Finite Element Analysis
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Module No. # 01
Lecture No. # 08

In the last lecture, we have seen how to solve 2 dimensional plane truss and 3 dimensional space truss problems. Basically, in both of these, knowing the orientation of a truss member and knowing the extreme ends of a truss member, we calculated direction cosines. Using the direction cosines and material property, geometric properties of a particular truss member, we assembled element stiffness matrices for each of the truss member. Then, based on the element connectivity, that is for each truss element, based on the node numbers of the extreme ends, we assembled the global stiffness matrix.

Later, we imposed essential boundary condition, that is a displacement boundary conditions. In addition, we made sure that the loads were applied at appropriate locations in the global force vector and we solved the reduced equation system after applying the essential boundary condition, that is, displacement boundary condition. We solved the reduced equation system. Once we obtained the unknown displacements, we went back to each element and we calculated strains, stresses, and the loads on that particular truss member. Knowing the global displacements, we calculated local displacements using transformation matrix.

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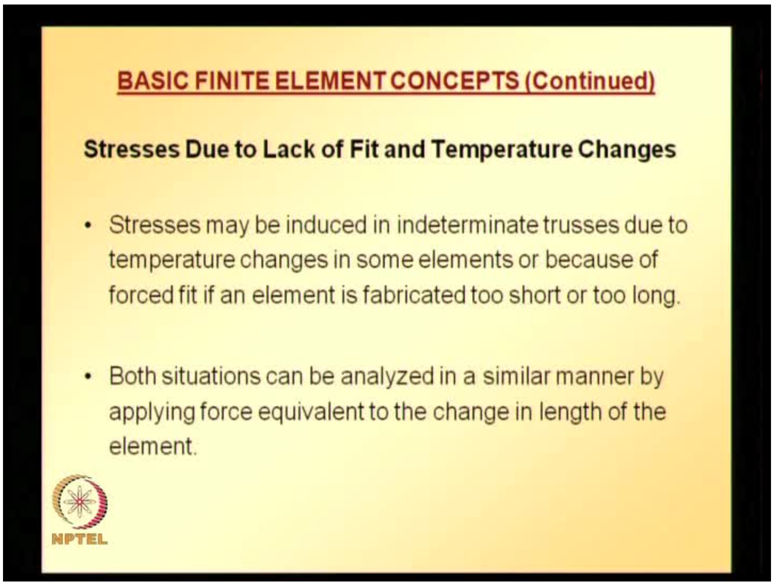

In this exercise, I want to emphasize two things. First thing is – we applied loads at node arbitrarily; that is, loads applied at a node can be arbitrarily assigned to any of the elements connected to the node. This is one thing. The other thing is – the terms in the global stiffness matrix corresponding to the nodes with zero displacements have no influence in the final system of equations. To minimize calculations, we ignored rows and columns corresponding to zero specified displacements. If the specified displacements are non-zero, then we actually need the columns corresponding to those specified degrees. Freedom – using those, we calculate some contribution and also with the help of that we calculate reactions.

With that, we will start looking at the stresses due to lack of fit and temperature changes in truss problems. Usually, the stresses are induced in the indeterminate trusses due to temperature change, changes in some elements, or because of forced fit if an element is fabricated too short or too long. So, both kinds of problems: that is, if you want to solve a problem to determine stress in the truss problem; to determine stresses due to lack of fit or temperature change, the procedure is almost similar. So, both situations can be analyzed in a similar manner by applying force equivalent to the change in the length of the element.

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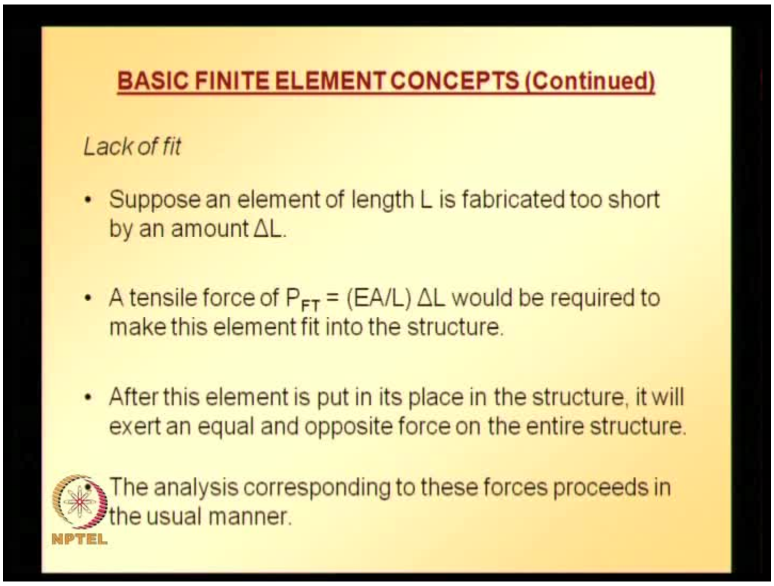

Let us see what happens. Suppose an element of length L is fabricated too short by an amount delta L; that is, first, we are going to look at the stresses – how to compute the stresses due to lack of fit in a truss member. Let us say an element of length L is fabricated too short by an amount delta L. If the element is fabricated too long by an amount delta L, we need to change the sign. Then, it becomes negative of this; that is, minus delta L. The tensile force would be required to make this element fit into the structure, because the member is long. So, to fit this truss member into the particular structure, we need to apply some tensile force. Because it is too short, we need to just stretch it. After this element is put in place in the structure – that means a member is too short; by some means it is stretched and placed in the structure. What this member will do when it is put in the structure? It will exert an equal and opposite force on the entire structure. So, this is what is expected.

Suppose instead of a member being too short by a certain amount, if it is little bit longer by certain amount, then we need to apply some compressive force to force fit into the given structure. What this will do is – when you force fit it, the member will be under compression and that member in turn will exert an equal and opposite force, which is going to be tensile force, if it is too long. Now, since the member is too short, the member will be under tensile force, whereas it is going to exert a compressive force in the structure. The analysis corresponding to these forces proceeds in the usual manner except that at the end, when we are calculating element forces, we need to add the forces due to the lack of fit.

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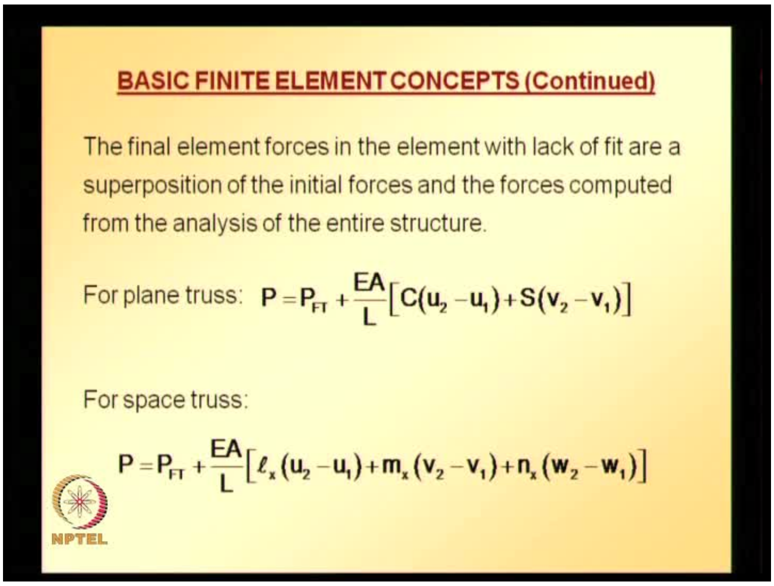

The final element forces in the element with lack of fit are a superposition of the initial forces and the forces computed from the analysis of the entire structure. So, for plane truss problems, the equation looks like this – the first term P FT is nothing but the force that is required to stretch a member or to compress a member to force fit into the given structure. This is for plane truss. However, rest of the analysis proceeds in the similar manner, which we already looked at in last class. For space truss, the equation looks like this – again the additional term here is P FT, which is nothing but the force due to force fit.

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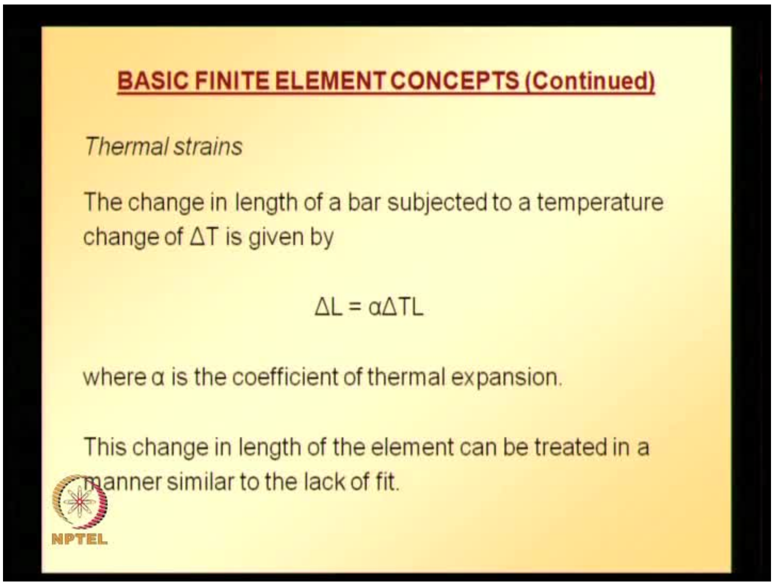

Now, how to solve problems because of change in the temperature? We know that change in the length of a bar subjected to a temperature of delta T is given by delta L is equal to alpha delta T times L, where alpha is the coefficient of thermal expansion; delta T is the change in the temperature; L is length of original length of the member. This change in length of the element can be treated in a similar manner as that of lack of fit. Now, because of temperature change, there will be change in the length of the member, which exerts some kind of force on a given structure. So, analysis proceeds in a similar manner as that of lack of fit.

