

Geosynthetics and Reinforced Soil Structures
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Lecture - 4
Strength Analysis of Reinforced Soils - I

Hello students, in the previous classes we discussed about more descriptive aspects of the geosynthetics, that is the strength and different types of materials and then their applications and so on. And now let us look at the some theoretical analysis of the strength of the reinforced soil, how the reinforcement increases the properties of the soil and so on.

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And before we go into the reinforced soil, let us look at the strength of the unreinforced soil itself. And for this purpose, normally we use triaxial compression test data, because it is more easy to explain the triaxial compression data through mathematical analysis.


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Triaxial Compression Tests

- Cylindrical soil samples with Height to Diameter ratio > 2 (e.g. $d=37$, $h=80$ mm)
- Soil is confined with different confining pressures (σ_3)
- While the confining pressure is kept constant, vertical stress (called deviator stress) $= (\sigma_1 - \sigma_3)$ is applied gradually
- Load applied by increasing axial strain at a constant rate
- The peak deviator stress is recorded
- Strength is interpreted from plots of Mohr-Circles prepared using σ_3 vs. σ_1 or mean normal stress $p = (\sigma_1 + \sigma_3)/2$ vs. shear stress

Rupture surface in triaxial test sample



$\tau = (\sigma_1 - \sigma_3)/2$

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And the triaxial compression test, they simulate the actual condition of the soil in the ground, that is the soil is confined laterally from all the directions. Then we apply some vertical loading that is simulate the load that comes from the foundations and so on. And typically what we do is, in the triaxial compression test, we take soil samples that are cylindrical in shape with a height to diameter ratio of at least 2. Some typical diameters are 37 millimeters, 50 millimeters, 100 mm and as much as 300 millimeters, and the heights are about 80 millimeters, 110, 220, and so on.

And to simulate the natural ground conditions, we apply some confining pressures, we take this triaxial soil sample in a cylindrical jar. And then, in case the soil sample itself in a rubber membrane so that, when we fill this cylinder with water, the water does not directly interact with the soil. And we pressurize the water to simulate the lateral confinement, that could act on the soil in the ground.

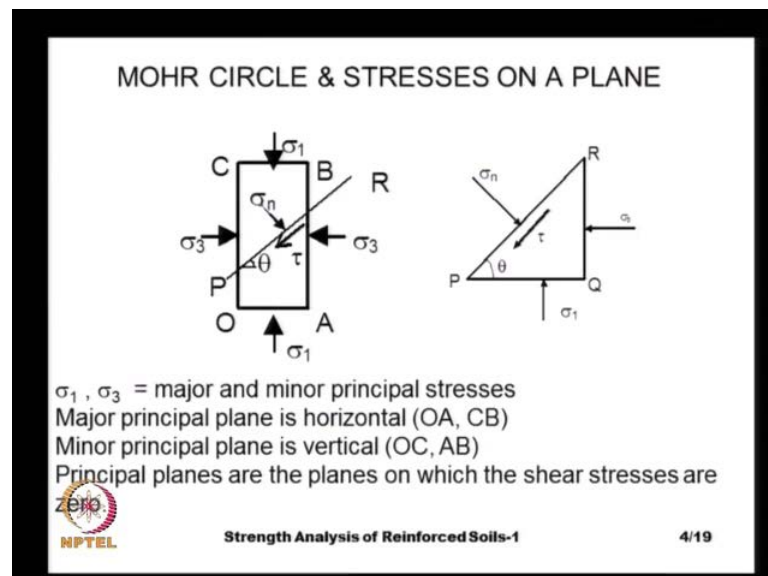
And while we keep the confining pressure constant, we go on increasing the vertical loading until the failure happens and this vertical stress, that we apply is also called as the deviator stress or that is the shear stress, that is actually causing the failure of the soil sample. And in the triaxial test, we apply the loading by increasing the axial strain at a constant rate. So, actually this test is called as a strain controlled test, because we apply axial strain and then, measure the axial load that is developed through some proving ring or a load cell arrangement.

And while when we do the test, we determine the peak deviator stress corresponding to different confining pressures and we perform several of these test at different confining pressures. And once we have the data, we can interpret for the strength, either by plotting the Mohr circles in terms of the σ_3 on the corresponding σ_1 maximum or in terms of the p and q and that we will see later on. And on the right hand side, at the top, you see a typical triaxial compression operators, so actually we have a cylinder perspex cylinder and inside we have the sample.

And you can clearly see that, the sample is placed inside a rubber membrane and then, we apply the axial loading by moving the entire cell upwards at a constant rate. While at the top, it is supported against a load cell to measure the applied load and the typical failure that happens inside a tri axial soil sample or something like this. At failure, we develop a very clear rupture surface and the two parts of the soil and both sides of the rupture surface, they separate out and there is a sliding failure that takes place.

And it is actually, it is more easy to analyze the triaxial test data, because we have only two principle stresses that is, the σ_3 that is confining pressure and σ_1 that is applied in the vertical direction.

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Before we actually analyze, let us look at the Mohr circle and the stresses that are acting on the cylindrical sample and on any arbitrary plane, that we have. And here, the stresses that are acting are the σ_3 that is, we call it as a minor principal stress because that is



smaller than the vertical stress that we apply. And then, the other one is the σ_1 , that we call as the major principle stress or the vertical stress.

And this horizontal plane BC and OA, they are called as the major principle planes and then, the vertical planes OC and AB they are called as the minor principle planes because the minor principle stress is acting on that particular plane. And as we know, the principle planes or the planes on which the shear stress is 0. And now, let us analyze the stresses that are acting on any arbitrary plane like this.


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Triaxial Compression Tests

- Cylindrical soil samples with Height to Diameter ratio > 2 (e.g. $d=37$, $h=80$ mm)
- Soil is confined with different confining pressures (σ_3)
- While the confining pressure is kept constant, vertical stress (called deviator stress) = $(\sigma_1 - \sigma_3)$ is applied gradually
- Load applied by increasing axial strain at a constant rate
- The peak deviator stress is recorded
- Strength is interpreted from plots of Mohr-Circles prepared using σ_3 vs. σ_1 or mean normal stress $p = (\sigma_1 + \sigma_3)/2$ vs. shear stress

Rupture surface in triaxial test sample


 $\tau = (\sigma_1 - \sigma_3)/2$

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Because, we know that, we have seen earlier that, the failure happens along a particular plane, on which the maximum obliquity between the shear stress that is acting on that plane and the normal stress happens.

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MOHR CIRCLE & STRESSES ON A PLANE

σ_1, σ_3 = major and minor principal stresses
 Major principal plane is horizontal (OA, CB)
 Minor principal plane is vertical (OC, AB)
 Principal planes are the planes on which the shear stresses are zero

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And let us take some arbitrary plane PR, that is inclined at an angle of theta to the major principle plane. And let us determine the sigma n, that is the normal stress acting on this plane and the shear stress tau, that is acting along the length of this shear plane.

