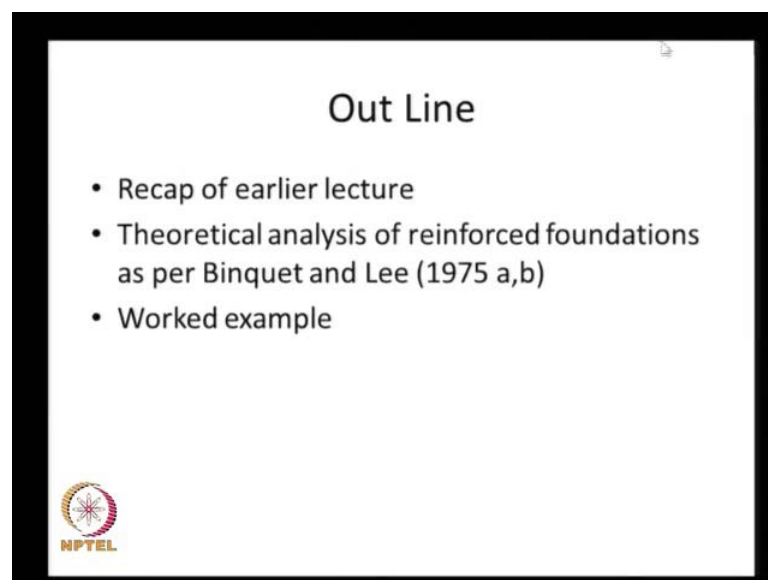


**Geosynthetics And Reinforced Soil Structures**  
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**Lecture - 29**  
**Bearing Capacity Analysis of Footings Resting on Reinforced Foundation Soils**

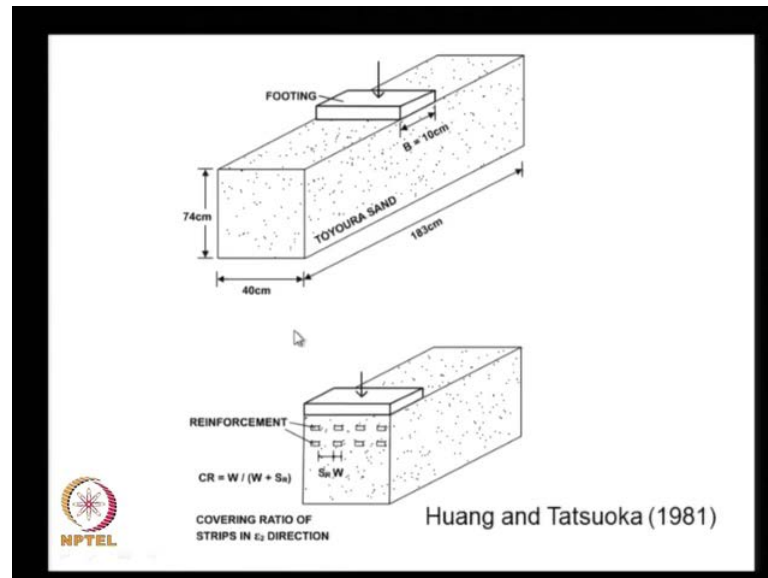
A very good morning students. In the previous class, we have seen some experimental data on the performance of footings that are resting on reinforced soil beds. And in today's class, let us look at the theoretical analysis of the improvement that we get by providing reinforcement layers, both in terms of the bearing capacity and also in terms of the settlements.

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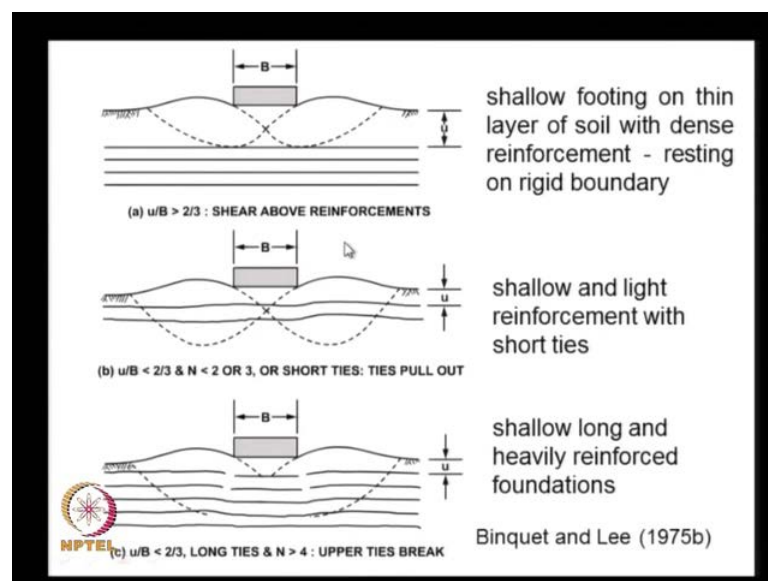
Just to give you a brief outline, the today's lecture will consist of a brief recap of the previous lecture. And then we will go through the theoretical analysis of the reinforced soil foundations. And this will be entirely as per the analysis that was proposed by Binquite and Lee way back in 1975. And then we will also look at one worked out example to illustrate all the equations that we derive.

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Just to recap, this is what Binquet and Lee have done. They have taken stiff footing that is resting on soil bed that is reinforced with number of ties, like this discreet ties. Each having a certain width and certain thickness, and that are provided at different vertical spacing. And then they have first observed the experimental performance. And then they try to explain the observed performance through some simplystick theoretical analysis. This particur picture is from Huang and Tatsuoka of just taken this picture because it is more easy to explain the cross section that was considered by Binquite and Lee.

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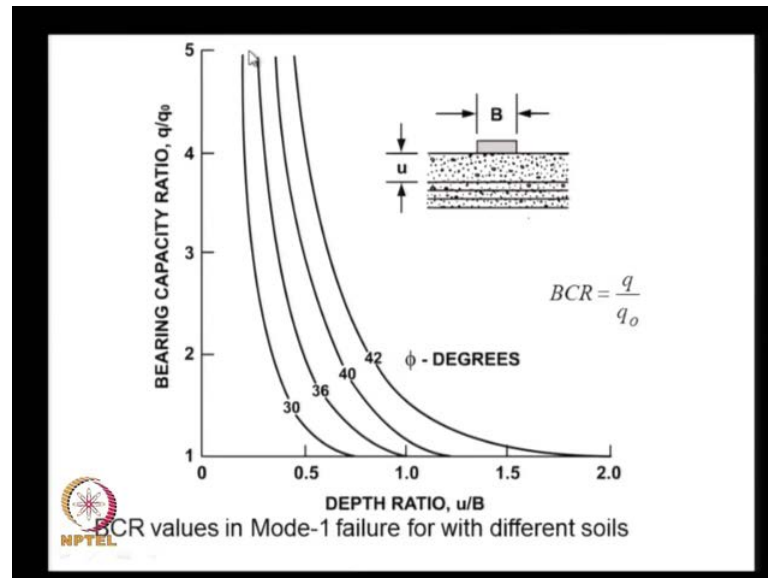
And based on the results that they have obtained, they have classified the bearing capacity failure into 3 different modes; mode 1, mode 2 and mode 3. And the mode 1 happens when the first reinforcement layer is at a sufficient depth where the  $u$  by  $B$ , where  $u$  is the distance to the top most reinforcement layer divided by  $B$  that is the width of the footing is greater than 2 by 3. And this particle reinforced soil is very densely reinforced.

