Geosynthetics and Reinforced Soil Structures
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Lecture - 5
Strength Analysis of Reinforced Soils - II

Hello students, let us continue the lecture from the previous lecture, where we discussed about the strength of the Reinforced soil and some experimental data on the strength of the reinforced soils. And we have seen that as the reinforcement layers are placed in soil, these the compressive strength increases, because of the interactions that take place between reinforcement and the soil.

And because of the placement of the reinforcement the soil is prevented from expanding laterally and because of that higher confinement stresses are generated in the soil. And because of that the vertical stress that we need to apply to fail a soil sample increases, and as we place more and more number of reinforcement layers, the strength of the soil has increased. And now in this lecture let us try to find out some theoretical equations that explain this observed phenomenon.

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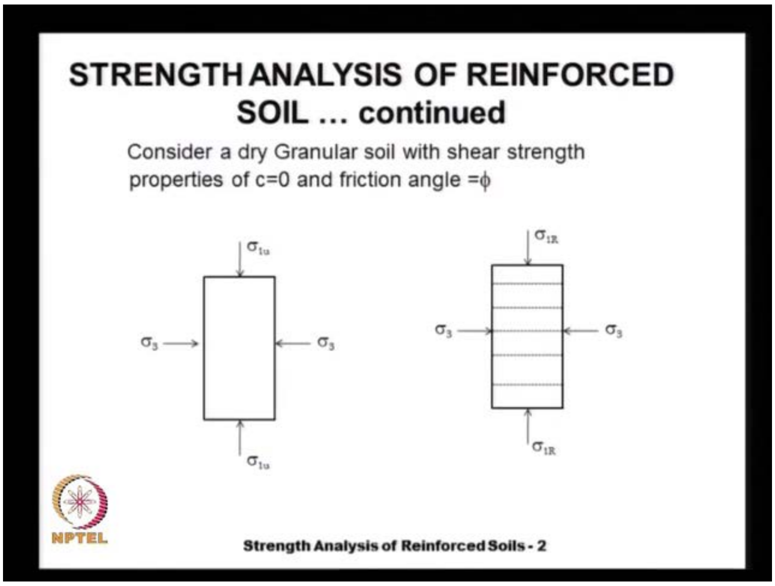

And for the purpose of analysis let us consider a dry granular soil and the reason why, we want to consider only dry granular soils is that, the actual construction, we prefer using dry granular soils, so that they are highly permeable. And then even if there is some cohesion, because the fines content, we neglect them to so that our designs are on the conservative side. So, the sheer stress properties that, we consider in all these analysis are c equal to 0, that is the cohesive strength is 0 and the friction angle is phi.

And the on the left hand side, we have the a sample of unreinforced soil, that is confined with a confining pressure of sigma 3 and it let us assume that, the vertical stress at, which it fails is sigma 1 u and the right hand side, we have the same soil, but with some reinforcement layers in this case, it fails at sigma 1 r, which is more than sigma 1 u.

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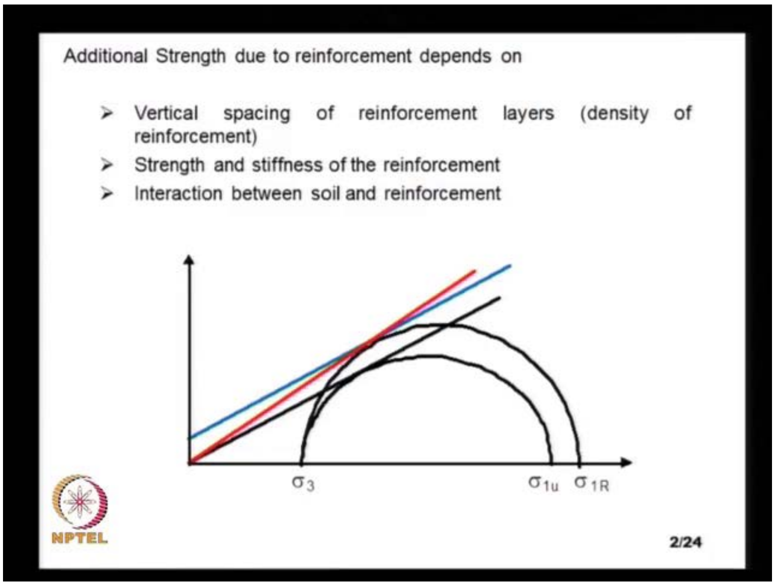

And as we have seen, in the previous class this additional strength that, we get because of the reinforcement layers depends on several factors, that is the vertical spacing between the reinforcement layers. The vertical spacing is very important, if the vertical spacing is very, very large then the rupture surface may develop in between the reinforcement layers and the soil may fail or if the reinforcement spacing is too small then they may not be able to have enough space.

So, that they can develop adequate bond strength between the reinforcement and the soil and the strength and the stiffness of the reinforcement itself, that is say if you use steel strips instead of plastic strips the strength and stiffness of steel is much higher than that of the plastic. So, because of that the inherent strength that, we get for the soil reinforcement composite could be much higher.

And then of course, the other factor that, determines the additional strength, because of the reinforcement is the interaction between the soil and the reinforcement, that is if there is a good bond between the reinforcement and the soil. Then it can develop a higher resistance force and contribute for the increase in the strength of the soil.

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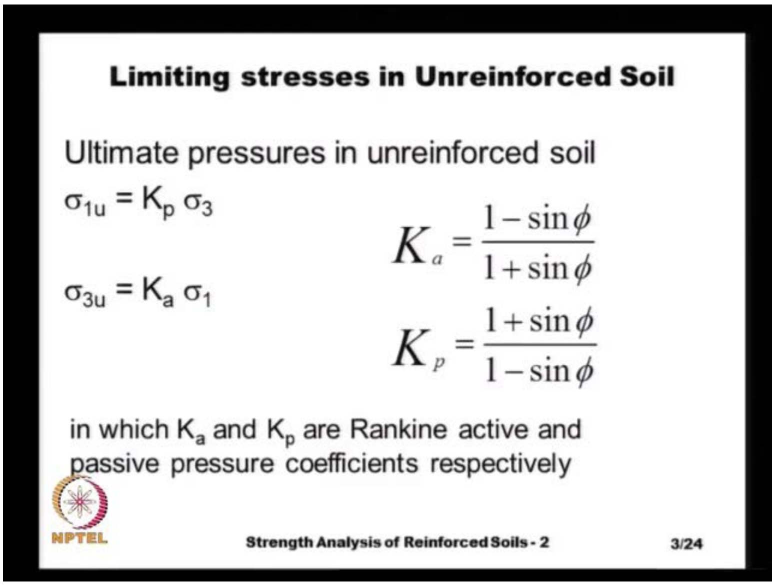

And just to recap the ultimate stresses in the unreinforced soil are related like this sigma 1 u, that is the maximum compressive stress that, we can apply is equal to K p times sigma 3 where, K p is the rankine's passive at pressure constant. And that is related to friction angle phi as K p is 1 plus sine phi by 1 minus sine phi and sigma 3 limiting stress sigma 3 u is K a times sigma 1 where, K a is called as the rankine active at pressure constant, that is equal to 1 minus sine phi by 1 plus sine phi.