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Stresses on the plane PR at angle θ to major principal plane

Resolving the forces normal to the plane PR

$$\sigma_n \times PR \times 1 = \sigma_1 \times PQ \times \cos \theta + \sigma_3 \times QR \times \sin \theta$$

$$\cos \theta = PQ / PR \quad \sin \theta = QR / PR$$

$$\Rightarrow \sigma_n = \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta$$

$$= \sigma_1 \cos^2 \theta + \sigma_3 (1 - \cos^2 \theta)$$

$$= (\sigma_1 - \sigma_3) \cos^2 \theta + \sigma_3$$

$$= (\sigma_1 - \sigma_3) \frac{(1 + \cos 2\theta)}{2} + \sigma_3$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

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And this particular wedge, it has the forces or the stress sigma 1, sigma 3 and then, the tau and sigma n and by resolving the forces normal to the plane of the PR that is, in the direction of the normal stress sigma n. And by considering a unit length in the perpendicular direction to the plane of the analysis, sigma n multiplied by PR, that is the

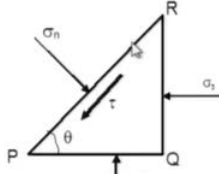
length of the failure plane times 1 is equal to sigma 1. That is, the vertical stress or the major principle stress times PQ times cosine theta that is, to resolve it in the direction normal to this plane of the PR.

And then, sigma 3 multiplied by this length QR multiplied by sin theta to resolve it in the direction of the sigma n or perpendicular to the plane of the PR. And realizing that, the cosine theta is PQ by PR and sin theta is QR by PR, we can write sigma n as sigma 1 cosine square theta plus sigma 3 sine square theta. And by further expanding sin square theta as 1 minus cosine square theta, we can simplify it like this. This sigma n is sigma 1 minus sigma 3 cosine square theta plus sigma 3 and we know that, cosine square theta is 1 plus cosine 2 theta by 2.

So, the sigma n that is, the normal stress acting on this arbitrary plane PR is the sigma 1 plus sigma 3 by 2 plus sigma 1 minus sigma 3 by 2 times cosine 2 theta. And here, the sigma 1 plus sigma 3 by 2 is called as the mean normal stress because that is the average of sigma 1 and sigma 3. And sigma 1 minus sigma 3 by 2, that is called as the shear stress or the q.

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Resolving the forces parallel to the inclined plane



$$\tau \times PR \times 1 + \sigma_3 \times QR \times \cos \theta = \sigma_1 \times PQ \times \sin \theta$$

$$\tau = \sigma_1 \sin \theta \cos \theta - \sigma_3 \sin \theta \cos \theta$$

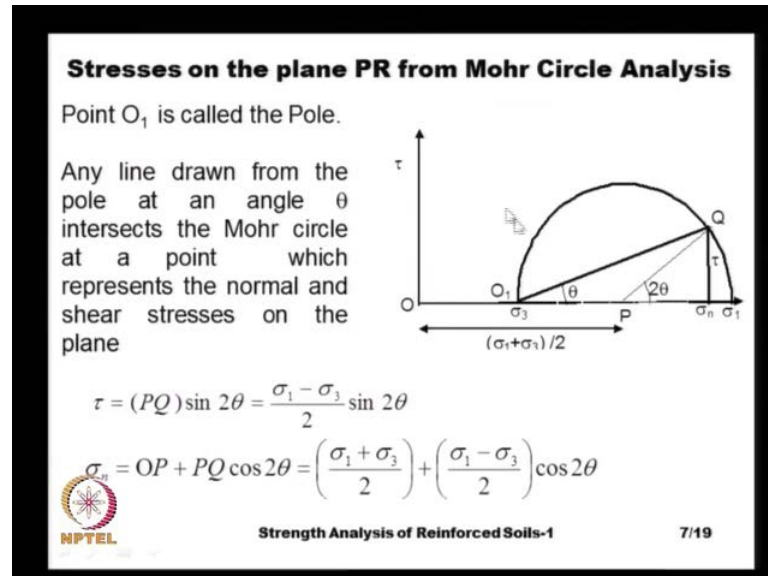
$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$


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And by resolving the forces parallel to the inclined plane PR, we get the tau times PR times 1 plus sigma 3 because that is acting in the direction of the tau, sigma 3 times QR times cosine theta that is equal to sigma 1. That is the major principle stress multiplied by PQ, the length on which the stress is acting multiplied by sin theta. To resolve it

parallel to the direction of this plane, we get tau is that is, the shear stress is sigma 1 minus sigma 3 by 2 times sine 2 theta.

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And is actually, we can get the same result by looking at the Mohr circle and using the principle of the pole, is actually the Mohr circle for the stress of sigma 1 and for the stress state of sigma 1 and sigma 3 is drawn like this. The x axis as called as the normal stress axis and the y axis is called as the shear stress axis, we plot sigma 1 and sigma 3 and then, we draw a circle that is called as the Mohr circle and this point O 1 is called as the pole. The property of the pole is that, a line drawn at an angle of theta intersects the Mohr circle at point, which represents the normal and shear stresses on the plane.

Say for example, if I draw a horizontal line at point O 1, it intersects the Mohr circle at sigma 1 and sigma 1 is nothing but the major principle stress. And then, the shear stress is 0 and that represents the horizontal plane and then, if I want to get the stresses on the vertical plane, I can draw a vertical line at O 1. Once again, I have this vertical line intersects the Mohr circle at sigma 3 and there is no shear stress because it is along the x axis.

And now, let us draw a line at an angle of theta and it intersects the Mohr circle at the point Q, which represents the shear stress and the normal stress on the plane inclined at an angle of theta to the major principle plane. And this ordinate is the shear stress tau and that can be determined as PQ multiplied by sin 2 theta from this triangle. And PQ is

nothing but the radius of this Mohr circle that is, $\sigma_1 - \sigma_3$ divided by 2. And so the τ can be written as $\frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$ and the x ordinate at point Q is the σ_n that is, $OP + P \times PQ$ or $PQ \times \cos 2\theta$.

And OP is nothing but the mean normal stress that is, $\frac{\sigma_1 + \sigma_3}{2}$ and PQ is $\frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$. And incidentally, these stresses are the same as what we determined earlier that is, the σ_n is this, the mean normal stress plus the radius multiplied by $\cos 2\theta$.

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CRITICAL FAILURE PLANE & FAILURE STRESS

On the inclined plane, θ

The normal and shear stresses are

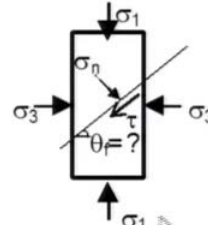
$$\tau = \left(\frac{\sigma_1 - \sigma_3}{2} \right) \sin 2\theta$$


$$\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2} \right) + \left(\frac{\sigma_1 - \sigma_3}{2} \right) \cos 2\theta$$

Maximum shear stress on plane PR

$$\tau = c + \sigma_n \tan \phi$$

$$\left(\frac{\sigma_1 - \sigma_3}{2} \right) \sin 2\theta = c + \left[\left(\frac{\sigma_1 + \sigma_3}{2} \right) + \left(\frac{\sigma_1 - \sigma_3}{2} \right) \cos 2\theta \right] \tan \phi$$





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So, once we determine the shear stress and the normal stress on this arbitrary plane, now we can analyze and then, try to determine the stress, at which the failure happens and also the angle of this failure plane. So, in the previous steps, we have determined the shear stress as $\frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$ and σ_n as $\frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$. And now, let us apply the Mohr coulomb rule on this failure plane which states that, the maximum shear stress τ that you can apply is $C + \sigma_n \tan \phi$.