So, that there is no failure within the reinforced soil bed. In that case the failure happens above the top most reinforcement layer. And this particle bearing capacity case is similar to the foundation beds that are thin relative to the footing width, which we have seen earlier. When we were designing the reinforced soil embankments that are resting on geocell matrixes and on a very thin soft clay layer.

And the second mode of failure is that involves the tie pull out. And this happens when you were reinforcement layers are provided at a shallow depth, where  $u$  by  $B$  is less than two-thirds and short ties and moderate number of reinforcement layers. Say, the number of reinforcement layers about 2 to 3. And then the third possible failure mode is the rupture of the reinforcement that happens, when you have the top most reinforcement layer at close enough depth that is  $u$  by  $B$  of two-thirds.

Very long reinforcement layers and very densely reinforced that is  $n$  greater than 4. In this case, because the length of the reinforcement is so high that they will not pull out and their failure is governed more by rapture. And what they have noticed is that as the pressure is increasing, the failure propagates from top most reinforcement layers to the bottom most reinforcement layers. As the upper layer breaks higher load is transferred into the bottom layers then you have rapture and then the progressively the failure continues.

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This is the data that they have given for the mode 1 type of failure, where the failure is in the upper unreinforced soil. And for different  $u$  by  $B$  values and bearing capacity ratios that is  $q$  by  $q_0$  and for different friction angles 30 degrees, 36, 40, 42. It is possible to get a bearing capacity ratio as high as 5. And at a sufficiently large depth where you provide the reinforcement layers. This BCR values comes down to about 1 that means that, it behaves more like an unreinforced soil. So, when we design the reinforcement layers we should aim to get BCR value of at least about 3 to 4 to have some cost economics.

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Bearing capacity improvement is expressed in terms of BCR

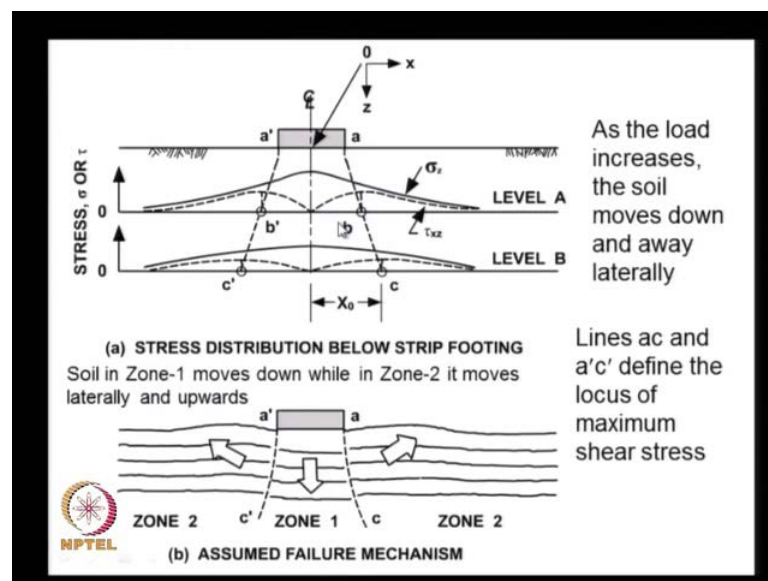
$$BCR = \frac{q}{q_0} \quad (1)$$

$q$  = footing pressure on reinforced foundation  
 $q_0$  = footing pressure on unreinforced foundation  
both measured at the same settlement

BCR was found to range from 1.5 – 4.0 depending on the settlement criteria and density of reinforcement

As I mentioned in the previous lecture, the entire premise of the bearing capacity improvement is defined in terms of this bearing capacity ratio BCR that is  $q$  by  $q_{naught}$  where,  $q$  is the footing pressure on the reinforced foundation. And then  $q_{naught}$  is the footing pressure of the on the unreinforced foundation at the same settlement. Both  $q$  and  $q_{naught}$  refer to the same settlement. And through the experimental means both Binquite and Lee and the other researchers like Tatsuoka Huang and others, they found that the BCR ranges anywhere from 1.4 to 4, depending on the settlement criteria. And then density of the reinforcement that we provide.

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The bearing capacity analysis itself was developed based on the assumed rupture surfaces  $a'c'$  and  $ac$ . And for this purpose they have looked at the variation of the vertical pressure  $\sigma_z$  and then the variation of the shear stress  $\tau_{xz}$ . And if a pressure of  $q$  is applied at the top surface obviously, the vertical pressure is maximum at the center line of the footing. And as you are going away, the vertical pressure intensity reduces.

And then the shear stress is zero at the center line or the symmetry line, because of the symmetry. And as you are moving away from the center the shear stress magnitude increases. And this is how the variation looks like the shear stress goes on increasing. And based on the observations Binquite and Lee they have divided the soil into 2 zones.

Zone 1 which is directly below the footing and zone 2 both on the left hand side and the right hand side away from the rupture surfaces a c and a prime c prime.

And what they have observed is that, as the load is increasing the soil within the zone 1 moves along with the footing and the reinforcement layers down. And the soil in the 2 zones away from the footing that is zone 2, it moves laterally away and upwards that results in some surface heaving. Depending on the type of soil, if it is dense and we have a good general shear failure with very significant ground heaving. If it more of a punching or local shear failure there is not much of heaving. And these two lines a c and a prime c prime they are observed to coincide with the locus of the maximum shear stress. And this is where they found most of the ties to rupture. And this is how they have interpreted their experimental data.


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From Boussinesq's theory, the stresses below a uniformly loaded strip footing can be determined using the following equations

$$\sigma_z(q, x, z) = \frac{q}{\pi} \left[ \tan^{-1} \left( \frac{z}{x-b} \right) - \tan^{-1} \left( \frac{z}{x+b} \right) - \frac{2bz(x^2 - z^2 - b^2)}{(x^2 + z^2 - b^2)^2 + 4b^2 z^2} \right]$$

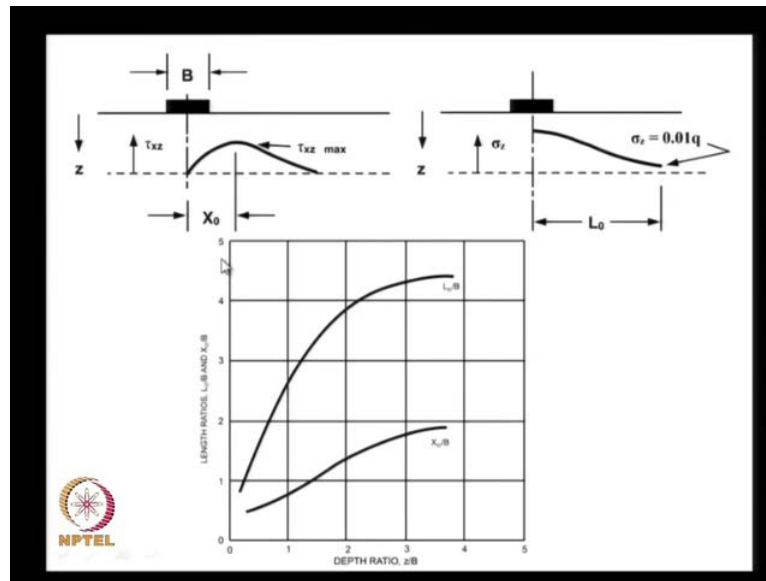
$$\tau_{xz}(q, x, z) = \frac{4bq x z^2}{\pi(x^2 + z^2 - b^2)^2 + 4b^2 z^2}$$