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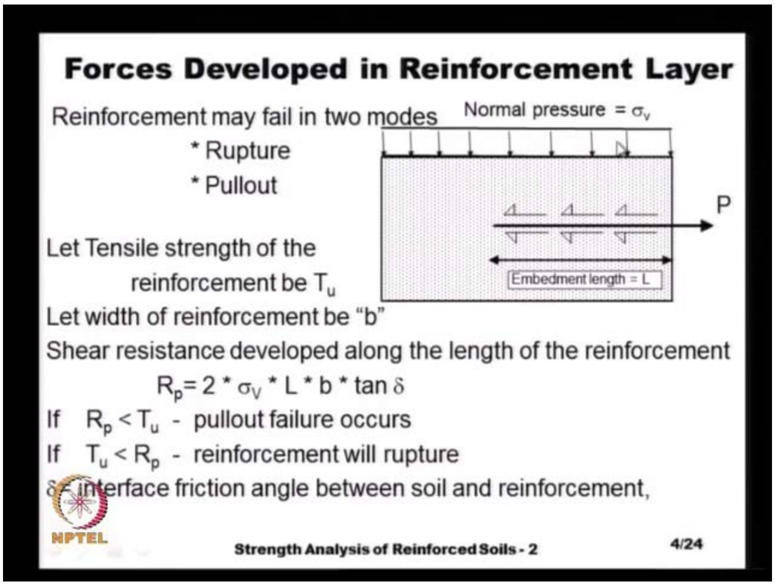

And before, we actually do the strength analysis let us consider, what are the forced that are developed in the reinforcement layer and how does it interact with the soil and how does it fail. See few place reinforcement layer In the soil, it could fail in 2 different modes that is by rupture, that is as you go on applying tensile force at some point, it will just simply snap or break. And the other mode is by pull out see few place some reinforcement layer in the soil and go on applying some load at some point it will just simply pull out, that happens when the when there is not enough bond strength. Either, because of lack of friction between the soil and the reinforcement surface or because of very low embedment length and the casing point is illustrated like this. Let us consider reinforcement of length L embedded in the soil and let us say that the width of the reinforcement layer is b.

And let us assume that the tensile strength of the reinforcement layers is T u and the applied pressure is sigma v that is we are applying some pressure and then let us say that on the system, we go on applying some force. So, that the reinforcement layer is pulled out. And as we are pulling the reinforcement layer, the sheer resistance forces, they develop along the length of the reinforcement and these resistance forces, they develop both on the top surface and the bottom surface as indicated here. And the pull out resistance R p is 2 times, the 2 is because we have the resistance force developed both on the top surface and also on the bottom surface.

And R p is 2 times sigma v, that is the vertical stress multiplied by L, that is the embedment length, b that is the width and this tan delta where, delta is the is called as the interfacial friction angle between the soil and the reinforcement. And this depends on the interaction that takes place between the soil and the reinforcement that will depend on the surface characteristics of the reinforcement.

So, if the reinforcement is very smooth, the delta could be small or if the reinforcement is rough or if it has good aperture openings, then there could be good interlocking between the reinforcement and the soil particles and because of that the delta could be high. And now the failure can happen in 2 ways, that is by rupture or pull out, see if the of the pull out or the pull out capacity is less than the tensile strength.

The pull out will the pull out failure will govern, that is the failure will happen by pull out or if the tensile strength is less than pull out capacity then the reinforcement will rupture, that is in even before the pull out failure happens the tensile strength of the of the reinforcement layer is exceeded. So, it will just simply break and the fail, so these are the 2 predominant modes of failure when it comes to the interaction between the reinforcement and the soil.

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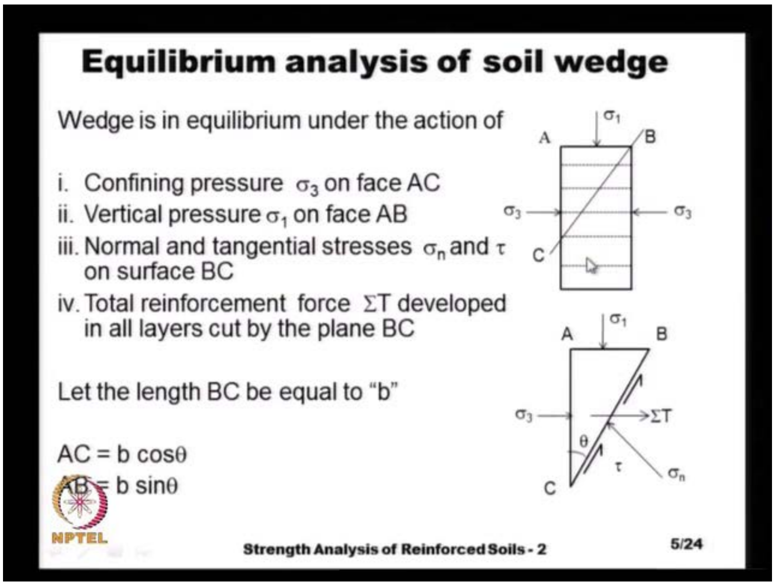

And now let us analyze the strength of the reinforced soil, let us consider ah reinforcement soil specimen like this with certain number of reinforcement layers. And let us consider an arbitrary wedge b c just as how we consider wedge in the previous case of unreinforced soil. And this the this triangular wedge A B C is in equilibrium under the actions of different forces that is sigma 3, that is the confining pressure, that is acting on this face A C.

And vertical stress sigma 1, that is acting on this face A B and the normal that is sigma n and tangential forces tau, that are acting on this surface B C. And then this the then of course, the reinforcement force, that is sigma T is actually, the reinforcement forces that, we consider for this equilibrium analysis are only coming from those reinforcement layers, that are cut by this rupture surface B C. And the angle between the vertical and this rupture surface is theta.

Is actually, please realize that the theta that, we used earlier was with respect to the horizontal plane, that is the maximum sorry, the major principle plane, whereas here just for convenience, we are considering the theta with respect to the minor principle plane. But, both results should be similar as long as the mathematical analysis is consistent and let us assume that, the length of this surface B C is B. And because of that the A C by resolving the B times cosine theta and A B that is the length of this horizontal surface is B times sine theta.

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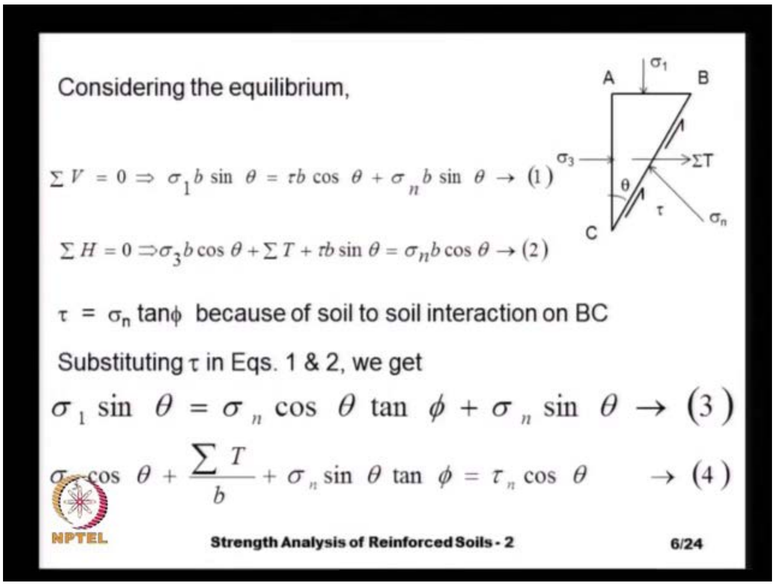

And let us consider the equilibrium of the wedge and then determine the force equilibrium in both vertical direction, that is the net sum of all the forces. And the vertical direction sigma V should be 0 and the net sum of all the forces in the horizontal direction should be 0.

So, this sigma V is 0 gives us that sigma 1 times A B and times B, that is the width that, we consider is sorry, the width that, we consider is a unit width 1 and then the sigma 1 times b sine theta. Because, this length is b times sine theta, that is equal to tau times that is the force, that is acting along this length times b times cosine theta to resolve into vertical direction. And then sigma n, that is the normal force multiplied by this length b times sine theta, because that is the vertical component of this force. And because the sigma 3 and then the reinforcement forces, they are acting in the horizontal direction, we do not consider any vertical component.

And this is 1 equilibrium equation and then sigma H that is the net sum of all the forces acting in the horizontal direction, that should also be 0. And so this sigma 3 times a c that is b times cosine theta plus sigma t, that is the net reinforcement force plus tau b times sine theta that is the horizontal component of the sheer forces, that is equal to that should be equal to sigma n times b times cosine theta, that is the horizontal component of this normal forces, that are acting on this rupture surface b c.

And the relation between this sheer stress on the surface b c and sigma n is through the morculum relation, that is the tau is c plus sigma n tan phi, but in this case as c is equal to 0, the tau is equal to just simply sigma n times tan phi, because it is interaction between the soil to soil interface. And substituting, this value of tau in equations 1 and 2, we get sigma 1 times sine theta is sigma n times cosine theta times tan phi plus the sigma n times sine theta and then sigma 3 plus sigma t by b plus sigma n sine theta tan phi is equal to tau n times cosine theta.