And because we have the expressions for τ and σ_n from the previous derivations, we can substitute them and then, try to do some mathematical analysis and determine what is the angle θ , at which the senior quality is satisfied. So, actually we have just written it in terms of an arbitrary angle θ and so I am writing this τ that is, σ_1

minus σ_3 by $\frac{1}{2} \sin 2\theta$ is equal to $C + \sigma_1$ plus $\frac{\sigma_3}{2} \cos 2\theta$ multiplied by $\tan \phi$.

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By re-writing the above equation


$$\sigma_1 = \frac{\frac{\sigma_3}{2} [\cos 2\theta \tan \phi - \tan \phi - \sin 2\theta] - C}{\left[\frac{\tan \phi}{2} + \tan \phi \frac{\cos 2\theta}{2} - \frac{\sin 2\theta}{2} \right]}$$

σ_3 is constant and σ_1 is gradually increased
 What is the minimum σ_1 at which failure occurs?
 Maximise the denominator quantity w.r.t. θ

$$\frac{d}{d\theta} \left[\frac{\tan \phi}{2} + \tan \phi \frac{\cos 2\theta}{2} - \frac{\sin 2\theta}{2} \right] = 0$$

$$\Rightarrow \tan \phi \tan 2\theta = -1$$

$$\Rightarrow 2\theta_r = 90^\circ + \phi = \frac{\pi}{2} + \phi$$

$$\Rightarrow \theta_{cr} = \frac{\pi}{4} + \frac{\phi}{2}$$


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And by simplifying this, we can get this the σ_1 that is, the vertical stress as σ_3 by 2 times this whole quantity that is, $\cos 2\theta \tan \phi - \tan \phi - \sin 2\theta - C$, this divided by $\tan \phi$ by 2 plus $\tan \phi \cos 2\theta$ by 2 minus $\sin 2\theta$ by 2. And now, what is the θ that satisfies this equation of the Mohr coulomb relationship that is, the τ is $C + \sigma_n \tan \phi$. And here, this equation σ_3 is the minor principle stress or the confining pressure, that is kept constant during the test.

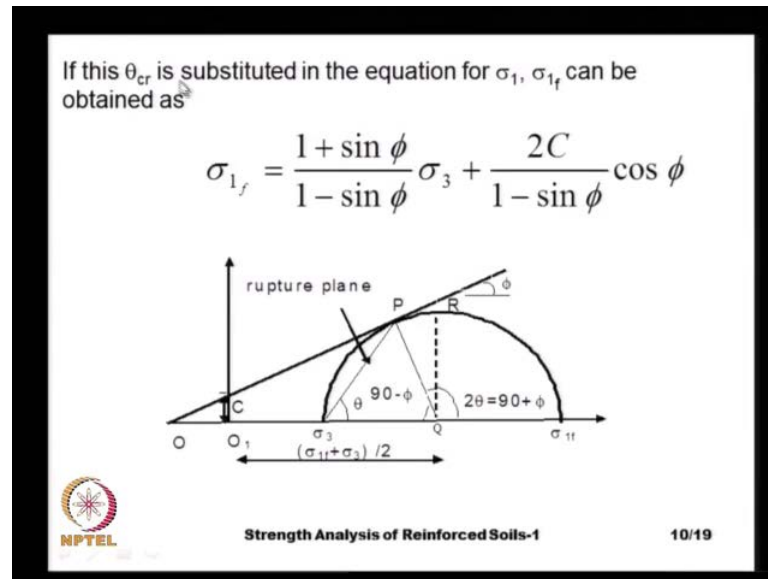
And the C and ϕ , they are the shear strength properties of the soil, C is the cohesive strength and ϕ is the friction angle of the soil and once again, those two are constants and the only quantity that is variable is the θ . And now, let us find out, as the vertical stress σ_1 is increased, what is the minimum stress at which the failure happens. And for that, the denominator should be maximized and we can determine the θ by taking the derivative of this denominator and setting it to 0.

And so if we do that, $\tan \phi \tan 2\theta$ is equal to -1 and we get this θ critical that is, the ruptured plane as $\frac{\pi}{4} + \frac{\phi}{2}$. And actually, it is a very important relation because this angle is used not only for the shear strength analysis but also for determining the rupture planes for the design of retaining walls or for design of

embedments and so on. And so for a undrained clay soil with a friction angle of 0, the rupture plane is at an angle of 45 degrees to the major principle plane.

Invariably the major principle plane is horizontal because our vertical stresses are much higher compared to the lateral stresses in most cases.

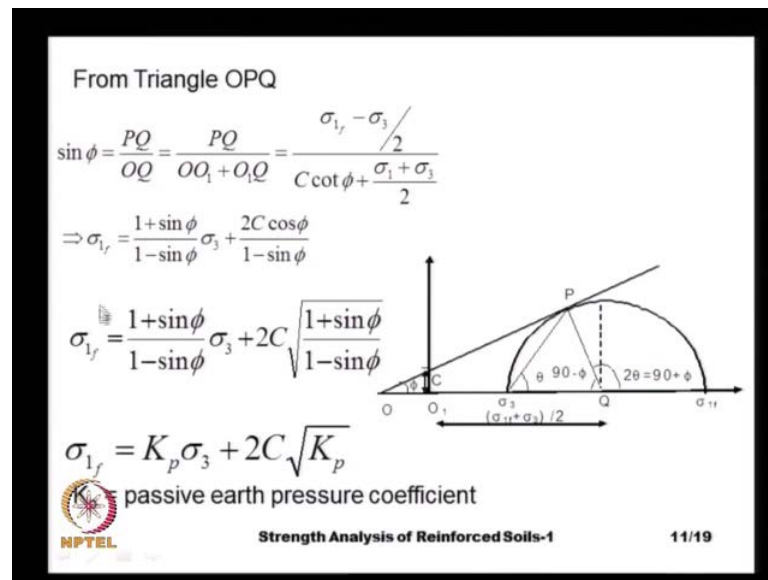
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So, and if this θ_{cr} is substituted in the equation for σ_1 that is, if we substitute this θ_{cr} in this equation for σ_1 and simplify, we get a relation that, σ_{1f} failure or σ_1 maximum is $\frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 + \frac{2C}{1 - \sin \phi} \cos \phi$. And here, the point P actually has the maximum obliquity, see in this Mohr circle, the maximum shear stress is QR and at point P, the shear stress is less than this but then, the failure happens here.

That is because the ratio between the τ / σ_n has higher value at this point as compared to this point or where the shear stress is much higher. So, in this Mohr circle, we call this line, that is drawn from the pole to point P as the rupture plane because that has the angle of $45 + \frac{\phi}{2}$. And now, we can actually analyze for the shear strength and other aspects through this Mohr circle.

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Actually, let us try to determine the same sigma 1 f by looking at this Mohr circle like this and let us draw a circle like this. And at the failure state, the Mohr circle is just tangent to the strength envelope, the strength envelope is this, this inclined line and that intersects the y axis or the shear stress axis at point C and it has a slope of phi to horizontal. And by considering the geometry of this Mohr circle with respect to this strength envelope, we can determine the relation between sigma 1 f and sigma 3.

