**Finite Element Analysis**
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**Module No. # 01**
**Lecture No. # 09**

In today's class, we will develop finite element formulation for two node beam element. Before we proceed with that, let me summarize what we have done in the last class. In the last class, we derived the governing differential equation for beam bending problem and while deriving this, we also looked at various sign conventions that we adopted, that is, for the nodal degrees of freedom, applied forces, moments, internal moments and shears. The sign conventions are very important, because the entire equation or the subsequent calculations will be based on the sign conventions. While deriving these governing differential equations for beam problem, what we did is, we used conditions of equilibrium; that is, sum of all forces in the transverse direction is equal to 0 and sum of all moments taken about a point is equal to 0. We got the relation between the shear force and the applied load, and also bending moment and shear force. In a way, we can find what the relation between bending moment and applied load is.

After that, what we did is, we assumed that these transverse displacements are small; based on the assumption that plane sections remain plane, we derived the relation between the displacement in the x direction and the derivative of transverse displacement. Then, we derived strain – what is the relation between strain and transverse displacement. Then, we used Hooke's law and got the relation between stress and strain. Then, taking moment equilibrium about a cross section, we developed a relationship between bending moment and transverse displacement. Once we got this relation, we can substitute what is the relation between bending moment and load applied. Through this, we got the governing differential equation. It turns out that this governing differential equation for beam bending problem is a fourth order differential equation. So, we require four boundary conditions to solve this problem. These boundary conditions can be a combination of any of the four boundary conditions; that is, transverse displacement, rotation and bending moment, shear force. Here I want to emphasize that rotation is nothing but first derivative of transverse displacement and bending moment is nothing but second derivative of transverse displacement multiplied by modulus of rigidity or EI. Shear force is related to third derivative of transverse displacement; it is equal to EI times third derivative of transverse displacement with respect to the spatial coordinate, which is x.

Now, with that, what we did is, later we applied variational approach and we got equivalent functional. While deriving this equivalent functional, we applied integration by parts or twice, because beam bending differential equation is fourth order differential equation. Also, we substituted the relationship between transverse displacement and shear force and transverse displacement and bending moment. Also, we used variational identities and we derived equivalent functional, which is going to the potential energy for beam bending problem for two conditions: first is essential boundary conditions specified and natural boundary conditions specified. This equivalent functional turns out to be the potential energy for beam bending problem. As you know, potential energy is nothing but strain energy minus work done by the applied forces. Here, strain energy is nothing but beam bending strain energy and work done by the applied forces are nothing but work done by the distributed loads and work done by the point loads – moments and forces.

That is what we have covered in the last class related to this beam bending. In today's class, what we will do is, we will develop a finite element two node beam finite element for later computational purposes. So, essential boundary conditions for a beam involves – as you know, it is transverse displacement and rotations – both of them specified at the ends. So, the trial solution must satisfy these boundary conditions. In finite elements, this can be easily done if both transverse displacements and rotations are chosen as independent degrees of freedom for each node.

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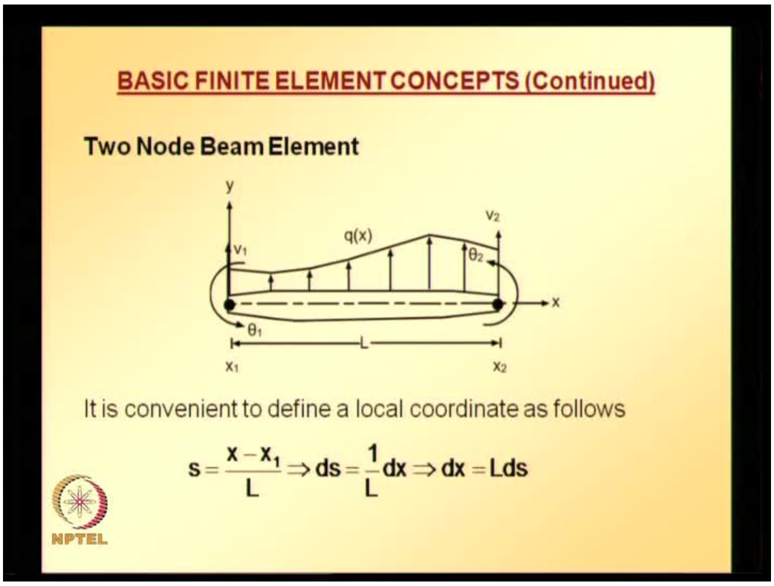


A simplest element, which is a two node beam element has 2 degrees of freedom per node. A two node straight beam element is shown here in this figure. Here EI is assumed to be constant over the entire length of the beam element and also, applied load along the element length is uniformly distributed. So, q is uniformly distributed and EI is assumed to be constant. We will be using these conditions at the end, when we are trying to simplify the stiffness equation and also load vector.

Now, to develop these shape functions and other things, it is convenient to define a local coordinate system as follows. Here you have two nodes: in the x-coordinate system, node 1 corresponds to x is equal to x 1, and node 2 corresponds to x is equal to x 2. The degrees of freedom at node 1 are v 1, theta 1; that is, transverse displacement and derivative of transverse displacement. The degrees of freedom at node 2 are v 2 and theta 2. This entire length, that is, x 1 to x 2 is mapped on to another coordinate system in s, where s is equal to 0 corresponds to x 1; s equal to 1 corresponds to x 2. So, the relation between x-coordinate system and s-coordinate system is given by s is equal to x minus x 1 over length of the element, which is nothing but x 2 minus x 1. This relation, that is, s is equal to x minus x 1 over L is obtained similar to how we arrived for the bar element; that is, we use linear interpolation formula; that is, y minus y 1 is equal to y 2 minus y 1 divided by x 2 minus x 1 multiplied by x minus x 1. We use that relation similar to what we did for two node bar element. Using that relation, we get the relationship between s-coordinate system and x-coordinate system.

Once we get this relation, we can find what is ds – a small differential element of length ds in s-coordinate system. How it is related to a differential element of length dx in x coordinate system? That is given by ds is equal to 1 over L dx. That can be rearranged in the manner dx is equal to L times ds. Suppose if there is an integral going from x 1 to x 2 and if you want to change the limits for integration to 0 to 1; that is, x 1 corresponding to 0, x 2 corresponding to 1, then the integral, which we need to integrate with respect to x, that is, dx term in that integral, we can replace with L times ds, when we change the limits of integration from x 1, x 2 to 0 to 1.