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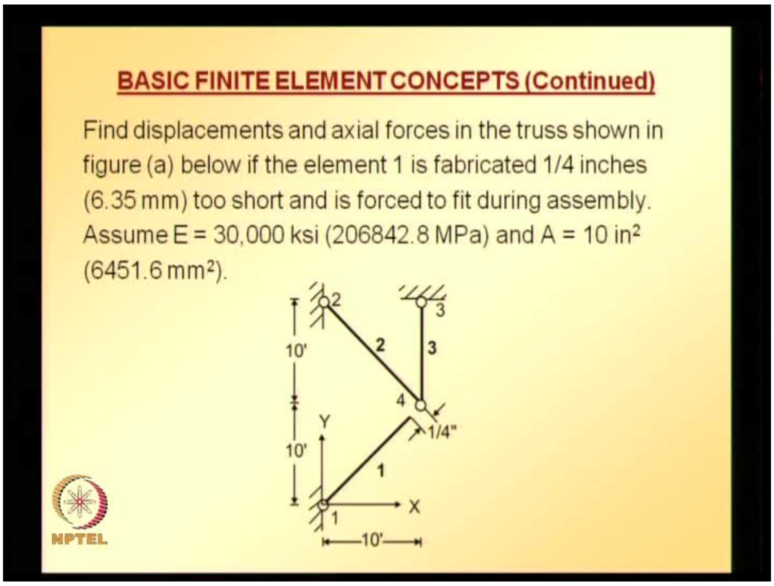

Now, let us take a problem to understand the various steps involved – a plane truss problem. Find displacement and axial forces in the truss shown in figure below if the element 1 is fabricated 4 and half inch – corresponding SI units in millimeters are also given – the element 1 is fabricated too short by an amount 6.35 millimeters or one-fourth an inch, and is forced to fit assembly. This member is too short and by some means it is stretched and force fitted. We need to analyze this plane truss for displacement and axial forces. Material properties are given – E and cross sectional area of members – all members are of same cross sectional area. They are given in the problem statement. Here, 10 is feet. The dimensions are indicated there.

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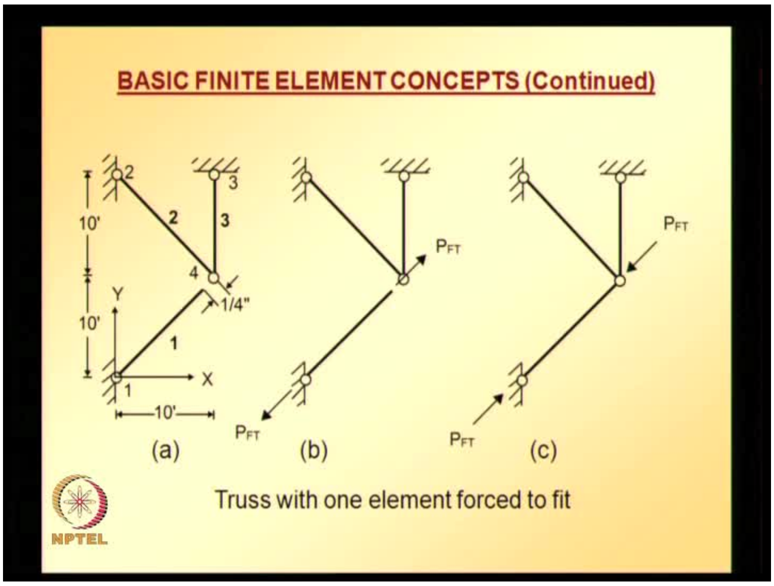

Let us see what this member is a going to do. Figure b shows that the truss member, which is short is by some means stretched. So, it will be under tensile force. This member in turn will exert an equal and opposite force, which is shown in figure c. So, this member is going to exert a force as shown in figure c on the structure. So, what we need to do is, we need to proceed and solve the problem, which is shown in figure c after calculating P FT. For those forces and the given orientation of truss members, we need to solve the problem and find stresses and displacements. Once we get those values, what we need to do is, we need to superpose the values that are given in figure b. So, that is how the solution proceeds.

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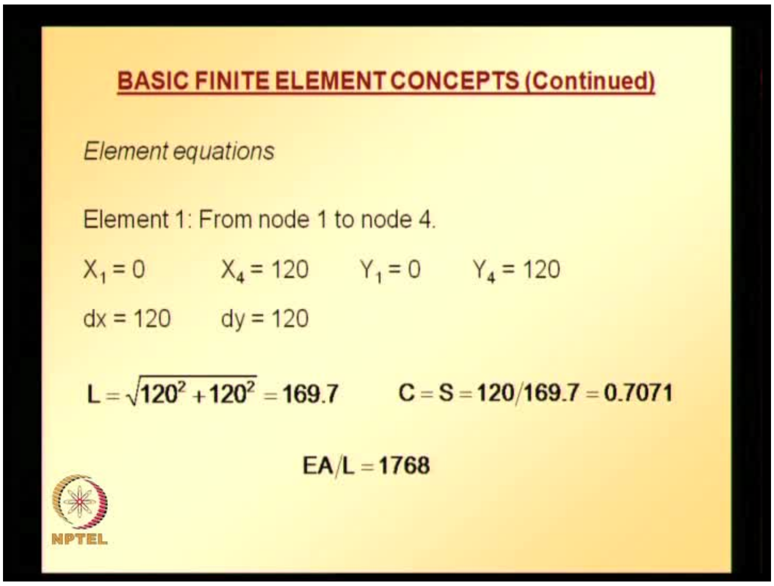

Now, let us look at the details – element equations. We need to start with element 1; element 1 – the local node 1 is same as global node 1; local node 2 is same as global node 4. So, element 1 goes from node 1 to node 4. Looking at the problem, we need to find the coordinates of node 1 – X 1 coordinate, Y 1 coordinate and X 4 coordinate, Y 4 coordinate. Once we have these values, we can find what is dx, which is nothing but X4 minus X1 and dy – Y4 minus Y1. All the quantities here are given in inches. The dimensions are given in inches and feet; feet is converted into inches. Length of the member is given by square root of dx square plus dy square. Once we know these values, we can find what is cosine alpha and sine alpha. Here, both are same and it is given there – 0.7071. EA over L is also calculated based on the material property, Young's modulus. Cross sectional area of the number is given. So, once we know length, we can calculate that quantity. So, this is for element 1, which is going from node 1 to node 4.

To have a better idea, let us go back to the figure and see (Refer Slide Time: 13:15). If you see figure a, element 1 is connecting node 1 to 4. Just now, we calculated the corresponding direction cosine values for that element. Similarly, element 2 is going from node 2 to node 4 and the element 3 is going from node 3 to node 4. So, we can calculate for those also in a similar manner. The global X coordinate and Y coordinate are defined at node 1. That is why, the nodal coordinates of node 1 are 0 0; the nodal coordinates of node 4 are 10 feet times 12, that is, 120 inches; X is 120; Y is also 120; nodal coordinates of node 2 are 0 and 240; nodal coordinates of node 3 are 120 and 240.

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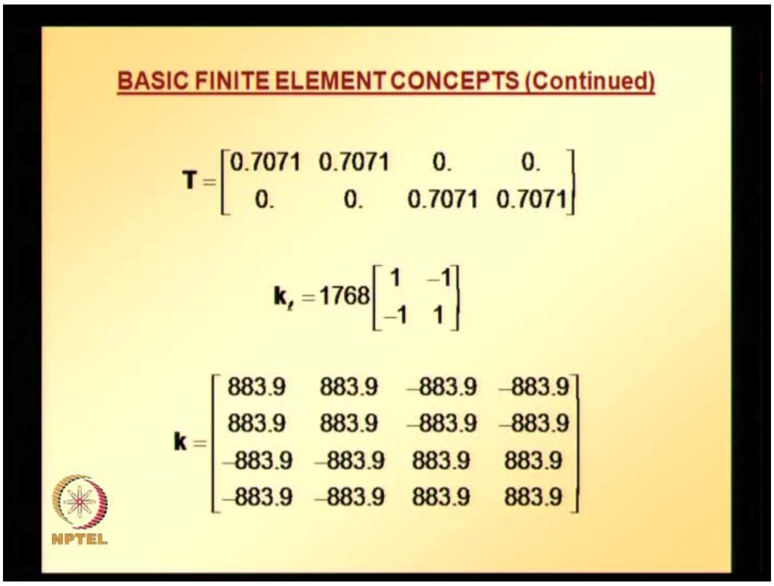

We already seen for element 1, what are the values. Once we have these values, we can assemble the element equations for element 1. Transformation matrix is given by these, which is c s 0 0 0 0 c s. Local stiffness is given by EA over L – 1 minus 1 minus 11. Once we have the transformation matrix and local stiffness matrix, global stiffness matrix is given by T transpose k l T. So, that is given by this one.

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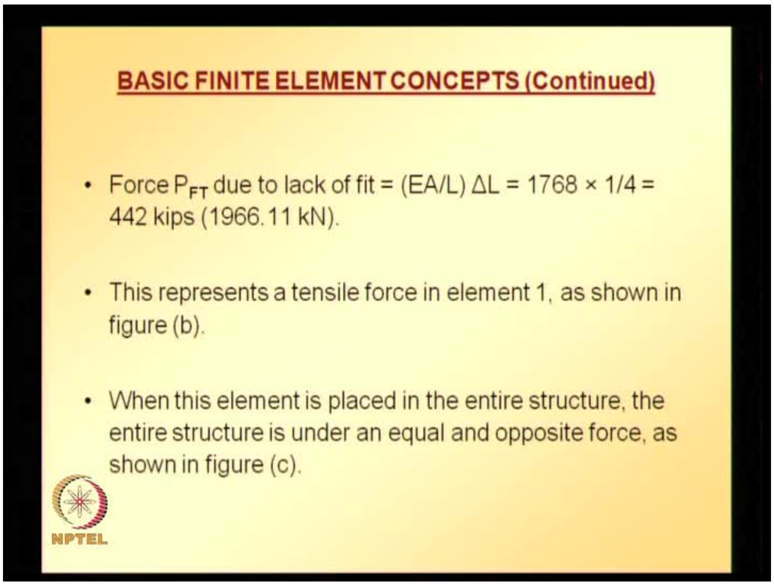

This member 1 is short by length one-fourth inch. So, we can .calculate the force P FT due to lack of fit for this member. EA over L delta L; delta L is one-fourth inch; all are in inches. So, one-fourth is substituted; that is, 0.25; it is calculated. The value both in FPS units and SI units kilonewtons are given there. This represents tensile force in element 1, as shown in figure b, which you already looked at. When this element is placed in the entire structure, the entire structure is under an equal and opposite force; we have already seen that; it is shown in figure c.