And by considering this triangle OPQ, the sin phi is PQ divided by OQ and PQ is nothing but the radius of this Mohr circle that is, sigma 1 f minus sigma 3 by 2. And OQ is O O1 plus O1 Q and O O1 is this small length that is, C times cot times phi and this O1 Q is the mean normal stress that is, sigma 1 plus sigma 3 by 2. And by simplifying this relation, we get sigma 1 f is 1 plus sin phi by 1 minus sin phi sigma 3 plus 2 C cosine phi by 1 minus sin phi.

And this quantity cosine phi by 1 minus sin phi can be written like this, the cosine phi is nothing but square root of 1 sin square phi and 1 minus sin square phi is 1 plus sin phi times 1 minus sin phi. And by taking this denominator 1 minus sin phi as square root of 1 minus sin phi times 1 minus sin phi, we can write this right hand side sigma 1 f as, 1 plus sin phi by 1 minus sin phi sigma 3 plus 2 C square root of 1 plus sin phi by 1 minus sin phi.

And this quantity $1 + \sin \phi$ by $1 - \sin \phi$ is called as K_p that is, passive earth pressure coefficient or we also call it as $n \phi$ that is, that represents the strength of the soil. And so now, we have a relation between the maximum vertical stress, that we can apply on the soil with shear strength properties of C and ϕ that is, the C is the cohesive strength and ϕ is the frictional strength and σ_3 is the confining pressure that is applied on the soil.

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Triaxial Compression Tests on Unreinforced and Reinforced Dry Sands

- Soil is a well graded angular coarse river sand
- Sand placed at 70% relative density by tamping
- Reinforcement Layers are soft mosquito meshes
- Diameter of the soil sample = 50 mm
- Height of the soil sample = 110 mm

Sample under preparation Reinforcement disks Triaxial apparatus

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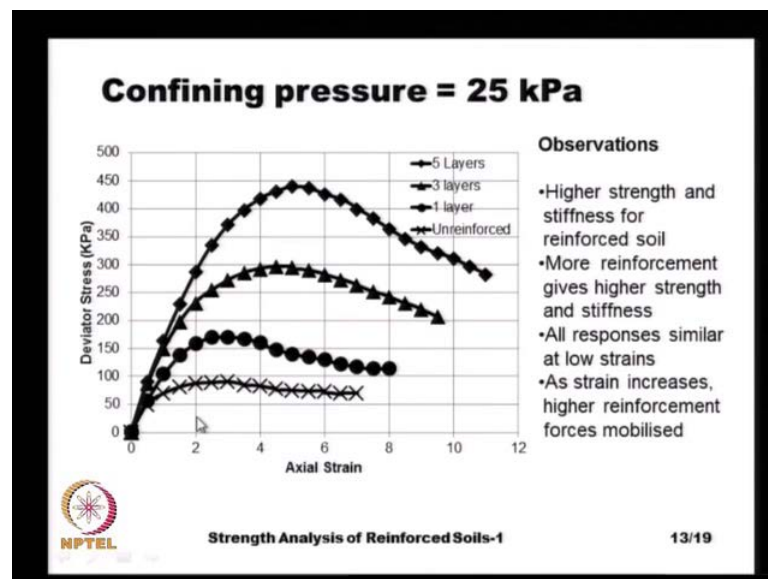
Let us look at some triaxial compression test data and both unreinforced and reinforced dry sands because here we use dry sands because typically in the reinforced soil structures, we use soils that are very well draining. That means, that we have to use coarse grained soils like sand or some aggregate or gravel and so on. And in this particular case, just for demonstration, we have used dry sand and the particular soil that is used is a river sand.

So, well graded sand and relatively coarse grained particles and angular, and the sand is placed in the triaxial compression sample at 70 percent relative density by adequate temping. And the reinforced test, they are performed by using soft mosquito meshes, they are shown here and the diameter of the soil sample is 50 millimeters and the height of the soil sample is 110 millimeters. And here, we can see the bottom of the triaxial compression cell and this is the sample that is being prepared.

And during the preparation, we support the soil sample in an aluminum former because when there is no confinement stress, the sand cannot be stable. And if we do it without the sample former, the soil will just simply collapse and in order to form it easily, we confine the soil in an aluminum foil or aluminum sample former. And here, we see number of these taps, some are to apply the cell pressure and some are to measure the pore pressures or to apply the back pressure and so on.

In this particular case, all the tests are done on a dry sand and in dry condition and so here we can see the close up, you can see the coarse grained nature of the soil. And these are the reinforcement disks that are used and the diameter of these reinforcement disks slightly less than the diameter of the soil sample that is, 50 millimeters. And on the right hand side, you can see some of the tri axial operators with the outside cell, the tri axial cell and inside the soil sample and so on.

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And the typical compression test data is shown like this and as I mentioned earlier, we perform the test at different confining pressures and this particular test data is obtained at a pressure of 25 kPa. And here, what we see is the data with axial strain on the x axis and the deviator stress on the y axis and where, the test was done on the unreinforced soil. This is the response, the peak stress of the unreinforced soil is nearly 90 kilo Pascal and when one layer of reinforcement was placed at mid height of the sample, the deviator stress increase to nearly 160 kPa.

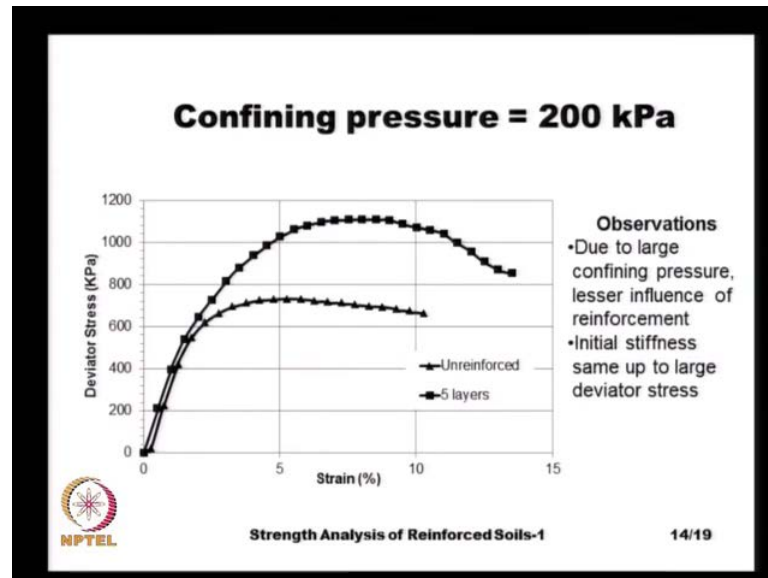
And when three layers of reinforcement were used, the one at one third height, one at mid height and other two thirds height. The response has further increased, the deviator stress is increased to 300 kPa and when five layers were employed in the triaxial compression test, the strength has further increased to nearly 450 kPa. And some of the observations that we can make are, the higher strength and stiffness for the reinforced soil, the higher strength that is, the peak deviator stress has increased when we put in more and more number of reinforcement layers.

Because basically when we apply some vertical stress, the soil wants to deform laterally and develop the rupture plane. But, because of the placement of the reinforcement soils, reinforcement layers, the reinforcement keeps the soil together by preventing the lateral expansion and it also abstracts the formation of the rupture plane and because of that, higher deviator stress develops. And the other interesting thing is, the initial slope of all the stress strain curves is almost the same that is, when we first start the test, the reinforcement layers may not have developed any tensile strain.

