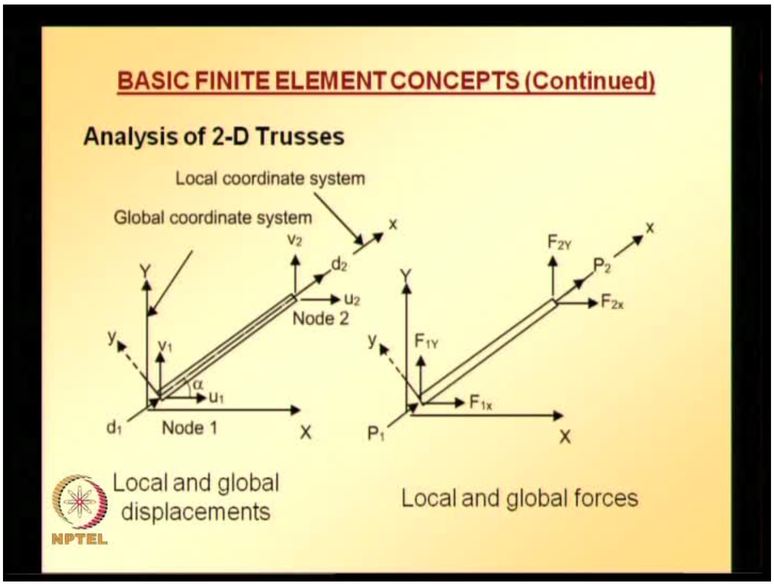
Finite Element Analysis
Prof. Dr. B. N. Rao
Department of Civil engineering
Indian Institute of Technology, Madras

Lecture No. # 07

In the last class, we have seen axial deformation of a bar. We have seen how to derive the differential equation and how to use variational approach or Gallerkin approach to get the element equation system. And once we get element equation system, what we need is we solved a problem, a stepped bar problem, subjected to axial loading and there, we have seen how to assemble, how to discretize the given problem and also, how to derive the element equations. And then, how to assemble the element equations to get the global equations system and how to apply the essential boundary conditions, which are displacement boundary conditions and then after solving the global equation system, again we go back to each of the element and get the solution of the stress strain and axial force in each element.

So, we have seen that for a stepped bar problem and also we have seen, what is the physical interpretation of stiffness matrix, ith column of a stiffness matrix is nothing but it gives us an idea about the force required to have unit displacement at ith degree of freedom with all other degrees of freedom values equal to 0. And also we have seen, how to get the spring the element equations. And also using the element equations that we derived for a spring and also axial, a bar under axial deformation. We also solved a coupled problem or a problem in, which we have spring and also a bar. A spring bar assembly problem, we have seen in the last lecture and today’s lecture, we are going to look at how to analyze truss members?

(Refer Slide Time: 02:38)

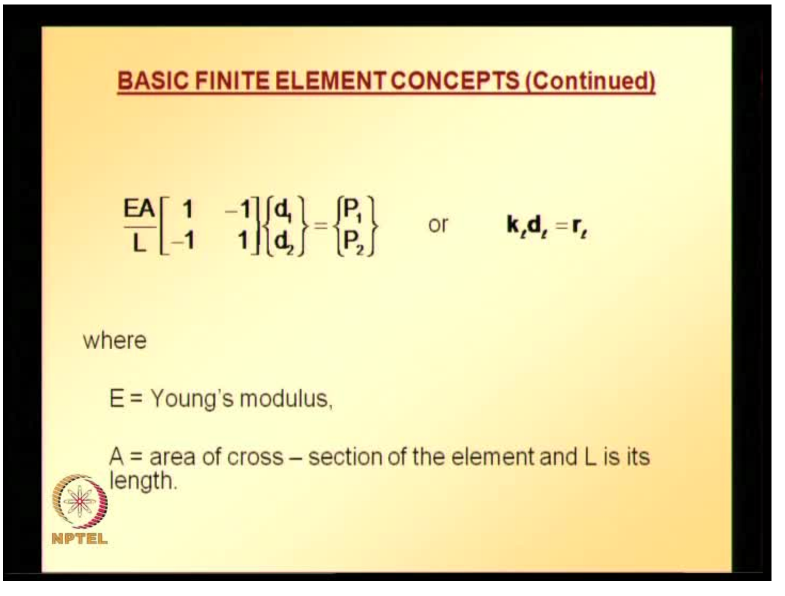


Analysis of 2-D and 3-D trusses, we will be looking in this lecture. And before we look at the derivation details, let us see what truss members are? Trusses are structural frame works, in which individual members are designed to resist axial forces only. They are commonly found in roof structures and bridges. The equations for these elements are well known in a structural engineering field and usually, are covered in traditional courses on matrix structural analysis.

Element equations can be derived from axial deformation element using simple coordinate transformation and if you look at the figure that is shown here, 2 dimensional truss element is shown there; 2 dimensional truss element is an axial deformation element oriented arbitrarily, in 2 dimensional spaces. And in the figure, you can see a local x coordinate system along member centroidal axis from node 1 to 2 and global coordinate system in Cartesian coordinate system defined as, or identified as capital X capital Y and the local coordinate system is denoted using a small letter x. All the truss elements in the model are referred to this global coordinate system that is capital X capital Y.

