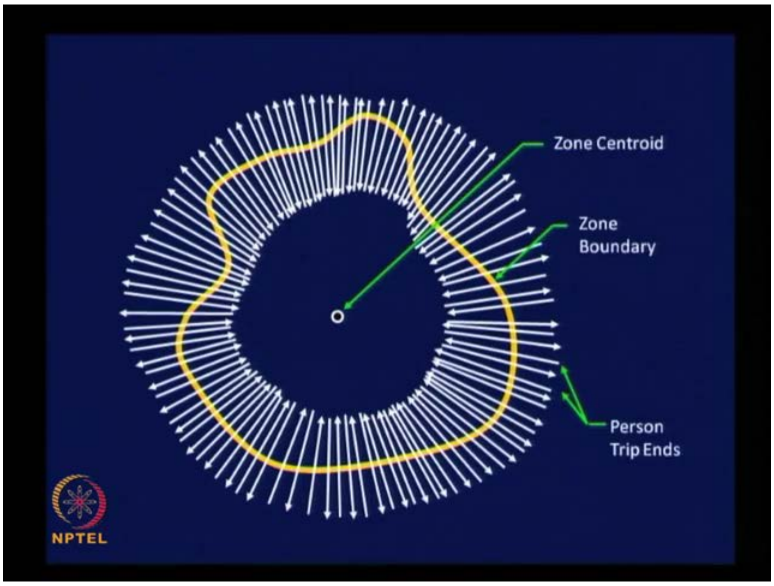
Urban Transportation Planning
Prof. Dr. V. Thamizh Arasan
Department of Civil Engineering
Indian Institute of Technology, Madras

Lecture No. # 20
Trip Distribution Analysis

This is lecture 20 on urban transportation planning. We will discuss on trip distribution analysis in this class, which is the third major step in the four step planning process. At the end of the first step namely trip generation analysis, we obtain this result.

(Refer Slide Time: 00:33)



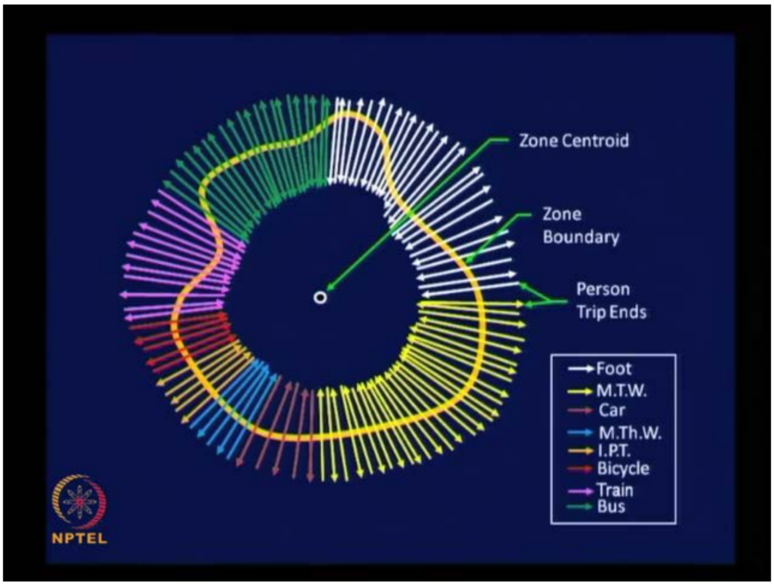
We considered zone centroids as points of origins and destinations for all the trips, and then we obtain a trip ends associated with each of the traffic zones. As person trip ends without identifying the mode used by individuals, we just obtain person trip ends in total associated with each of the traffic zones at the end of step one, namely trip generation analysis, which includes both trip production as well as trip attraction analysis.

In step two namely mode choice analysis or modal split analysis what we did is this, we try to assign modes for these trip ends, we try to find out how many of these trip ends will be associated with the mode food, how many will be associated with the motorized two wheeler, how many will be associated with the another mode say bicycle, and so on; that is what we did.

In that process, we found that disaggregate mode choice modal which considers each individuals as separate entity for the purpose of mode choice analysis is more appropriate, because other methodologies assume average behavior of travelers at zonal level which may not accurately replicate the reality.

Then we also assume that utility of alternatives modes cannot be treated as a constant quantity, because the perception of utility of modes varies from individual to individual. So, we treated utility as the random quantity and considered each, and every individual as independent entity for the purpose of mode choice analysis based on these two aspects, we found that logic modal of mode choice is a right analytical procedure which can provide appropriate result for mode choice analysis. And we have also seen a set of case studies in the application of logic modal of mode choice.

(Refer Slide Time: 03:12)

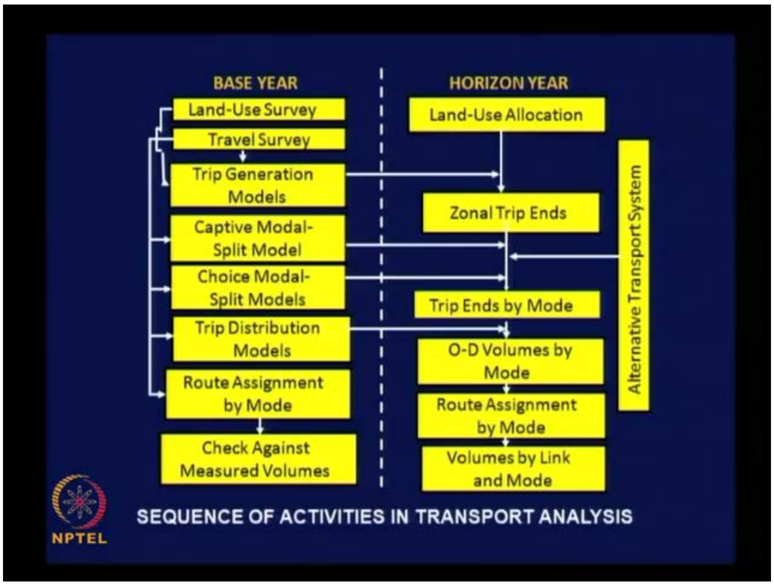


And at the end of mode choice analysis this is what we will get these white lines will assume different shades with different proportions. Each color pertains to a particular mode of travel this is what we essentially did at the end of mode choice analysis. You can see we have indicated white colored once as trip made by foot, yellow colored once trip made by two wheeler, and red once trip made by car, blue one motorized three wheeler, and then the other one by I P T, then trips made by bicycle, train and bus these were the different alternative modes that we discussed under Indian conditions.

So, this is what we do at the end of mode choice analysis still, we are dealing with trips ends only which are zone based the only difference we have made is that we have associated modes for the trip ends. Otherwise data still is zone based and these are all person trips.