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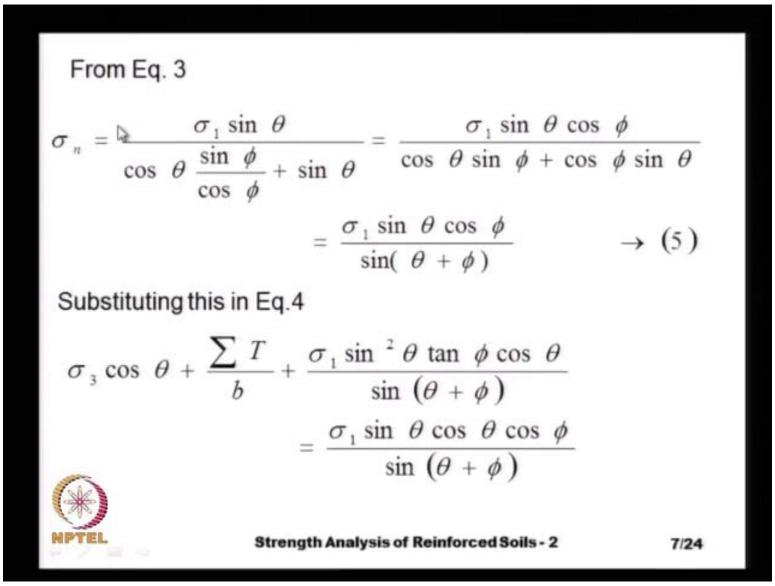

And then by further simplifying the normal stress on the on this failure plane can be obtained as sigma 1 sine theta divided by cosine theta sine phi by cosine phi plus sine theta. And so by further simplifying, we can get the sigma n as sigma 1 sine theta cosine phi by sine theta plus phi. And if you substitute this sigma n in equation 4, that is this equation, we get a the relation between this sigma 3 and sigma 1 like this, sigma 3 cosine theta plus sigma t, that is the net sum of the reinforcement forces by b plus sigma 1 sine square theta tan phi cosine theta divided by sine of theta plus phi, that is equal sigma 1 sine theta cosine theta cosine phi by sine theta plus phi.

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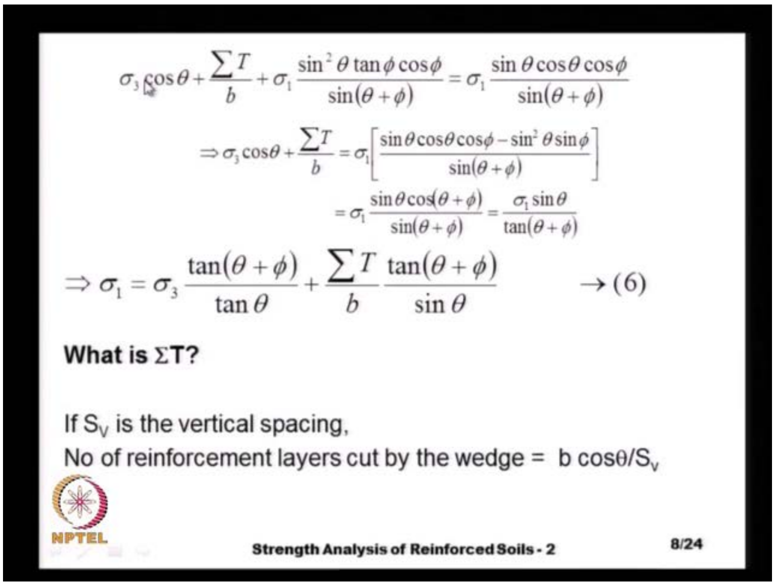

And we can simplify it like this and this the entire equation and we can simplify that and write the equation like this. And finally, we get sigma 1 as sigma 3 times tan theta plus phi by tan theta plus sigma t by b tan theta plus phi by sine theta. And what is this reinforcement force, that is the sigma of t.

And is actually, that depends on several factors and let us try to estimate the force, that is developed in the reinforcement layers and let us say that the vertical spacing between the different reinforcement layers is S v. And so the number of reinforcement layers cut by the wedge becomes b cosine theta that is the length of this wedge a c is just simply b times cosine theta and that b cosine theta divided by the vertical spacing S v gives us the number of reinforcement layers.

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If T_U is the tensile strength of a single reinforcement layer

$$\sum T = T_U \frac{b \cos\ \theta}{S_V} \text{ (rupture capacity)} \rightarrow (7)$$

If pullout is the failure mode, the net tensile force generated at the verge of failure is,

$$\sum T = 2\sigma_1 \mu b_r \frac{b \cos\ \theta}{S_V} \rightarrow (8)$$

μ = friction factor = tan δ

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And once, we have the number of reinforcement layers, we can calculate the tensile contribution of the reinforcement layers separately, if the failure is by rupture or the failure is by pull out. And if the failure is by rupture and if T u is the tensile strength of a single reinforcement layer the sigma t is T u times the number of reinforcement layers, that are cut by the wedge that is b cosine theta, that is the vertical height divided by S v, that is the vertical spacing between different reinforcement layers.

And if pull out is the failure mode, the net tensile force generated at just at the wedge of failure sigma t is 2 times, because we have the sheer force, the sheer resistance developed both at the upper surface and also the bottom surface. That is 2 times sigma 1, that is the vertical stress mu that is the friction factor b r is the width of the reinforcement layer and b cosine theta by S v, that is the number of reinforcement layers. And here the mu is the friction factor, that is equal to tan delta and we will see how to determine this friction factor later on through large scale pull out stress that, we perform on the reinforce systems.

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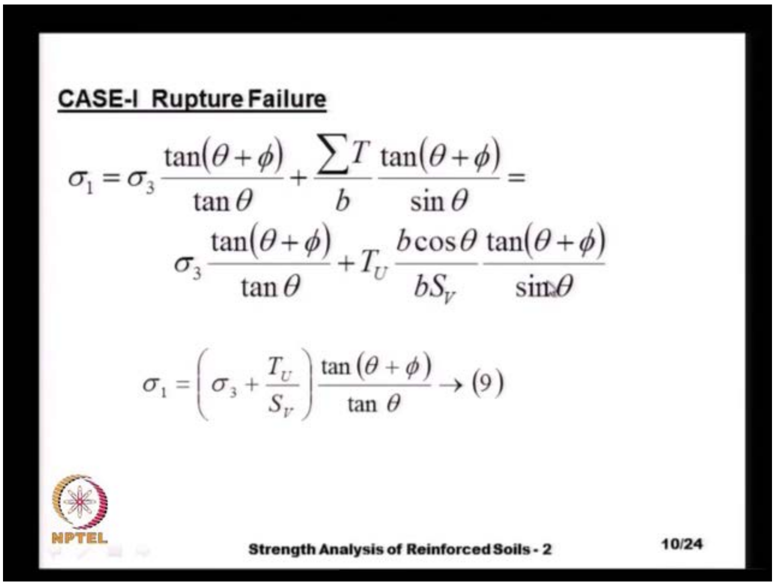

And let us consider 2 separate cases, the case 1 for rupture failure, wherein our sigma 1 is sigma 3 tan theta plus phi by tan theta plus sigma t, that is the net sum of the reinforcement forces by b tan theta plus phi by sine theta. And that is equal to sigma 3 tan theta plus phi by tan theta plus T u b cosine theta the T u is as, we discussed earlier the strength of 1 single reinforcement layer. And the number of reinforcement layers is b cosine theta by S v divided by b, that is coming from here from the denominator times tan theta plus phi by sine theta. So, the sigma 1 can be written as sigma 3 plus T u by S v tan theta plus phi by tan theta, that is our limiting equation. So, if there is a confining pressure of sigma 3 and reinforcement strength of T u spaced at vertical spacing S v, we can determine the sigma 1 like this.