And once if there is no tensile strain, the reinforcements will not have any stiffness and they will not be able to contribute any strength or stiffness to the reinforced soil composite. And because of that, the slope of all these four graphs is very similar at the start and as the axial strain increases, the reinforcement layers start developing some strains. And they start developing some force and develop some resistance against the lateral deformations.

And here, we see that, upto about 50 or 60 kPa, the stress strain curve of both unreinforced soil and one layer reinforcement is almost the same. And as the number of layers is increased the stiffness, the slope of the stress strain curve is increasing and upto about 160 kPa, the response of both the three layers and five layer curves are almost the same. So, that once again supports the contention that, the reinforcement forces, they develop only after a certain amount of deformation that takes place in the reinforcement layers.

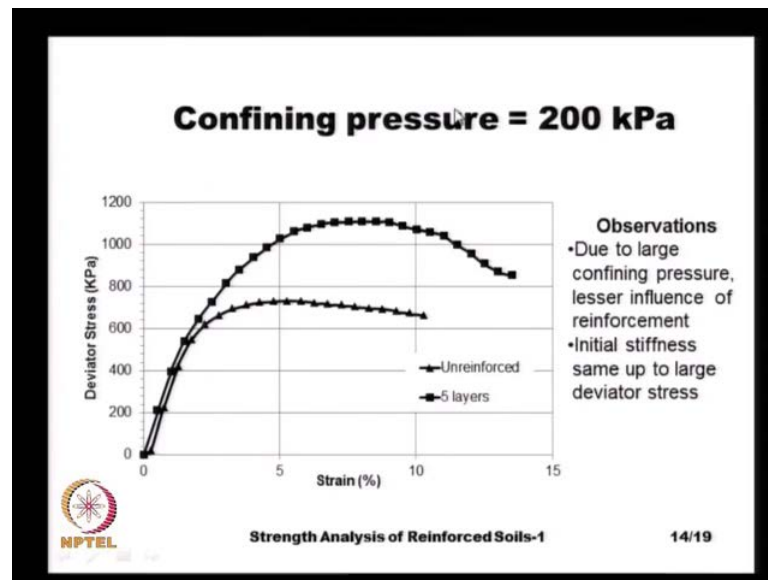
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And now, let us look at the same data when the test was performed at much higher confining pressure 200 kPa. And here, just for illustration purpose I am showing only two graphs, one corresponding to the reinforced soil and the one other corresponding to five layers. And here, the one thing that we notice is that, the slope of the unreinforced soil and the reinforced soil with five layers is almost the same upto nearly 600 kPa pressure.

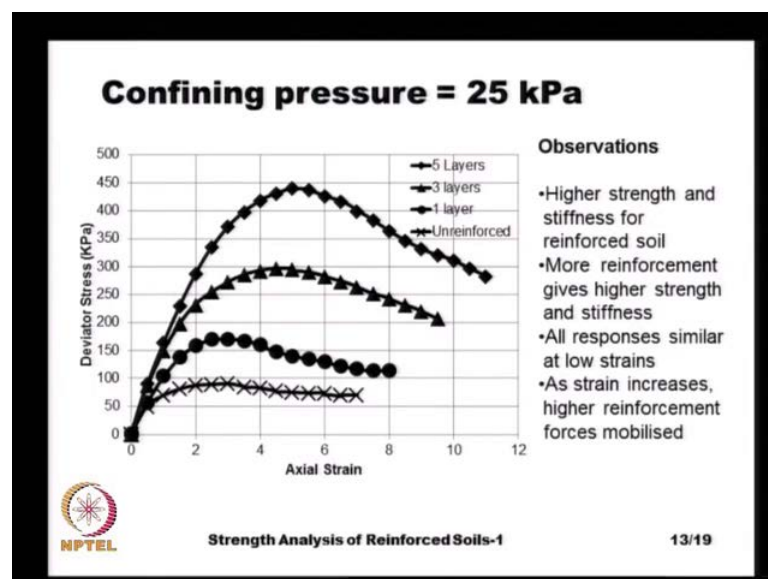
Because, the confinement is so high that, it takes very large amount of strain for the reinforcement layers to develop strain, because as you apply the axial compression stress, the lateral deformations may not have taken place, because of the higher confining pressure from the cell pressure. And because of that, the slope of the both the reinforced soil and unreinforced soil is almost the same upto about 600 kPa as compared to very small stresses in the earlier cases, when we did the test at a lower confining pressure of 25 kPa.

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And then, at some stage, the peak stress has developed and in this case, the difference between the peak stress of the unreinforced soil and the reinforced soil is not as much lesser as compared to the earlier case.

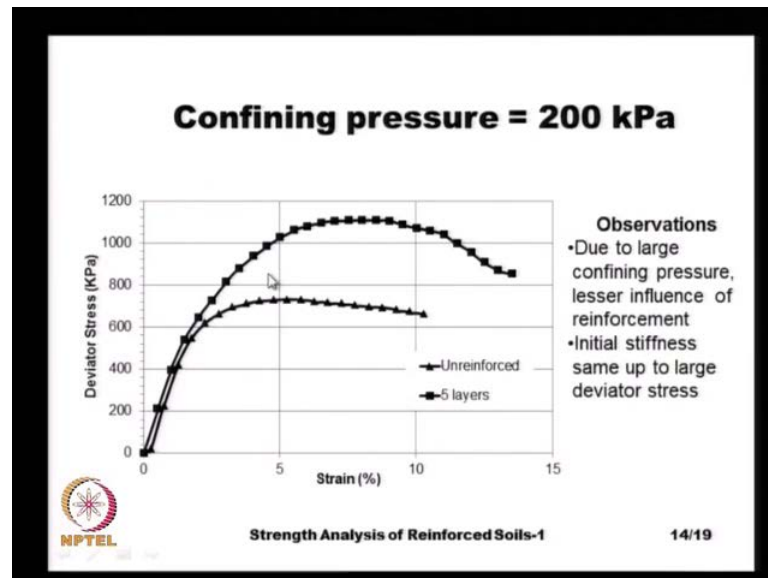
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That is, in this case, when the test was done at 25 kPa and the difference between the unreinforced soil and the reinforced soil is say, here the unreinforced soil is failed at a deviator pressure of nearly 90 kPa and the reinforced soil with five layers, it has failed at

a deviator stress of nearly 450. That means that, the deviator strength is almost 5 times higher compared to that of the unreinforced soil.

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Whereas, at a higher confining pressure, the difference is only is much lesser say, for example, the unreinforced soil has failed at a pressure of nearly 750 kPa whereas, the reinforced soil, it has failed at about 1100 kPa. So that means, the ratio between these two is only 1.5 whereas, in the previous case, the pressure difference was very, very high almost 5 times. So, just to summarize once again, because of the very large confining pressure, the influence of the reinforcement is lesser and the initial stiffness is also very similar upto very large deviator stress.

So, by comparing the results from the low confining stress that is, 25 kPa and a very high confining pressure of 200 kPa we can learn that, the soil requires reinforcement only when there is not much confinement stress. That is, when the soil is at the surface or when the shear stresses are very high at very low confining pressures then, we can put in the reinforcement layers and so that the soil will develop strength. That is say, when you have a footing, the soil requires reinforcement closer to the footing than at a very deep depth.