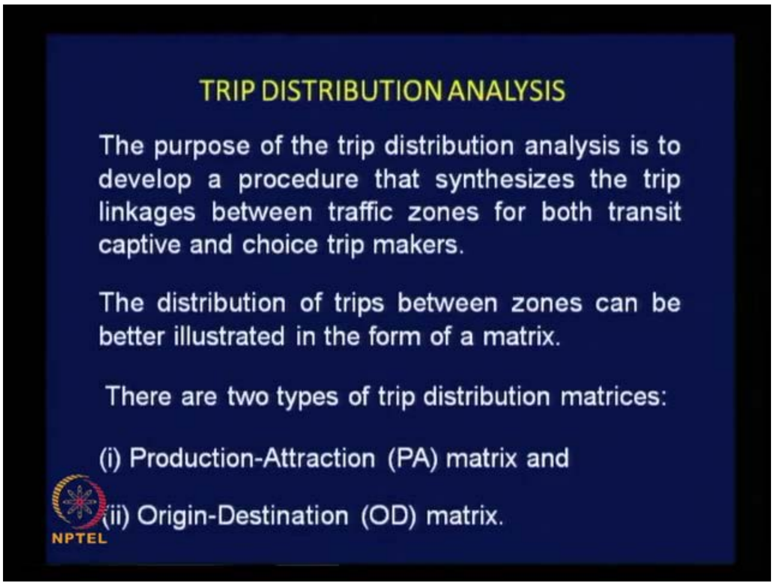
Not vehicular trips person trips with identification of the modes used for the trips that is it with this background, let us proceed with the next step. And let us also identify in the master flow chart where exactly we are now when we start our third step of the analysis.

(Refer Slide Time: 5:00)



We have completed as you can recall trip generation modals, captive modal, split modal, choice modal, split modals; these three boxes are completed now, we are into the forth box namely trip distribution modeling exercise.

(Refer Slide Time: 05:24)



And trip distribution is aimed at developing a procedure that synthesizes a trip linkage between traffic zones for both transit captive and choice trip makers. Please recall when we started mode choice analysis, we segregated transit captive riders, and did the analysis only for choice riders, when we do the third step we put both the categories of travelers together and do the analysis for both the groups.

Both for transit captive as well as choice riders all the trips are to be distributed and the distribution of trips between zones can be better illustrated in the form of a matrix. Why in the form of a matrix, because we are going to study distribution of trips from one zone to all the other zones.

If there are one hundred zones in an urban area we are going to study how the trips or trip ends at zone one going to be distributed at all the other zones including zone one there could be trips distributed within the zone also. So, it is if you when you consider zone by zone it become 100 by 100 analysis. So, that way it will be easy for us to present trip distribution data in the form of a matrix that is the idea.

And there are two types of trip distribution matrices the first one is Production-Attraction P A matrix because as you may recall any traffic zone will produce trips or attract trips or simultaneously, there will be productions as well as attractions in a particular traffic zone. So, when you develop a matrix it is going to be production attraction matrix, but please remember later on when you get into step four for traffic assignment we need to know the direction of movement also the exact origin and destination.

To give input for route assignment analysis, we may have to prepare another matrix name OD matrix, Origin-Destination matrix, this OD matrix can be prepared based on P A matrix production attraction matrix. So, we can say here there are two types of matrices number one production attraction or PA matrix number two origin destination or O D matrix.

(Refer Slide Time: 08:44)

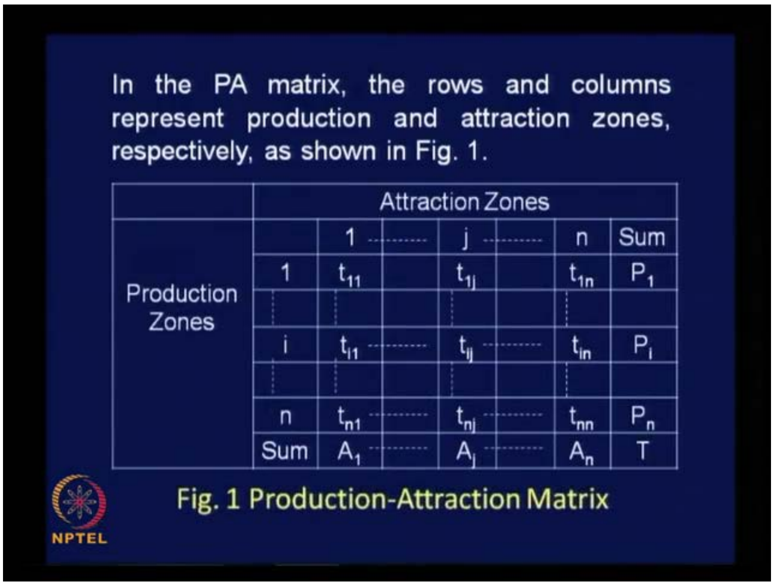


Fig. 1 Production-Attraction Matrix

Let us first take production attraction matrix, and see how this matrix is prepared and how we understand the data set given in a PA matrix this is the PA matrix we are taking production zones along the row in the table and attraction zones along the car, production zones along rows and attraction zones along cars. I have just indicated the number of zones as one two etc; up to n both along the row as well as along the column.

And the cell values in the matrix gives a particular information to us the cell value t i j means what? This cell value will give the number of trips produced at zone i and attracted to zone j that number will be given in this cell t 1 j; for example, will give us the information about number of trips produced at zone one, and attracted to zone j right and if you sum up all the cell values along the row.

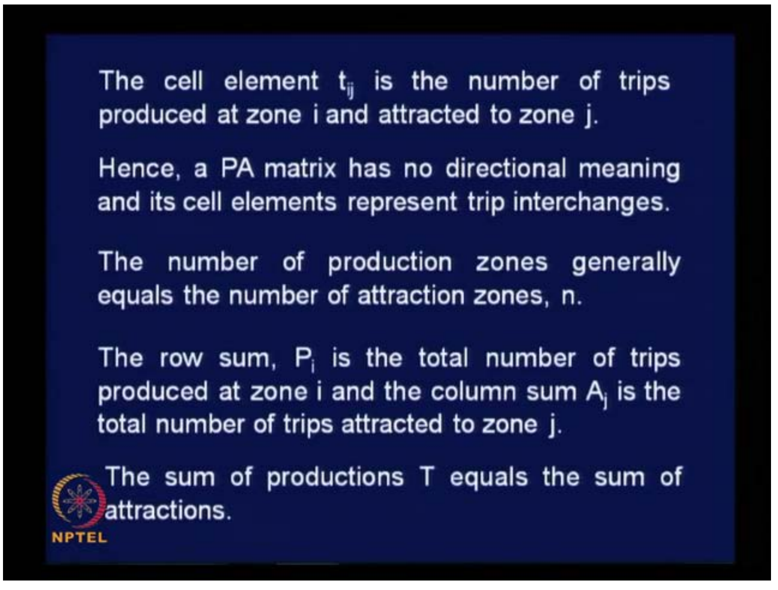
What would be the result? The sum should give us for example, if we take zone one the first cell value gives t 1 1 trips produced at one and attracted to one itself then we will have one tow one three and so on up to 1 n; that means, the distribution pattern pertaining to zone 1 is given along one row. So, if you sum all the cell values you must get the total of the trip production.