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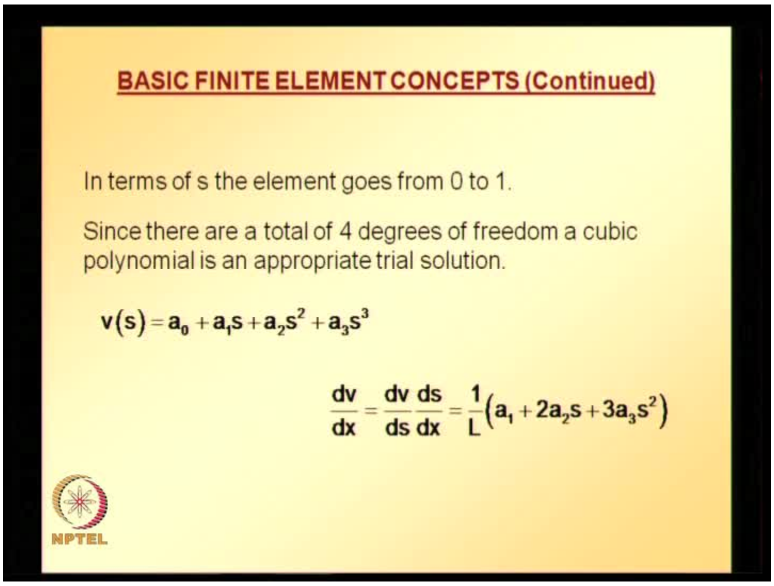


In terms of s element goes from 0 to 1; that is what I just explained. Since here at each node you have 2 degrees of freedom and total 4 degrees of freedom are there, we need to choose a cubic polynomial. A cubic polynomial is an appropriate trial solution. If you recall – when we are deriving bar element with 2 nodes and with 1 degree of freedom at each node, what we did is – since we have 2 degrees of freedom total for two node bar element, what we did is, we started out with a linear trial solution. So, depending on the number of degrees of freedom, if N number of degrees of freedom, it is better to start out with the trial solution having N minus 1 order or degree. So, that is what is done here. Since there are 4 degrees of freedom, we are starting with a cubic polynomial for trail solution. As we did for bar element, you start out with this kind of polynomial, v is equal to a naught plus a 1 s plus a 2 s square plus a 3 s cube.

Now, our job is to find a naught, a 1, a 2 and a 3. Back substitute these – a naught, a 1, a 2, a 3 into this equation and group the terms containing the coefficient v 1 theta 1, v 2 theta 2. Whatever is left in the brackets – that turns out be the shape functions or interpolating functions for this beam element. While deriving this beam element – shape functions, we are using both transverse displacement and rotation. These kinds of shape functions are called hermite shape function.

Now, let us start. What is theta? Theta is nothing but derivative of transverse displacement with respect to x. Using chain rule, dv by dx can be written as dv by ds ds by dx. We just learnt that the relation between dx and ds is dx equal to L times ds. So, we know what is ds over dx, which is going to be 1 over L. v is already given in terms of s. So, one can easily take derivative of v with respect to s. So, dv over dx can be obtained.

Now, at a each extreme point, that is, at s equal to 0, which corresponds to x 1, we know what are the degrees of freedom – transverse displacement is v 1 and derivative of transverse displacement is theta 1. Similarly, at the other extreme end, which corresponds to s is equal to 1, which is nothing but x 2, we know what is transverse displacement and also we know what is the derivative of transverse displacement, which are v 1 and theta 2 respectively.

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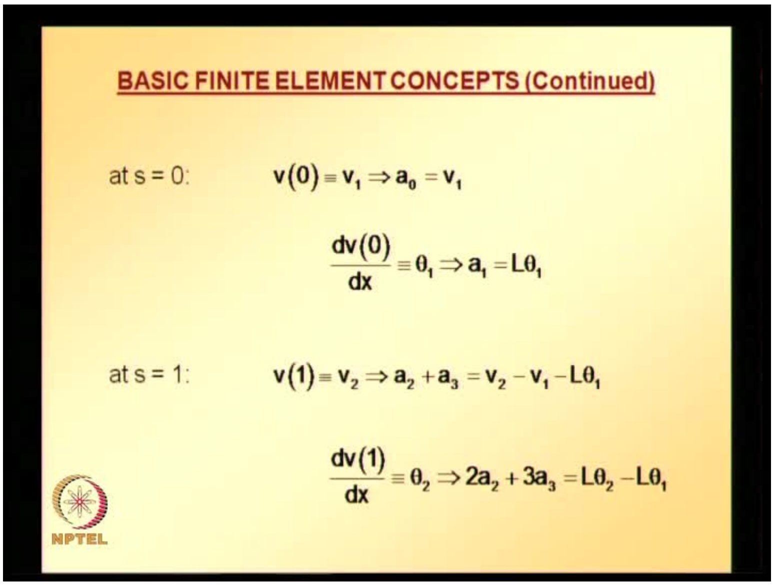


We will make the substitutions. Now, let us say at s is equal to 0, substitute s is equal to 0 into previous equation; that is, v is equal to a naught plus a 1 s plus a 2 s square plus a

3 s cube. In that, you substitute s is equal to 0, we will be ending up getting a naught is equal to v 1. Next, we need to substitute at s is equal to 0; derivative of v with respect to x is equal to theta 1. When you do the substitution, we get a 1 is equal to L theta 1. So, out of unknown coefficients – a naught, a 1, a 2, a 3, we already obtained what is a naught and what is a 1 – by these two equations.

Now, let us substitute the other condition that at s is equal to 1, v is equal to v 2. Substitute s is equal to 1 in the equation v is equal to a naught plus a 1 s plus a 2 s square plus a 3 s cube. Then, we obtained this equation, which is a 2 plus a 3 is equal to v 2 minus v 1 minus L theta 1. Similarly, now, what we need to do is, we need to substitute s is equal to 1 in the derivative of v with respect to x, which is equal to theta 2. Substituting s is equal to 1, in the derivative of v with respect to x, we get this; equating it to theta 2, we get this equation, which is 2 a 2 plus 3 a 3 is equal to L theta 2 minus L theta 1. So, we got two equations in terms of a 2 and a 3 and we have two unknowns to be determined, which is a 2 and a 3. We can solve these two equations and get a 2, a 3 in terms of v 1, v 2, theta 1, theta 2, and L.

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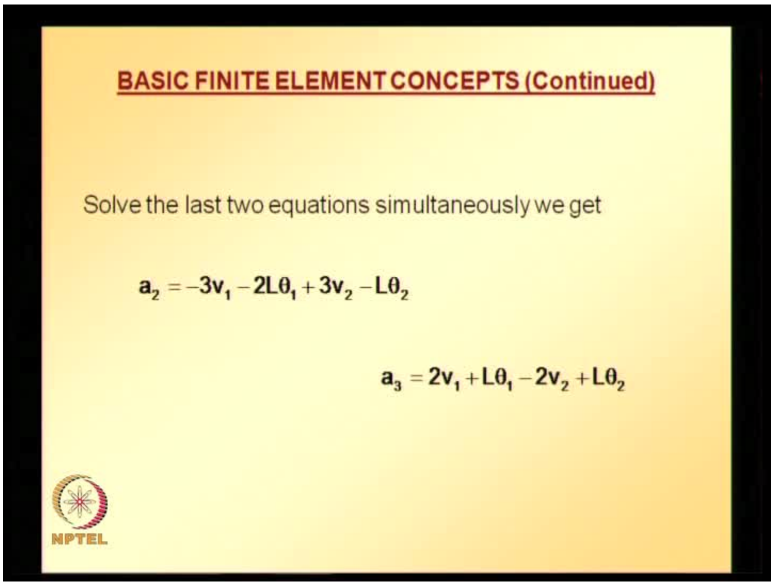


To solve these two equations for a 2 and a 3, we get a 2 to be this one – a 2 is equal to minus 3 v 1 minus 2 L theta 1 plus 3 v 2 minus L theta 2 and a 3 as 2 v 1 plus L theta 1 minus 2 v 2 plus L theta 2. So, we got all the coefficients a naught, a 1, a 2, and a 3 in terms of v 1, theta 1, v 2, theta 2, and L, which is length of the beam element.