$b = B/2 = \text{half-width of footing}$



The variation of the stresses sigma z and tau x z can be obtained from the most Boussinesqs equation like this. The sigma z at any location x and z in terms of the q is given by this, q by pi multiplied by tan inverse z by x minus b minus tan inverse z by x plus b minus this whole quantity. And tau x z is also given by this formula and this can be found in any of the standard soil mechanics text books. And in here the small b is called as the half width of the footing that is B by 2. So, if you plot this variation it looks something like this. Sigma z is maximum at x is equal to zero and the shear stress is maximum somewhere along these 2 lines a c and a prime c prime.

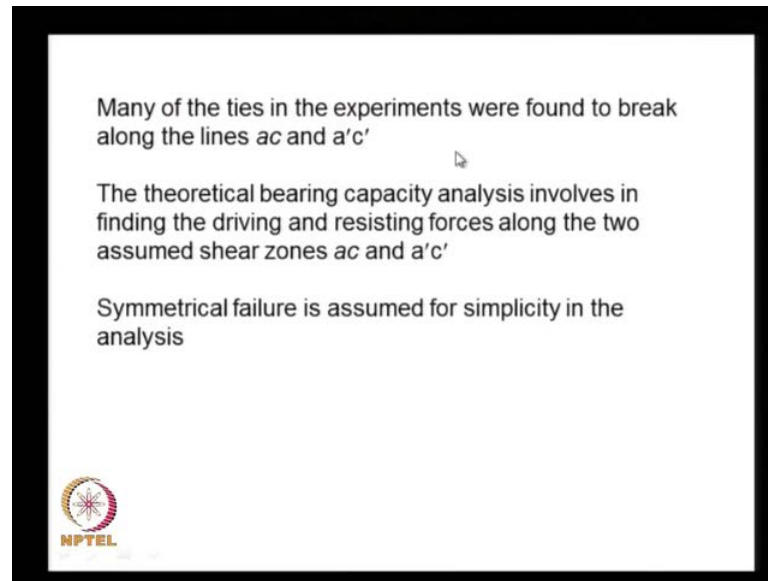
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This is how the variation looks like. Let us define 2 quantities  $x_0$  where the shear stress is maximum. The  $x_0$  distance itself is measured from the center line of the footing. And this  $x_0$  depends on the depth, cause as you go down the  $x_0$  increases. And then the  $\sigma_z$ , it has the maximum value at the mid section and then goes on reducing. For the purpose of analysis, the point where the  $\sigma_z$  because of the applied loading can be limited to a location where, this  $\sigma_z$  is equal to 1 percent of the applied load beyond that nothing much is going to happen, because there is not much of over burden pressure.

The reinforcement layers may not be able to provide sufficient resistance. And this  $L_0$  is defined as the distance from the center line, where the vertical pressure is equal to 1 percent of the applied pressure  $q$ . And they have given a chart that expresses the  $z$  by  $B$ , and  $x_0$  by  $B$ , and  $L_0$  by  $B$ , to determine the locations of the maximum shear stress and the location of the place where  $\sigma_z$  is 1 percent of the applied pressure.

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As I mentioned earlier, many of the ties in the experiments were found to break along the lines  $a c$  and  $a \text{ prime } c \text{ prime}$ . And the theoretical bearing capacity analysis involves in finding the driving. The resistance forces along the 2 assumed shear zones  $a c$  and  $a \text{ prime } c \text{ prime}$ . And for simplicity they have assumed a symmetrical failure that is in terms of the center line. Whatever happens on the right hand side is also assumed to happen on the left hands side.

Although, most of the experimental observations, they show that the footing fails by rotation towards the end. That is when the soil reaches the plasticity limits wherever, there is a small weak part the footing settles more. And then it turns over, but for the purpose of theoretical analysis it is assumed that the entire soil is homogenous. So, that we have only vertical uniform settlements and there is no rotation.



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In the failure modes 2 and 3, the bearing capacity depends on the strength of the reinforcement ties.

$$T_{z \leq} \left( \frac{F_y}{FS_y}, \frac{F_f}{FS_f} \right) \quad (2)$$
$$T_D(z, N) = \frac{T_D(z, N=1)}{N} \quad (3)$$


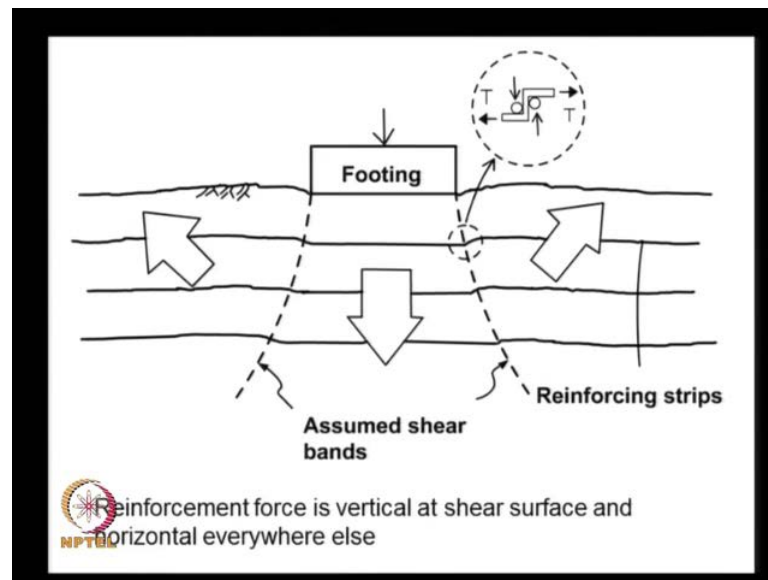
When we are doing the analysis we have to limit the reinforcement force to some safe value. And as we have seen earlier in the design of reinforced soil retaining walls and then the design of reinforcement embankments, the failure of the reinforcement could be by yielding, or the rapture, or by pull out. And 2 strength values are determine or defined  $F_y$  and  $F_f$ .  $F_f$  is the yield strength of the ties divided by  $F_{s_y}$  that is the factor of safety against rapture.

Because we should not allow any of the ties to rapture under working loads. And similarly,  $F_f$  is the pull out resistance and that divided by  $F_{s_f}$  that is the factor of safety in the pull out mode of deformation. And the minimum of these 2 is defined as the allowable load within the reinforcement layer. In other words, we have to compare the force transferred into each of the reinforcement layers against the minimum of the rapture capacity or the pull out capacity as we have done earlier. And to further simplify the analysis, what Binquite and Lee have assumed is that.

The force that is transferred into each of the reinforcement layers, when we have multiple numbers of reinforcement layers is equal to  $T_D$  of  $n$  is equal to 1. That is the reinforcement force when there is only 1 layer of reinforcement divided by  $n$ , where  $n$  is the number of reinforcement layers. And this particle assumption is mainly to simplify the analysis, and also because of lack of experimental data on the reinforcement forces in the case of multiple reinforcement layers.