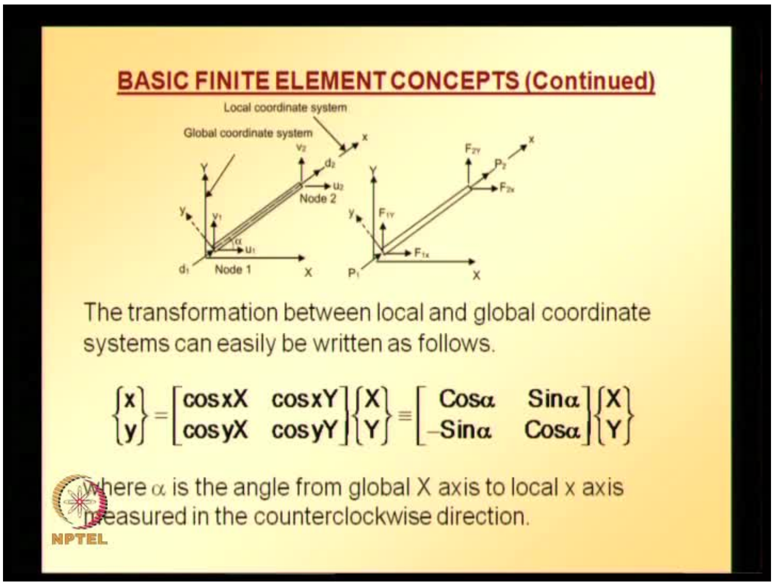
The nodal displacements in the local coordinate system are denoted using small d 1, small d 2 and the corresponding applied forces are denoted using capital P 1 and capital P 2; these are in the local coordinate system and these displacements and forces they get resolved in the global coordinate system or the global degrees of freedom are denoted using small u 1, small v 1 at node 1; and small u 2, small v 2 at node 2. Similarly, the force at node 1; the global coordinate system is denoted using capital F 1x 1x is subscript capital 1 F y in the global Y direction; similarly, at node 2; capital F 2x, capital F 2y. So, at node 2 these are the forces and displacements. So, now since, the element is based on axial deformation equations, which are derived earlier, the same sign convention will be following here that is, if a tensile force is assumed to be positive and a compressive force is assumed to be negative. In the local coordinate system, the displacement vector is d 1, d 2 and the global coordinate system; displacement vector will have 4 components u 1, v 1, u 2, v 2.

(Refer Slide Time: 06:39)



Similarly, the force the local coordinate system consists of 2 components: capital P 1, capital P 2. In the global coordinate system, the force vector consists of 4 components: F 1x, F 1y, F 2x, F 2y. And now, let us see the element equations for axial deformation element, which we already learnt in the last lecture, EA over L 1 minus 1 minus 1 1 and d 1 d 2 is a local displacement vector; capital P 1 capital P 2 is local force vector.

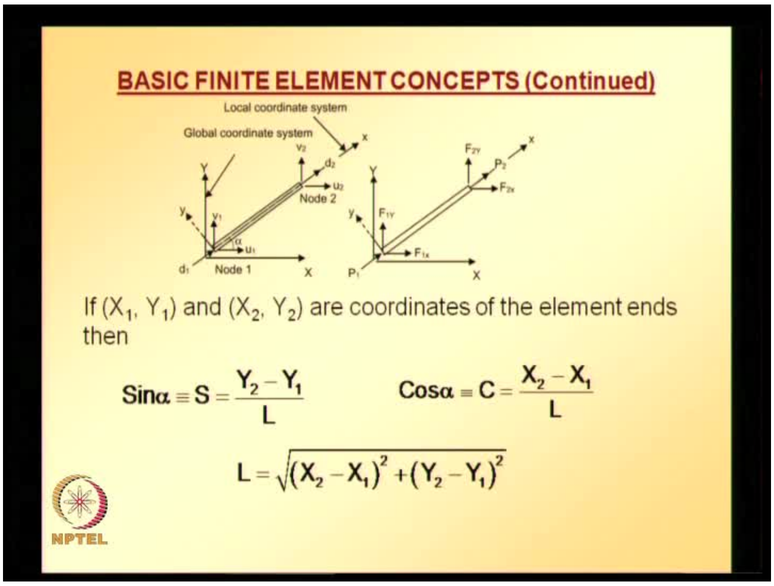
(Refer Slide Time: 07:27)



And this can be compactly written as, k subscript l, l stands for local coordinate system and d subscript l is equal to r subscript l. And here in this element equation, capital E is Young’s modulus; capital A is area of cross section of element, L is length of element and now let us see, what is the transformation? Transformation between local and global coordinate system is nothing but it is simply relation consisting of direction cosines. You can see their small x small y is related to capital X capital Y through, the angle between local x axis to the global x axis, that is cosine of angle small x capital x, that is angle between local x axis and global x axis. Similarly, the first component is cosine of small x capital x, second component similarly it is cosine of small x capital Y, which is angle between local x coordinate system to the global y coordinate system.