Of that particular zone, because we just distribute the total trip production between the zone under consideration and the all other zones so; obviously, when you add up all the cell values along a row you need to get the total trip production of that particular zone.

Similarly if you sum all the cell values along the column, let us say t 1 1 2 and so on, and n 1.We have indicated that value as A 1 that is nothing but the total of the trips attracted by zone one the sequence will be like this 1 1, then what will be the cell value here?

This is produced in one attracted to one here it will be produced in two attracted to one produced in three attracted to one right; all these things are trips attracted to one. See, last value is produced in n attracted to one. So, that is how the column total should give you the total trip attraction, in a particular traffic zone right. So, this is how presentation of the trip attraction distribution data in the matrix form is very helpful.

(Refer Slide Time: 12:47)



To summarize the same thing we can say the cell element t i j means the number of trips produced at zone i and attracted to zone j. Hence, a P A matrix has no directional meaning and its cell elements represent trip interchanges only. It is a very important statement, you cannot find out the direction of movement by looking at a PA matrix it just gives you information about total number of trip interchanges between zonal pairs.

There is no information provided with regard to the direction of movement. So, this as to be understood very clearly and the number of production zones generally equals number of attractive zones in practice. The row sum p i is the total number of trips produced at zone i as we understood, and the column sum A j is the total number of trips attracted to zone j.

The sum of productions T equals the sum of attractions. Finally, we found that some of the sums along the row and column should also be equal, because any way total number of trip should have got attracted somewhere within the study area normally both are equal.

(Refer Slide Time: 14:28)

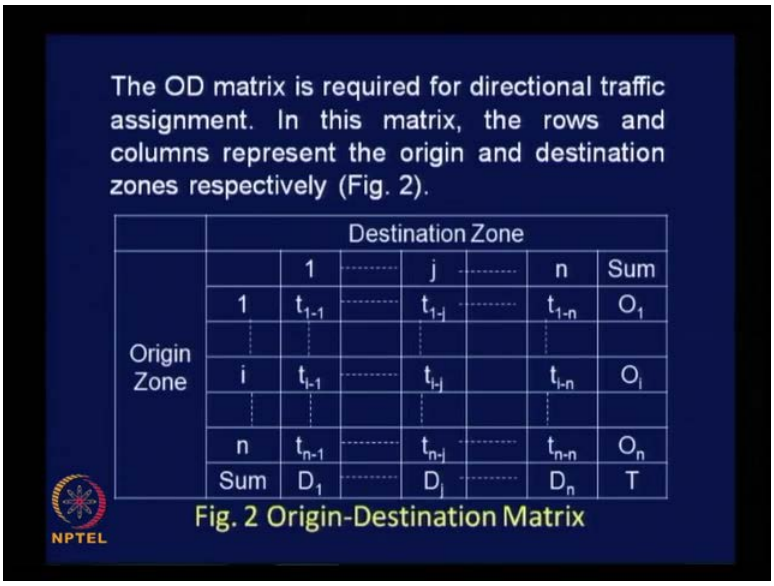


Fig. 2 Origin-Destination Matrix

Then OD matrix as I said OD matrix is necessary to have information about the direction of movement origin, and destination; once you have this information you can perceive the direction of movement of trips from one zone to all other zones. That is the purpose of preparing a OD matrix. Now here the destination zones are given along, columns and origin zones are given along rows.

That the only difference between P A matrix and O D matrix and in the notation there is a minor difference earlier we named a cell value as t i j; for example, in this case it is named as t i dash j to differentiate between P A matrix and O D matrix.

T i dash j means trips originating from i distinct to j that is the meaning number in that cell will give you information about the number of trips originating from i and distinct to j; that means, the direction of movement is clearly indicated that is the point and t1 dash 1 means number of trips originating from one distinct to one itself intra zonal trips.

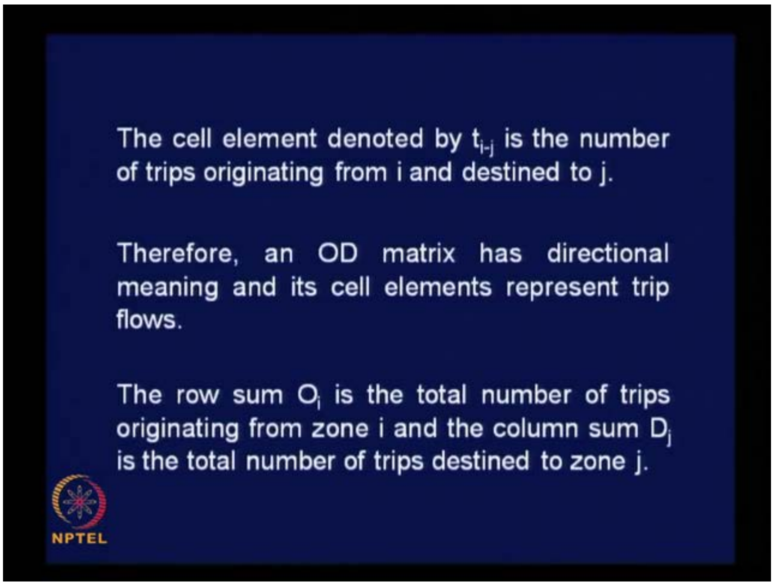
T 1 dash 2 number of trips originating at one distinct to two they are inter zonal trips, you may wonder why should we bother about intra zonal trips are we really interested in intra zonal trips, because right from beginning we understood trip as a movement between zone centroids, we never discussed about a movements with in a traffic zone.

So, in this case also for completion sake we have information about intra zonal trips for subsequent analysis also we will consider only inter zonal trips otherwise, we will be deviating from the earlier assumption of defining trip as a movement between zone centroids it may not have any draw back, because as I said earlier zones are small urban areas, and there is no need to vary about the road network within a smaller area our concern is a major roads developed or adjusting in an urban area.

Right if you sum up the cell values along row for a particular zone what will be the result will give information about what? Let us say you are adding t 1 dash 1, t 1 dash 2, 1 dash j,1 dash n, along this row and it is indicated as O 1 the row sum is indicated as O 1. O 1 means what?