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And that is illustrated here. So, if you have the reinforcement layer like this, when the set footing is settling down, the reinforcement is assumed to undergo a bending like this, just at this rupture surface the reinforcement is vertical. That means that the reinforcement force acts vertically. Then at all other locations the reinforcement is assumed to act horizontal. And that is once again one of the simplifying assumptions. So, that with very minimal calculations, we can get some theoretical estimates for the reinforcement capacity.

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In order to simplify (and due to lack of experimental data), the tie forces are assumed to be inversely proportional to the number of reinforcement layers provided (N)

$$T_D(z, N) = \frac{T_D(z, N = 1)}{N} \quad (3)$$

The equilibrium of the element ABCD in the unreinforced soil and reinforced soil with single layer of reinforcement (N=1) may be expressed as follows,

$$F_{v_{top}}(q_o, z) - F_{v_{bot}}(q_o, z) - S(q_o, z) = 0 \quad (4)$$

$$F_{v_{top}}(q, z) - F_{v_{bot}}(q, z) - s(q, z) - T_D(z, N = 1) = 0 \quad (5)$$

As we have discussed earlier the  $T_d$ , that is the reinforcement force. That is developed, when the number of reinforcement layers is  $N$  is the  $T_d$  of  $z$  comma  $n$  is equal to 1. That is the reinforcement force when there is only 1 single layer of reinforcement divided by  $N$ . In other words we can say that the reinforcement force is inversely proportional to the number of reinforcement layers. And we can look at the equilibrium of the element A B C D and the unreinforced soil with a single layer of reinforcement as given below.

And here we have the different forces the vertical force that is acting down. And then the vertical acting upwards at the bottom surface B c. And then we have some shear force that is coming from the soil. And then the  $T_d$  that is the reinforcement force the vertical component of the reinforcement force. From equilibrium, we can say that downward forces are exactly equal to the upward forces so that we maintain the equilibrium. In the case of unreinforced soil  $F_{v\ top} - F_{v\ bot} - S$ , that is the shear force is equal to 0. In the case of reinforced soil beds  $F_{v\ top} - F_{v\ bot} - S - T_d$  is equal to 0. And because of this reinforcement force by that much the shear resistance that needs to be provided by the soil can be reduced to achieve similar response.

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The force developed in the soil at the bottom surface  $F_{v\ bot}$  is the same in both reinforced and unreinforced cases

$$F_{v\ bot}(q_o, z) \equiv F_{v\ bot}(q, z) \quad (6)$$

Combining Eqs 4, 5 and 6 leads to,

$$F_{v\ top}(q, z) - F_{v\ top}(q_o, z) = S(q, z) - S(q_o, z) + T_d(z, N=1) \quad (7)$$

The only term in the above equation that cannot be evaluated directly for any given value of  $q_o$  or  $q$  and  $z$  is the reinforcement force  $T_d(z, N=1)$ . The other terms may be evaluated by integrating the stress equations



It is fair enough to assume that, the  $F_{v\ bot}$  that is developed in the unreinforced soil is exactly equal to that is developed in the reinforced soil, because we have the same soil with the same friction angle. And so it is written that  $F_{v\ bot}$  under the unreinforced

soil  $q$  naught  $z$  is exactly equal to  $F$  bottom  $q$  comma  $z$ . That is the  $q$  is refereeing to the reinforced foundation pressure. So, if we combined the previous equations 4 5 and 6.

We can derive this relation  $F$  v top minus  $F$  v bottom is  $S$  of  $q$  comma  $z$  minus  $S$  of  $q$  naught comma  $z$  plus  $T$  d. Where the shear resistance of the reinforced soil, and the shear resistance of the unreinforced soil plus this reinforcement force. And all these quantities we can determine by integrating this stresses. And the only quantity in the above equation, that cannot be evaluated for any given load of  $q$  naught or  $q$  and  $z$  is the reinforcement force  $T$  d. The other terms can be evaluated by integrating the stress equations and as given here.

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$$F_{vtop}(q, z) = \int_0^{x_0} \sigma_z(q, x, z) dx \quad (8)$$

$$S(q, z) = \tau_{xy}(x_0, z) \Delta H \quad (9)$$

$$F_{vtop}(q, z) = J\left(\frac{z}{B}\right) q B \quad (10)$$

$$J\left(\frac{z}{B}\right) = \int_0^{x_0} \sigma_z\left(x, \frac{z}{B}\right) dx \quad (11)$$

$$S(q, z) = I\left(\frac{z}{B}\right) q \Delta H \quad (12)$$

$$I\left(\frac{z}{B}\right) = \frac{\tau_{xz} - \max\left(\frac{z}{B}\right)}{q} \quad (13)$$


The  $F$  v top that is the vertical force acting at a depth of  $z$ , because of a surface pressure of  $q$  is integral of  $g$  zero to  $x$  naught  $\sigma_z$   $q$   $x$   $z$   $d$   $x$ . And if we integrate this, we can get the downward acting force at the top surface. And the  $S$  that is the shear resistance that is offered by the soil is  $\tau$   $x$  times  $\Delta$   $h$ , where  $\Delta$   $h$  is vertical spacing between the different reinforcement layers. And this  $F$  v top, we can define in terms of a function  $j$  which is defined in terms of  $z$  by  $B$  and multiplied by  $q$   $b$  where,  $z$  is nothing but this integral quantity.

And the Binquite and Lee, they have given a theoretical solution in terms of graph. If we know the  $z$  by  $B$ , we can always find the  $j$  value. And similarly, the  $S$  that is the shear resistance, at any depth  $z$  because of surface pressure of  $q$  is written as  $I$  of  $z$  by  $B$  times


$q \Delta h$ . Where  $I$  of  $z$  by  $B$  is given in terms of the  $\tau$  x  $z$  maximum that is developed at the rupture surface.

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Substituting Eqs. 3, 11 and 12 into Eq. 5 gives us an expression for the tie-force  $T_D$  developed at a depth  $z$  due to applied load on the footing expressed in terms of the bearing capacity ratio  $q/q_o$ .

$$T_D(z, N) = \frac{1}{N} \left[ J \left( \frac{z}{B} \right) B - I \left( \frac{z}{B} \right) \Delta H \right] q_o \left( \frac{q}{q_o} - 1 \right) \quad (14)$$

Tie-break or yield resistance gives us the allowable total tensile force in the ties

$$F_y = \frac{w N_r t f_y}{F S_y} \quad (15)$$


And substituting the equation 3, 11 and 12 into equation 4 gives us an expression for the tie force  $T_d$  developed at a depth  $z$ , due to applied load on the footing be expressed in terms of bearing capacity ratio  $q$  by  $q$  naught. The reinforcement force  $T_d$  at a depth of  $z$ , when the number of reinforcement layers is  $n$  can be written as  $1$  by  $n$ . Because that is a simplifying assumption that we have seen earlier multiplied by  $J$  of  $z$  by  $B$  times  $B$  minus  $I$  of  $z$  by  $B$  times  $\Delta h$  times  $q$  naught times  $q$  by  $q$  naught minus  $1$ .