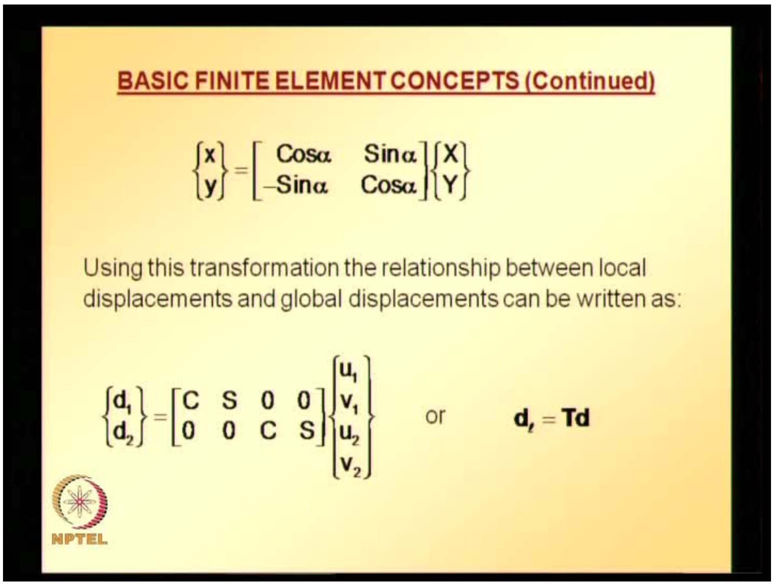
Similarly, the component at 2 1 location, that is, second row first column location is cosine of small y capital X and at 2 2 location, that is, second row second column; it is cosine of small y capital Y; if alpha is the angle from global x axis to the local x axis measured in counter clockwise direction, these direction cosine matrix, becomes cos alpha sine alpha minus sine alpha cos alpha.

(Refer Slide Time: 09:12)



So, the relation between the global coordinate system and a local coordinate system is given by this equation. For an arbitrarily oriented element, if capital X 1 capital Y 1, capital X 2 capital Y 2 are coordinates of element ends, then these the components of this transformation matrix can be obtained using the coordinates of the element ends like, which is given here in this equation, sine alpha is Y 2 minus Y 1 over L cos alpha is X 2 minus X 1 over L. Where, L is length of that particular element, which we can easily obtained using the once we know the coordinates of element ends. In this manner, using this, we can easily get the transformation from global X axis Y axis to the local X axis the Y axis or in the vice versa. Now, we interested in the relationship between the local displacement vector and the global displacement vector.

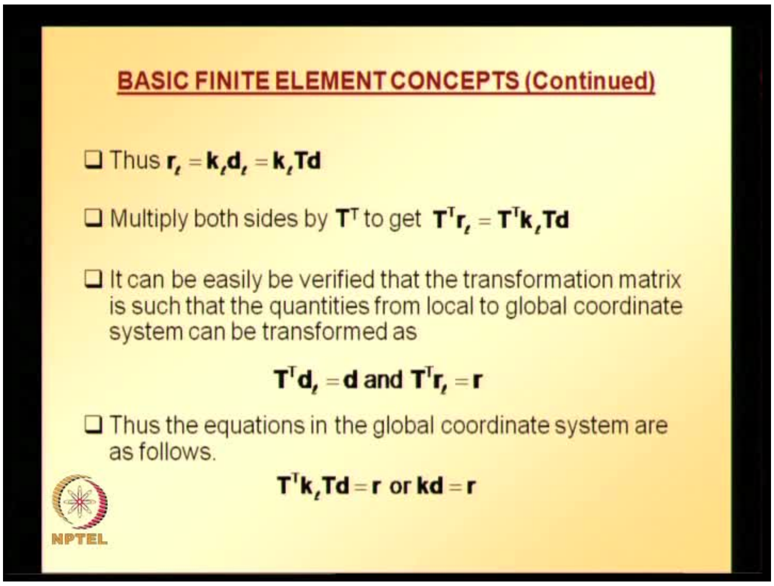
(Refer Slide Time: 10:18)



And please note, that local displacements gets transformed in the same manner as coordinates spatial coordinates. So, we know that the transformation between local X axis Y axis and global X axis Y axis. So, this d 1, which is a local displacement at node 1 in the X direction is related to the global displacements u 1, u 2 in the capital X capital Y direction in the same manner as, small x is related to capital X capital Y, that is d 1 is equal to cos alpha u 1 plus sine alpha v 1.

Similarly, d 2 is cos alpha u 2 plus sine alpha v 2, which can be written in a matrix and a vector form in the manner shown in the slide and this can be compactly written as, d l is equal to T times d; d l is nothing, but displacement vector in the local coordinate system; d is displacement vector in the global coordinate system and T is transformation matrix, which is cos alpha sine alpha 0 0 0 0 cos alpha sine alpha. Please note that, this transformation matrix is an orthogonal matrix, that is, T transpose is same as it can be easily verified, that T transpose is same as T inverse or T transpose T T transpose is equal to identity matrix. So, the inverse relation, that is, d is related to d l via d is equal to T transpose d l since, T is an orthogonal matrix.

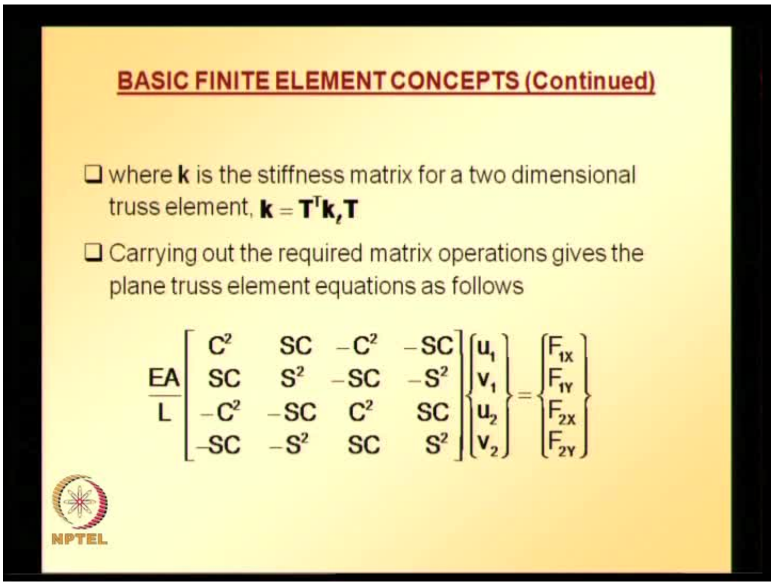
(Refer Slide Time: 12:11)



So, this local equation system that is r l equal to k l d l; now, we can replace d l with T times d and now multiplying on both sides of this equation with T transpose, we get T transpose r l is equal to T transpose k l Td and note that - T transpose r l is nothing, but r because T is a an orthogonal matrix. So, it can be easily verified that transformation matrix is such that the quantity is from local to global coordinate system can be transformed as, T transpose d l is equal to d or and T transpose r l is equal to r.