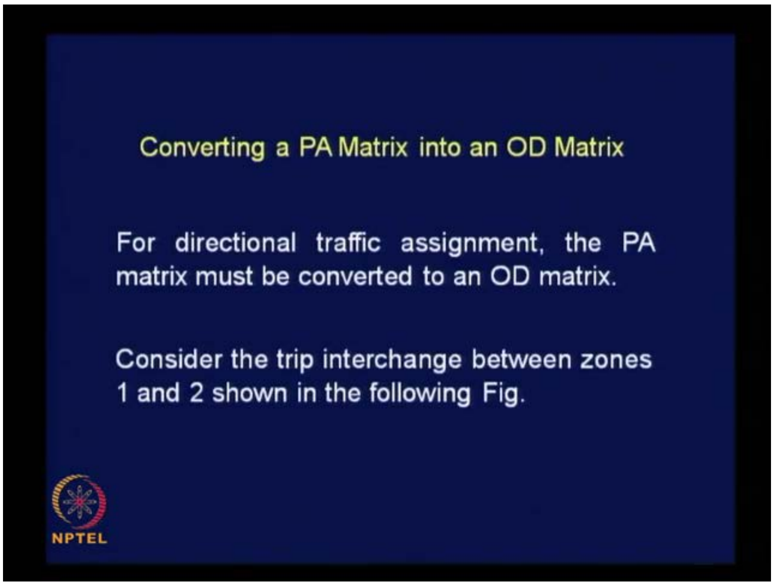
Total trips originating from zone one. Similarly the column totals will give information about total of the trips distinct to a particular zone you can clearly see here you consider zone one is going to be one dash one then it will be 2 dash 1, 3 dash 1 and so on n dash 1, so all pertaining to one zone. So, d 1; obviously, gives information about total of the trips distinct to zone one.

(Refer Slide Time: 18:57)



And the cell element denoted by t i dash j is the number of trips originating from i and distinct to j as we understood earlier and. Therefore, an OD matrix has directional meaning it gives you information about direction of movement, and its cell elements represent trip flows actually, you will have a feel of the trip flow from one zone centroid to another zone centroid. The row sum O i is the total number of trips originating from zone i, and column sum D j is the total number of trips distinct to zone j.

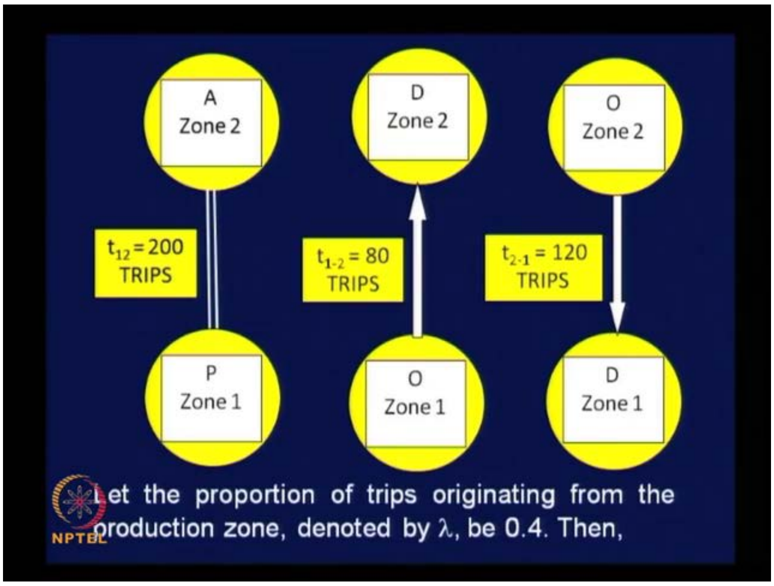
(Refer Slide Time: 19:51)



This is our concern how to convert a P A matrix into a O D matrix initially you would have estimated the total trip production attraction zone wise; that information is available to us through step one, and step two we have done the mode choice analysis. Now while presenting the data in the form of P A matrix, we have no problem, because we have information in terms of P and A production and attraction, but we need to have OD information for route assignment.

So, in this context it is better to understand how to convert a PA matrix into a O D matrix, it is not difficult its very simple for directional traffic assignment, as I said the P A matrix must be converted to an OD matrix. Let us consider a very small example trip interchange between zones one and two as shown in the following figures.

(Refer Slide Time: 20:59)



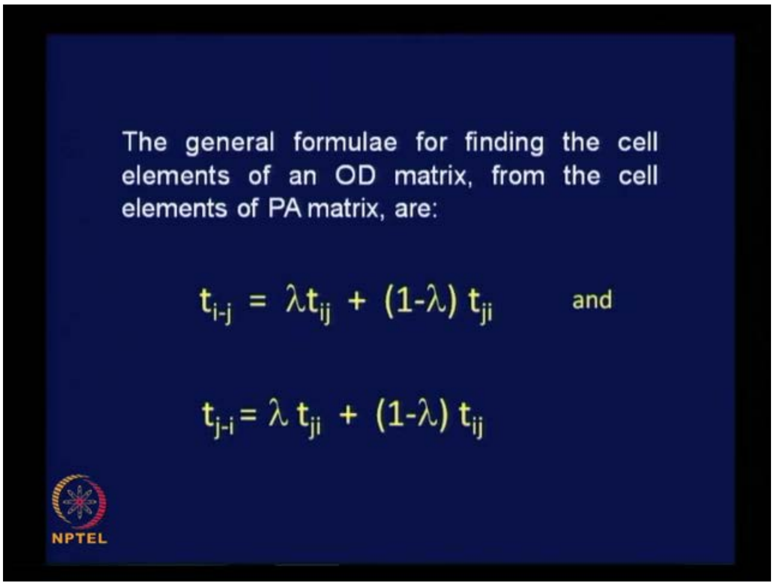
Let us say this is the production attraction information PA information zone one is a production zone two is an attraction zone, and total number of trip interchanges between these two zones is 200 trips are interchanged between zones one and two. We have no information about the direction movement P A matrix will not give you the directional information.

Let us say the proportions of trips are the originating from the production zone denoted by the letter lambda be 0.4. Let us say forty percent of the trips of the production zone are originating from that zone arrows tending outwards, and 60 percent distinct to that zone obviously.

Once you have this information then putting this information here to get OD information. Then I can say that number of trips originating from one, and distinct to is 0.4 into 200 which is eighty trips eight trips or originating from one and distinct to two.

And then; obviously, the number of trips originating from two distinct to one is going to be 1 minus 0.4 that is 0.6 into 200 which is 120 trips are originating from two, and distinct to one. So, first we had information about the sum only 120 plus 80, now we are able to split by knowing a proportion of trips originating from a particular zone. So, based on this principle, we can just write a generalized formula for converting PA information into O D information.