And this is our governing equation to determine the tensile force that is developed in each of these ties at different depths  $z$  by  $B$ . And at different pressures  $q$  and  $q$  naught, and different  $n$  values, and the footing width  $B$ , and vertical spacing of the reinforcement  $\Delta h$ . So, the actual force that we can allow within the reinforcement layer is controlled by both the rupture and the yielding. The pull out force and the yield capacity of  $y$  can be written as  $w$  time  $N_r$  times  $t f_y$  by  $F S_y$ .

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$$F_y = \frac{w N_R t f_y}{F S_y} \quad (15)$$

w = width of a single tie, t = thickness of the tie,  $N_R$  = no. of ties per unit length of the footing,  $f_y$  = yield strength of the reinforcement material,  $F S_y$  = factor of safety on the yield strength of the material.

$w N_R$  represents the total width of ties per unit length of the footing – it is represented as the Linear Density of Reinforcement

$$LDR = w N_R \quad (16)$$

The tie force  $T_D$  needs to be compared to the yield strength  $F_y$  to assess the tie-failure



Where  $w$  is the width of a single tie,  $t$  is the thickness of the tie and  $N_r$  is the number of ties per unit length of the footing in the plane perpendicular direction to the plane of analysis. And  $F_y$  is the yield strength of the reinforcement material and  $F S_y$  is the factor of safety on the yield strength of the material. And this is fairly straight forward analysis. Once we assume some properties for the ties, we can always find the yield strength.

And this  $w N_r$  this quantity in the numerator represents the total width of the ties per unit length of the footing. And it is represented as the linear density of reinforcement LDR that is  $w$  times  $N_r$ , that is for 1 meter length of the footing the perpendicular direction to the plane of analysis. We have a width of reinforcement ties equal to  $w$  times  $N_r$ . And the tie force  $T_d$  that we have calculated earlier, from equation 14 needs to be compared against the tie force from equation 15. So, that we can assess whether there is a rupture in the reinforcement layer or the factor of safety against rupture.


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Pullout frictional resistance  $F_f$  requires the evaluation of the total vertical normal force  $F_v$  on the length of reinforcement beyond the shear rupture surface

$$F_{v-EF}(q, z) = LDR \int_{x_0}^{L_0} \sigma_z(q, x, z) dx \quad (17)$$

$L_0$  is distance from centre line where  $\sigma_v$  reduces to 1% of applied pressure

$$F_{v-EF}(q, z) = LDR B M\left(\frac{z}{B}\right) q \quad (18)$$

$$M\left(\frac{z}{B}\right) = \frac{\int_{x_0}^L \sigma_z\left(\frac{z}{B}\right) dx}{qB} \quad (19)$$


Similarly, we need to also assess the pull out capacity or the pull out frictional resistance of the reinforcement layers. And the pull out frictional resistance  $F_f$  requires the evaluation of the total vertical normal force  $F_v$ . And the length of the reinforcement beyond the shear rupture surface. As we have done in the case of retaining walls and embankments. We only consider the length of reinforcement that is embedded beyond the active rupture plane.

Similarly, in this case because we have assumed that our rupture is taking place along this line  $a-c$  and  $a'-c'$ . Only this length of reinforcement is assumed to provide the pull out resistance. And so our  $F_v$  that is developed over a length of  $f$ , that is beyond our rupture plane  $x$  naught up to a distance of  $l$  naught where,  $l$  naught is the distance from the center line, where this  $\sigma_z$  reduces to 1 percent of the applied pressure. And by integrating this quantity, we can evaluate this  $f$ . And in order to simplify the analysis, the Binquet and Lee they have given a simple equation for this resistance force is  $LDR$ . That is the reinforcement density multiplied by  $B$  that is footing width multiplied by  $m$  of  $z$  by  $B$  times  $q$ , where  $m$  is integral of this quantity.



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$$F_{nD} = F_{v-EF}(q, z) + LDR \gamma (L_o - x_o)(z + B) \quad (20)$$

$$f = \frac{\tan \phi_f}{FS_f} \quad (21)$$

Combining the equations 18, 20 and 21 lead to the tie pullout capacity  $F_f(z)$

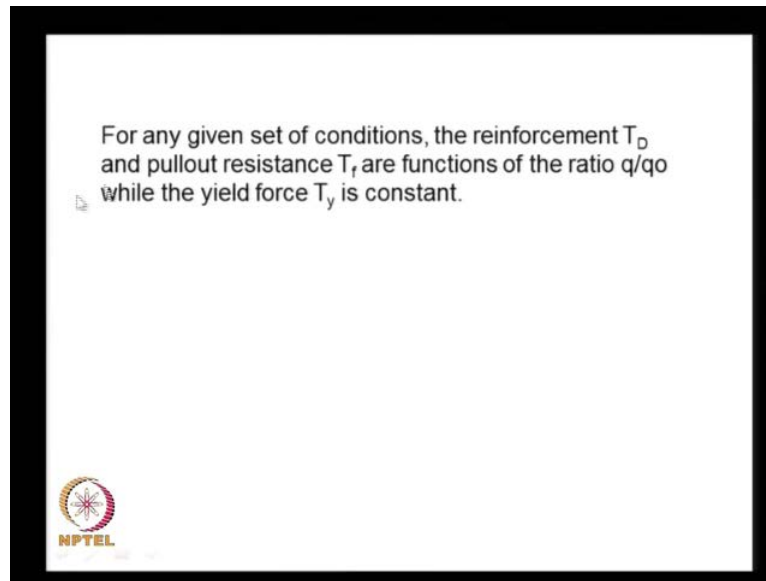
$$F_f(z) = 2 f LDR \left[ M \left( \frac{z}{B} \right) B q_o \left( \frac{q}{q_o} \right) + \gamma (L_o - x_o)(z + D) \right] \quad (22)$$

The force developed in reinforcement layer  $T_D(z)$  is compared to the pullout resistance  $T_f(z)$  given above



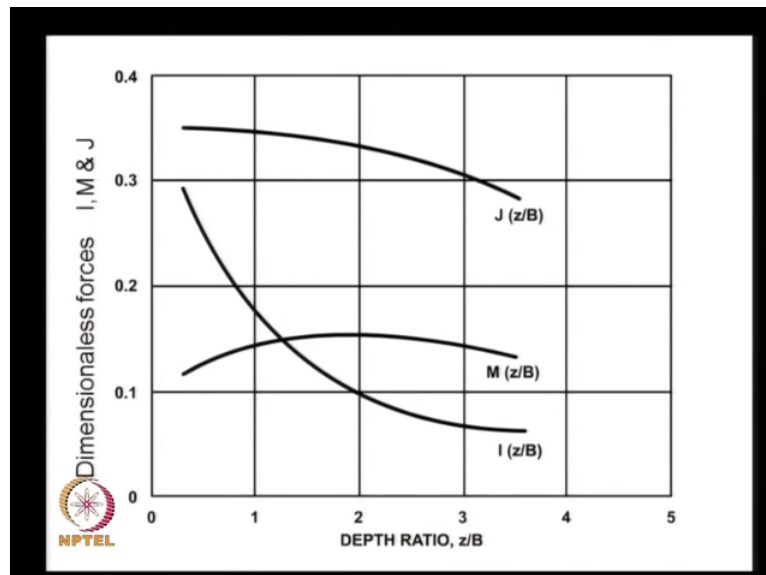
Our  $F_n$  that is the pull out capacity is equal to  $F_{v-EF}(z) + LDR \gamma (L_o - x_o)(z + B)$ , where this quantity defines the over burden pressure. And I think a factor of  $f$  is missing here. And  $f$  is  $\tan \phi_f$  by  $FS_f$ . And combining equations 18, 20 and 21 lead to the tie pullout capacity at a depth of  $z$ . The tie pullout capacity is  $2 f LDR \left[ M \left( \frac{z}{B} \right) B q_o \left( \frac{q}{q_o} \right) + \gamma (L_o - x_o)(z + D) \right]$ .  $f$  is the friction factor time  $\tan \phi_f$  by  $FS_f$  times,  $LDR$  times the function  $m$  of  $z$  by  $B$ , times  $B q_o$ , times  $q$  by  $q_o$  plus the over burden pressure over the length of  $L_o - x_o$ . And the force that is developed in the reinforcement layer  $T_D$  should also be compared against the pullout resistance this  $F_f$ . And so that we can get the factor of safety against the pullout.