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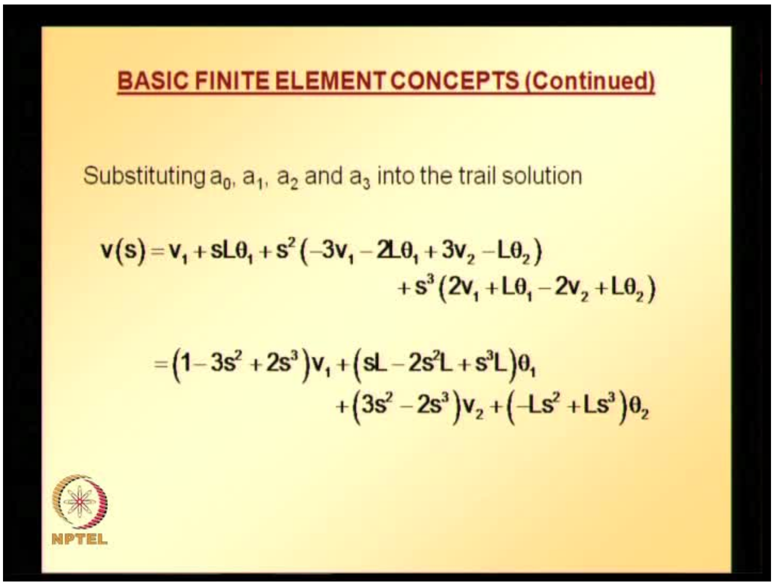


Now, what we need to do is, substitute all these coefficients – a naught, a 1, a 2, a 3 into the equation v is equal to a naught plus a 1 s plus a 2 s square plus a 3 s cube. Substituting a naught, a 1, a 2, and a 3 into the trail solution, we get this one. Now, rearranging this equation such a way that we group all the terms having v 1 as coefficient separately, theta 1 as coefficient separately, v 2, theta 2 as coefficients in a same manner separately, we can rewrite this equation in this form. So, whatever is acting like a coefficient to v 1 is nothing but N 1, whatever is acting like a coefficient to theta 1, is nothing but N 2, whatever is acting like a coefficient to v 2 is nothing but N 3, and whatever is acting like a coefficient to theta 2 is nothing but N 4.

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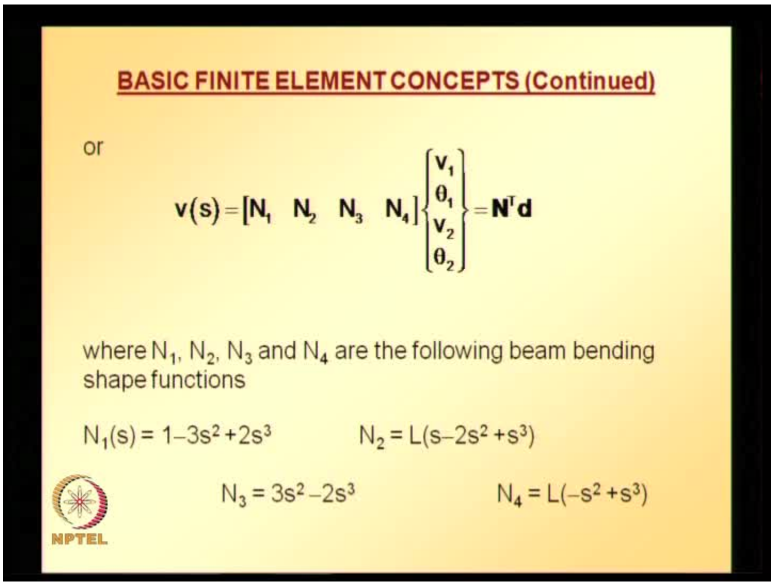


With this understanding, we can write this equation in a matrix and vector form by defining N 1, N 2, N 3, N 4, which are going to be the shape functions or interpolating functions for this two node beam element in this manner, which can be compactly written as N transpose d. Please note that the N vector is defined as N 1, N 2, N 3, N 4; whereas, earlier, for bar element, it is defined as N1, N2. At each stage, you should be checking what the shape function vector corresponds to. To avoid confusion, the displacement vector d consists of transverse displacement and rotation components at node 1 and node 2; that is, v 1, theta 1, v 2, theta 2. As I mentioned, N 1, N 2 and N 3, N 4 are nothing but shape functions or interpolating functions. If you write these separately, they look like this.

It can be easily checked that N 1 is equal to 1 at s is equal to 0 and N 3 is equal to 1 at s is equal to 1. If you plot these shape functions by normalizing N 2 and N 4 with respect to L; that is, the equation corresponding to N 2 is divided on either side with L. Then, it becomes a function of s alone. So, we can plot how these shape functions look like, when s goes from 0 to 1.

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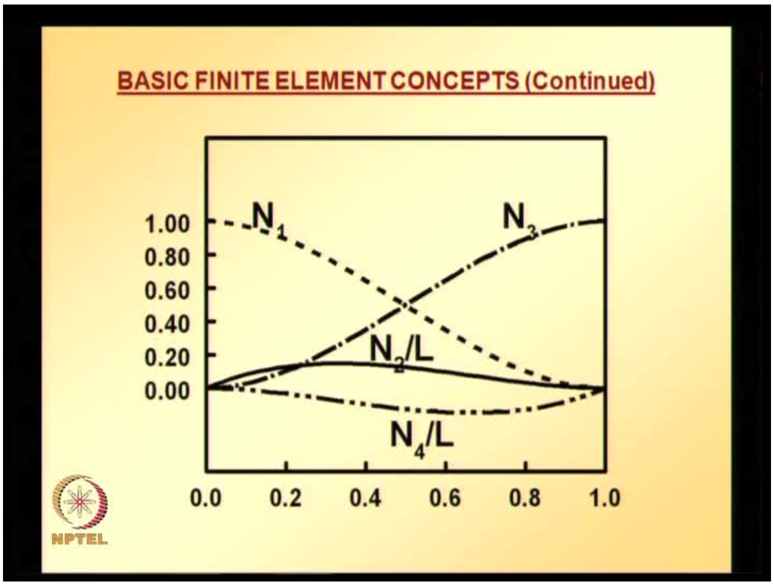


This plot shows how shape functions vary with respect to s; that is, s going from 0 to 1. You can easily observe that N 1 is equal to 1 at s is equal to 0; N 3 is equal to 1 at s is equal to 1; N 2, N 4 are normalized with respect to L. We learnt that the trail solution v, now, can be written as N 1 v 1 plus N 2 theta 1 plus N 3 v 2 plus N 4 theta 2.