So, in the previous equation, where we have T transpose r l we can replace that with r; and we can define k as T transpose k l times T. So, we get k d equal to r. So, what is k here; k is t transpose k l T; k l is nothing but it is stiffness of an axial element and T is nothing but transformation matrix consisting of direction cosine components.

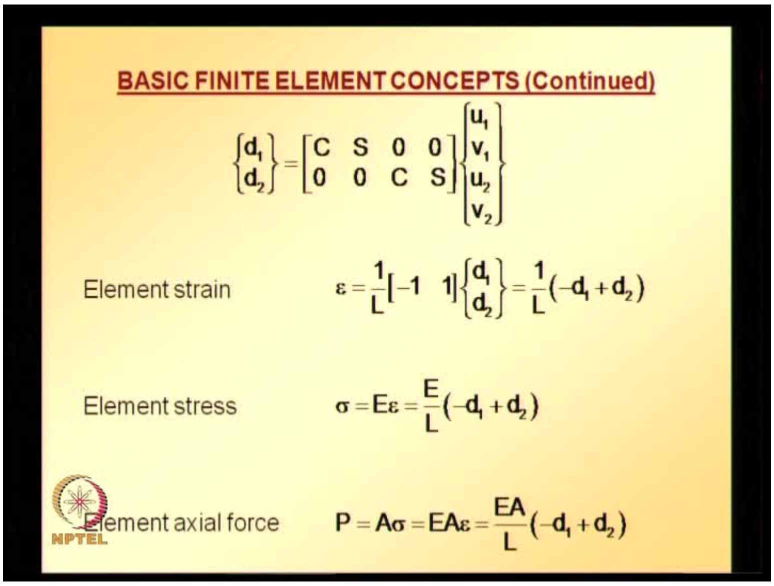
(Refer Slide Time: 13:39)



So, where k is the stiffness matrix for 2 dimensional truss element. So, carrying out the multiplication; carrying out the required matrix operations, we get for the plane truss element the equation system in this manner. So, once we know the ends, the spatial coordinates of ends of a plane truss element, we can easily assemble. When once we know the material properties and geometric properties, we can easily assemble the element equations for a truss element using this equation system.

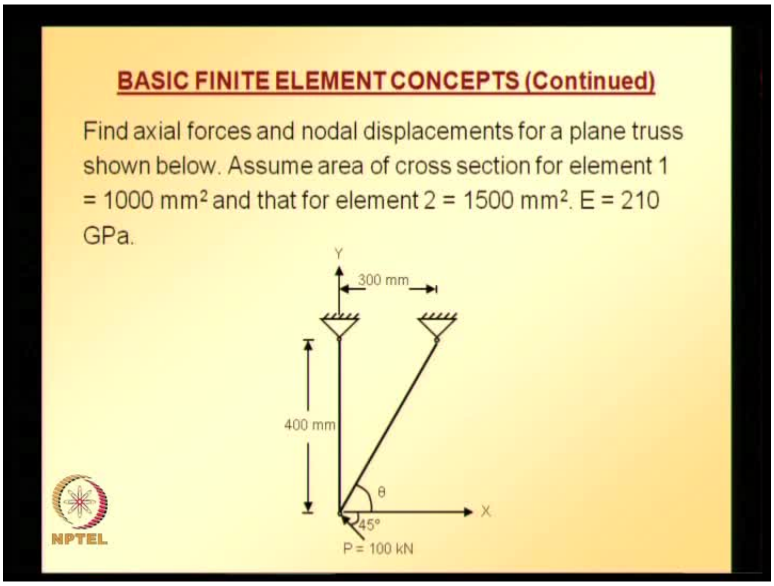
And the nodal displacements for any two dimensional truss can be computed using this equation. Once nodal displacement are nodal displacements are known, the axial forces stresses strains in the elements can be computed by first calculating the local element deformations, that is, here when we solve this equation we get, u 1 v 1 u 2 v 2. Please note - that u 1 v 1 u 2 v 2 are the displacement components in the global coordinate system global x, global y coordinate system. We need to convert them back into the local coordinate system, before we calculate strains. So, that is why, we need to first compute the local element deformations and then, using shear function for axial deformation element, we can calculate strains and then from strains, we can calculate stresses and then from stresses, we can calculate the axial forces elemental axial forces.