Because at a deep depth, the confinement pressures are higher than at a shallow depth and if the confinement pressures are high, the natural strength of the soil itself could be very high and it may not require much of reinforcement.

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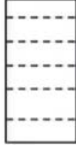
Data from Triaxial Compression Tests on Unreinforced and Reinforced Dry Sand

Unreinforced sand

σ_3 (kPa)	$(\sigma_1 - \sigma_3)_u$ (kPa)	σ_{1u} (kPa)	$p = (\sigma_{1u} + \sigma_3)/2$	$q = (\sigma_{1u} - \sigma_3)/2$
25	89.1	114.1	69.6	44.6
100	468.4	566.4	333.2	234.2
200	731	931	565.5	365.5

5-layer Reinforced sand

σ_3 (kPa)	$(\sigma_1 - \sigma_3)_u$ (kPa)	σ_{1u} (kPa)	$p = (\sigma_{1u} + \sigma_3)/2$	$q = (\sigma_{1u} - \sigma_3)/2$
25	439.8	464.8	244.9	219.9
100	828.7	928.7	514.4	414.4
200	1109.3	1309.7	754.9	554.7



Strength Analysis of Reinforced Soils-1

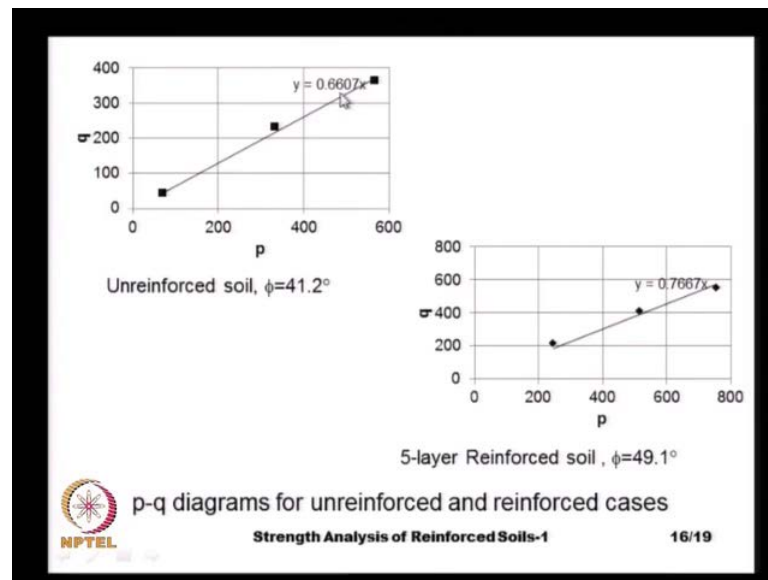
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So, this is the consolidated data for both unreinforced and the five layer reinforced soils and just for brevity, only some test data is given corresponding to 25 kPa, confining pressures of 25 kPa, 100 and 200 kPa. And unreinforced soil, it has failed at 89.1 and deviator stress at a confining pressure of 25, at nearly 470 at a confining pressure of 100 and at 730 at a confining pressure of 200. And so the sigma 1 is this deviator stress, sigma 1 minus sigma 3 plus sigma 3 that is, this 14.1.

And the p is the mean normal stress or sigma 1 plus sigma 3 by 2 that is, 69.6 and then, the q is the shear stress that is, the deviator stress by 2, sigma 1 minus sigma 3 by 2. And so we need to do the similar calculations for the other confining pressures of 100 and 200 and the data for the five layer reinforced sand is like this. At 25 kPa, the soil has failed at deviator stress of 439.8, which is almost 5 times more than that of the unreinforced soil.

And the five layer reinforced soil has five layers of reinforcement, as shown here at equal spacings and sigma 1 is sigma 1 minus sigma 3 plus sigma 3. And p is the average stress sigma 1 plus sigma 3 by 2 and the q is the shear stress sigma 1 sigma 3 by 2. And by using this data, we can determine the shear strength properties the C and phi.

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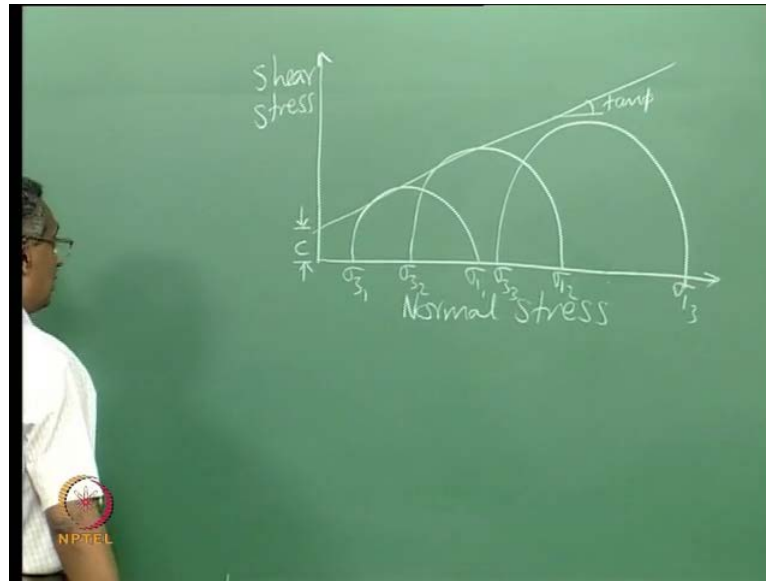


It is actually here, we see the p q diagram that is, the p is the mean normal stress on the x axis and the q is the shear stress on the y axis. And here, just for simplicity, I am showing only three data points corresponding to confining pressures of 25, 100 and 200, and the slope of this line is 0.66 and the friction angle, the phi is sin inverse of this. And it is actually, this line is forced to go through the origin because we have used basically dry sand, which has no cohesive strength.

And so the C is 0 and the phi is 41 degrees and the same data for reinforced soil has a higher slope. So, once again on the x axis, we have the p that is, the mean normal stress and the y axis, we have the shear stress q and the slope of this line now is much higher 0.77 and sin inverse of this is 49 degrees. So, actually we see that, the shear strength of the reinforced soil is much higher than that of the unreinforced soil.

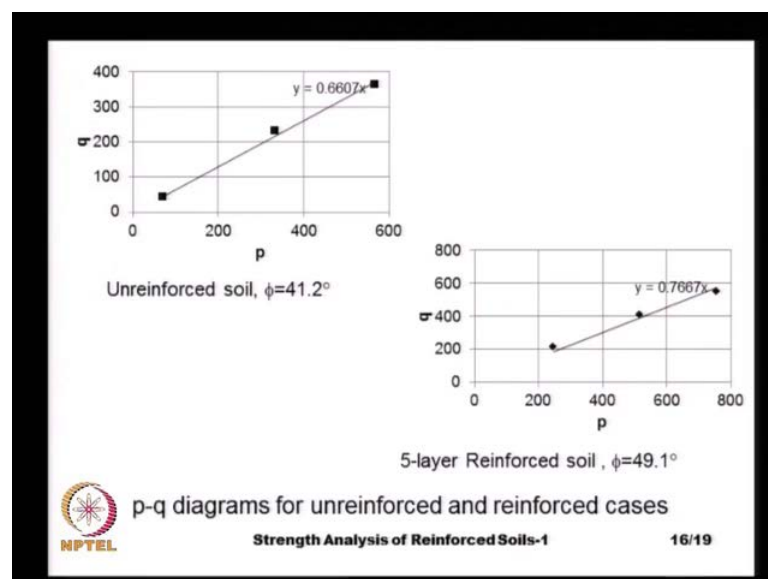
And the other way of interpreting this data is by drawing Mohr circles, I will just briefly show them to you on the blackboard.