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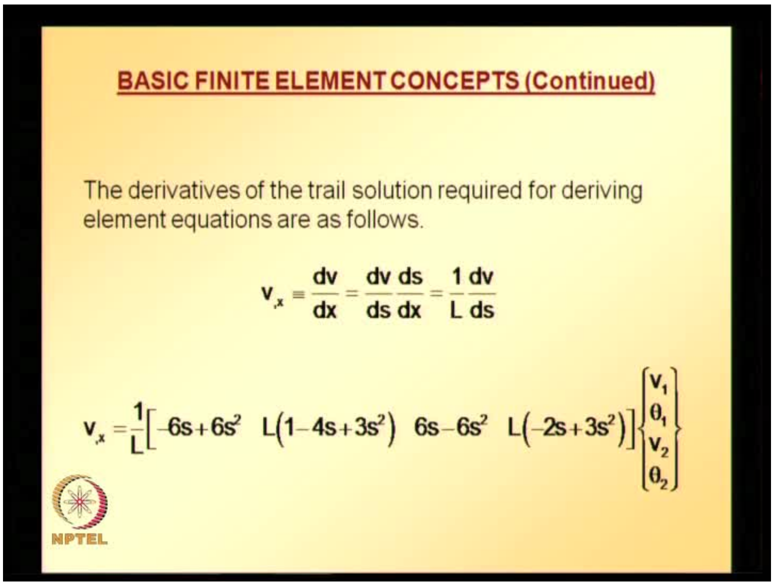


Now, we are ready to find the derivatives of trail solution, which are subsequently required for deriving the element equations. What is derivative of v with respect to transverse displacement with respect to x? Using chain rule, we can write like this – dv over dx can be written as dv over ds ds over dx, which can be written as 1 over L dv over ds. Substituting what is dv over ds, we get this relation (Refer Slide Time: 22:14). While arriving at this relation, v is equal to N transpose d; that is, N 1 N 2 N 3 N 4 values are substituted and d is nothing but v 1, theta 1, v 2, theta 2. After making the substitution and taking derivative with respect to s, we get this relation.

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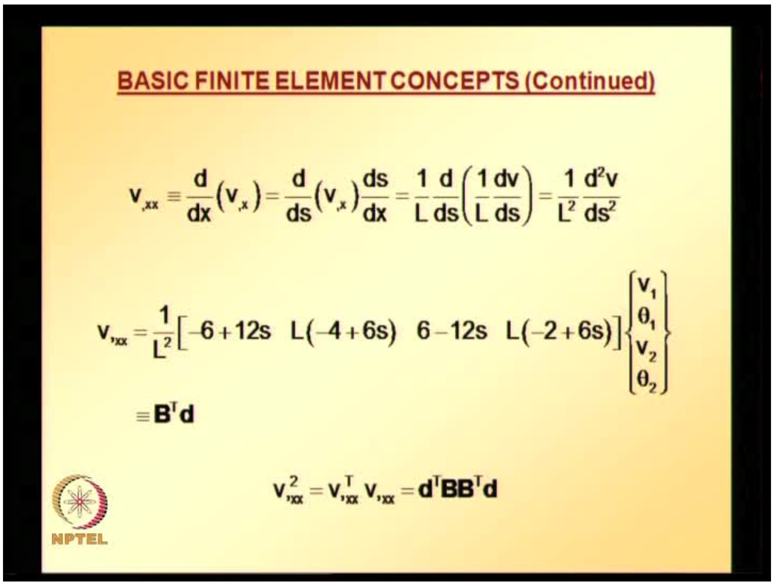


What is the second derivative of transverse displacement with respect to x? Taking one more time derivative of this, second derivative of transverse displacement with respect to x can be written as derivative of theta value, which is nothing but derivative of transverse displacement. Using chain rule, this can be written in a similar fashion as we did in the previous equation. Finally, we get second derivative of transverse displacement with respect to x as 1 over L square d square v over ds square.

Now, substituting what is v, which is N transpose d and noting that the nodal values, which are nothing but v 1, theta 1, v 2, theta 2 are constants, when we are taking derivative with respect to s. Taking second derivative of shape functions, we get this relation (Refer Slide Time: 23:57). If you recall, while deriving the relationship between bending moment and transverse displacement, we noted that bending moment is nothing but second derivative of transverse displacement times modulus of rigidity, which is EI. So, bending moment is equal to EI times second derivative of transverse displacement.

Now, we obtained what is a transverse displacement relation once we know the nodal values v 1, theta 1, v 2, theta 2. If you observe this equation, it turns out that this equation is linear, whatever two node beam element that we just developed – that gives us that captures – bending moment accurately if the variation of bending moment is linear along the element length, because s is going from 0 to 1. This relation, that is, second derivative of v with respect to x can be compactly written as B transpose d, where B is defined as the first row vector along with 1 over L square; as you already know, d is nothing but v 1, theta 1, v 2, theta 2. However, if you recall, in the potential energy, we require square of second derivative of v with respect to x. Because second derivative of v with respect to x is a scalar quantity, we can write this as second derivative of v with respect to x – transpose second derivative of v. We just noted that second derivative of B is nothing but B transpose d. Substituting that, we get square of second derivative of v as d transpose BB transpose d.

Now, we are ready; we got the trail solution in terms finite element shape functions. Also, we just obtained what is second derivative of v and square of that in terms of finite element shape functions.