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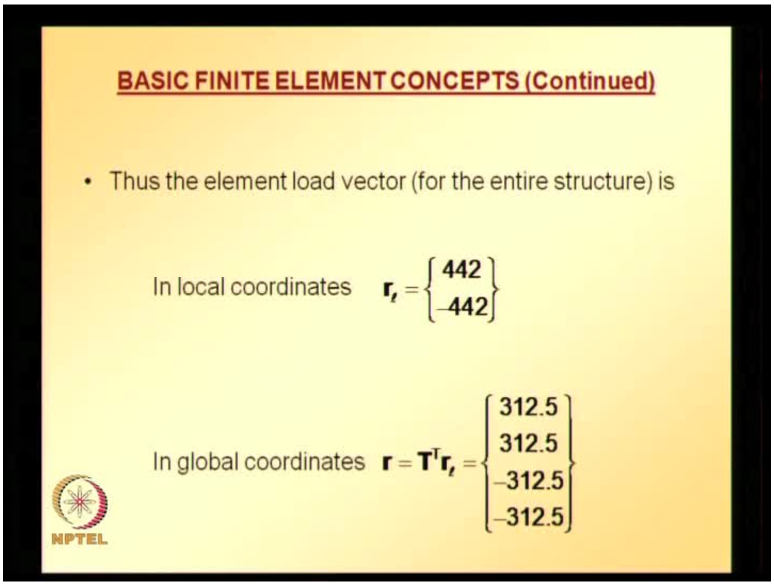

Element load vector is in the local coordinate system at node 1. P FT is acting in the direction of the member; that is, it is acting in the direction of local node 1 to 2 or it is acting in the direction from node 1 to 4. The first component is positive. The second component is acting in the opposite direction; so, negative sign is appended to it. In the local coordinate system, r l is given by this. We already know what is transformation matrix. So, we can calculate in the global coordinate system, what is the load vector. Please note that here we are assigning given loads to member 1. So, all the loads were assigned to only member 1; they are given here.

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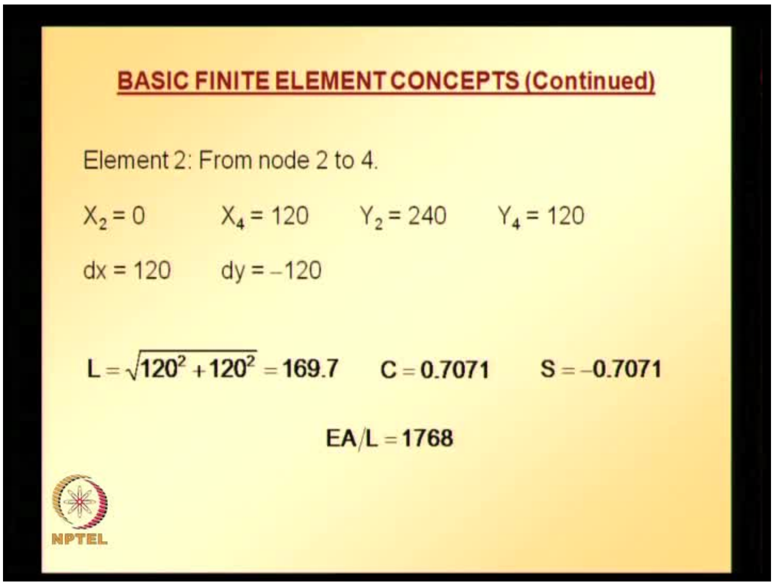

For element 2, local node 1 is global node 2, local node 2 is global node 4; it is assumed to be going from node 2 to node 4. Noting down the coordinates of node 1 and node 4, we can calculate what is dx, dy, and length of this member. Also, once these values are known, we can calculate what is cos alpha and sine alpha, and also EA over L.

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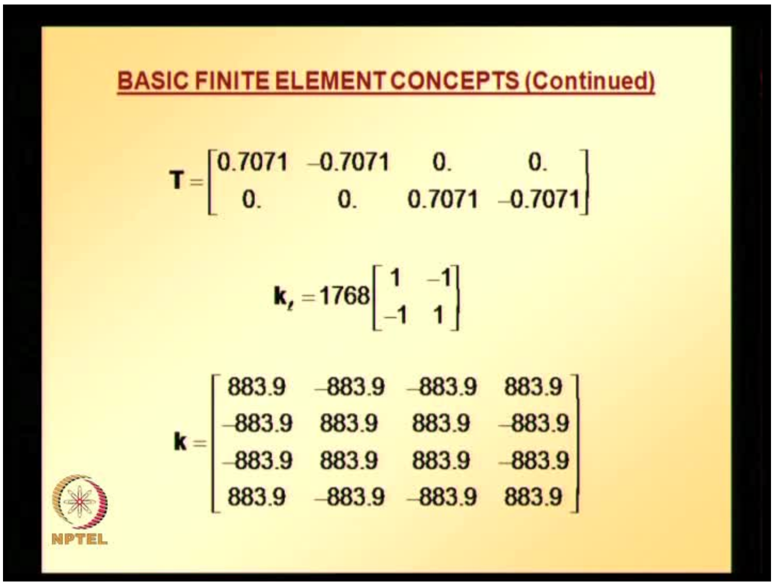

Once we have all these quantities, we can calculate what is transformation matrix and what is local stiffness matrix. Global stiffness matrix k is given by T transpose k l T; that is, given by this one.

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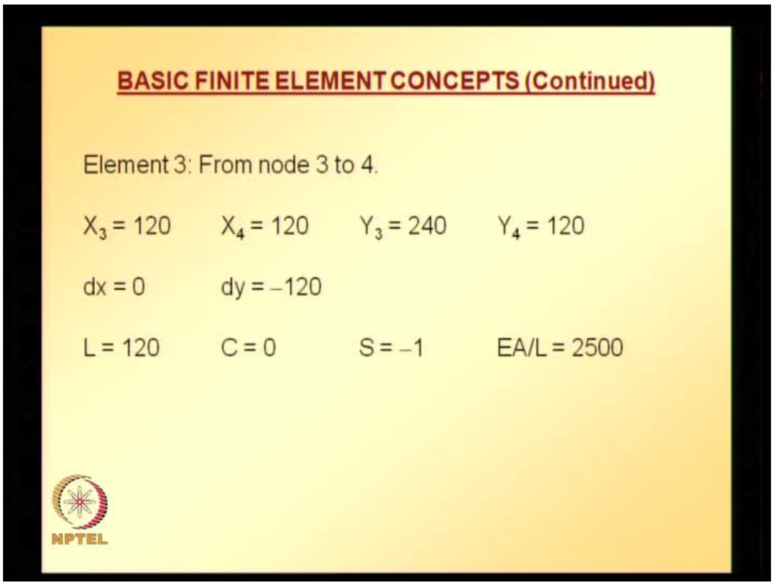

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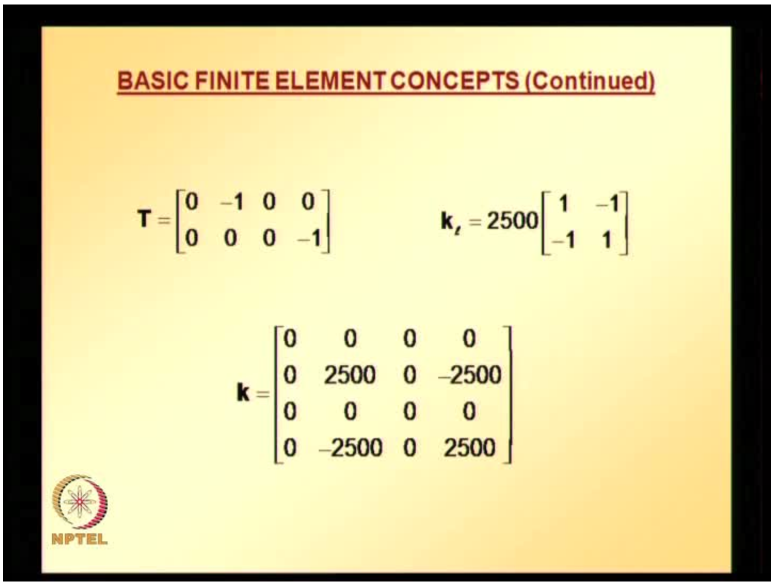

Next is element 3. Element 3: local node 1 is global node 3; local node 2 is global node 4. Element 3 is going from node 3 to node 4. Noting down the coordinates of node 3 and node 4, we can calculate what is dx, what is dy. Length of the member can be calculated in a manner similar as we have seen for element 1 and element 2 – cos alpha, sine alpha and EA over L value. We can calculate what is transformation matrix, local stiffness matrix and the element stiffness matrix. The global coordinate system is given by this.