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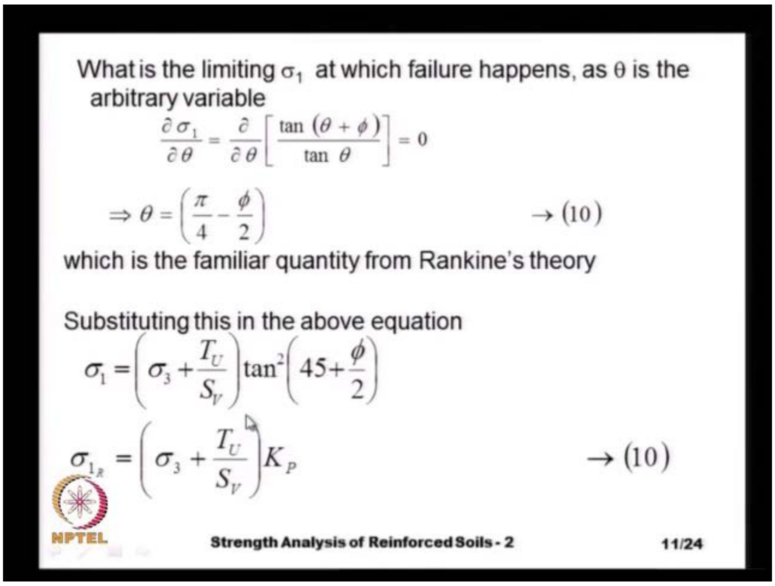

And because our phi sorry, our theta is an arbitrary angle, that we have taken for different thetas, we may get different sigma 1 and how do we know at, which exactly at, which sigma 1 the failure happens and we can determine that by differentiating, that equation with respect to theta and setting it to 0. So, here delta sigma 1 by tau sigma 1 by tau theta is tau by tau theta of tan theta plus phi by tan theta and by setting that to 0, we get theta is phi by 4 minus phi by 2, in the earlier case, we have got it as pi by 4 plus phi by 2. But, that is with respect to horizontal plane, that is major principle plane whereas, here this is with respect to minor principle plane, that is the vertical axis.

So, in fact, the result now that, we have pi by 4 plus phi by 2, if we add it to the earlier equation of pi by 4 plus phi by 2, we get pi by 2, that is the angle between the major principle plane and the minor principle plane, that is 90 degrees. So, here once again our theta is the familiar quantity that, we can get from Rankine's failure theory as we discussed in the earlier lecture.

So, our theta is pi by 4 minus phi by 2 and if we substitute this, in the above equation that is sigma 1 is sigma 3 plus t by S v times tan theta plus phi by tan theta. We can, we get a relation that is sigma 1 is sigma 3 plus T u by S v tan square phi 45 plus phi by 2. And this tan square plus 45 by 2 is nothing but K p that K p is the Rankine's passive at pressure constant. So, this sigma 1 reinforced soil is sigma 3 plus T u by S v times K p.

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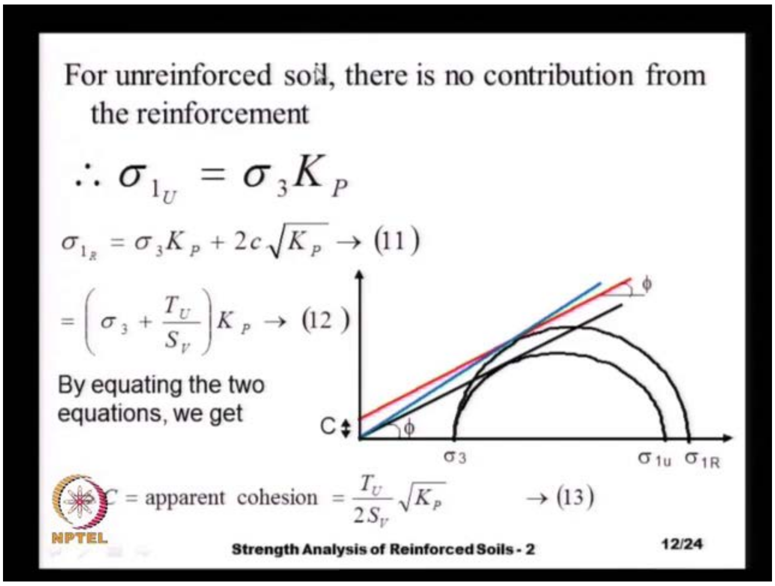

And if we have an unreinforced soil, there is no contribution from the reinforcement or we can say T u as 0 and in that case, the sigma 1 u is just simply sigma 3 times K p, which is the same as what we got earlier and if there is a contribution from the reinforcement this is that sigma 1 is sigma 3 plus T u by S v times K p. And we can also write it, in terms of this sigma 3 and apparent cohesions C like this sigma 3 K p plus 2 C square root K p. And from the earlier theory, we can also write that is equal to sigma 3 plus T u by S v K p and by equating, these 2, we can get an equation for the apparent cohesions t as 2 C by sorry, the C is t by 2 S v times square root K p and this is the equation to determine the apparent cohesions C.

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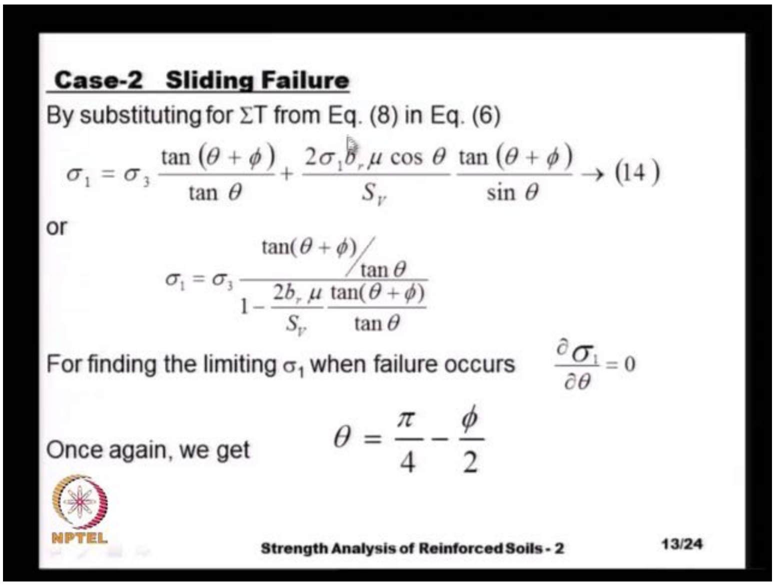

And now, let us consider the other failure mode that is by pull out. The pull out failure happens when the pull out capacity r p is less than the tensile strength and by substituting sigma 3 from equation 8, that is the sigma t in terms of the sigma 1. The width of the reinforcement and then the friction factor mu and so on. This sigma 1 is equal to sigma 3 plus tan theta plus phi by tan theta plus 2 sigma 1 b r, that is the width of the reinforcement mu is the friction factor, times cosine theta divided by S v, that is the vertical spacing between the reinforcement layers.

And multiplied by tan theta plus phi by sine theta or we can simplify this equation and write sine sigma 1 as sigma 3 times tan theta plus phi by tan theta, this whole divided by 1 minus 2 b r mu by S v tan theta plus phi by tan theta. And for finding the limiting sigma 1, when the failure happens, we can determine dau sigma 1 by dau theta and set it to 0 and once again, we get the familiar quantity that the rupture plane is equal to pi by 4 minus phi by 2, that is the rupture plane is at an angle of 45 minus phi by 2 with respect to minor principle plane.