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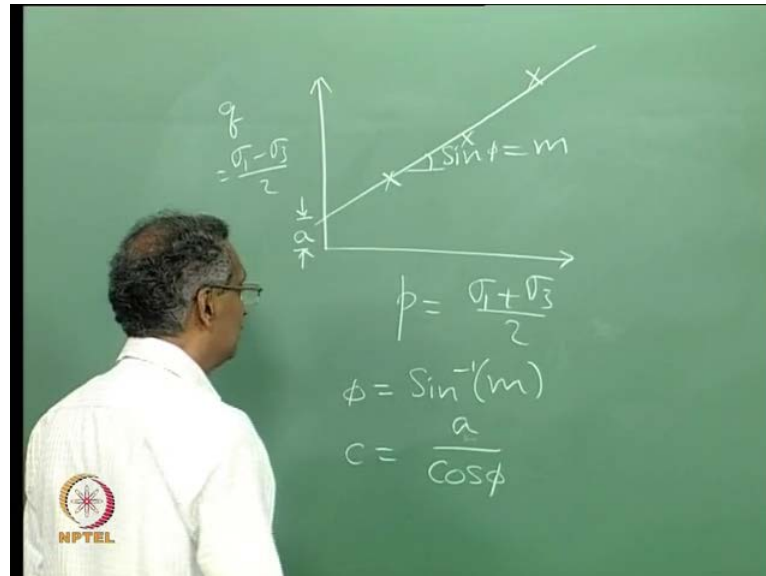
So, we draw the different Mohr circles corresponding to the different confining pressures $\sigma_3 1$, $\sigma_3 2$ and $\sigma_3 3$. And we try to draw a common tangent for all the three circles and invariably, it becomes difficult to draw a common tangent, because of some small experimental errors. And because of that, we prefer $p-q$ diagrams, rather than drawing this the common tangent but here, we get the cohesive strength C as the intercept on the y axis. And this slope is $\tan \phi$ and if we have the $p-q$ diagram, the interpretation becomes more simpler.

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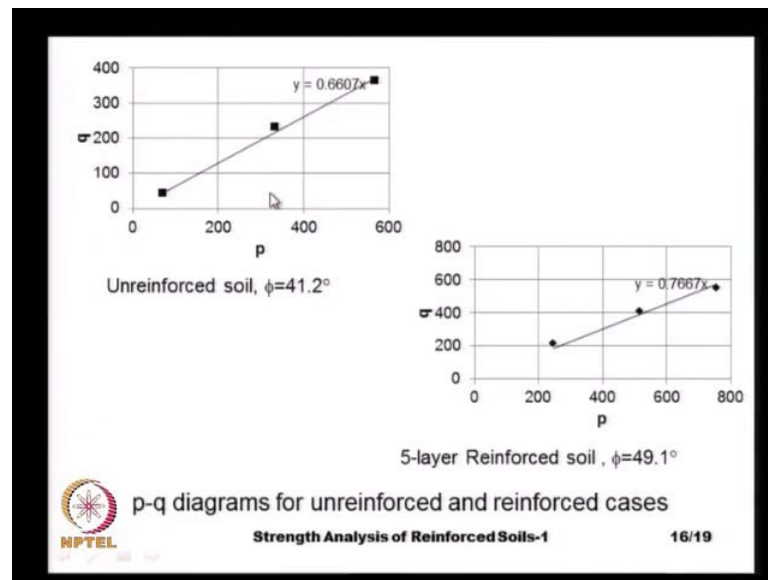
Because, as we see here, this is a regression line and this regression can be either done in a computer program or even now a days, even in the small calculators also, we can do the regression and the interpretation of the p q diagram is slightly different.

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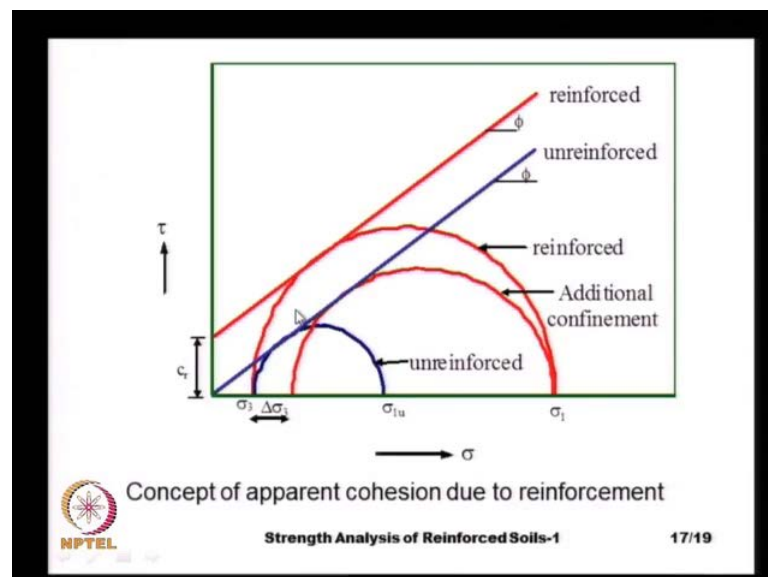
Let us say that, we have some test data and we can always do some regression and plot a best fitting line through all the data points. And they intercept is a and this slope is $\sin \phi$ say, if this slope of the line is m , the ϕ can be interpreted as \sin inverse of m . And if a is the intercept, this c is a by \cos ine ϕ , these theoretical relations can be derived from the fundamentals by drawing the p q line or the Mohr circles.

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And here, we have only interpreted by drawing the p q diagrams for simplicity and now, we see that, the friction angle of the unreinforced soil is 41 degrees and the friction angle of the reinforced soil is 49 degrees.

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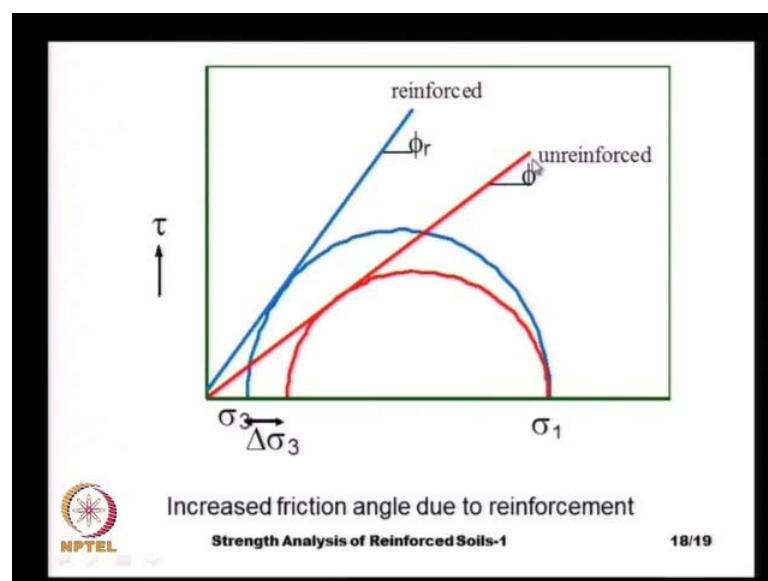


And now, let us do some analysis on this increased strength, this blue circle is the Mohr circle for unreinforced soil and this red circle is for the reinforced soil, which has failed at a much higher axial stress σ_1 . And we can also think of the increased axial stress, as because of the increase in the confinement pressure. And there are different concepts

that explain the increase in the strength of the reinforced soil, one concept is we draw a let us say that this blue line is the strength envelope of the unreinforced soil with $c = 0$ and some slope of ϕ .