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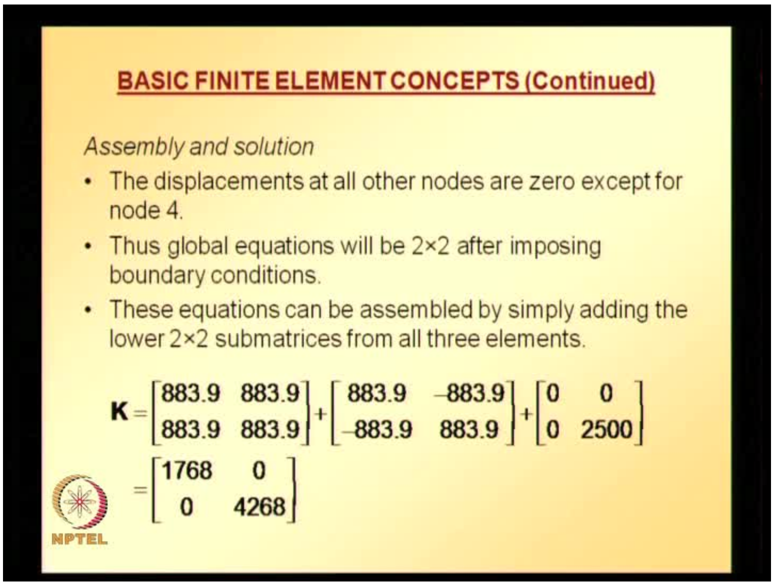

Please note the displacements at all nodes except node 4 or 0. This is what is required – instead of assembling, the entire global stiffness matrix. We can assemble directly the reduced stiffness matrix or reduced equation system. So, the global equations will be 2 by 2 after imposing the boundary condition. This is a 2 dimensional or 2D plane truss problem. There are 4 nodes; at each node, we have 2 degrees of freedom. So, final global equation system or global stiffness matrix will be of dimension 8 by 8. However, instead of assembling entire global stiffness matrix of 8 by 8, we can directly assemble the reduced global stiffness matrix by deleting rows and columns. Corresponding to the nodes at which displacements are 0 – here node 1, node 2, node 3, all degrees of freedom are fixed. So, the corresponding rows and columns can be avoided.

The contribution to the reduced stiffness matrix will come from the 4th quadrant. If you divide the stiffness matrix of element 1 into 4 quadrants; similarly, stiffness matrix of element 2 into 4 quadrants and stiffness matrix of element 3 into 4 quadrants… If you take the 4th quadrant and add the corresponding locations in the 4th quadrant of each of the element stiffness matrices, we will get the global reduced global stiffness matrix. These equations, that is, the global system equations can be assembled by simply adding the lower 2 by 2 submatrices for all the three elements. Please note that this is true only for this case, if you take element 1, it is going from node 1 to node 4; element 2 is going from node 2 to node 4; element 3 is going from node 3 to node 4. So, for all elements – 1, 2, 3, 4th node is local node 2. So, that is why this is applicable only for this particular case; or, if you carefully number the node numbers similar kind of thing can be done for other problems also. So, the reduced global stiffness matrix is given by this one. Adding up the 4th quadrant of all – the element 1, element 2, element 3, stiffness matrices, we get the reduced global stiffness matrix.

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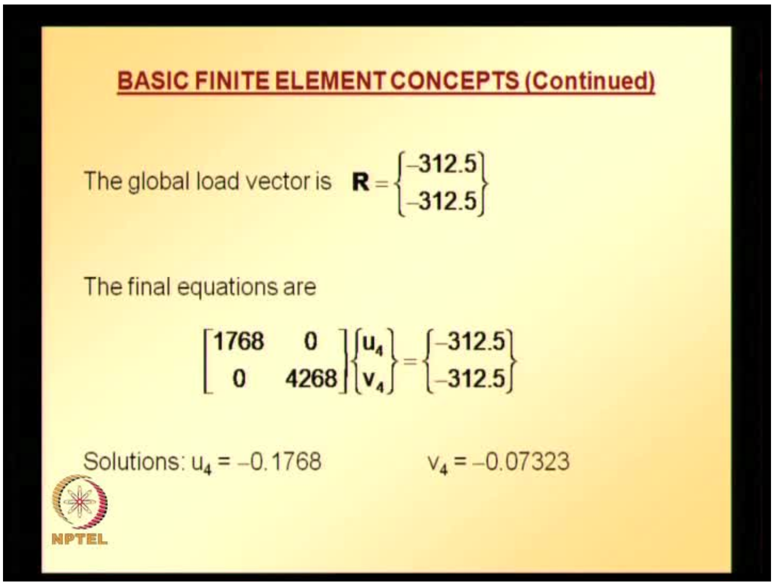

The global load vector: global load vector is nothing but the load vector; the components of the load vector at location 7 and 8. That means we are removing all the components in the global load vector, which are corresponding to node 1, node 2, node 3, and which corresponds to the locations 1 to 6. Removing those, we get the reduced global load vector, which is shown there. So, we have reduced stiffness matrix and reduced load vector. So, we can write final reduced equation system, is given by this, which is a 2 by 2 system. We can solve for u 4, v 4 and we get the values like this – all are in inches. Now, we got the nodal values – u 1, v 1; u 2, v 2; u 3, v 3. All these are 0. Whatever is non-zero, that is, u 4, v 4 – we just obtain.

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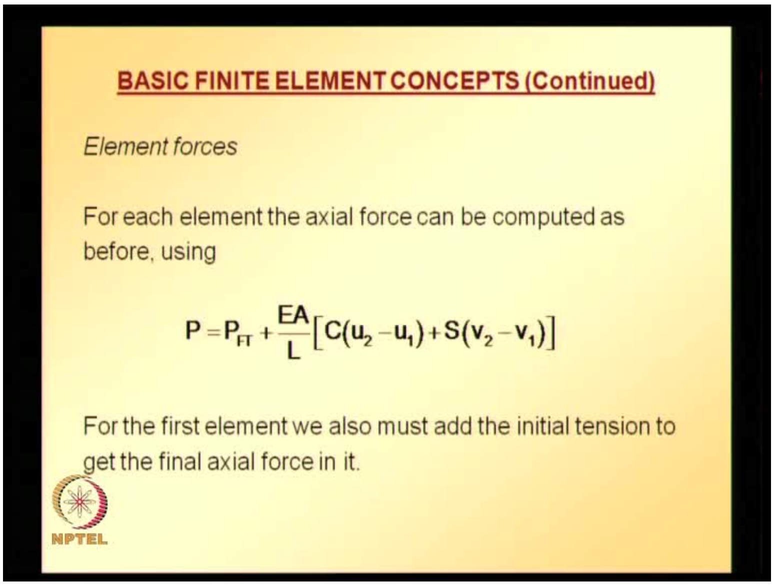

Now, we are ready to calculate or compute element forces. For each element, axial force can be computed as before using this one. Only the first term is additional one, which is contribution from lack of fit. For element 1, we have this term; for element 2, element 3, we do not have this term. For the first element, we must add initial tension to get the final axial force in it.

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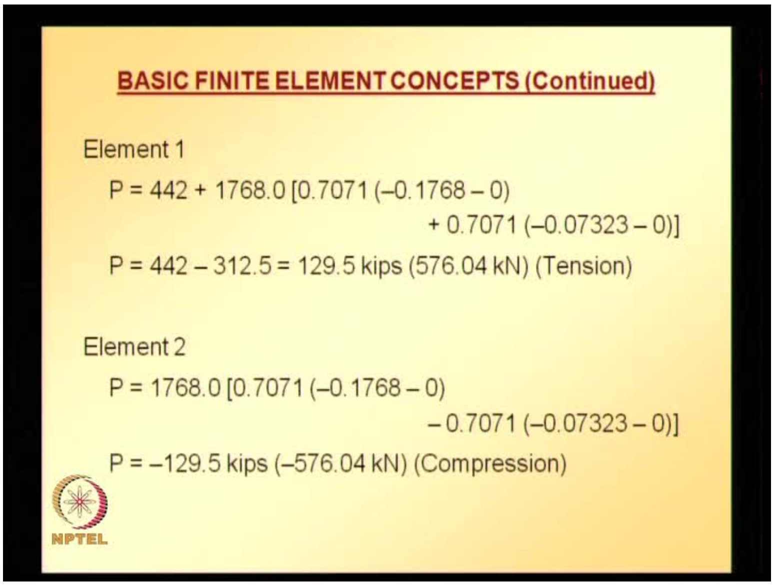

For element 1, the axial force is given by the value here. As expected, this value turns out to be positive, which means the member is in tension. That is what is expected because element 1 is short by certain amount. Force is stretched and force fitted is a structure. So, that particular member will be in tension. Element 2: as I mentioned, element 2 is not going to have P FT contribution, because there is no forced fit for element 2. So, P FT is 0. The calculations were similar to what we have seen in the earlier class, where there is no forced fit, such kinds of things. It turns out that the value of force in element 2 is negative, which means it is compressive. It is expected because when element 1 is force fitted, it is exerting compressive force over the rest of the structure.

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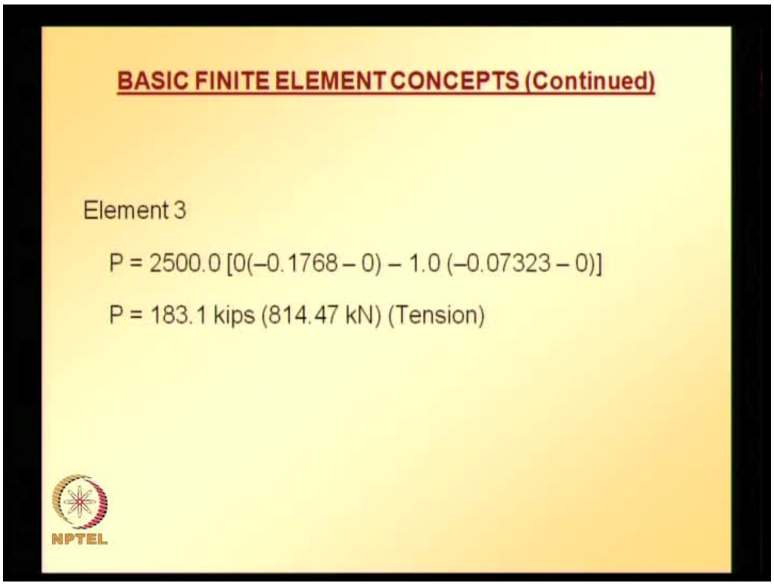

Element 3 calculations are given here, which is also in tension. This example demonstrates how to calculate stresses and displacements, because of lack of fit. The procedure will be similar even if the problem is instead of lack of fit. It is due to change in the temperature. So, the procedure is similar.