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And for any given set of conditions, the reinforcement force  $T_d$  and the pullout resistance  $T_f$  or functions of the ratio  $q$  by  $q_0$  while, the yield force  $T_y$  is constant. Because the yield capacity is dependent on only the reinforcement material properties.

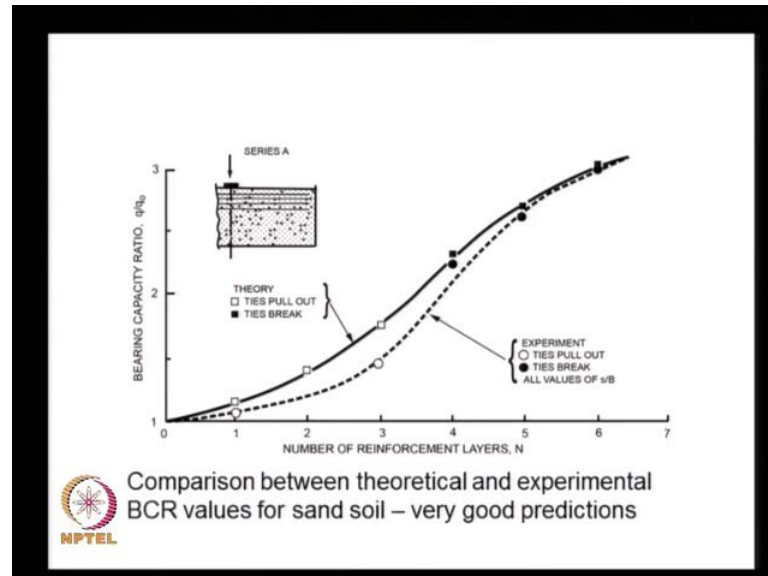
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This is the chart that was given by Binquite and Lee, to determine the 3 functions. The values of the 3 functions J, M and I in terms of the depth ratio  $z$  by  $B$ . And for any given

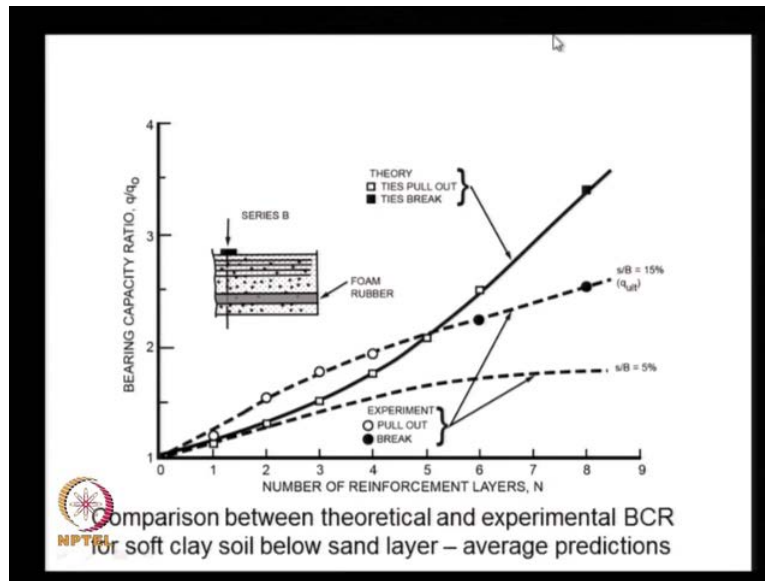
reinforcement layout, based on the point of the reinforcement  $z$ , we can evaluate these functions  $J$ ,  $M$  and  $I$ . And can use them to predict the pullout force and other parameters.

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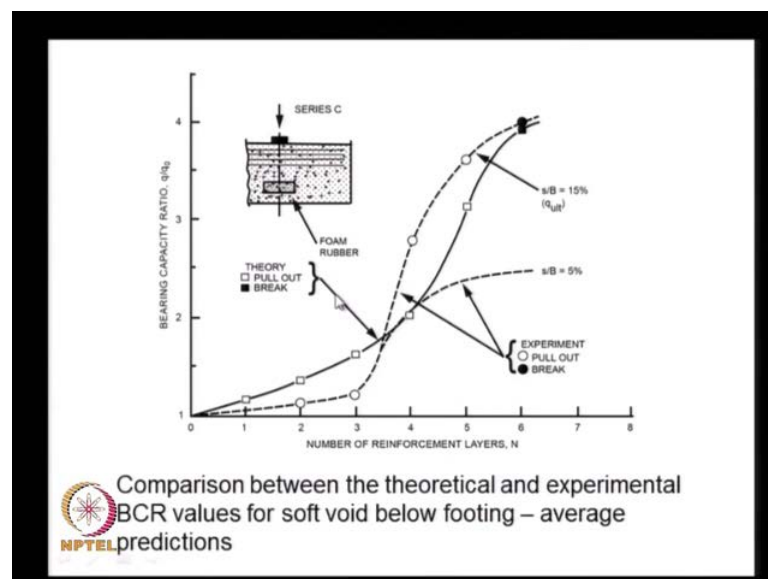
And the Binquite and Lee they have also compared their experimental results against the predictions that, they have obtained from this analysis that we have briefly discussed. And they have done 3 series of test. One is where the reinforcement soil is fully sand for the whole depth. And the x axis we have the number of reinforcement layers and the y axis, we have the bearing capacity ratio  $q$  by  $q$  naught. The experimental value is given by this dashed line whereas; the theoretical result is given by this solid line. The failure that they have observe d is shown by the hallow square for the pullout failure. And then the solid symbol for ties break and the comparison is excellent as we can see.

(Refer Slide Time: 30:22)



A similar comparison was also done for the other cases where, the soil that they have considered that is the sand is underlined by a thin layer of clay. And for this particular case the comparison is not so good, but then the trends are there. All the trends that they have seen in the experiment are also predicted by theoretical analysis. So, we can say that the prediction is not bad or average.