(Refer Slide Time: 23:30)

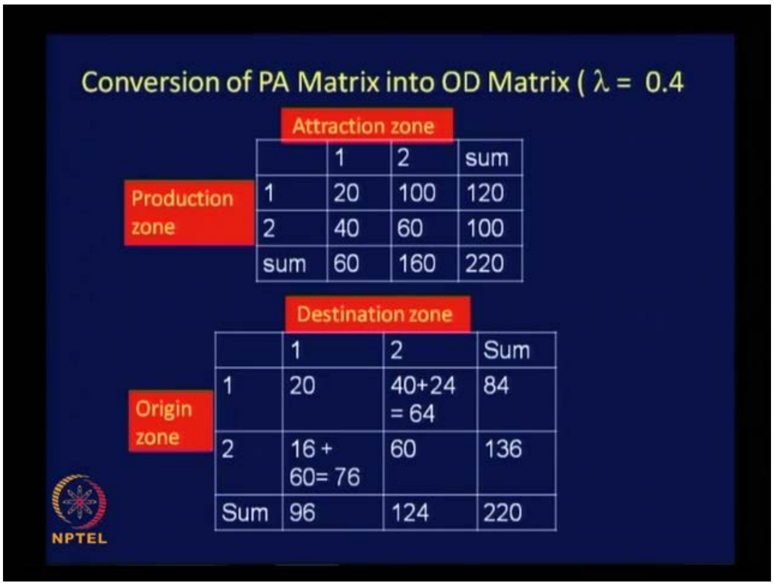


The general formulae for finding the cell elements of an O D matrix, from the cell elements of P A matrix, are; as follows you can say t i dash j is equal to lambda times t i j plus 1 minus lambda times t j i or in other words the number of trips originating from i distinct to j is equal to the proportion of trips originating at i multiplied by the number giving information about trips produced at i and attracted to j that is what is mean by t i j plus 1 minus lambda times the number of trips produced at j and attracted to i. That is what we mean by the simple equation.

And similarly you can write the other directional movement namely t j dash i number of trips originating at j and distinct to i is equal to lambda timest j i plus 1 minus lambda times t i j. So, using these two equations you should be able to convert any cell value of a P A matrix into an equivalent cell value of O D matrix.

So, given a matrix we should be able to apply this formula and convert the cell values that is what we want conversion of P A matrix into the O D matrix is that clear let us take very small numerical example to understand these two equations much better

(Refer Slide Time: 25:42)



Example of conversion of P A matrix into O D matrix let us say lambda value given is 0.4. This is the P A matrix given to us production attraction there are two traffic zones involved right t 1 1 is 20, t 1 2 is 100 t 2 1 is 40 t 2 2 is 60 and p 1 is 120, and p 2 is 100,a 1 is 60 and a 2 is 160.

That is the information given to us let us say we are interested to convert the cell value pertaining to t 1 1 what will be the value of t 1 dash 1, t 1 1 means trips produced at zone one and attracted to zone one itself and the corresponding cell value in OD matrix will be trips originating from one and distinct to zone one itself in that case what will be the cell value for this particular cell in OD matrix. Same there will not be any change at all whereas, there will be difference in respect of the other cell value 1 2 lambda is 0.4.

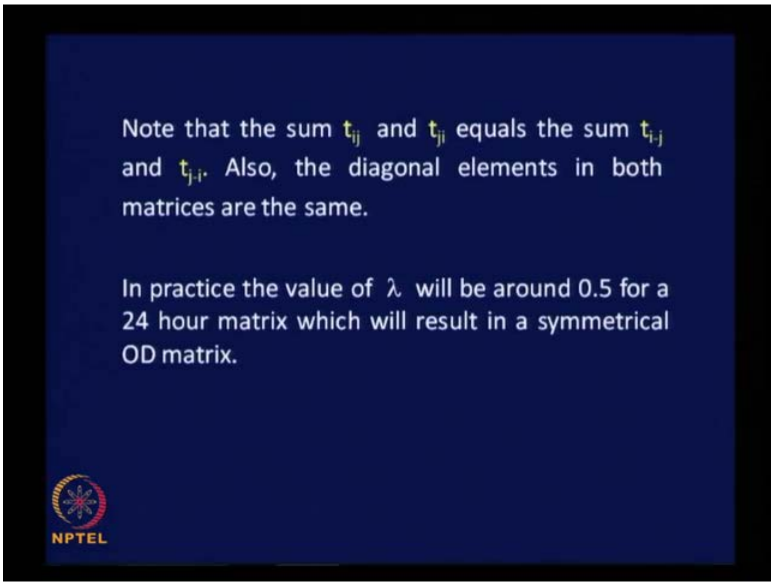
Let’s say we are interested to know the value of t 1 dash 2 t 1 dash 2 what do we do lambda times t 12 plus 1 minus lambda times t 2 one. So, that will give you the value for this particular cell in OD matrix. So, that is what I have done here 20 reminds unchanged and 116 becomes 64, here cell value of 100 has become 64 how it is 40 plus 24,100 is t i j lambda value is 0.4, times 100 is 40plus 1 minus lambda is 0.6 times t 2 1 is 400.6 time 40 is 24. So, we get 64 as a cell value in O D matrix, and similarly you can get the values for other cells interestingly you can see the diagonal cell values are unchanged because they represent intra zonal movements, and they they will remain unchanged.

So, you can apply the same principle to convert any size of P A matrix into O D matrix, and you may wonder whether lambda is going to be having a constant value or it may have different values and so on that is very important question to be answered what is the likely value of lambda in practice.

You take a traffic zone let us say about certain number of trips are originating from the zone people moving out of the zone are likely to come back most of the times, and even if there is some deviation there could be some compensating error; there could be some one way a movement outward some one way movement inward also.

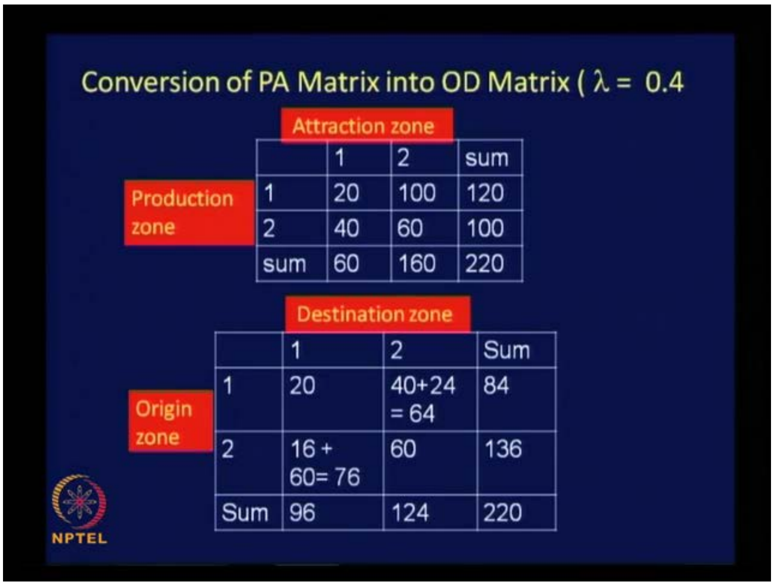
And in general it has been observed the lambda is mostly 0.5 for most of the traffic zone in an urban area. The movement two way movements are almost balancing and let us take lambda to be 0.5 and do the same exercise, and check there is any major difference between this result, and the result the practical case of the lambda being 0.5.