Now, we will go to the next concept, which is beam bending. So far, we have looked at trusses and the next one is beam bending.

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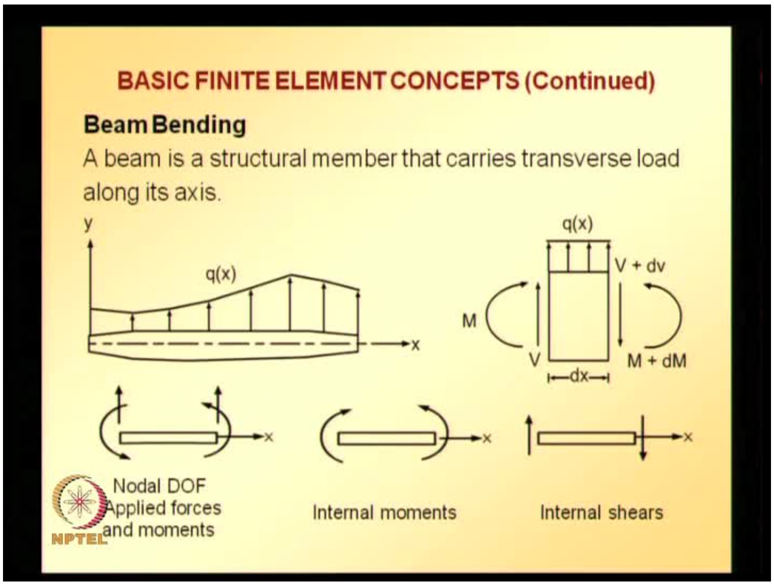

Beam bending problem is governed by a 4th order differential equation. What we will do is – we will first derive the governing equation for beam bending problem. Then, the 2 node element for bar element, which we are using so far, the same element we have adopted even for truss problems – that element at each node, you have 1 degree of freedom. So, that element is not applicable for solving beam bending problem. So, what we will do is – we derive finite element formulation for a two node beam element. As we did for two node bar element, we will be doing for two node beam element. However, if somebody is interested to derive the finite element formulation for three node beam element, similar procedure can be adopted to get the higher order beam elements.

A beam is a structural member that carriers transverse load along x axis. Here, a typical beam is shown. Length of beam is large compared to its cross section dimensions. In general, cross section can be of any arbitrary shape and also it can vary along the length of the beam. However, what we will do is, whatever equations we are deriving in this course, we will limit ourselves to cross sections, which are symmetric with respect to the plane of loading. x-axis passes through the centroid of the section. A typical beam lying along x-axis and loaded in x y plane is shown there. One important thing is sign conventions, conventions adopted for positive directions for transverse displacements rotations, applied forces, applied moments, are shown there in the slide – nodal degrees of freedom and applied forces and moments. Also, sign convention for internal moments and internal shears are also shown there.

Now, as we did for a bar under axial deformation, when we are deriving the governing differential equation, what we have done is, we have taken a differential element and based on the free body diagram of the differential element we derived the governing differential equation. Similar manner, a differential element is shown on which all the forces are indicated shear force moment on both the sides of the element. Element is of length dx. Now, as we did for bar under axial deformation, when it applies the equations of statics – here, forces are acting in the y direction. So, sum of all the forces in the y direction should be equal to 0; sum of all the moments taken about a point should be equal to 0.

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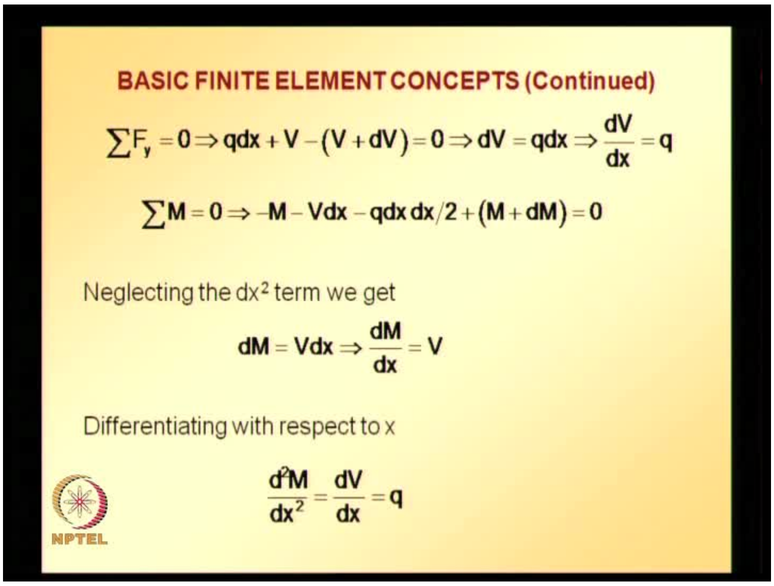

Let us apply the first condition: sum of all forces in the y direction is equal to 0. That gives us derivative of shear force; special derivative of shear force is equal to the distributed load applied. Moment: sum of moments is equal to 0. That gives us that equation. Neglecting dx is small, dx square is going to be very small. So, neglecting dx square, we get derivative of moment; special derivative of moment is equal to shear force. The first equation gives – special derivative of shear force is equal to the applied distributed node. The second equation gives – special derivative of moment is equal to shear force. So, what we can do is, we can differentiate second equation one more time; differentiating second equation with respect to x one more time, we get this equation. Here, this equation gives us relation between moment and shear force and load applied, but we want to solve for displacements because of the transverse load. So, we need a relation between moments and transverse displacements.

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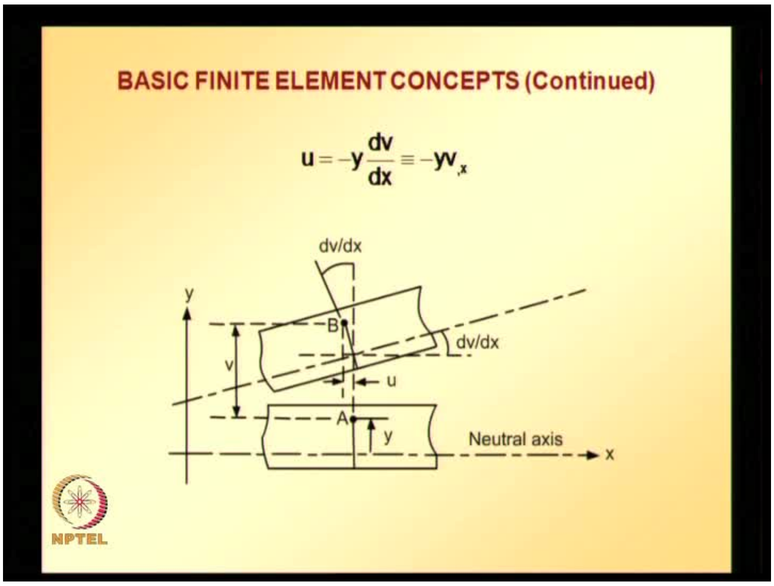

Moments can be related to transverse displacements by considering beam deformation shown here. Deformation of beam is shown here. Here it is assumed that the displacement v is small; transverse displacement, v is small and plane sections remain plane after bending. If you look at the figure, point A located at a distance y from the neutral axis is displaced or shifted to point B. Initially, the vertical plane section has rotated by an amount equal to first derivative of transverse displacement with respect to the x. So, that is equal to theta; that is, it is rotated by an angle equal to dv over dx. Assuming u, here two displacement components are shown: u and v. u is in the x direction, v is in the y direction. Assuming u to be positive in the positive x direction, the point A because of application of load and bending beam deformation, it has moved in the negative x direction.

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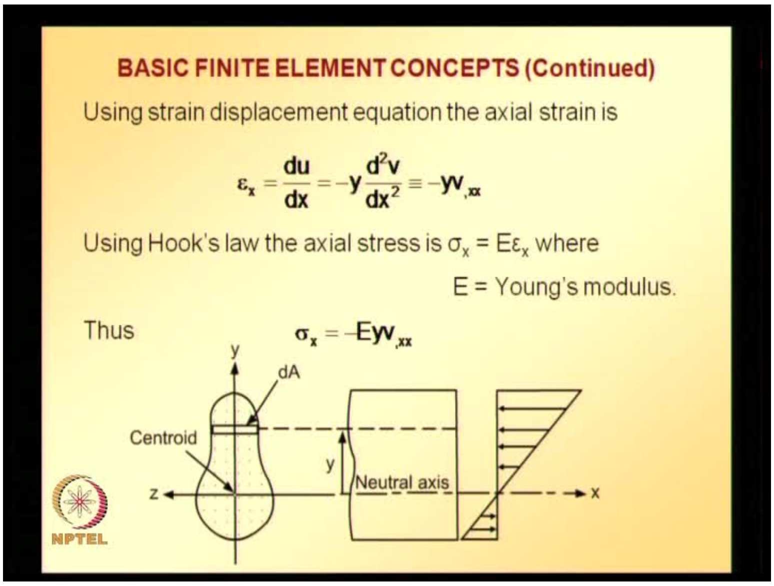

u component of displacement of point A is given by u is equal to minus y dv by dx – minus is because it is moved in the opposite direction – to the positive x direction. This gives us relation between displacement with x direction and displacement in the y direction or rotation. Using strain displacement equation, axial strain is given by epsilon is equal to derivative of u with respect to x. Just now, we derived what is u – u is minus y dv by dx. So, taking derivative of that, we get this.