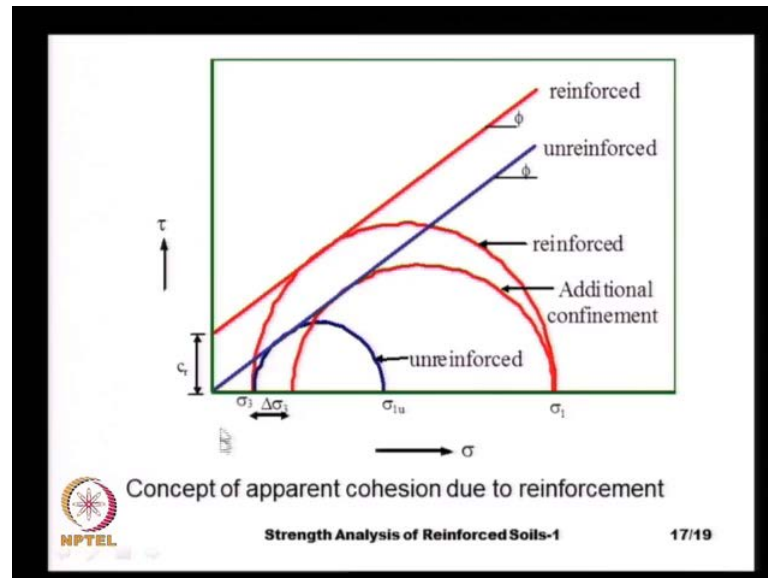
And this increased, this larger circle the red circle is for the reinforced soil and we can draw a line that is, tangent to this strength envelope of the unreinforced soil with the same slope of ϕ but with a vertical intercept of C and this C is called as the apparent cohesion.

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The other way of interpreting this result is like this, the ϕ that is the strength of the unreinforced soil and the increased slope is the slope of the reinforced sand, that is with a friction angle of ϕ_r . And see these actually, what we do is, we assume that the reinforcement is increasing the confinement pressures, just as what happens when we construct an external retaining wall that laterally supports the soil.

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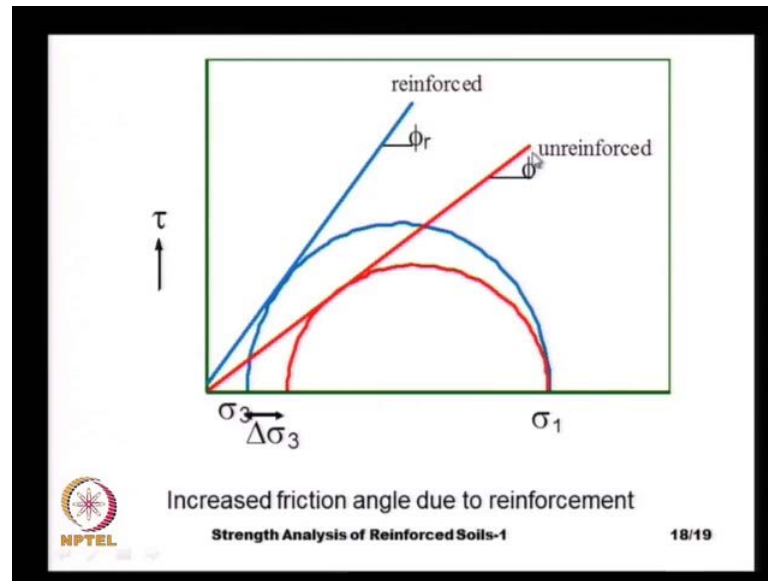


The internal reinforcement layers, they support the soil and because of that, the confinement pressure σ_3 has increased to $\sigma_3 + \Delta\sigma_3$ and that leads to a higher σ_1 that is, the higher axial stress. So, we get a larger Mohr circle or other way of looking at it is, you assume that, the σ_3 remains constant but then, there is some apparent cohesion the C , because of that which the axial stress increases to σ_1 .

And these aspects, we will study them in the next lecture and just summarize this lecture, we have seen the triaxial compression device for shear testing of the soils. And then, we have analyzed the stresses on an arbitrary plane, that is inclined at an angle to major principle plane. And then, theoretically, we have derived a relation between the σ_1 that is, the vertical stress or the deviator stress and the σ_3 in terms of the shear strength properties, that is the C , that is the cohesive strength and the ϕ , that is the friction angle by using the fundamental soil strength relation that was developed by Mohr coulomb that is, the τ is $C + \sigma_n \tan \phi$.

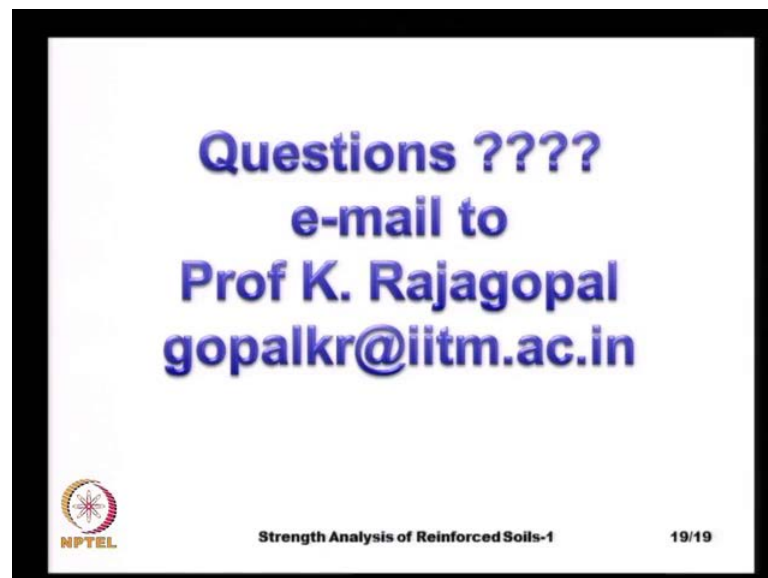
And by analyzing the stresses on the arbitrary inclined plane, we could find the nice relation that, σ_1 is $K p$ times σ_3 plus $2 C$ times square root $K p$ and the rupture plane is at an angle of $45 + \phi/2$. And that is a very important relation, that we are going to use them, use even in the future analysis when we go to design of the retaining walls and embedment's, and so on.

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So, that is a brief about today's lecture and in the next lecture, we will see the influence of the reinforcement and how to account for it.

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So, if you have any questions, you can send an email to me and I will try to response to them to you as early as possible.

Thank you.