(Refer Slide Time: 30:23)



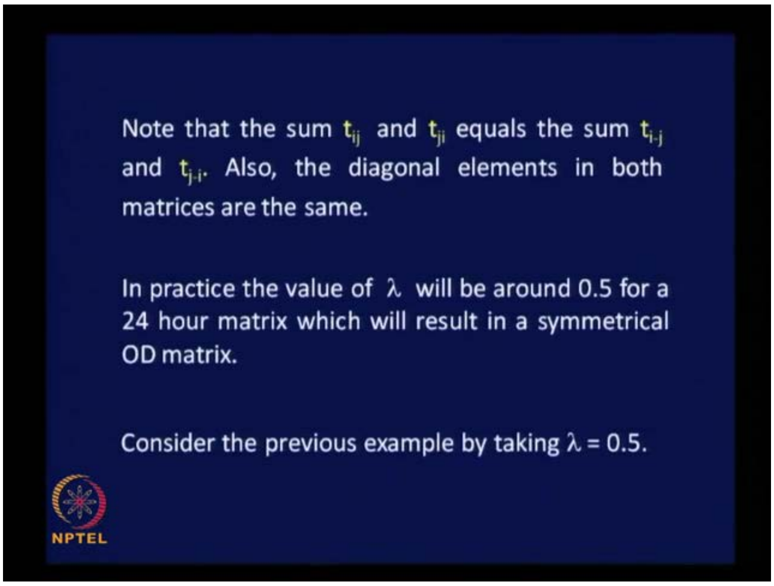
Note that the sum t i j and t j i equals to sum t i dash j and t j dash i of course, that is obvious from the previous example, also the diagonal elements in both the matrices are to are the same they remain and change that is what we have seen. In practice as I said the value of lambda will be around 0.5 for a 24 hour matrix, we do normally twenty four hour matrix one day travel is our interest which will result in a symmetrical OD matrix.

(Refer Slide Time: 31:23)

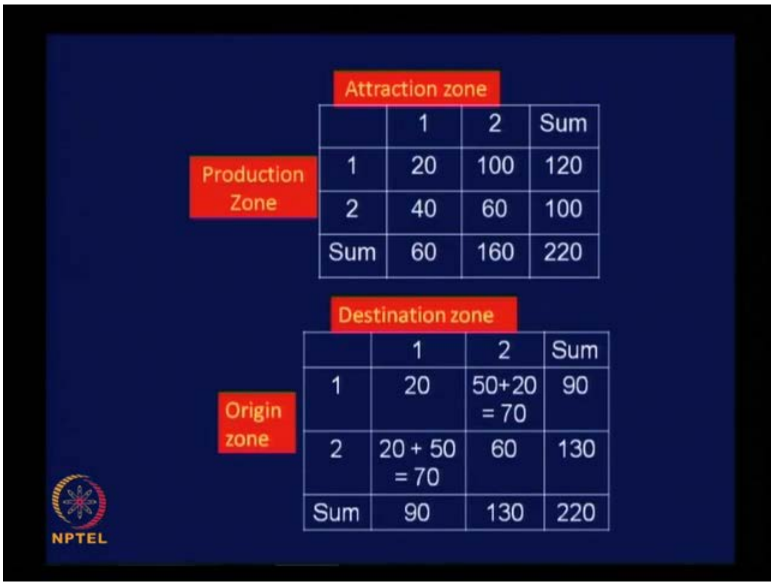


That is the difference the earlier case the symmetric symmetry was not there were changes in the cell values of the matrix, if you look at the O D matrix here it is 64 here 76. And let us see, what happens in these cell values when we take lambda to be equal to 0.5. Consider the previous case with lambda to be equal to 0.5.

(Refer Slide Time: 31:32)



(Refer Slide Time: 31:37)

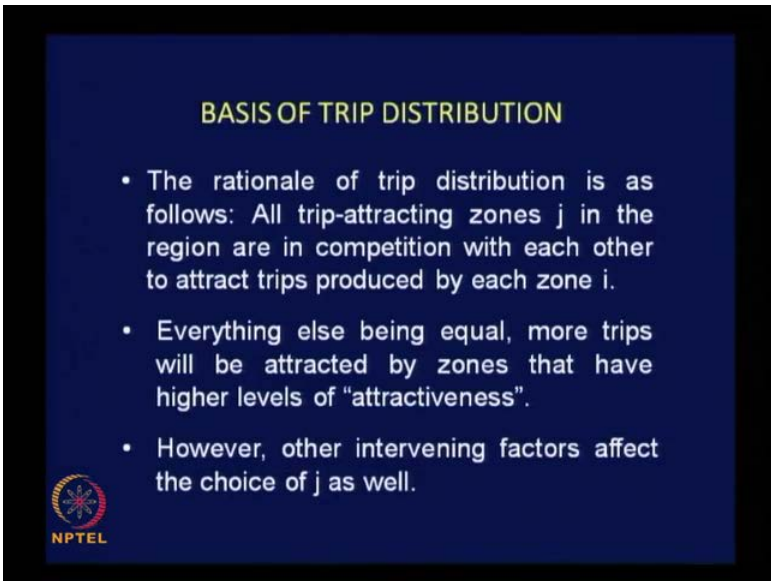


This is the P A matrix and O D matrix would be like this you will have 70 and 70 for 1 2 and 2 1 cell values t 1 dash 2 and t 2 dash 1, because of the symmetry that is the main difference between the previous case. And this is what we come across mostly in practice there could be of course, I would like to constitute we need not blindly assume that lambda is going to be only point five there could be situations where you may have deal with different lambda values, since the general formula is known to us there is no problem, you can always apply the formula and get resale values.

While converting P A matrix into O D matrix. Please note there we have discussed. So, for about just a presentation of the data available to us and we have not started the process of trip distribution model. Now, we need to study the process of modeling trip distribution any modeling process involves first a clear understanding of the causal variables, can you just guess the possible factors that may influence distribution of trips between traffic zones. Unless we identify factors will not be able to develop a model. We extract or derive variables from the identified factors and then formulate an analytical frame work to model any process.

What are the factors that might influence distribution of trips, between traffic zones a land use type, we have different types of land uses and it is very difficult to say that distribution is influenced by land use distribution is two way movement of trips between traffic zones for example, trips between zone 1 and zone 5 might be more compare to trips interchanged between zone 1 and 100, and we should be able to theoretically explain why it is different.

(Refer Slide Time: 34:34)



That is what we mean by trip distribution model let us try to understand certain basic aspects of trip distribution. The rational of trip distribution is as follows: All trip attracting zones j in the region or in the urban area are in competition with each other to attract trips produced by each zone i.

This is the basis we assume that all the traffic zones in the urban area are competing to one another to attract as many trips as possible from a particular zone i or from each of the other zones. There is a kind of competition to attract trips let us perceive that way first and then.

Everything else being equal, more trips will be attractive by zones that have higher level of attractiveness. It is obvious, when there are competing for attracting more trips from trip producing zones when the attractiveness of a particular zone is relatively high that particular zone is likely to attract more trips from all other producing zones.

Now our point or the objective should be to define the attractiveness of traffic zones how, to define the attractiveness, so that we will be able to quantify the same, and then put that as a factor in the modeling process.