(Refer Slide Time: 30:54)



The third series of test that they have done is by putting a void below the footing. And this is the comparison, the theoretical analysis the solid line. And then the experimental

analysis, that is shown by dotted line for 2 different S by B values. And the comparison is once again it seems to be average or reasonable. So, we can conclude that the bearing capacity analysis that was proposed by Binquite and Lee 1975 is valid for reasonable design.

(Refer Slide Time: 31:37)


## WORKED EXAMPLE

Design a strip footing on reinforced soil bed to carry a line load of 1700 kN/m. The data for the design is as follows:

**Soil parameters:**  
 $c=0$ ,  $\phi=35^\circ$ ,  $\gamma=17$  kN/m<sup>3</sup>,  $E_s = 30,000$  kPa,  $v_s = 0.35$

**Reinforcement parameters:**  
 $F_y = 2.5 \times 10^5$  kPa,  $\phi_\mu=28^\circ$ ,  $FS_y = 3$ ,  $FS_r = 2.5$ ,  
width of ties = 75 mm, LDR = 65%

Depth of Foundation = 1m  
Permissible settlement = 25 mm  
Design life = 50 years



Now, let us apply the equations that we have just derived to solve an example design problem. And let us try to design a stiff footing on a reinforced soil bed to carry a line load of 1700 kilo Newton per meter. And the data for the design is as follows. And the soil is pure granular soil with  $c$  of 0 and friction angle  $\phi$  of 35 degrees. And the unit weight is 17 kilo Newton's per cubic meter. The Young's modulus of the soil is 30,000 and the Poisson's ratio is 0.35.

And the reinforcement parameters the yield strength is 2.5 times 10 power of 5 kPa. And the interface friction angle  $\phi_\mu$  is given as 28 degrees. And then the factor of safety that is required against a rupture is 3. The factor of safety that is required against pullout is 2.5. And let us assume that the width of the ties is 75 millimeters. And then the LDR that is the density of reinforcement layers is 65 percent. Let us also assume that the footing is provided at a depth of 1 meter and the permissible settlement is 25 millimeters. The design life is also given as 50 years. And let us try to work out solution for this.

(Refer Slide Time: 33:16)


Let width of the footing  $B=1\text{m}$   
Depth to top most reinforcement layer ( $u$ ) = 0.5m  
Vertical spacing ( $\Delta H$ ) = 0.5m  
No. of reinforcement layers ( $N$ ) = 5  
LDR = 65%

$w \times N_R = \text{LDR} \Rightarrow N_R = 0.65/0.075 = 8.67/\text{m}$   
8.67 number of ties will be present at each layer per unit length of the strip footing

Bearing capacity of unreinforced foundation

$$q_u = \gamma D_f N_q + \frac{1}{2} \gamma B N_\gamma$$

For the soil properties of  $\phi=35^\circ$   $N_q=33.30$   $N_\gamma = 48.03$   
 $c_u = 975 \text{ kPa}$   
 $c_s = 975/3 = 325 \text{ kPa}$



As such the design involves in assuming some reinforcement layout. And then checking against the predicted reinforcement forces. And then the ties break capacity and then the pullout capacity. And see whether our factors of safety are achieved or not. And let us assume that the width of the footing is 1 meter. And the depth to top most reinforcement layer below the footing  $u$  is 0.5 meters. Let us say that we provide the reinforcement layers at a vertical spacing of 0.5 meters.

And let us assume that we provide 5 numbers of reinforcement layers. And the LDR that is the reinforcement densities 65 percent, that is the  $w$  times  $N_r$  is LDR, that is 0.65 or 65 percent. The  $N_r$ , the number of reinforcement tires per unit length of the footing perpendicular to the plane of analysis is 0.65 divided by the width of the tires that is 0.075, that is 8.67 per meter.

So, we have 8.67 numbers of ties per unit length of the strip footing in the plane perpendicular to the analysis plane. And so the bearing capacity analysis of the unreinforced foundation by using the Terzaguis equation. We can find the  $q_u$  as  $\gamma D_f N_q$  plus 1 half  $\gamma B N_\gamma$ . And for soil properties of 35 degrees  $N_q$  is 33.3 and  $n_\gamma$  is 48 and we get a  $q_u$  of 975 k P a. And to have a factor of safety of 3 against the bearing capacity failure. We can define the  $q_s$  as 975 by 3, that is 325 k P a. The actually all these numbers are rounded so when you calculate you may see a slight difference.


(Refer Slide Time: 35:32)

Settlement of strip footing is

$$\delta = \frac{q_{np} B}{E_s (1 - \mu^2)} I_f$$

Substituting all the values and influence factor  $I_f$  of 2,  $q_{np}$  is obtained as 329 kPa  
 As  $q_s$  is  $< q_{np}$ ,  $q_{all} = q_s = q_o = 325$  kPa

Determination of applied pressure on reinforced foundation  $q = 1700/1 = 1700$  kPa  
 Reinforcement force TD =

$$T_D(z, N) = \frac{1}{N} \left[ J \left( \frac{z}{B} \right) B - I \left( \frac{z}{B} \right) \Delta H \right] q_o \left( \frac{q}{q_o} - 1 \right) \quad (14)$$


This is one part of the analysis that is bearing capacity and the other part is we have to make sure that our settlements are within permissible limits. And the settlement equation for shallow foundations is given like this delta is  $q_{np}$  times  $B$  where,  $q_{np}$  is the safe bearing pressure divided by  $E_s$  times  $1 - \mu^2$  times  $I_f$ , that is the influence factor. The influence factor as you know that depends on the shape of the footing and for flexibly loaded strip footing  $I_f$  is 2.


And if you substitute all the numbers here, that is delta of 25 millimeters  $B$  of 1 and the  $E_s$  of 30,000 kilo Pascal's. And the poisons ratio and the  $I_f$  value, we can get  $q$  and  $p$  as of 329 k P a. And the allowable bearing pressure is the lower of the safe bearing capacity  $q_s$  and the safe bearing pressure  $q_{np}$ . So, the lower of 325 and 329 comes out as 325 k P a. Now, the unreinforced soil will have a bearing capacity of only 325 k P a. Whereas, our applied pressure is 1700 k P a.

So, we need to defiantly come out with some reinforcement layers so that we have a safe bearing capacity or the factor of safety against bearing capacity failure. And the reinforcement force  $T_d$  at any depth  $z$  for  $N$  number of reinforcement layers was earlier derived as  $\frac{1}{N} J \left( \frac{z}{B} \right) B - I \left( \frac{z}{B} \right) \Delta H$  times  $q$  naught times  $q$  by  $q$  naught minus 1, where  $q$  is 1700 and the  $q$  naught is 325.

(Refer Slide Time: 37:43)

The tie forces can be estimated using the above equation for each layer

Layer	$\frac{q_s}{N} \left( \frac{q_s}{q_s} - 1 \right)$	Z (m)	z/B	J B	I ΔH	JB-IΔH	T <sub>D</sub> (kN/m)
1	275	0.5	0.5	0.35	0.125	0.225	61.9
2	275	1	1	0.34	0.09	0.25	68.8
3	275	1.5	1.5	0.34	0.065	0.275	75.6
4	275	2	2	0.33	0.05	0.28	77
5	275	2.5	2.5	0.32	0.04	0.28	77



If we substitute all these numbers in this equation because we have seen that we wanted to have 5 reinforcement layers starting from a depth of 0.5. So, we can tabulate all these results we have totally 5 number of layers 1 2 3 4 5. One being the top most reinforcement layer 2 below and so on. The  $q$  naught by  $N$  times  $q$  by  $q$  naught minus 1 comes out as 275 by substitute  $N$  of 5  $q$  naught of 325  $q$  of 1700. And the  $z$  for the top reinforcement layer is 0.5 and it increases by 0.5 as you go down and the  $z$  by  $B$ .