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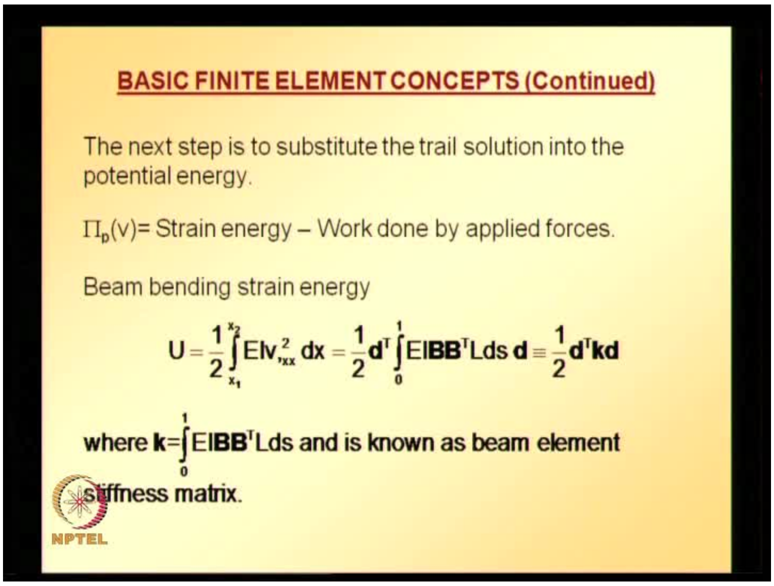


Now, the next step is to substitute the trail solution – all these quantities into the potential energy equation that we derived. We learned that potential energy is nothing but strain energy minus work done by the applied forces. Strain energy here corresponds to beam bending strain energy. We already derived this; it is nothing but this – u is equal to half integral x 1 to x 2 EI times second derivative of v with respect to x square times dx. As I mentioned earlier, we can change the limits of integration from x 1, x 2 to 0 to 1 in s coordinate system. By making that substitution, that is, x 1 is equal to 0, x 2 is equal to 1, dx is replaced with L times ds, and EI is modulus of rigidity. Substituting what is the second derivative of transpose displacements – square of that; that is, d transpose B B transpose d. Noting that d vector, which is nothing but v 1, theta 1, v 2, theta 2 they are not functions of s, we can take them out of the integral and we can write this bending strain energy in a compact form, which is denoted with u there as half d transpose k d. Now, k is defined as integral 0 to 1 EI BB transpose L ds and is known as beam elements stiffness matrix. This is a general equation. Even modulus of rigidity is not constant, this equation can be used and integrated, and we will get the stiffness matrix.

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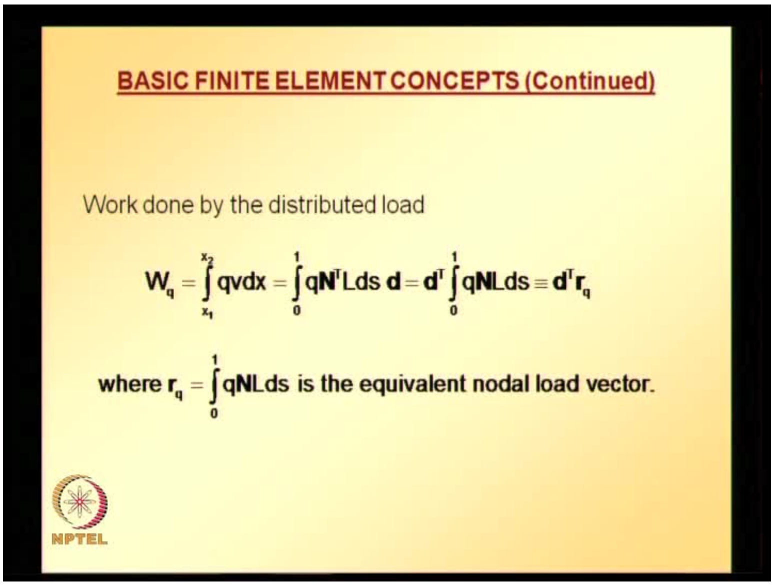


Now, work done by the distributed load; that is given by – we already derived this equation also earlier – W q is equal to… – that is, W is because of distributed load q – that is why subscript q there. So, W q is equal to integral x 1 x 2 q v dx. Changing the limits of integration, x 1 is replaced with 0 and x 2 is replaced with 1, dx is replaced with L ds, B is substituted – v value in terms of shape functions, and transpose d – noting that d is a dependent of s, it is taken out of the integral, the entire thing W q is compactly written as d transpose r q; where, r q is defined as integral 0 to 1 q N L ds. This is equivalent nodal load vector. This equation is also a general equation even if q, the distributed load is the function of x or the function of spatial coordinate that can be included inside the integral and integrated, which gives equivalent nodal vector. The previous equation and this equation are general equations, which are applicable even if E I and q are functions of spatial coordinate.

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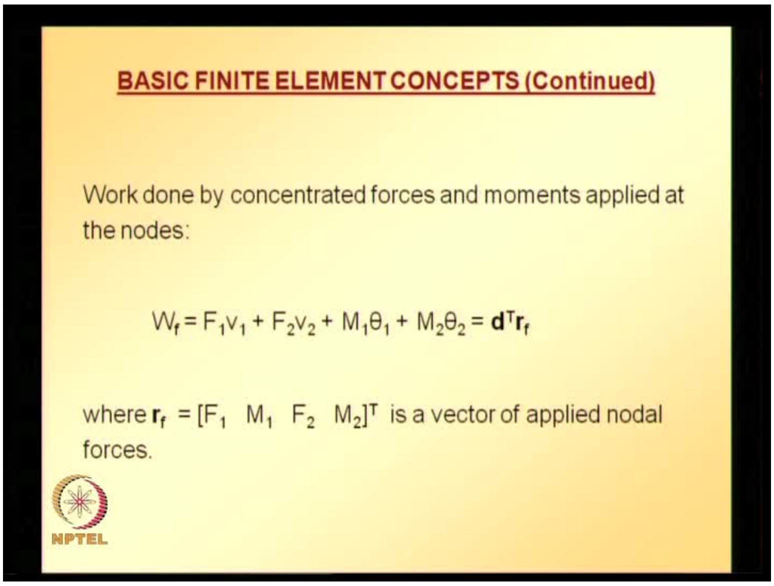


Now, work done by the concentrated forces and moments, which are applied at the extreme points of the two node beam element, which are nothing but nodes is given by – it is nothing but force times displacement and moments times rotation. This can be compactly written as d transpose r f, where r f just comprises of all the forces and moments – F1 M1 F2 M2 – is a vector of applied nodal forces.

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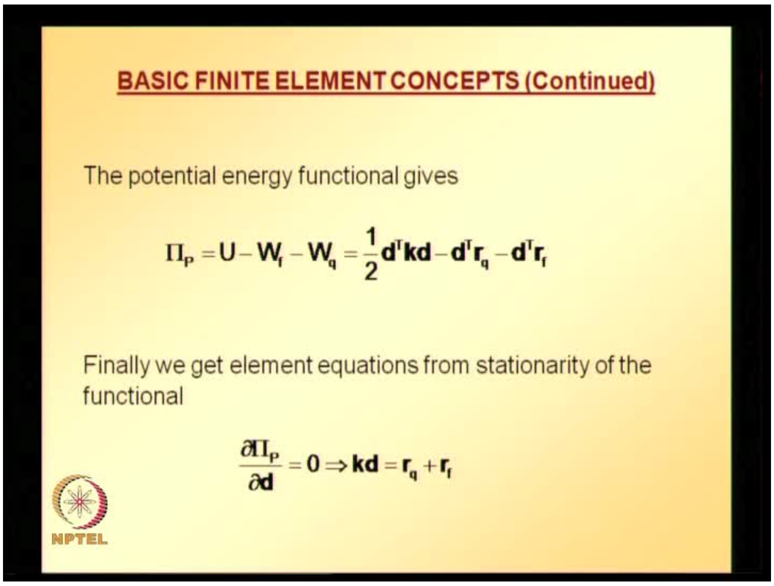


Potential energy, pi can be written by substituting U W f, that is, work done by the concentrated forces, which includes forces and moments, loads and moments, and W q, which is the work done by the applied distributed forces, is given by half d transpose k d minus d transpose r q minus d transpose r f. Now, applying the stationarity condition, if you recall, we got potential energy is nothing but equivalent functional in variational approach. So, variation of this functional should be equal to 0, which is nothing but partial derivative of pi with respect to the unknown parameters, which are here – partial derivative of pi with respect to d, gives us element equations. Finally, we get element equations from stationarity of the functional.