(Refer Slide Time: 15:22)



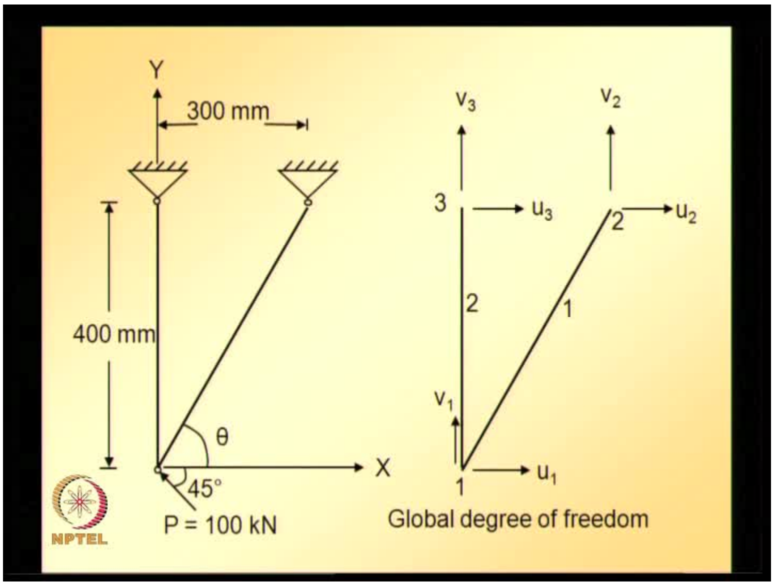
So, this is the relation between local displacement vector and global displacement vector. So, using the values of u 1 v 1 u 2 v 2 , we can back calculate what is d 1 d 2 using this relation? Once, we get d 1 d 2; we can calculate element strain and once we get element strain, we can calculate element stress and we can calculate element axial force.

(Refer Slide Time: 16:06)



So, now we can solve an example. We can solve an example to understand the concepts, which we learnt now. Find axial forces and nodal displacements for a plane truss, shown below. And assume cross section area of cross section of element 1 to be 1000 millimeter square and that for element 2 to be 1500 millimeter square and material property, Young's modulus is 210 GPa. And the truss is shown there and all the dimensions are indicated and the first thing, we need to do is the choice of a local node numbers and the choice of local node numbers defining elements called element connectivity is arbitrary.

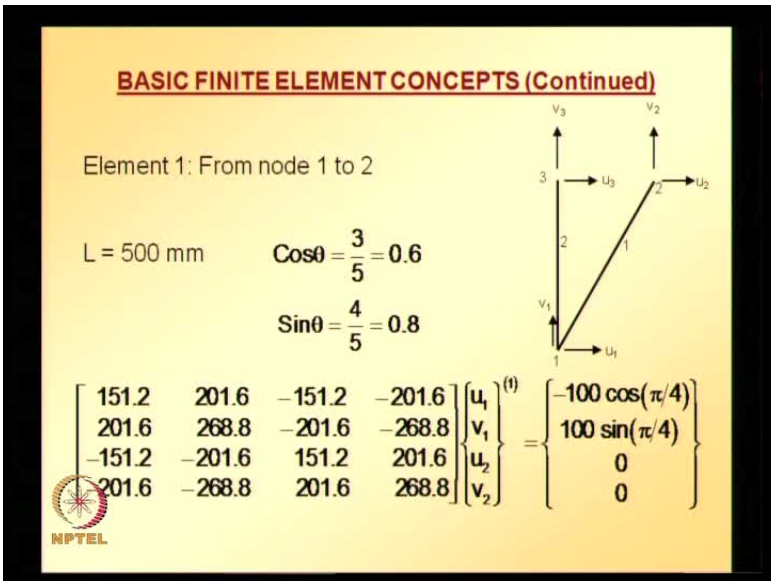
(Refer Slide Time: 17:13)



However, this choice determines the direction of local small x axis and is important to remember for proper interpretation of element quantities. So, definition of local x axis is important for example, for element 2, if we choose first node to be node 1 and second node to be node 3, then positive direction of local x axis for this element is same as global y axis. Hence angle alpha which is nothing but angle between the local x axis and the global x axis it turns out to be 90 degrees. However, if we choose to define element 2 to go from node 3 to node 1, then positive local axis for element is along negative global y axis; giving alpha value is equal to 270 degrees. A consistent pattern for defining element connectivity is helpful, when analyzing stresses with large number of elements.

So, it is very important. The choice of local node numbers is very important. Now, for this particular problem, the node numbers are shown there in the figure and also global degrees of freedom are indicated. In the figure that is at node 1; we have 2 degrees of freedom u 1 v 1; in the global x axis direction displacement is u 1 global y axis direction displacement is v 1. Similarly, at node 2, u 2 v 2 and at node 3, u 3 v 3 so, looking at the element connectivity, element 1: the first node is local node 1 is node global node 1, local node 2 is global node 2; for element 2: local node 1 is global node 1; local node 2 is a global node 3.

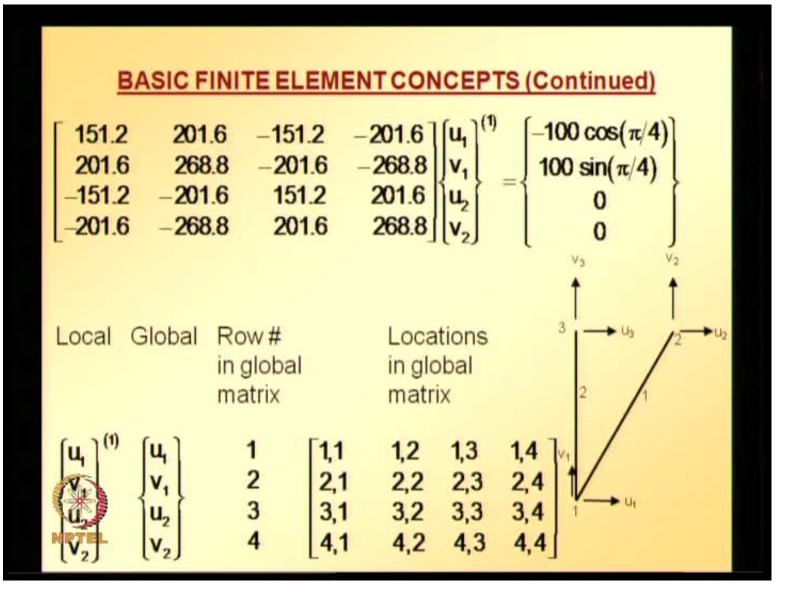
(Refer Slide Time: 19:10)