However, other intervening factors affect the choice of j as well. What are the other intervening factors first we have said that the zone which is relatively more attractive will be able to attract more trips, but it cannot be taken to be hundred percent correct there are other intervening factors affecting the choice of j as well we cannot say that j is going to be the zone which attracts more trips what are the other possible factors.

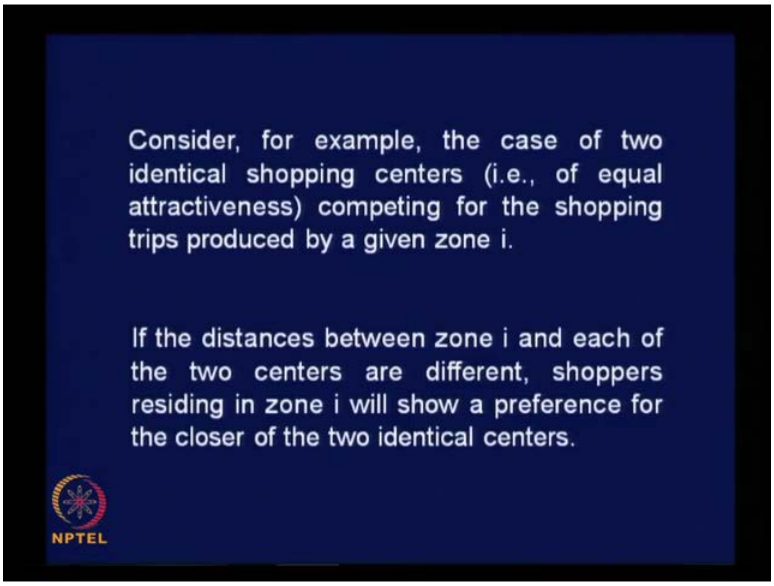
Let us say a zone hundred is the most attractive zone as for as trip attractive concern it is relatively an attractive zone compare to all other zones, zone 50 is relatively less attractive, and we are trying to understand the distribution of trips between zone one and 50 and one and 100.

Let us say zone number hundred is at 10 kilometers away from zone one zone 50 is just 5 kilometers away from zone one that zone 100 is more attractive compare to zone 50, what will be the equal distribution pattern.

People have to travel long distance to reach zone one hundred and lesser distance to reach zone fifty even though zone fifty might be relatively less attractive, let us say shopping trips, more trips maybe attracted from one to fifty then from one to one hundred because of this intervening factor of travel distance.

Now, we are able to identify another factor to understand distribution of trips namely intervening spatial separation between zones.

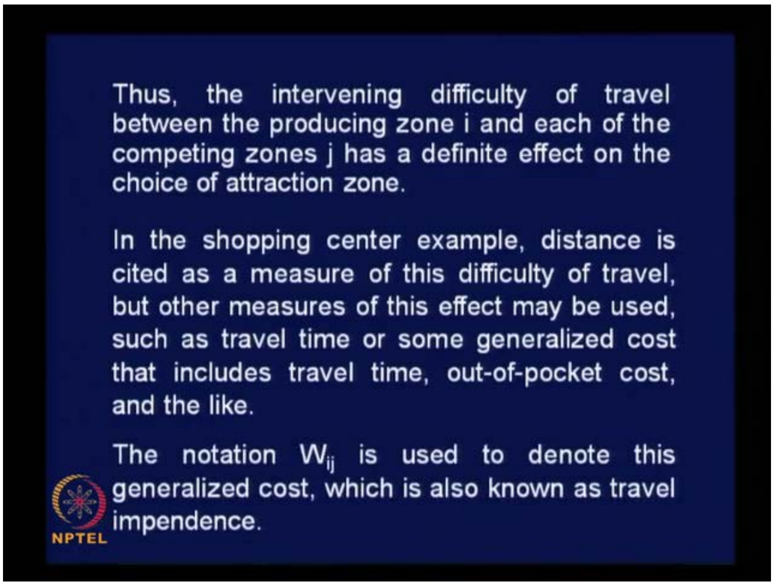
(Refer Slide Time: 38:43)



Consider for example, the case of two identical shopping centers .Of equal attractiveness for example, competing for the shopping trips produced by a given zone it here are two attraction zones with good shopping facility both the zones are competing to one and other to attract trips from zone i.

Right if the distances between zones i and each of the two centers are different shoppers residing in zone I, will show a preference for the closer of the two identical centers that what I said attractiveness is same, but distance makes a difference shoppers; obviously, prefer the one which is closer to them that is how spatial separation comes into play in understanding distribution of trips.

(Refer Slide Time: 39:50)



Thus intervening difficulty of travel between a producing zone i and each of competing zones j as definite effect on the choice of attraction zone intervening factor. Now should we have to understand the intervening factor namely spatial separation, in terms of distance, or time, or some other factor.

Can we just take distance as a factor intervening factor, if you take distance it may or may not work well normally in urban travel time of travel is understood better as real effect of spatial separation then the actual distance. Distance may be less in a particular case, but travel time may be more, because there may not be good road network available to connect these two zonal pairs right.

So, travel time appear to be more attractive, but travel time maybe less, but the available mode may be expensive compare to another choice where travel time is likely more, but cost of travel is less than people may perceive differently after all they have to spend to make the trip.

So, you see the distance travel time and cost of journey all work together simultaneously while choosing a destination point. So, we have to think on these lines and take a decision to really define the intervening factor in terms of spatial separation related variables; the shopping center example distance is sighted as a measure of this difficultly of travel, but other measures of this effect maybe used such as travel time, or some generalized cost that includes travel time out of pocket cost, and the like.

Are you may not have to consider only this things only travel time, you can have a combination of factors and put them all together and give some name for that and use that as a factor influencing trip distribution, and this factor is known as the travel impedance the notation w i j is used to denote this generalized cost. So, all these factors together is termed as generalized cost of travel which is also known as travel impedance.

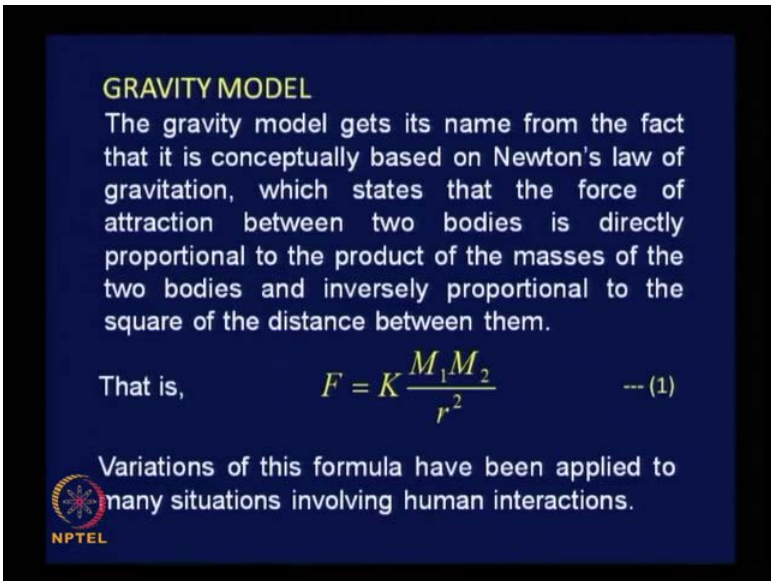
What you understand by generalized cost? The aggregate money value of making a trip is called generalized cost in one trip you may spend less time, and out of pocket cost and you enjoy a better comfort in travel. And if you assign money value for each of these elements including your travel time you can get aggregate money value for that particular trip. In another case you can spend more time, but spend less of out of pocket money maybe you may be subjected to little more discomfort try to assign money value for all this elements and then aggregate all the elements to get the total.