Using Hook's law, axial stress is given by Young's modulus times epsilon – E is Young's modulus there. So, axial stress is given by this (Refer Slide Time: 36:52). If you plot this variation of stress over a cross section, it looks like this. So, this equation shows a linear variation of stress over beam section. The equilibrium at a section requires stress resultant over the cross section – must be 0 because there is no applied forces in the x direction. Since x-axis is assume to pass through the centroid of a section, it is clear that this requirement implies that neutral axis pass through centroid of the section. Also, moment equilibrium condition requires that moment of forces acting on cross section must be equal to the applied moment at the section. So, the resulting moment from the stress distribution shown in figure given here results in the following moment curvature relationship. So, taking moment of stress over area, stress over area dA gives you force over the differential element. So, sigma times dA. If the force is in the positive x direction, it is a tension; if it is in the negative x direction, it is assumed to be compression.

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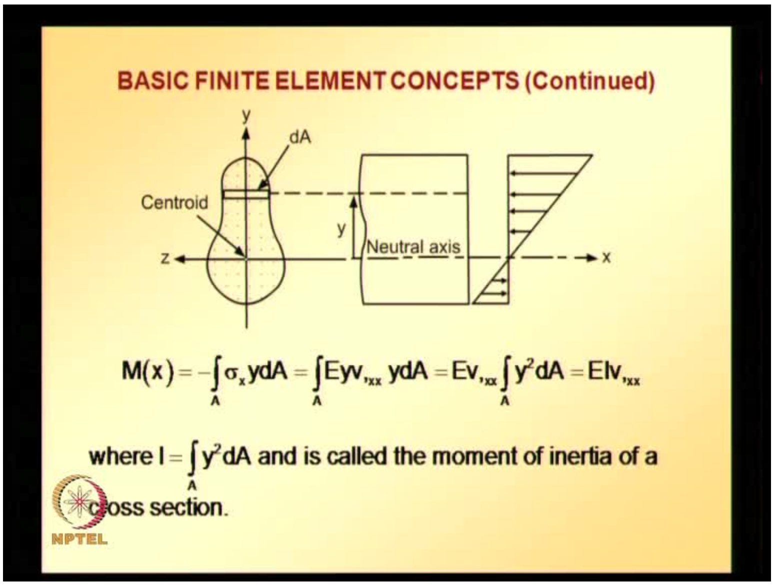

Here the sigma times dA is acting in the negative x direction and because of that, it gives a moment, which is opposite to the sign convention. That is why the relation here is, M is equal to minus sigma y – y is the distance of the differential element from the centroid or the neutral axis – this is for a differential element dA. We need to sum up over the entire cross sectional area. So, it is minus integral sigma x y dA. Sigma value is substituted there. Finally, it is equal to EI second derivative of transverse displacement; moment is equal to EI second derivative of transverse displacement. So, this is the relation between moment and transverse displacement. I – here is defined as integral y square dA – over the cross section; it is called the moment of inertia of a cross section. So, now, we got the relation between moment – please note that this is internal moment – moment and transverse displacement. Earlier, we have seen what is the relation between moment and load applied.

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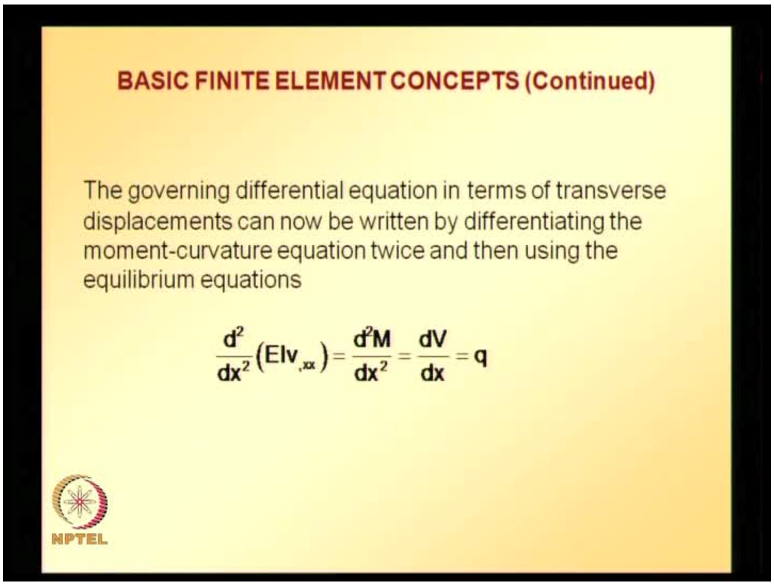

We can rewrite the previous equation and obtain the governing differential equation in terms of transverse displacement. Here basically, M is equal to EI second derivative of transverse displacement with respect to the special coordinate is substituted and we obtain this equation.

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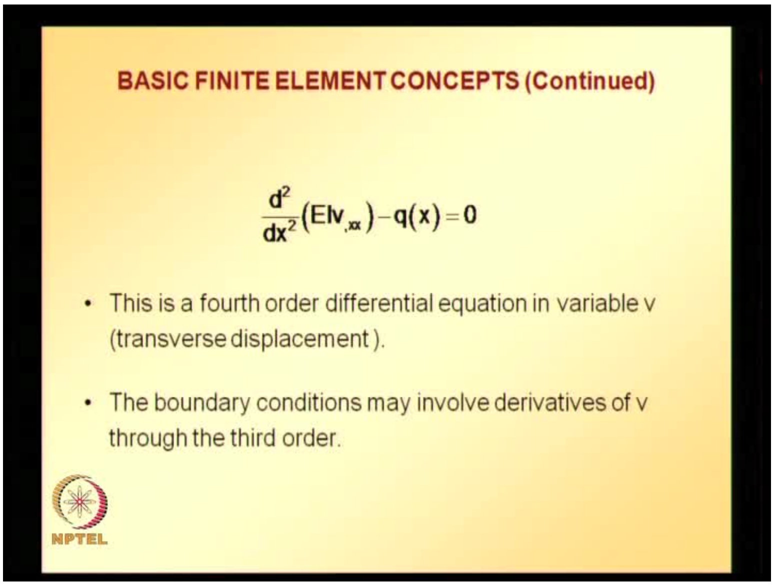

The governing differential equation is this. It can be observed that this is a 4th order differential equation in transverse displacement v. So, we require four boundary conditions to solve this equation. The boundary conditions may involve derivatives of transverse displacement up to third order.

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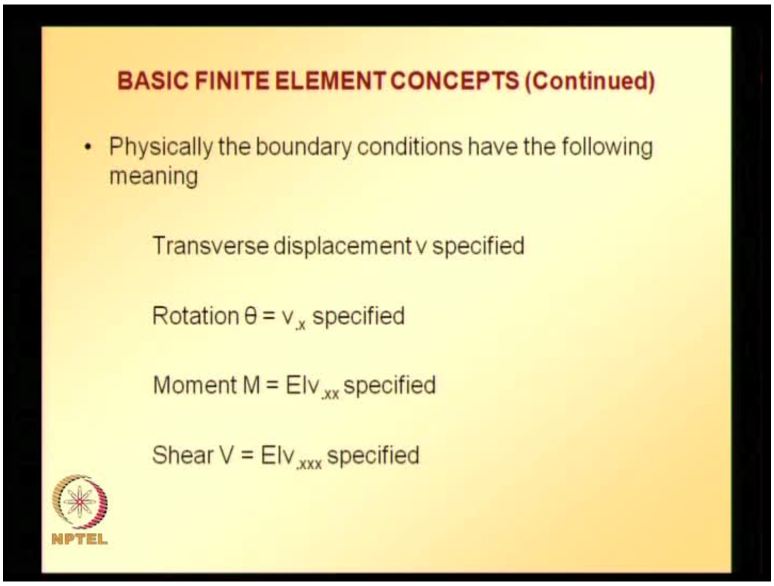

Physically, boundary conditions have the following meaning: v is transverse displacement; derivative of v is nothing but rotation; second derivative of v times modulus of rigidity, that is, EI gives you moment; EI times third derivative of transverse displacement gives shear. So, any combination of these boundary conditions can be given. Four boundary conditions are required to solve this problem. Looking at this physical meaning of the terms, the boundary conditions involving v and derivative of v are essential, and the boundary conditions involving second derivative of v and third derivative of v are natural boundary condition. This can be verified using the thumb rule that I gave in the earlier classes. So, if a differential equation is of order 2 p, those boundary conditions of order 0 to p minus 1 are essential boundary condition and those boundary conditions of order p to 2 p minus 1 are natural boundary conditions. Based on the thumb rule, we can classify the boundary conditions involving v and first derivative of v are essential, and boundary conditions involving second and third derivatives of transverse displacement are natural boundary conditions. So, now, we got the differential equation, the boundary conditions.

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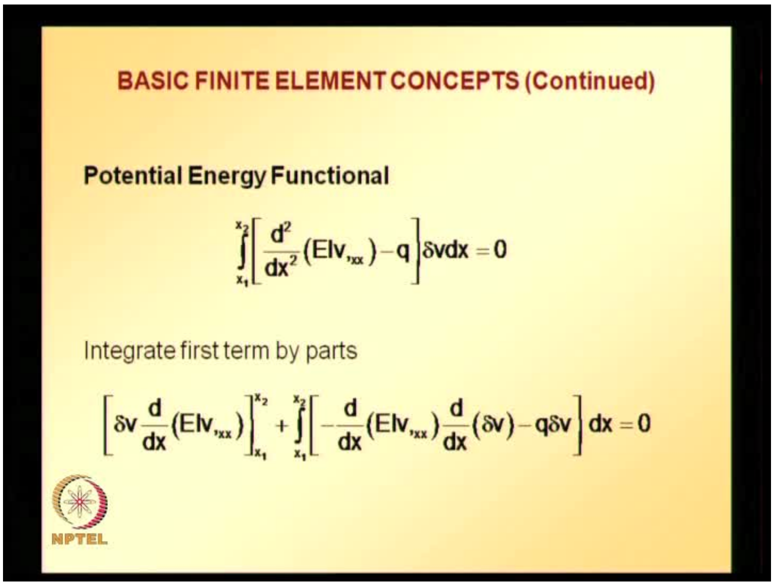

Now, we are ready to apply variational approach to get the equal and functional. Now, to find that, the procedure is as follows: multiply the given differential equation with variation of transverse displacement, integrate over the problem domain. Here, the problem domain is taken as x 1 to x 2. Next step is, any of the higher terms having higher order derivatives, we can reduce the order of derivative by using integration by parts. Also, please note that integration by parts can be applied as many number of times as one wishes to reduce the order of differentiation.