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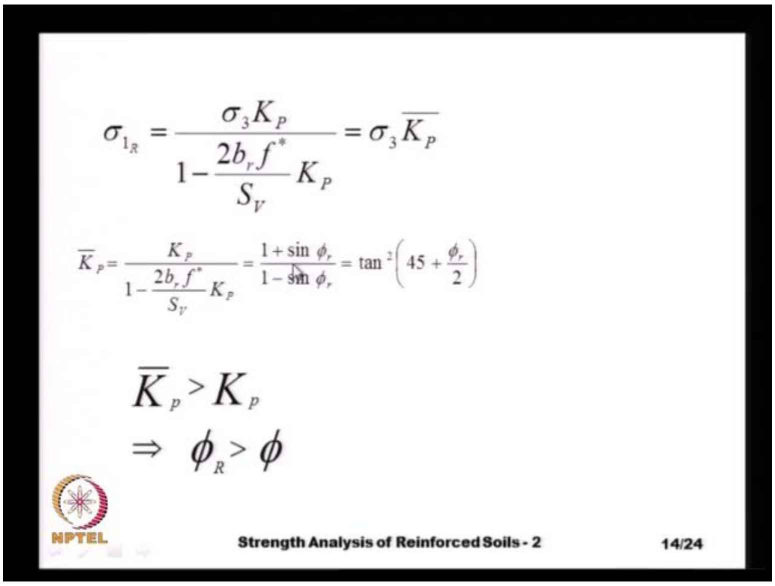

And if we substitute this in the previous equation, we get sigma 1 r as sigma 3 K p by 1 minus 2 b r f star that is f star is nothing but the friction factor divided by S v times K p that is I am combining this entire quantity. And write it as K p bar, that is the modified passive at pressure coefficient wherein K p bar is equal to K by 1 minus 2 b r by S v f star times K p. And let us write it in a very simple form in terms of phi r, that is the friction angle of the reinforce soil as 1 plus sine phi r by 1 minus sine phi r, that is equal to tan square 45 plus phi r by 2.

And as long as our reinforcement does develop some pull out resistance, that is through this quantity 2 b r f star by S v times K p our K p bar will be greater than phi K p and phi R, that is the reinforcement, sorry the friction angle of the reinforce soil will be greater than the friction angle of the unreinforced soil.

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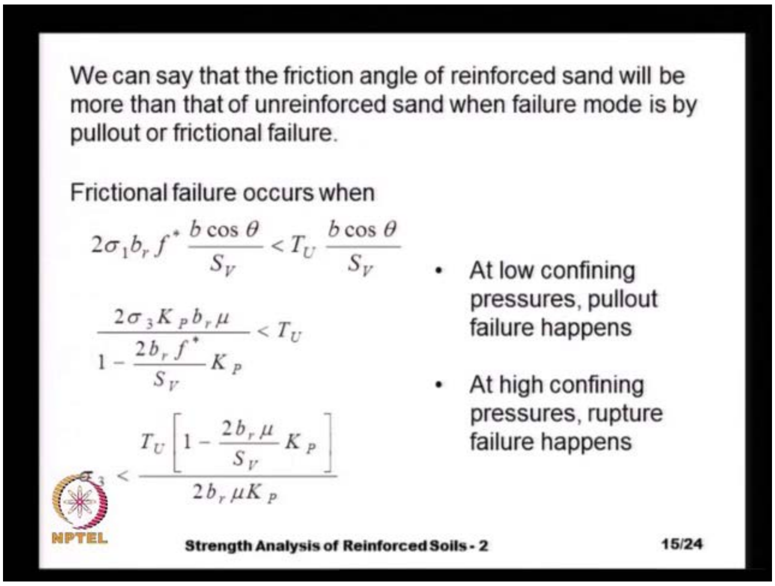

So, actually we can say that the friction angle of the reinforce sand will be more than that of the unreinforced sand when the failure is mode is by pull out or frictional failure. Thus actually, we can see that the sorry, the denominator here is less than 1 and sorry, the denominator here is less than 1, so because of that the K p is always K p bar is always greater than K p. And so our friction angle phi r is always greater than friction angle phi.

So, our is actually, we can determine the critical confining pressure when the failure is by pull out or when the pressure is by rupture, that is we can equate, the pull out capacity to the tensile strength capacity like this. The 2 sigma 1 b r times f star b cosine theta by S v is less than T u b cosine theta by S v, that is the on the left hand side, we have the pull out capacity, because of the interaction between the soil and the reinforcement layer.

And the right hand side, we have the total reinforcement strength, that is the T u the strength of 1 single layer of reinforcement multiplied by the number of reinforcement layers, that is b cosine theta by S v. And so by simplifying, we can write that the 2 sigma 3 K p b r, that is the sigma 1 is related to sigma 3 as like this at the limiting state b r times mu is actually. Mu is nothing but the f star, that is the that is the friction factor divided by 1 minus 2 b r f star by S v K p, that is less than T u or we can actually, write sigma 3 as T u 1 minus 2 b r mu by S v times K p, that whole thing divided by 2 b r mu K p.

So, at very low confining pressures, we can say that, the pull out failure is governing that is the pull out failure is the failure pattern and at very high confining pressures the rupture failure governs, that is at very high confining pressures the frictional force, that is developed between the soil and the reinforcement is so high that the pull out capacity will be much higher than the reinforcement capacity. And because of that the reinforcement strength is less than the pull out capacity and there the failure is governed by the by the rupture.

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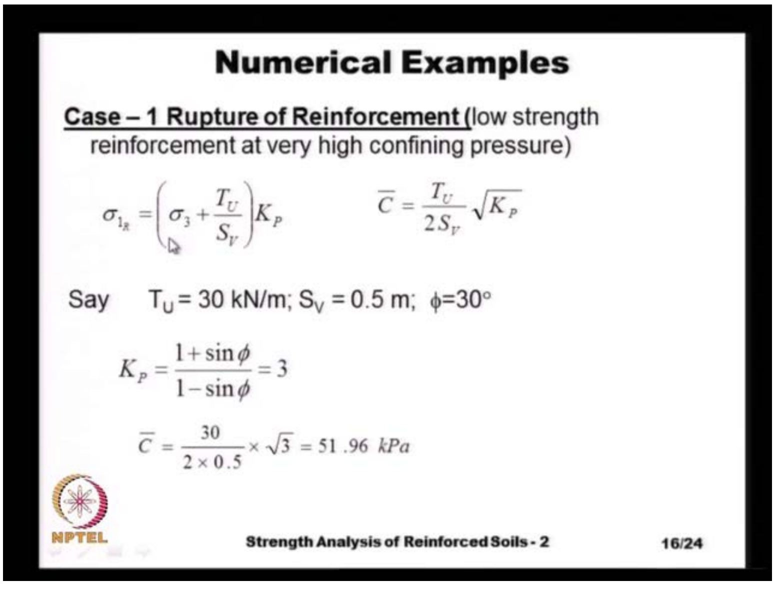

Well let us look at some small numerical example, we have seen that, the sigma 1 r is equal to sigma 3 plus T u by S v times K p, that is equal to C bar, we have seen that the apparent cohesion C bar is T u by 2 S v times square root K p. And let us substitute some numbers the T u, let us say that the tensile strength is 30 kilo Newtons per meter and the vertical spacing is half a meter 0.5 meters. And let us say that the friction angle phi is 30 degrees and for this case the K p is 1 plus sine phi by 1 minus sine phi, that is 3 and our C bar, that is apparent cohesion is 30, that is the tensile strength and divided by 2 times 0.5, that is the 2 times the vertical spacing S v multiplied by square root of 3 that comes to nearly 52 k P a 52 kilo pascals.

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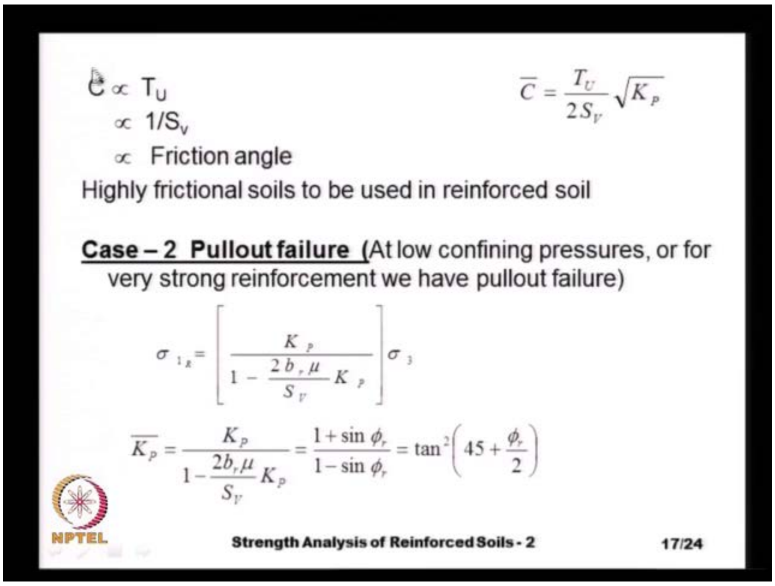

That is 5 tones per meter square and here actually from this equation C bar is T u by 2 S v square root K p, we see that the apparent cohesion is directly proportional to tensile strength of the reinforcement or inversely proportional to the vertical spacing between the reinforcement layers or directly proportional to the friction angle. Because, the K p is in the in the numerator and because of this reason, we prefer using highly frictional soils. So, that we get a higher influence of the reinforcement.