And the  $J$  times  $B$  is the  $J$  can be read off from this graph that we that we have, the  $J$  can be read off multiplied by  $B$  that is the width of the footing is here  $I$  times  $\Delta h$ . Once again  $I$  is obtained from that graph and  $J B$  minus  $I \Delta h$  is this. And substituting all the numbers the  $T_d$  that is the reinforcement force developed in the first reinforcement layer is nearly 62 kilo Newton's per meter. And as we go down the reinforcement force is increasing slightly. And the reinforcement layers 4 and 5, they have the same force 77 kilo Newton's per meter because both  $J$  and  $I$  they reach a constant value beyond a certain depth.



(Refer Slide Time: 39:33)

Resistance forces of ties in pullout

$$F_f(z) = 2 f LDR \left[ M \left( \frac{z}{B} \right) B q_o \left( \frac{q}{q_o} \right) + \gamma (L_o - x_o) (z + D) \right] \quad (22)$$

$$f = \tan \mu$$

quantity	Layer-1	Layer-2	Layer-3	Layer-4	Layer-5
$2 \tan \mu LDR$	0.691	0.691	0.691	0.691	0.691
M	0.125	0.14	0.15	0.15	0.15
$M B q_o (q/q_o)$	225	252	270	270	270
Z	0.5	1	1.5	2	2.5
$z/B$	0.5	1	1.5	2	2.5
$L_o$ (m)	1.55	2.6	3.4	3.85	4.2
$X_o$ (m)	0.55	0.8	1.1	1.4	1.65
L-X	1	1.8	2.3	2.45	2.55
Z+D	1.5	2	2.5	3	3.5
$(L-X)(z+D)$	25.5	61	97.5	124.7	151.3
$F_f$ (kN/m)	169.9	212.9	250	268.9	287.4
Factor of Safety	2.7	3.1	3.3	3.5	3.7

Similarly, we can calculate the pullout resistance by using this formula  $F_f(z)$  is 2 times  $f$  LDR times all this quantity. And for different layers, layer 1 2 3 4 5, we can determine all these quantities, the 2 times  $\tan \mu$  times LDR, where  $\mu$  is given as 28 degrees. And M is that function which is related  $z$  by  $B$ . And the  $Z$   $z$  by  $B$  1 naught is the distance from the center line of the footing up to a location, where our vertical pressure is equal to 1 percent of the applied pressure. This 1 naught  $x$  naught 1 minus  $x$  that is 1 naught minus  $x$  naught  $z$  plus  $d$  and so on.

And these are the friction the pullout resistances that we have calculated. And the factor of safety against pullout is just simply this divided by pull the force that is developed. That is for example, for the top most layer the force that is developed is 61.9. So, 170 divided by 61.9, it gives factor of safety of 2.7. So, these are all the factors of safety against pullout for different reinforcement layers, and for the bottom most reinforcement layer the factor of safety is 3.7.


(Refer Slide Time: 41:16)

The minimum factor of safety against pullout of 2.5 is achieved in all layers

Thickness of ties,  $t = FS_y T(N) / (LDR * F_y)$

Substituting all the values in the above equation, we can obtain the thickness of different layers as,

Layer-1,  $T_1 = 1.15$  mm  
Layer-2,  $T_2 = 1.27$  mm  
Layer-3,  $T_3 = 1.4$  mm  
Layer-4,  $T_4 = 1.4$  mm  
Layer-5,  $T_5 = 1.4$  mm

 As per BS-8006, the corrosion loss on each face for 50 year service life is 1.35 mm on each face. Hence the above thickness values should be increased by 2.7 mm

The minimum factor of safety that we need against pullout is 2.5. And we see that the least factor of safety that is obtained is obviously for the top most reinforcement layer that is 2.7 which is more than 2.5. So, we are safe against the pullout and the thickness of the ties can be determined by using the yield capacity. Then the force that is generated in each of these layers and divided by the factor of safety. And by substituting this  $F_s y$  of 3 and the force that is developed that is in each of these layers 62 68 75 77 and so on. And the divided by LDR that is 0.65 and  $F_y$  that is the yield capacity, we get the thickness of the different reinforcement layers.

The layer 1 because it has the least force, has the least thickness of 1.15 and the layer 2 has 1.27 millimeters, layer 3 4 and 5 have the same thickness of 1.4 millimeters. And these being as the steel ties, we need also account for the corrosion loss. And if you go back to the B s 8006 code they say that, the corrosion loss on each face for 50 year design life is approximately 1.35 millimeters on each face. That means that 1.35 millimeters at the top surface and 1.35 millimeters of the bottom surface. So, we need increase the thickness of each of these layers by 2.7 millimeters so that we achieve a factor of safety of 3 against rapture.

(Refer Slide Time: 43:26)


Minimum length of ties

Minimum length of ties should be at least  $2L_o$ .

Layer-1 = 3.1m  
Layer-2 = 5.2 m  
Layer-3 = 6.8 m  
Layer-4 = 7.7 m  
Layer-5 = 8.4 m

If foundation soil is unreinforced, the width of the concrete footing should be at least  $1700/325 = 5.25$  m

As the width of footing increases, its thickness also will increase.



Minimum length of the ties because the  $l_{naught}$  is calculated from the center line of the footing. So, the length of the tie should be equal to 2 times  $n_{naught}$ . And the layer 1 is 3.1, layer 5 0.3 and so on. So, these are the different lengths of the reinforcement that we need to provide. And let us go back to our unreinforced soil and we have seen that the maximum bearing pressure allowable bearing pressure is through 325. If we do not provide any reinforcement within the soil, the minimum width of the footing that we need to provide is 1700 divided by 3.23 25 that is 5.25 meters against the 1 meter width that we have provided.

The thickness of the foundation that we provide is also a function of the width. So, we need to provide a very massive reinforced concrete footing because our width instead of being 1 meter, it is nearly 5 meters in the case of unreinforced foundation. So, we can compare the cost economy of this much of concrete. And that is provided with steel reinforcement against our 1 meter wide, and may be the thickness will be very small because our width is very small and the number of reinforcement layers.

So, this design has given as adequate factor of safety against the tensile rupture reinforcements and then the factor of safety against the pullout. And this entire analysis was obtained by assuming some reinforcement layout. And this may or may not be the most optimum because we can actually reduce our reinforcement quantity because the minimum factor of safety that we had is 2.7. And so we can slightly increase the vertical

spacing. And then come out with some other design to see, what is the most optimum reinforcement configuration that we can provide.

So, that is a brief description of the bearing capacity analysis that was developed by Binquite and Lee. And in this lecture, we have seen the theoretical derivation of the bearing capacity equations that Binquite and Lee have developed. And then we have seen a simple worked out example, to illustrate the procedure of the calculations.

Thank you very much.

And if you have any questions you can send an email to me.