Integrate first term by parts. That gives us this equation (Refer Slide Time: 44:43). If you see the second term, which is integral, the first term in integral is actually having derivative of second derivative of transverse displacement, which turns out to be the third derivative of transverse displacement. So, we can still use one more time, this integration by parts and reduce the order of differentiation of that term. Also, note that derivative of variation of v with respect x is same as variation of derivative of v with respect to x.

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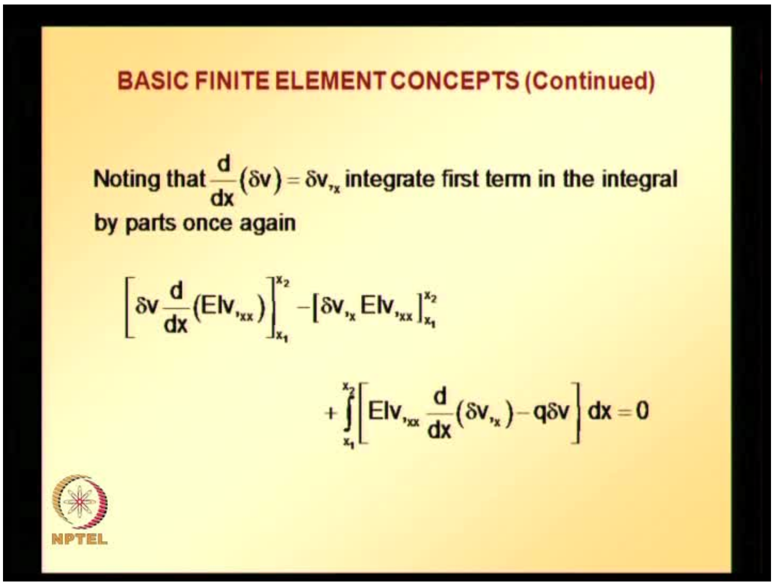

Since differentiation and variational operators are interchangeable, we can write that in that manner. Noting these points, this equation can be written in this manner. Now, wherever E times second derivative of v appears, we can replace it with M, internal moment. Wherever E times third derivative of v appears, we can replace that with internal shear. Also, we can apply variational identity.

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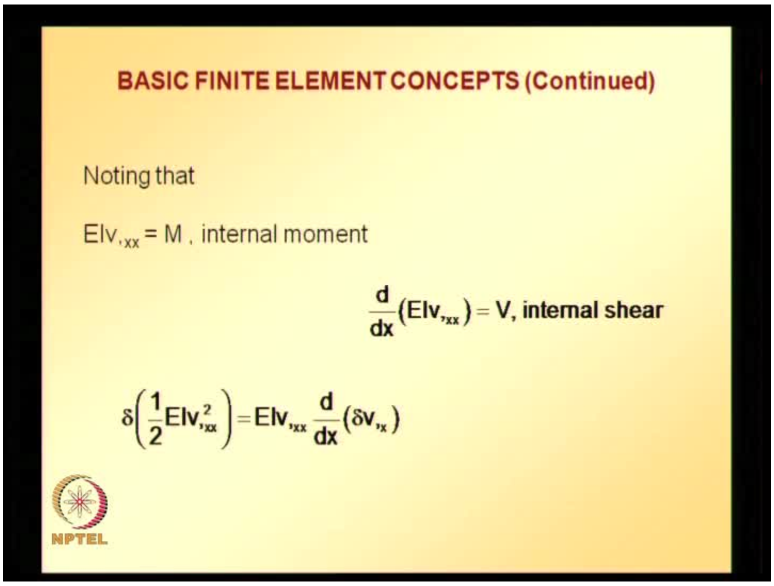

We can apply variational identity to reduce or to rewrite this equation. So, this is the relation between internal moment and transverse displacement, internal shear and transverse displacement. In addition to these, we will be using this variational identity, which one can easily verify. So, wherever the right-hand term appears, we can replace it with the left-hand side term.

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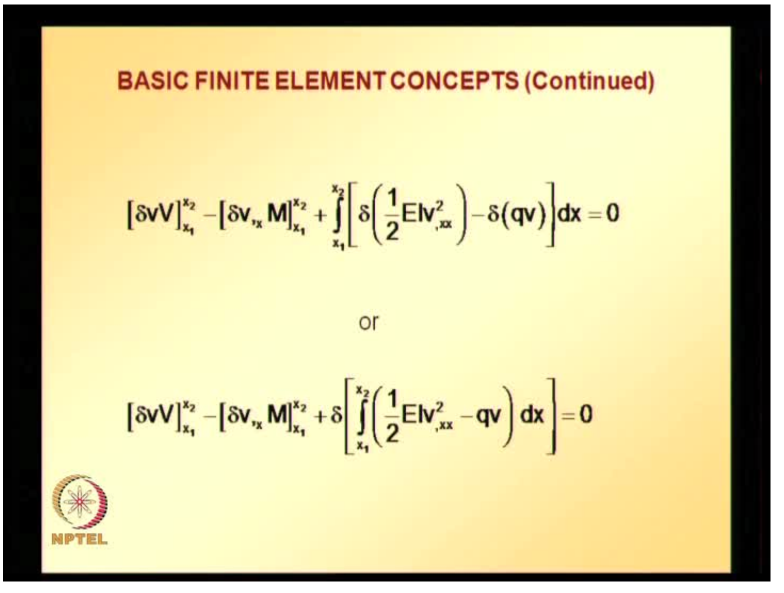

Using these definitions of internal moment, internal shear and variational identity, the previous equation can be rewritten in this manner. Here, inside integral integrant is in the form variation of u plus variation of v. So, we can bring that variational operator out; I sort the integral, since variational operator and integral operator are interchangeable. So, we get this one.

Now, to simply this equation further two conditions arises: if essential boundary conditions are specified and if natural boundary conditions are specified. If essential boundary conditions are specified, please note that for a beam problem, displacement and rotations – both are essential boundary conditions. So, wherever displacement and rotations are specified, at those locations, variation of transverse displacement, variation of rotation should be equal to 0. That is what we learnt earlier. So, with that reasoning, the first two terms get cancelled if essential boundary conditions are specified and the equivalent functional…

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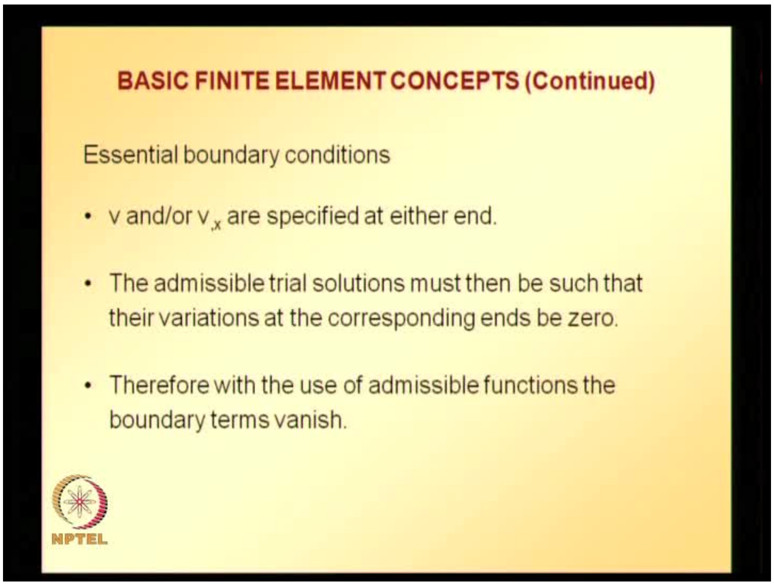

This is the reason. The admissible trial solution must be such that their variations at the corresponding ends be 0. Therefore, with the use of admissible functions, the boundary terms vanishes.

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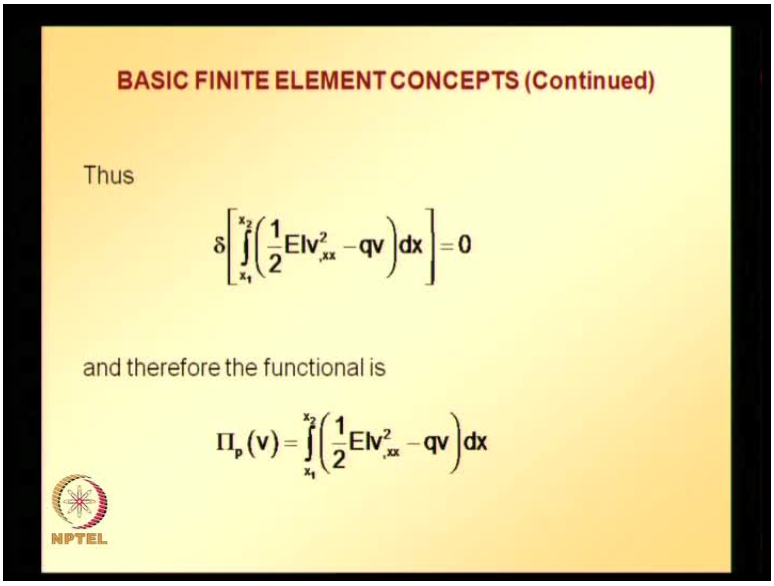

The equivalent functional becomes this one. Whatever is there inside variation that is nothing but potential energy. This is only applicable when essential boundary conditions are specified at both ends. Suppose if natural boundary conditions are specified; that is, moments and shear forces are specified, we need to substitute those values and simplify the functional that we have seen earlier.