Then let us look at the other case of failure, that is the pull out failure at very low confining pressures or for very, very strong reinforcement layers, we will have pull out pull out failure. The relation between the sigma 1 r and sigma 3 when the failure is by pull out is through this equation sigma 1 r is K p by 1 minus 2 b r mu by S v K p times sigma 3. And we can actually, write this whole thing as K p bar as K p by 1 minus 2 b r mu by S v times K p, that is 1 plus sine phi r by 1 minus sine phi r or that is equal to tan square 45 plus phi r by 2.

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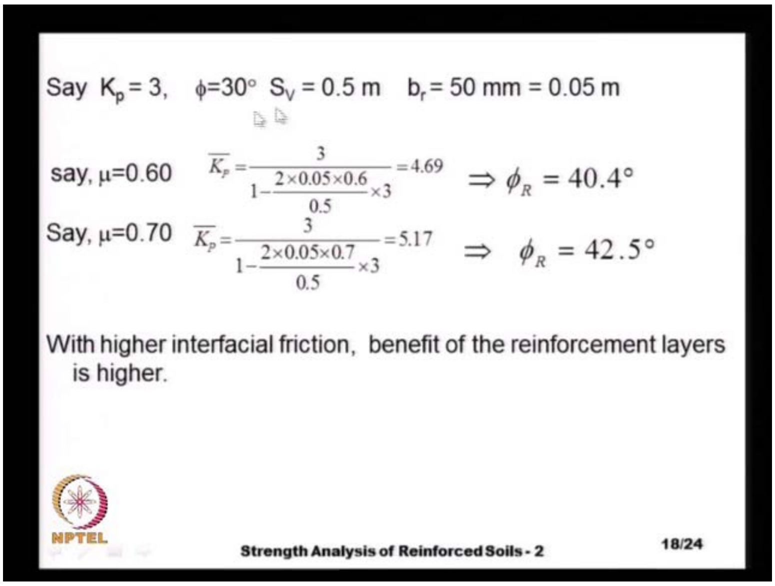

And now let us, substitute some typical numbers, let us say that our friction angle is 30 degrees and for that k p is 3 and vertical spacing, once again let us assume, that it is at 500 millimeters, that is 0.5 meters. And the b r, that is the width of the reinforcement layer that is 50 m m, that is equal to 0.05. And now, let us work out for 2 different friction factors, one for a friction factor of 0.6 where, K p bar is 3 divided by that is the K p by 1 minus 2 times b r, that is 0.05 times, the friction factor 0.6 divided by vertical spacing 0.5 multiplied by K p, that is equal to 4.69. And the by taking the inverse of this, we get the phi for the reinforce soil is 40.4 degrees. And now let us assume that the friction factor between the soil and the reinforcement is slightly higher at 0.7 and if you do the calculation for K p bar, it comes out as 5.17 as opposed to only 4.69 in the previous case.

So, in this case the friction angle of the reinforce soil comes out as 42.5. So, from this simple example, we see that as the interfacial friction angle between the soil and the reinforcement increases, we get higher benefit of the reinforcement layers. So, this is the effect of better interaction between the reinforcement and the soil.

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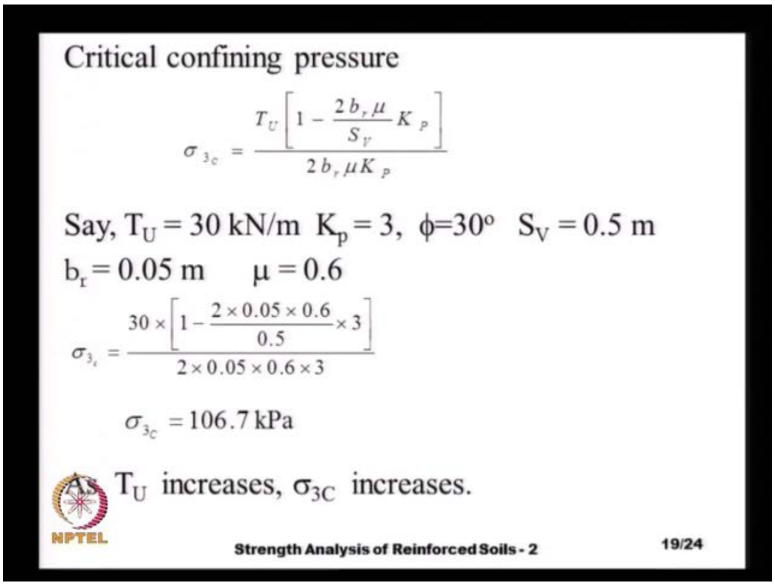

And let us calculate the critical confining pressure corresponding to the pull out failure mode sigma 3 C is T u multiplied by 1 minus 2 b r mu by S v times K p divided by 2 b r mu times K p. And by substituting all our parameters, that is T u is 30 kilo Newton per meter K p is 3 phi is 30 degrees S v vertical spacing is 0.5 meters and the b r, that is the width of the reinforcement layers as 0.05 meters. And the friction factor mu is 0.6. So, our sigma 3 critical is 30 times 1 minus 2 times 2 times 0.05 times 0.6 by 0.5 times K p that is 3 divided by 2 times b r, that is 0.05 mu is 0.6 K p is 3.

And so the critical confining pressure comes out as 1 naught 6.7 and actually from this equation, we see that as the T u increases our sigma 3 increases. And that means, that for this particular properties that, we have if your confining pressure is less than this value of 1 naught 7 kilo Pascal, your failure is by pull out and if your confining pressure is higher the failure is by rupture. That is actually, this is an important relation or important fact to be considered in design, because if your pull out is going to govern then we need to give higher reinforcement length. So, that the failure does not happen or the reinforcement strength is higher.

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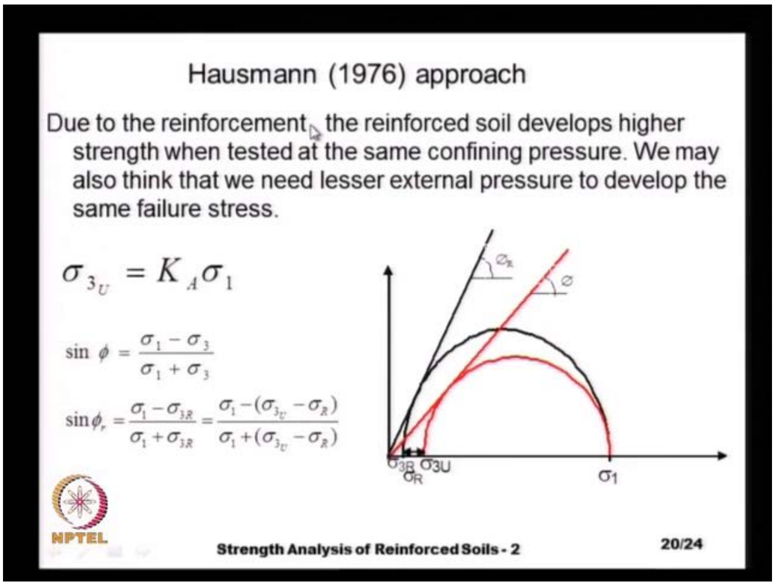

See the previous analysis is directly based on equilibrium between the different forces on the failure wedge and Hausmann in 1976, he has proposed a slightly different approach, but then the results are similar to what we get earlier. But conceptually, the Hausmann approach is superior, because it is directly based on the more circles and then the conceptual understanding of the development of the apparent cohesion and then the friction angle.