Then compare these total two aggregate values which are termed as generalized cost for deciding the choice of destination for making a trip. You may wonder how to assign money value for your time there are methods available it is possible to assign money value for time of any individual traveler and how do we assign money value for level of comfort.

There are methods available to assign money value for level of comfort or discomfort and these theories are discussed in the subject transportation economics in detail. So, as of now you can assume that it is possible assign money value for all these elements and get aggregate money value, and that is how we need to understand travel impedance.

It is some of all the related factors; now take w i j as the basis and proceed further to model trip distribution. So, we have identified two important factors the attractiveness of the zones, and then the travel impedance defined as w i j which includes distance travel time out of pocket expenditure related inconvenient discomfort all together.

(Refer Slide Time: 45:32)



And the model that is used to theoretically explain trip distribution is named as gravity model this model is based on the Newton's law of gravitation the gravity model as degreed here gets its name from the fact that it is conceptually based on Newton's law of gravitation which states that the force of attraction between two bodies in space in general is directly proportional to the product of the masses of the two bodies, and inversely proportional to the square of the distance between them.

This is the very well known the Newton's law of gravitation now the question before us is how to use this creative concept to explain trip distribution it is concerned with two bodies in space separated by the sub distance; once you know the masses of two bodies in the distance between them you will be able to quantify the attractiveness between the two bodies.

How to apply this principle in understanding distribution of trips consider two traffic zones as two bodies zones i and j and the spatial separation between the two zones will be known to us, because we know they are fixing zone centroids we know a spatial separation. So, distance between the two zones are also known right and then how to define the mass of these zones attractiveness of these zones, let us say one zone is production zone another zone is attraction zone or mix up production attraction.

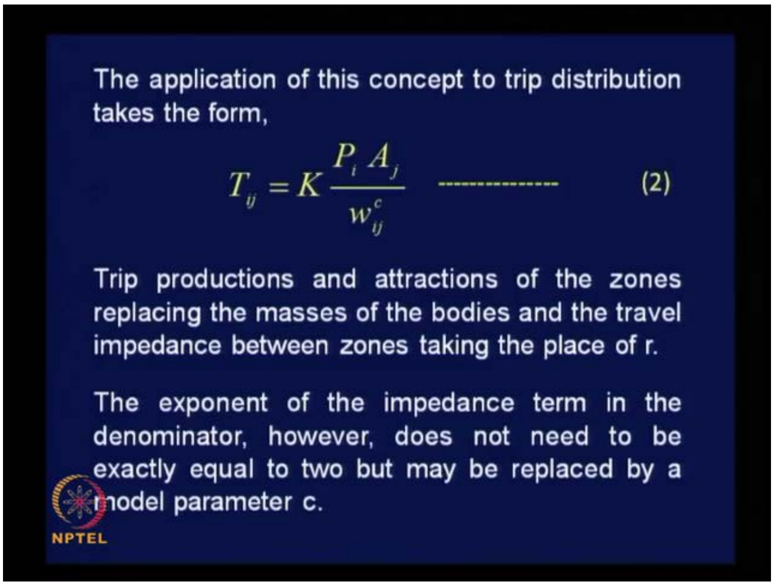
In that case it is both in the extend of production as well as attraction p i and a j right, that is how you must understand the mass of zones it could be the quantum of production or mix up production attraction in another case quantum of attraction or mix up both total trip ends associated with the traffic zones can be considered as the mass associated with the traffic zones.

Then you can apply this law provided, they accept this law can be applied for understanding trip distribution can we accept this law, because we have already agreed that the trip interchanges are directly proportional to more or less the attractiveness of the traffic zones. So, that concept can be incorporated very clearly here, and the spatial intervening factors spatial separation expressed as travel impedance.

So, it is possible to apply this principle to understand trip distribution that is Newton's law can be written like this force of attraction between the two bodies can be written like an equation using a proportion i d constant K as F to be equal to K into M 1 M2 divided by r square M 1 M 2 are the masses of the two bodies, and r is the distance between the two bodies. And the square of the distance is the denominator, and K is the proportionality constant, and variations of this formula have been applied to many situations involving human interactions also about which we are not going to discuss in detail, but just for information.

It is not that this law is confined to a particular aspect very general principle applied for different situations to explain, and understand the effect of attractiveness could be between individuals also.

(Refer Slide Time: 50:06)



The application of this concept to trip distribution takes a form as given here you can say instead of force the total number of trips interchanged between i and j is equal to K times P i A j divided by W i j raise to power c. We have just generalized the exponent part instead of two we are just giving a notation c. So, M 1 and M 2 are represented as simply P i and A j and f is understood here as total number of trip interchanges between this particular zonal pair

Between zones i and j that how we can understand the force total number of trips interchange between a given zonal pair of zones taken (( )) right, and trip productions and attractions of the zones replacing the masses of the bodies and the travel impedance between zones taken into place of hard that what we have done.

We are just substituted P A and A j for M 1 and M 2 and r square is replaced by w i j raise to power c. The exponent of the impedance term the denominator how are that is not need to be exactly equal to two as I said, but may be replaced by a model parameter that is what we have done.

And let us hold at this point and just try to recapitulate what we did. So, for in this class we started our discussion with the understanding that on completion of mode choice analysis, we were able to quantify the association of modes related to the trip ends at the zonal level that is how we just saw the trip ends with different colors indicating different modes.

In the choice process at the zonal level then we discussed about presentation of a data available for analyzing trip distribution in the form of PA matrix and OD matrix. In fact, we can first present the data in the form PA matrix and then convert PA matrix into OD matrix for the purpose of applying the information for the last step namely traffic assignment.

Now, we are very clear about the procedure of converting PA matrix into OD matrix then we try to understand the factors that might influence distribution of trips between traffic zones, we are now clear that the quantum of trip production and attraction. And the spatial separation between the zones expressed in the form of generalized cost of travel or the factors that will influence trip distribution between zones, and to provide a mathematical formulation for these factors to be put into a formula we found that Newton’s law of gravitation could be a good basis. And we have also substituted the variables involved in the Newton's law of gravitation with the variable that we know in the form of production attraction and travel impedance. And we are able to now formulate an equation to explain distribution of trips between zonal pairs. We will stop here and continue our discussion in the next class.