So, for element 1: the node numbering is given such a way, that element goes from node 1 to node 2 and length of this element is 500 millimeters. The distance between node 2 and node 2 is given as, 300 millimeters and the distance between node 1 and node 3 is given as a 400 millimeters. So, it turns out the distance between node 1 and node 2 is 500 millimeters. So, length of this element is 500 millimeters. Since, the element, the coordinates of ends of element 1 can easily be calculated or looking at the geometry, we can easily find the angle between the global x axis and local x axis for element 1, is given by cos of that angle is equal to 0.6 and cos of the angle between local x axis and global x axis for element 1, is 0.6 and sine of angle between global x axis and local x axis for element 1, is 0.8.

And using these values, we can assemble the element equations for element 1 and here if you see the element equations actually, load is applied at node 1; the loads applied at node 1 can arbitrarily assigned to any of the elements, that is, you can assign those loads 2 element 1 or element 2, but here it is assigned to element 1. Alternatively, the nodal loads can be assigned to global load vector directly. In this example, the applied load at node 2 is arbitrarily assigned to element 1, it is that a load is the load is applied at node 1 so, it is arbitrarily assigned to element 1, that is minus 100 cos of pi over 400 sine pi over 4 and this is the equation system for element 1.

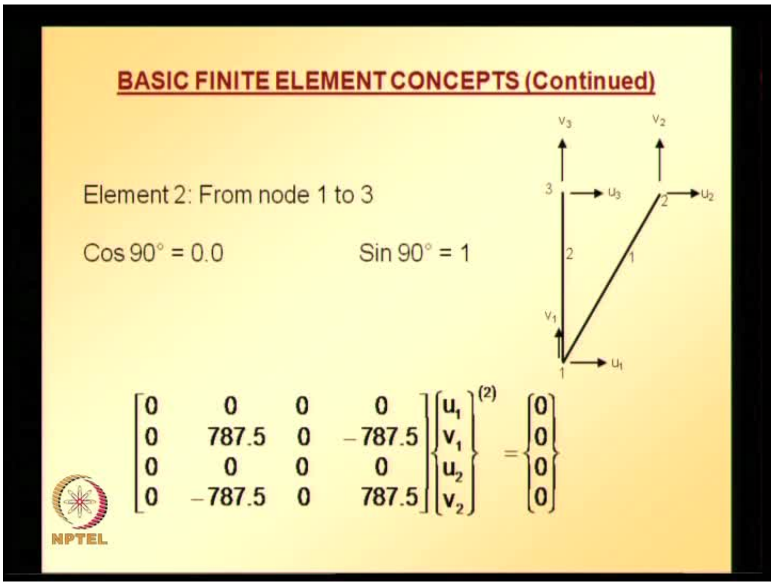
(Refer Slide Time: 22:03)



Similarly, we can assemble the equation system for element 2, noting down the angles which the local coordinate system, local coordinate system makes with global x coordinate system and if you look at element 1 equation system, we should now make a note, where the contribution for element 1 goes into the global matrix. Please note - that the local coordinate system, the degrees of freedom or the displacement vector u 1 v 1 u 2 v 2 corresponding to element 1 is same as global u 1 global v 1 global u 2 global v 2 and the row number and column number in the global matrix are nothing but 1 2 3 4.

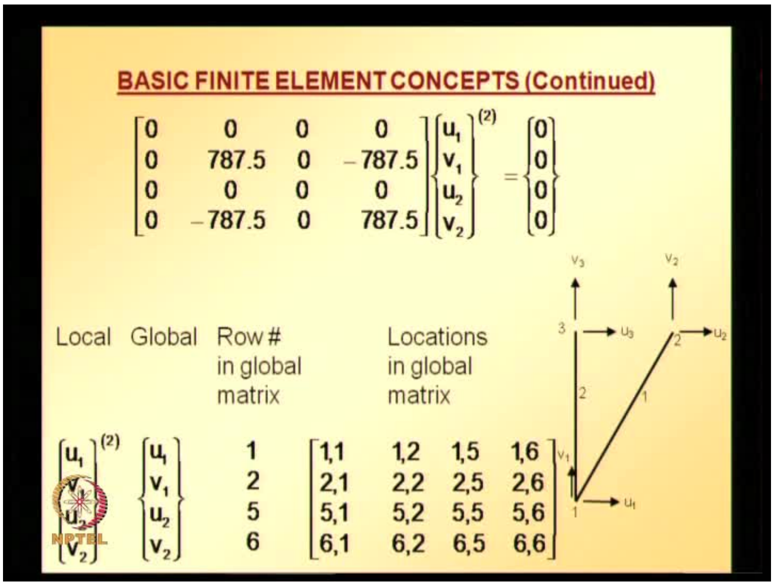
So, the locations into which the contribution from element 1 equations goes at the global equation system is given by, the global matrix locations in global matrix given in this slide. So, 1, 1 location whatever, is there at 1, 1 location the element equation for element 1; it goes into the location 1, 1. Similarly, whatever is there at location 4, 4; it goes into the 4, 4 location of the global equation system.