Please note that these equations are applicable to any beam element regardless of how modulus of rigidity E I and q vary along beam length. Once these values, that is, E I and q are specified, numerical integration can be carried out to obtain element equations. We will see what is numerical integration in a while or in the later classes.

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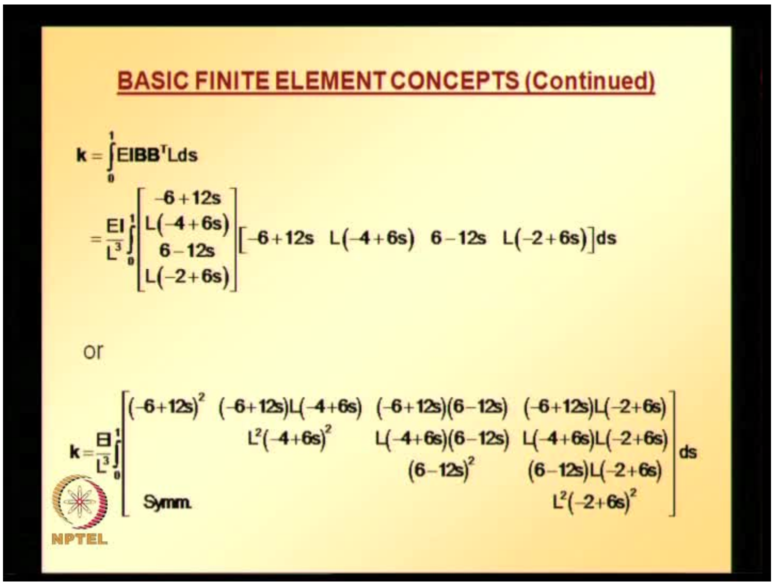


However, we will see a specific case because we started out assuming that modulus of rigidity EI is constant and q is uniformly distributed load over the entire element. We will take that specific case when E I and q are constants. Then, what we can do is, we can explicitly write what is the stiffness matrix and what is the load vector because once we take out EI, integrant becomes very simple and we can easily integrate it manually. So, by multiplying those two vectors, we get this.

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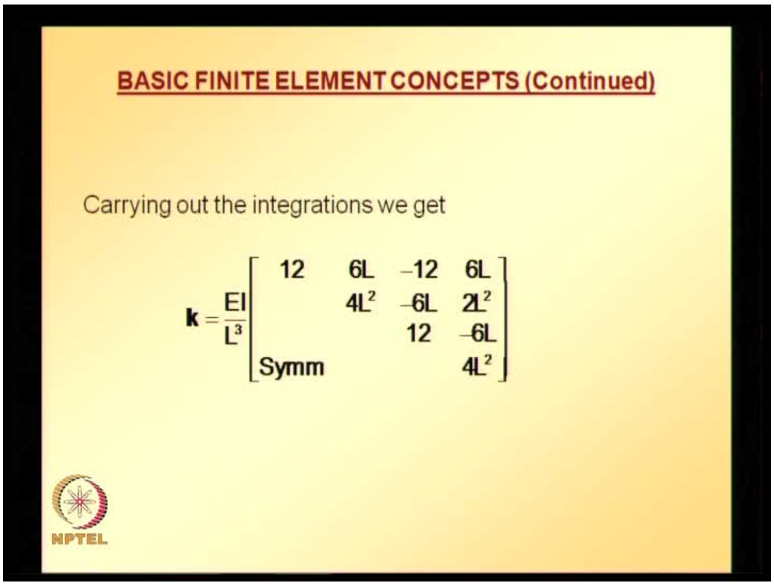

Now, integrating all the components, we get this. If you recall, from your matrix analysis of structures course, which you must have taken, this is the kind of stiffness matrix that you arrive even using any of the matrix methods. The same thing we obtained using starting with a finite element beam formulation. Also, starting with variational approach, we derived the potential energy equation. Then, we substituted derivative of transverse displacement in terms of finite element shape functions. Finally, after simplifying, we arrived at this one. The stiffness matrix is a symmetric matrix and all the diagonal terms are always positive. Here, only upper triangular part is shown and lower triangular part is nothing but reflection of that.

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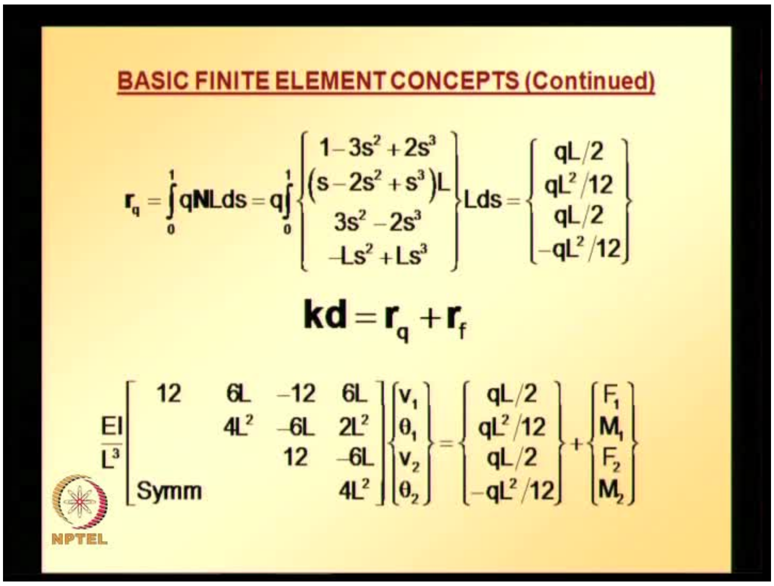


We can also simplify r q. Now, assuming q to be constant, you can take that out of the integral q and integrating the rest of the terms, we get this. There is no need of integration for r f because r f is already obtained as F1 M1 F2 M2. So, the finite element equations for a prismatic beam element that is having constant modulus of rigidity or EI with uniformly distributed load on the span and concentrated forces and moments applied at the nodes, is given by this one – k d equal to r q plus r f. If you substitute what is k, d, r q, r f, this is how it looks. We obtained what is r q, which is q L over 2, q L square over 12, q L over 2, minus q L square over 12.