And what he said is that, because of the placement of the reinforcement layers reinforced soil develops higher strength when tested at the same confining pressure. Because of the additional contribution by the reinforcement or on the other hand, we can say that, if you want to get the same ultimate compressive strength, we can test reinforce soil at a lesser confining pressure, because of the effect of reinforcement layers and get the same the same compressive strength.

And actually, that is illustrated here, this red circle is for the unreinforced soil, which has a friction angle of phi and this black circle is for the reinforced soil and because here in both the cases. The sigma 1 is the same and the because of the reinforcement effects, we can test the reinforced soil at a lesser confining pressure, that is equal to sigma 3 R. And then the is actually the friction angle of the reinforce soil is higher than, that of the phi that is phi r.

And because we know that sine phi is sigma 1 minus sigma 3 by sigma 3 by sigma 1 plus sigma 3, that is the limiting relation is actually, K a is 1 minus sine phi by 1 plus sine phi and we can transform the relation. And by using this equation sigma 3 as K a times sigma 1, we get this relation and the sine phi R is actually, this sigma 1 minus sigma 3 R, because that is the confining pressure, that we need to test reinforce soil. So, that the same vertical stress sigma 1 is obtained.

So, the sine phi R is sigma 1 minus sigma 3 R by sigma 1 plus sigma 3 R and let us say that at the difference between this sigma 3 u and sigma 3 R is sigma R. And that is the sigma 1 minus sigma 3 u minus sigma R, that is sigma 3 R divided by sigma 1 plus sigma 3 u minus sigma R, and by expanding, this relation and by talking out sigma 1 as a common factor.

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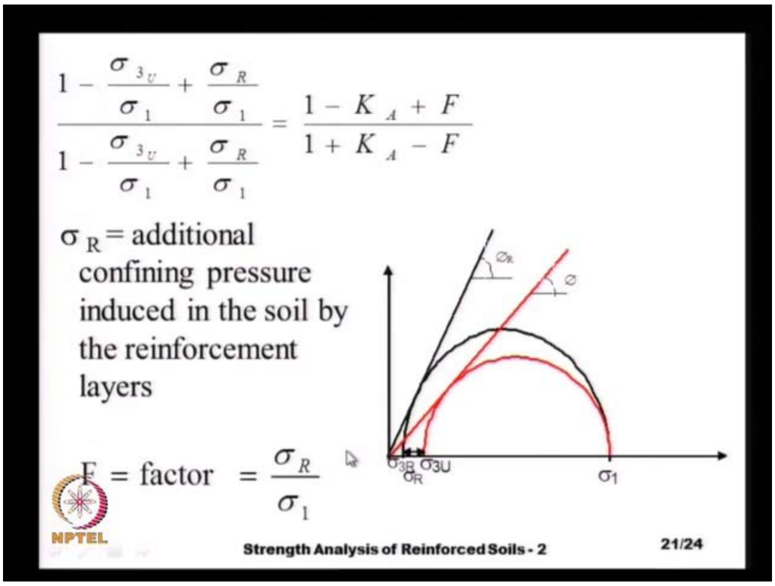

We get 1 minus sigma 3 by sigma 1 plus sigma R by sigma 1, that is 1 plus K a, that is sigma 3 by sigma 1 is nothing but rankine active at pressure constant K a plus some factor F. Let us say that sigma R by sigma 1 is some factor F that divided by 1 plus sorry, this should be plus and this should be minus 1 plus K a minus F. And sigma R is the additional confining pressure induced by the soil by reinforcement and as F is greater than 0. This ratio is going to be very high and because of that, your friction angle phi R is growing to be greater than phi, because our sine phi is 1 minus K a by 1 plus K a. And sine phi or is 1 minus K a plus F, that is we are adding something in the numerator and 1 plus K a by 1 minus F, that is we are subtracting something F from the denominator.

So, because of that the ratio of the numerator divided by the denominator is going to be greater than that for unreinforced soil. So, our the friction angle for the reinforce soil phi R is going to be greater than phi. So, this sigma R is actually, the additional confining pressure induced in the soil by the reinforcement layers. And this additional confining pressure, because of the reinforcement layers, once again it depends on several factors that is the vertical spacing of the reinforcement, that is S v and then the strength and stiffness of the reinforcement layers, that is if you use metallic reinforcement, they will be stronger and they will be stiffer.

So, because of that they may induce higher confinement in the soil and then the interaction between the reinforcement and the soil that is the friction factor mu say, if the mu is higher. Then obviously, mobilize higher confinement forces and your you have higher additional confinement, because of the reinforcement layers. So, this F is just simply written as sigma R by sigma 1 and that explains the relation between the phi and phi R.

And Hausemann is actually, he has also considered another approach, that is the pseudo cohesion approach or apparent cohesion approach. See initially, we call it as apparent cohesion, because we have taken a dry granular soil that has 0 cohesion, that is the C is 0, because the particles, they do not have any inter granular attraction forces unlike in the clays.

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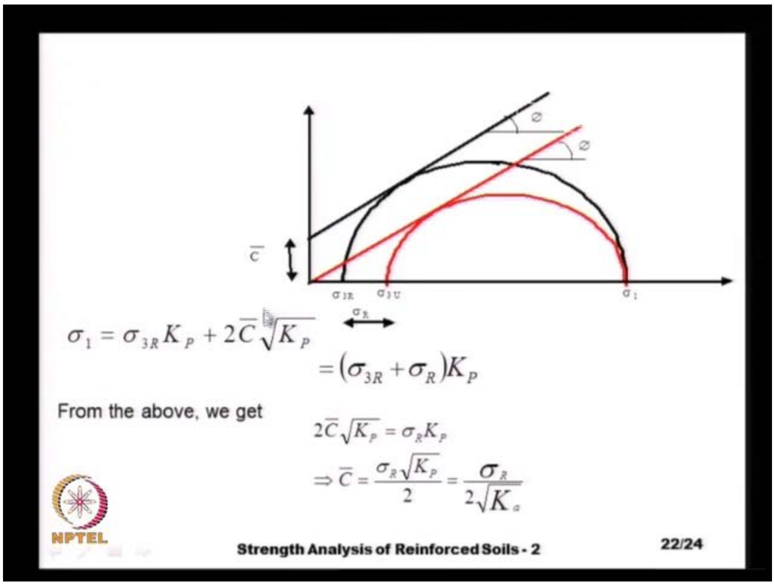

And so we can develop a strength envelop, that is parallel to this line phi, but intersecting the vertical axis at some intercept C bar. And the and if we assume that the friction angle for both reinforce soil and the unreinforced soil are the same at phi the sigma 1 is sigma 3 R K p plus 2 C b 2 C bar K p square root of K p, that is equal to sigma 3 R plus sigma R.

That is actually, the additional confining pressure times K p that is by looking at this equation that is I am sorry, the sigma the additional confinement pressure, that is the by equating, these 2 quantities because this sigma 3 R K p cancels out with sigma 3 R K p. We can equate 2 C v 2 C bar square root K p as sigma R times K p or C bar is sigma R by 2 times square root K a.

And this is actually, this is the that is apparent cohesion is equal to the increase in the confining pressure the sigma R divided by 2 times square root K a. And is actually, this is a beautiful relation that gives us relation between the developed cohesion, that is apparent cohesion C bar and then the confinement, that is provided by the reinforcement layers.