(Refer Slide Time: 23:34)



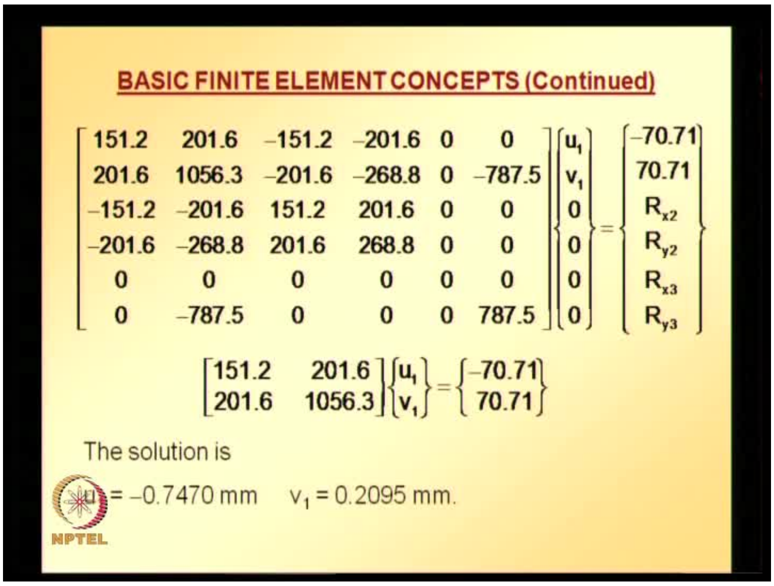
So, now let us look the equation system for element 2. Element 2 local node 1 is 1; local node 3 2 is 3 and the angle between local x axis and global x axis, based on that cos of that angle is 0, cos of the angle between local x axis and global x axis is turns out to be cos 90, which is equal to 0; sine 90 is equal to 1 and substituting, the direction cosine values into the element equations along with the length of the element Young’s modulus of the element length of this element is 400 millimeters. Substituting, all these values the element equation system for element 2 turns out to be this. As you can see here since, we already assigned load which is applied at node 1 to element 1. We have here the again it is not assigned to this element. Now, let us see where the contribution from element 2 goes into the global equation system.

(Refer Slide Time: 24:45)



This is the elemental equation system for element 2 and based on the local displacement vector u 1 v 2 u 2 v 2 corresponding to element 2; its corresponds to the global locations or the row number in the global matrix 1 2 5 6. So, the location in global matrix is based on the row number given there row number in the global matrix. So, the corresponding locations in the local corresponding values in the local equation system, the contribution of those goes into the global locations shown in the slide there, that is whatever value at 1 1 location, the local elemental equation for element 2 the contribution goes into the global 1 1 location.

(Refer Slide Time: 26:08)

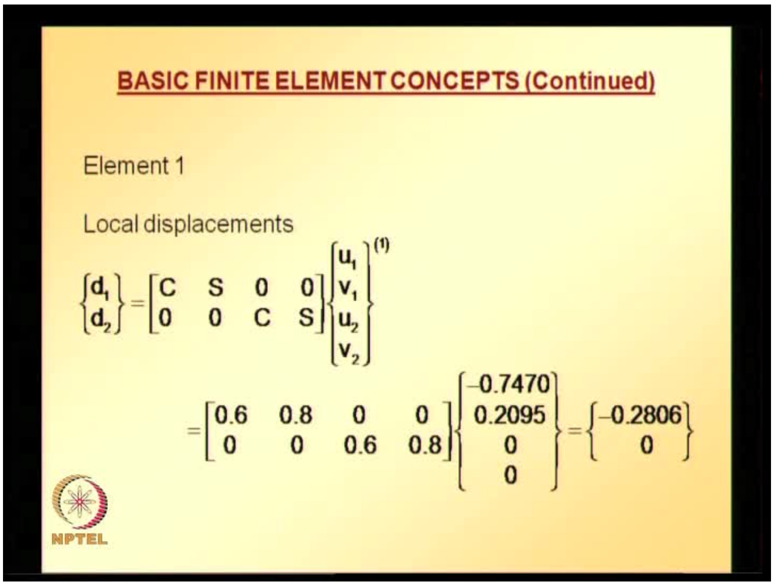


Similarly, whatever value is there at the location 4 4 in the local stiffness matrix; its goes into the 6 6 locations global equation system. So, based on this we can assemble the global equation system and here it is indicated. You can see here for the given problem node 2 and 3 are fixed. So, the corresponding degrees of freedom values will be equal to 0 and the only unknowns are u 1 v 1, which corresponds to the degrees of freedom for node 1 and wherever 0 value for the degrees of freedom is applied, reactions will be developed at those points so, since displacement degrees of freedom values at node 2, node 3 are 0, corresponding locations in the force vector reactions terms will be there.