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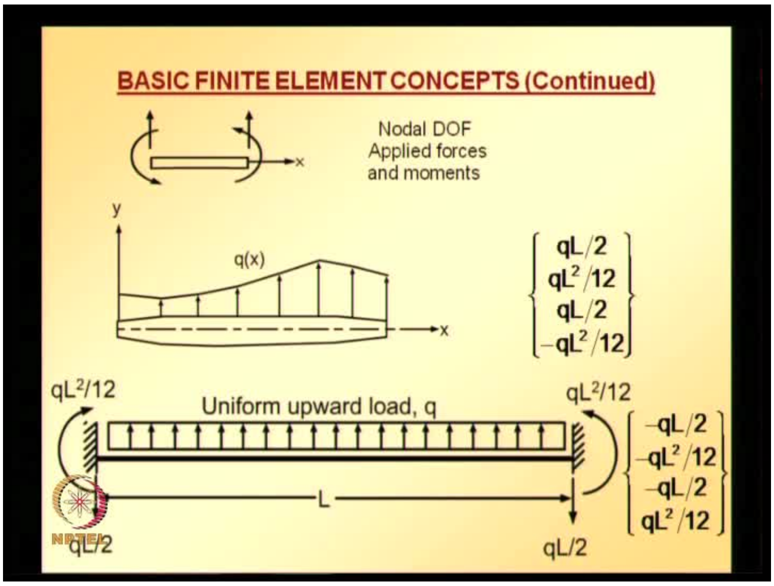


What is the physical interpretation of this load vector or equivalent nodal loads? Please note that this is a sign convention that we are using for nodal degrees of freedom and applied forces and moments. With this sign convention for a two node beam element, for a constant value of q, that is, uniformly distributed load, we obtained load vector to be this one or the equivalent nodal loads at… Nodal loads here includes loads and moment at node 1 as q L over 2 and moment as q L square over 12 and q L over 2, minus q L square over 12. Here, if you see, it is not just enough to look at this vector as it is, but we should also make a note of what is this positive negative means. Here, positive, negative means whatever we are using based on the sign convention, that is, for nodal degrees of freedom, applied forces and moments – following that convention, this is the vector we got.

Now, let us consider a fixed end beam subjected to uniform upward load. The solution for this problem is well known; it is presented in most elementary mechanics of deformable bodies text books. Here the figure shows fixed end beam subjected to uniform upward load, q along with the end moments and shear forces obtained using any of the solution procedures that are given in elementary mechanics of deformable bodies text books. Now, all the forces and moments are indicated for this fixed end beams subjected to uniform upward load. Following the sign convention for nodal degrees of freedom, applied forces and moments, all these forces end moments on this fixed end beam with uniform upward load, q – all the moments and forces we can write in a vector like whatever is shown there, that is, minus q L over 2. If you see this fixed end beam at the left end, their shear force is acting in the negative direction to the sign convention – that is why minus q L over 2. Again, at the left end moment is, q L square over 12, which is acting in a negative direction. If you follow the sign convention for the applied forces and moments, it is going to be minus q L square over 12. Similarly, the rest of the quantities.

Now, if we compare this; comparing the moments and shears with equivalent nodal loads, it can be observed that in the both vectors, the magnitude is same except that the sign is opposite. Wherever minus sign is there here for fixed end beam at that location, we have positive moment; wherever a positive value is there, we get a negative value here in this fixed end beam case. So, a physical interpretation is useful in developing equivalent nodal load vectors.

What is physical interpretation? Physical interpretation is the finite element equivalent nodal loads are simply fixed end forces with sign reverse. So, if we have a distributed load, instead of finding equivalent nodal loads, using finite element shape function substitution and doing all the integration and all that kind of stuff – for that corresponding span, if you know this fixed end moments and shear, we can put them in a vector form. We can reverse the sign – that gives us the equivalent nodal loads for finite element calculations. So, this physical interpretation is useful in developing equivalent nodal load vectors for more complicated distributed load patterns. Formulas for fixed end forces are available in already existing hand books. Using these formulas, the equivalent load vectors can be written directly without going through the integration. Not only that, we will be using this physical interpretation when we look at a method called superposition method in the later lectures.

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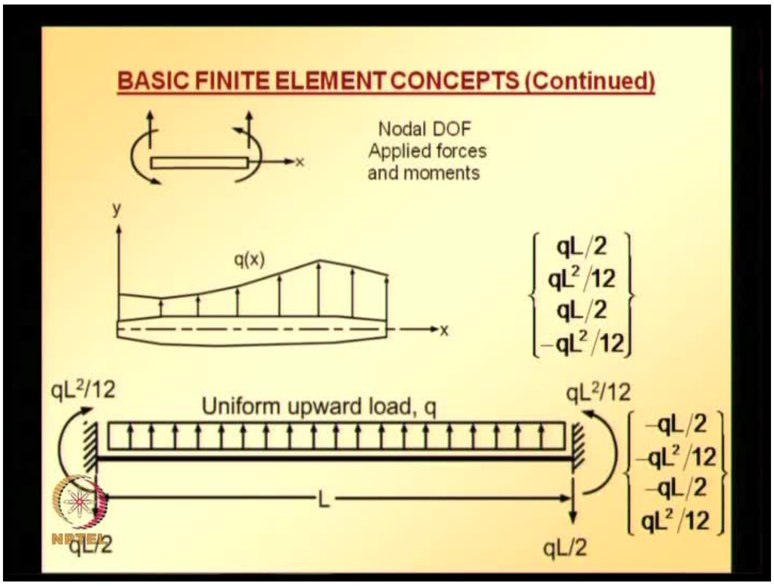


Now, this is about physical interpretation of equivalent nodal loads. In summary, finite element equivalent nodal loads are simply fixed end forces with sign reversed. Now, once we calculate for the nodal values, that is, v 1, theta 1, v 2, theta 2, what we do is – to analyze any beam problem, the element equations are written and then assembled in the usual manner depending on the element connectivity. Whatever element equations we derived that using those equations, we have to assemble for each of the element depending on how many elements are there. Then, assemble these element equations to get the global equation system using element connectivity. The solution of global equation gives nodal displacements and rotations. So, we will get v 1, theta 1, v 2, theta 2 for each of the element. After solving for these nodal unknowns, element quantities such as bending moment and shear forces in each of the elements, can be computed from the shape functions. How we do that?