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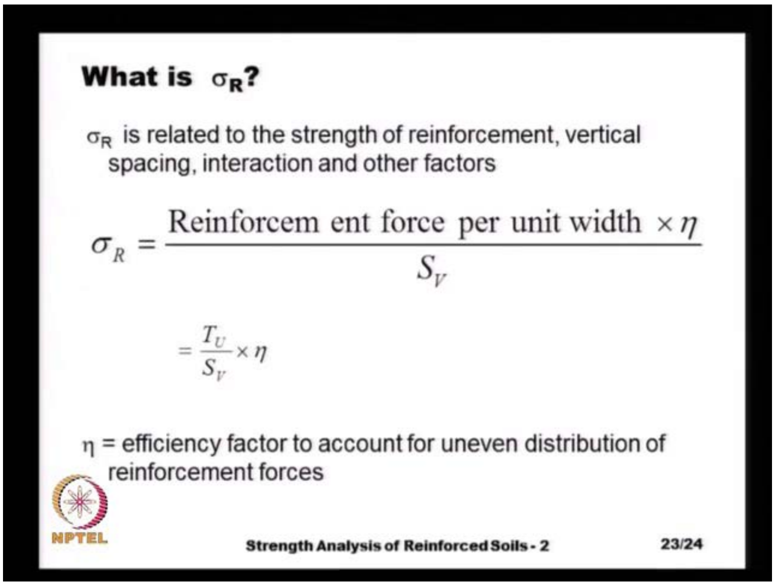

And this sigma R, that is related to the strength of the reinforcement vertical spacing interaction and other factors that govern the force, that is developed by the reinforcement layers that are cut by the rupture wedge. So, the sigma R, we can actually, if the reinforcement failure is by rupture, reinforcement force per unit width multiplied by some eta, that is the efficiency factor divided by S v. The sigma R, we can write as T u by a by S v times eta or if the failure is governed by pull out, we can write the other equation and then the we can develop an equation for sigma R. The reason for placing the eta is to account for uneven distribution of reinforcement forces. Is actually, the we have assumed the extreme relation, that the failure is either by pull out or by rupture, but if we go back and see the fundamental.

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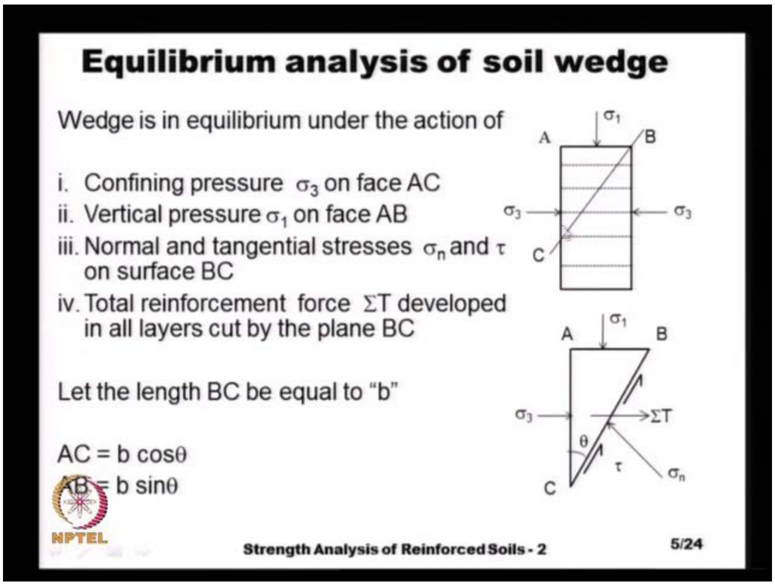

Fact that not all the reinforcement layers have equal embedment length say for example, the this particular layer, it has got, so much of length within the failure wedge whereas, this has only this much. And this particular one has much lesser length the embedment length. So, it is possible that some layers may undergo rupture failure, whereas some other layers may undergo pull out failure and because the contribution that we get by rupture and pull out are not the same, it is very difficult to quantify.

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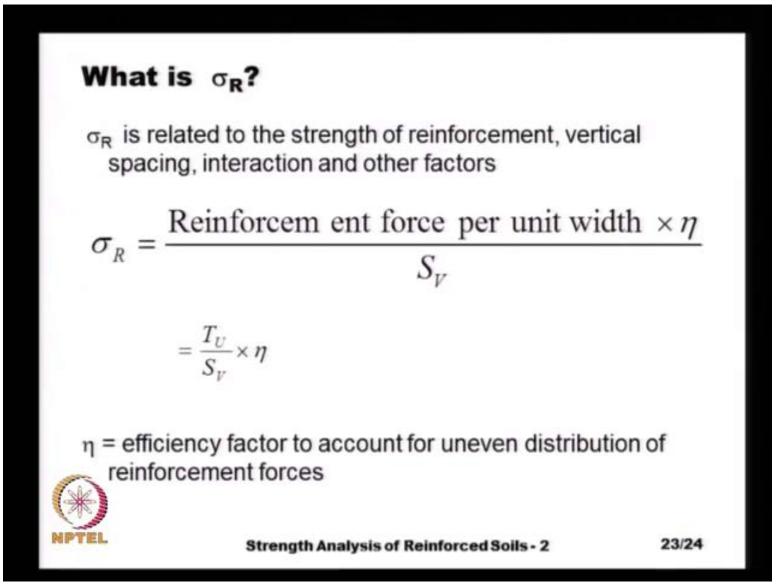

What exactly is the is the force, that is developed. So, we put in some efficiency factor eta and is actually, when it comes to the design, we consider each and every reinforcement layer and then calculate the reinforcement contribution either as the tensile strength or the pull out force, whichever 1 is lesser is actually, considered for the design purposes.

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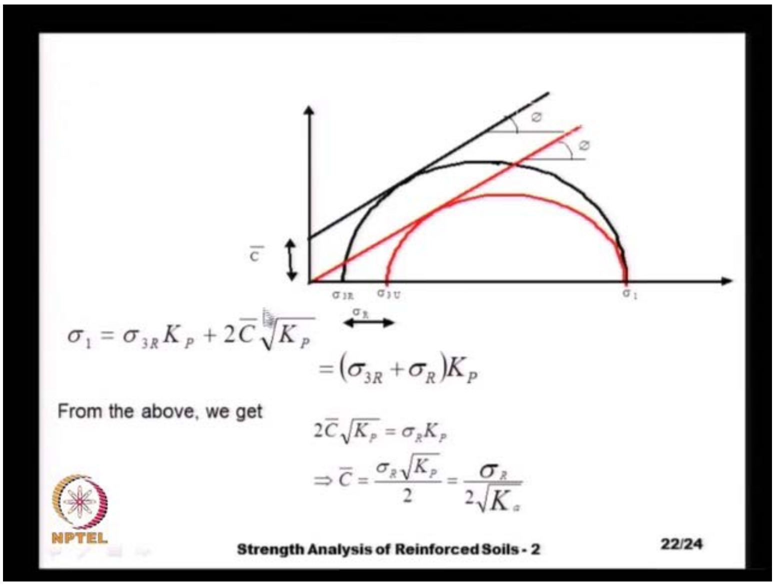

So, this is actually the Hausemann concept is very similar to that of the equilibrium of the failure wedge, but it is a bit more philosophical, because the entire analysis of the strength is through the more Culoumb relation through this more circles.

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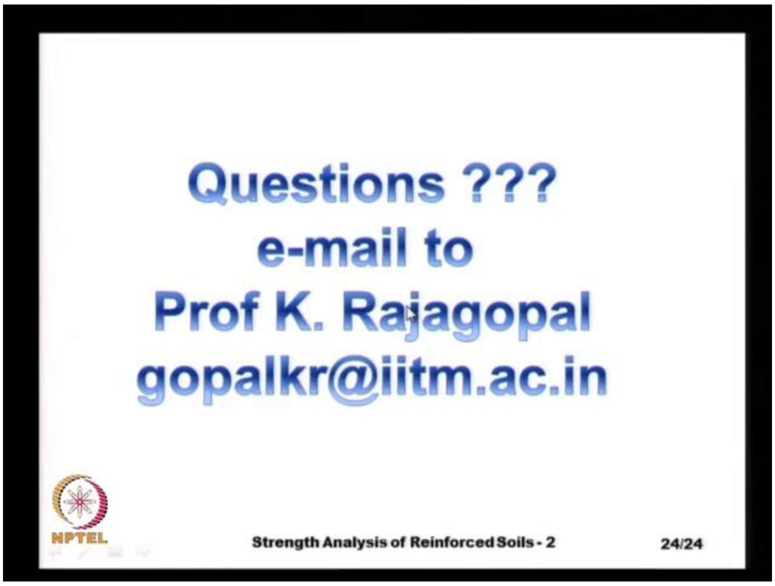

So, this these concepts, we are going to employ for our design purposes and if you have any questions, you can send an email to me and I will try to respond back to you as early as possible.

Thank you.