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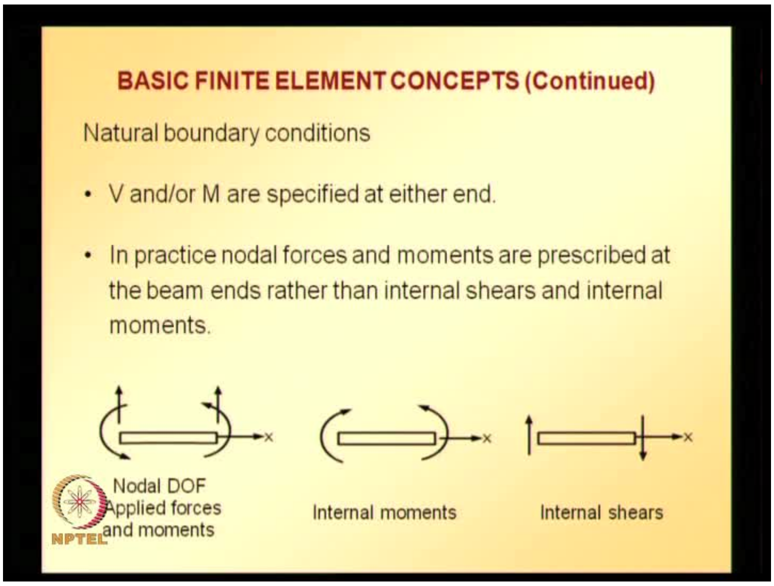

Natural boundary conditions V and M are specified at either end. Please note that the equation so far we have derived, V is internal shear and M is internal moment. However, in practice, nodal forces and moments are prescribed at beam ends rather than internal shears and internal moments. So, we should note down the relation between the nodal forces and moments and internal shears and internal moments. If you go back and see the sign convention that we started out with – this is what we started out with, the first figure shows the sign convention for nodal degrees of freedom, that is, transverse displacements and rotations. Also, that is the sign convention for applied forces and moments; whereas, internal moments and internal shear figures are given – the second figure shows for internal moment, third figure is internal shear.

If you compare the sign conventions there, the applied moments at the left-hand end is in opposite direction to the internal moment at the left-hand end; whereas, at the right-hand end, the applied moment is in the same direction as internal moment. Similarly, at the right-hand end, the applied force is in opposite direction to internal shear; whereas, at the left-hand end, internal shear is in the same direction as applied force. So, noting these relations between internal moments, internal shears and applied forces and moments…

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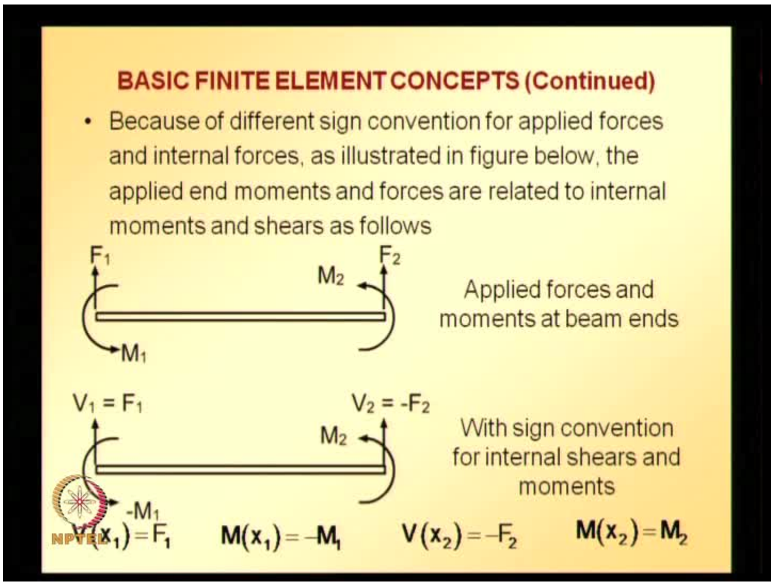

Because of different sign convention for applied forces and internal moments, as illustrated in the figure below, the applied end moments and forces are related to internal moments and shears as follows. These are the quantities that we need to substitute into the equivalent functional that we have earlier and simplify it further to get the corresponding potential energy functional, when natural boundary conditions are specified.

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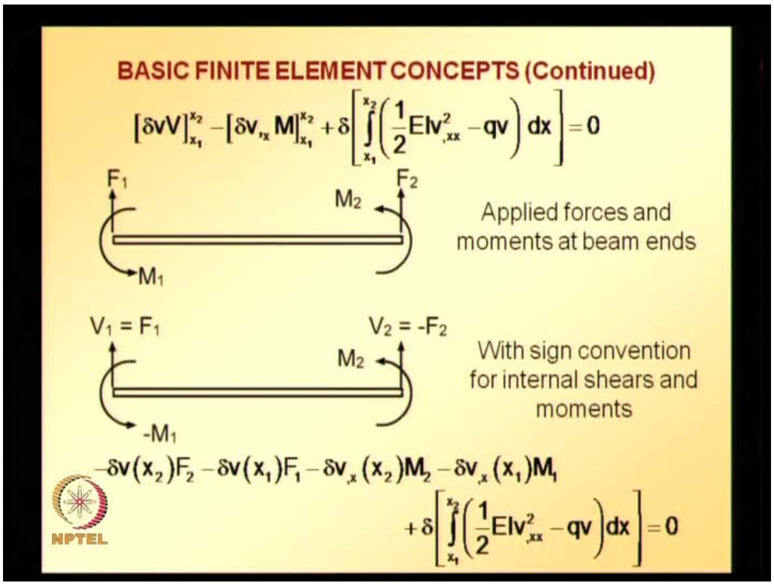

Substituting what are the values of v at x1, x2; what are the values of M at x1, x2, we get this relation. Here, please note that F1, F2, M1, M2, acts like constants as far as variational operator is concerned. So, the terms: variation of V times F and variation of derivative of V with respect X times M – those terms can be grouped together.

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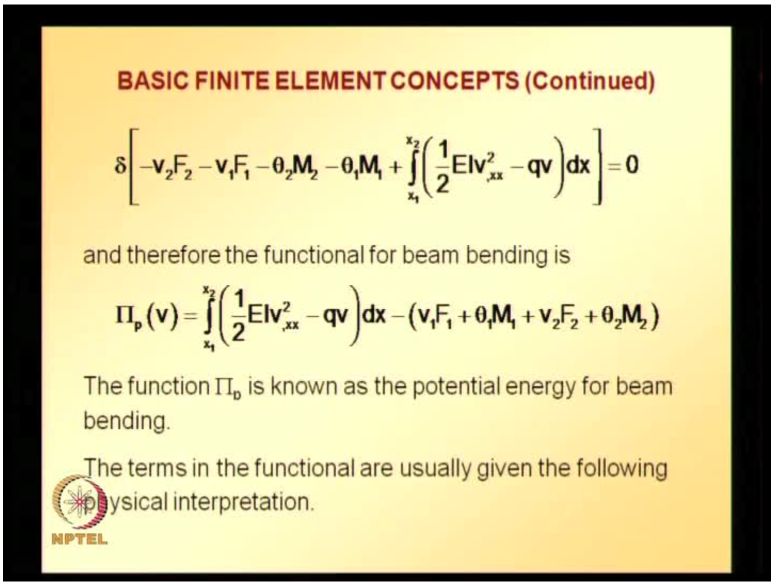

The total functional can be written in this manner. So, the potential energy functional for beam bending problem looks like this. The functional is known as potential energy for beam bending. The first term in the functional is nothing but bending strain energy. The terms in the functional are usually given by the following physical interpretation.

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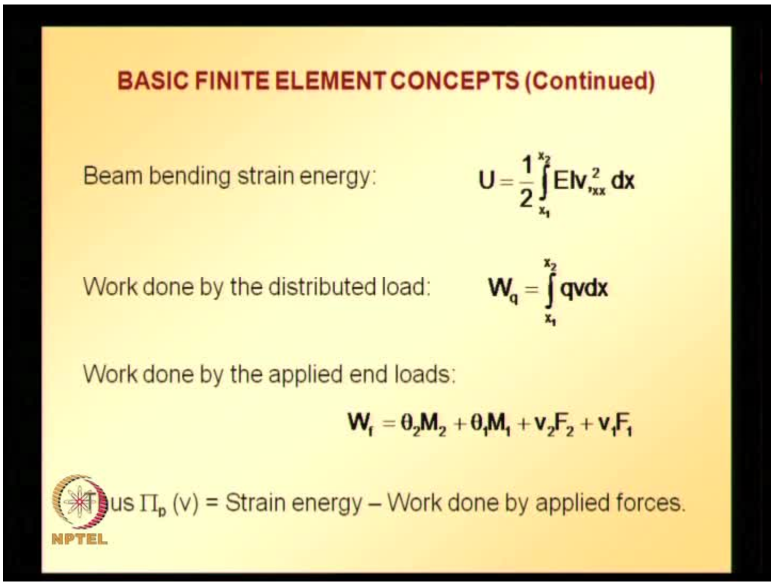

The first term is beam bending energy, second term is work done by the distributed load q, and the third term is work done by the applied end loads, which is given by rotation times moment at both ends, plus transverse displacement times force at both ends. So, the potential energy can be written as strain energy minus work done by the applied forces.

What we have done is, we have looked at governing differential equation for a beam bending problem. Starting with governing differential equation, we have derived equivalent variational functional or equivalent functional or potential energy functional for beam bending problem. So, to proceed further, what we need to do is, we need to develop a finite element for beam formation, which we will continue in the next class.