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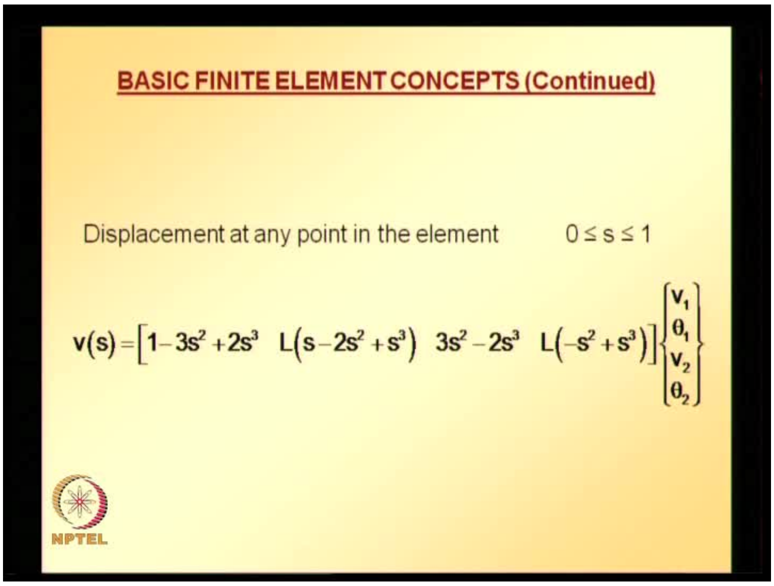


Once we get v 1, theta 1, v 2, theta 2, we go back to the relation v is equal to N transpose d; where, N is nothing but N 1, N 2, N 3, N4. Into that, we substitute v 1, theta 1, v 2, theta 2. Then, we get displacement at any point in the element, where s goes from 0 to 1. So, s is equal to 0 corresponds to one extreme point of the element; s is equal to 1 corresponds to the other extreme point of the element. So, we can sweep over the entire element length and get displacement at any point in the element using this equation.

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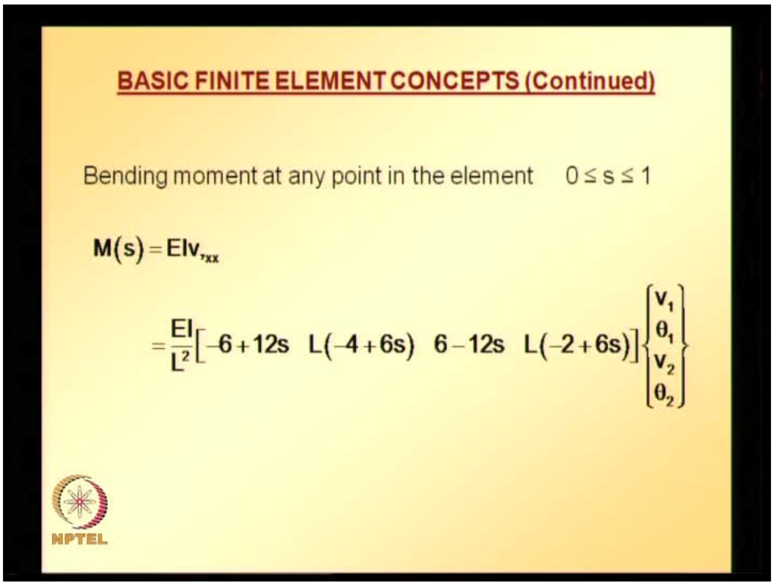

Similarly, bending moment at any point in the element; once we calculate v 1, theta 1, v 2, theta 2, we can use this relation – that is, bending moment is nothing but EI times second derivative of v with respect to x. Please note that second derivative of v with respect to x is nothing but B transpose d. So, we get bending moment at any point in the element by using this relation – by sweeping from s is equal to 0 at s is equal to 1. So, if you just see this equation, it is linear in s. So, as I mentioned earlier, the two node beam element that we developed – that gives us accurate prediction of bending moment, if the bending moment in that particular beam problem is linear with respect to the spatial coordinate.

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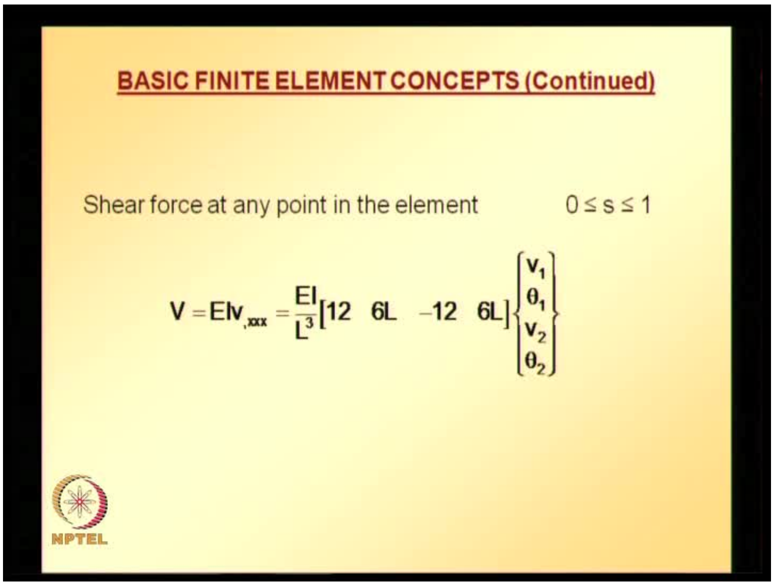


Now, what about shear force? Taking one more time derivative of this, we get shear force at any point in the element, is given by this. Once we know v 1, theta 1, v 2, theta 2, we can substitute in to this equation and sweep from s is equal to 0 to s is equal to 1 to get the shear force value at any point in the element. So, if you look at these two equations for bending moment and shear force, bending moment is linear and shear force is constant. If you see this equation, you can notice that shear force is constant.

From mechanics of deformable bodies, we know that exact solution for a uniform beam subjected to concentrated loading involves constant shear and linear bending moment. If you have a uniform beam, that is, where E I is constant and it is subjected to concentrated load, we know that shear force is constant, bending moment is linear. So, the two node element that we just developed gives exact solution when it is used to analyze prismatic beams; that is, EI is constant and subjected to concentrated loads.

What about if the beam is not prismatic, that is, non-uniform beams? This particular element may not give exact solution. So, in that case, we have to use more number of elements per span for better accuracy. Not only that, if we have distributed loads, the solution may not be accurate and we may need to use some special technique what is called a superposition technique. Using superposition, exact solution can also be obtained for uniform beams subjected to distributed loading. Therefore, analysis of continues beams in which cross-sectional properties do not change in span, requires nodes only at the supports and under concentrated loads. So, wherever concentrated load is there, better we have a node at that point.

Whatever beam element that we looked at, is applicable for prismatic beam subjected to concentrated loads. If the beam is non-uniform and if we have distributed load, we may have to use more number of elements – that is one option. For the case of distributed loads, the other option is we can go for superposition method, which we will be looking at in the later part of the lecture. In the next class, we will look at a continuous beam problem, which is subjected to concentrated loads and we will see how the two node beam element that we developed performs.

Thank you.