So, they are indicated using R x2 R y2 R x3 R y3 and to solve this equation system what we can do is, we can actually ignore rows and column corresponding to 0 specified displacement. If the specified displacements are non-zero, then we need the columns corresponding to these specified degrees of freedom which are non-zero. But in this case, what happens is we can actually ignore the rows or delete the rows and columns corresponding to the degrees of freedom, for which the value is 0. So, we can eliminate the rows and columns 2 to 6 2 3 4 rows and 3 4 5 6 rows and columns can be deleted, deleting the rows 3 to 6, we get reduced equation system and solving this equation system. We can find what are the values of u 1 v 1. So, we obtained this equation system by deleting the rows and columns at the location 3 4 5 6, which corresponds to the global degrees of freedom of node 2 and 3. Since, the global degrees of freedom of node 2 and

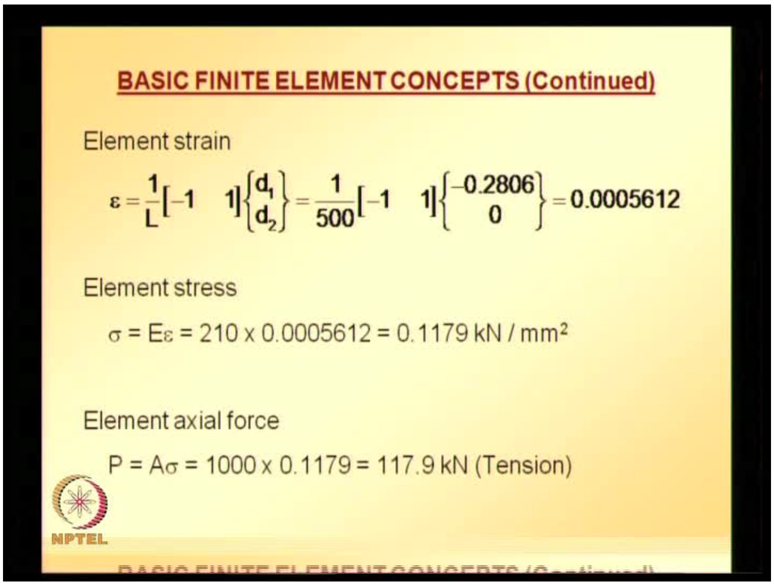
3 are 0, we can delete those the corresponding rows and columns and we obtained this equation system and solving this equation system we get u 1 v 1.

(Refer Slide Time: 29:10)



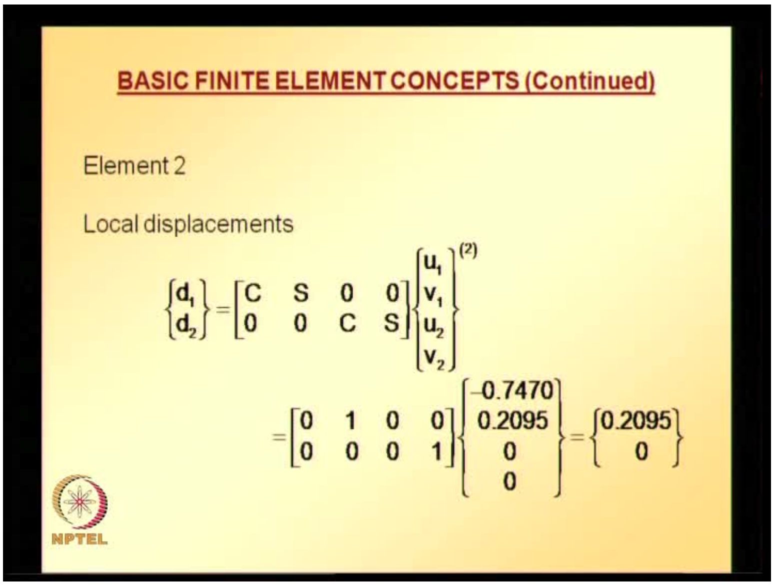
So, once we get u 1 v 1, we can go back to each element and we can calculate stresses, strains stresses, axial forces in that element, but before we do that we need to find, what are the corresponding local values? So, for element 1, local displacements using the global displacement that is, u 1 v 1 we just calculated and u 2 v 2 are 0.

(Refer Slide Time: 29:32)



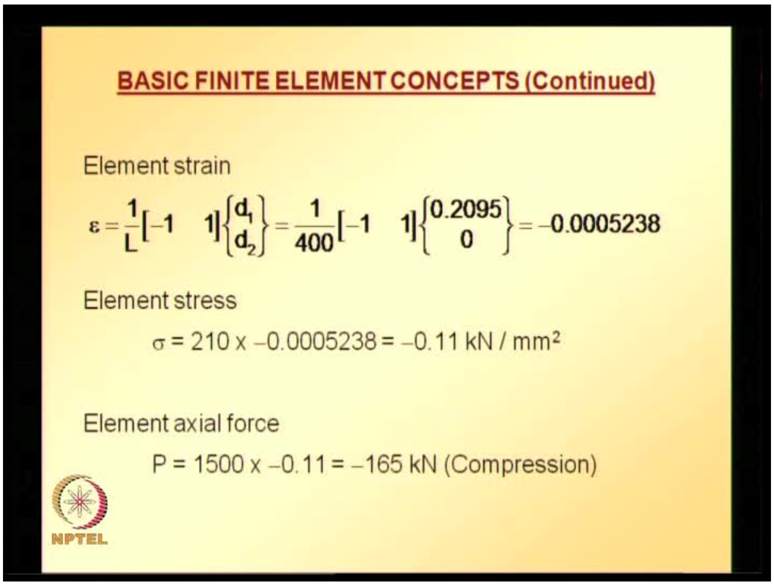
So, using these values and also for element 1, we know, what is cos alpha sine alpha? So, we can find what the local displacements and using the local displacements, we can calculate the element strains and element stress and axial force and axial force turns out to be positive.

(Refer Slide Time: 29:55)



So, it is tensile force. Similarly, for element 2, local displacements can be obtained using global displacements u 1 v 1; u 1 v 1 just we calculated and the values that we calculated using though they are nothing, but u 1 v 1; u 2 v 2 global displacements u 2 v 2 for element 2 corresponds to u 3 v 3 which are 0.

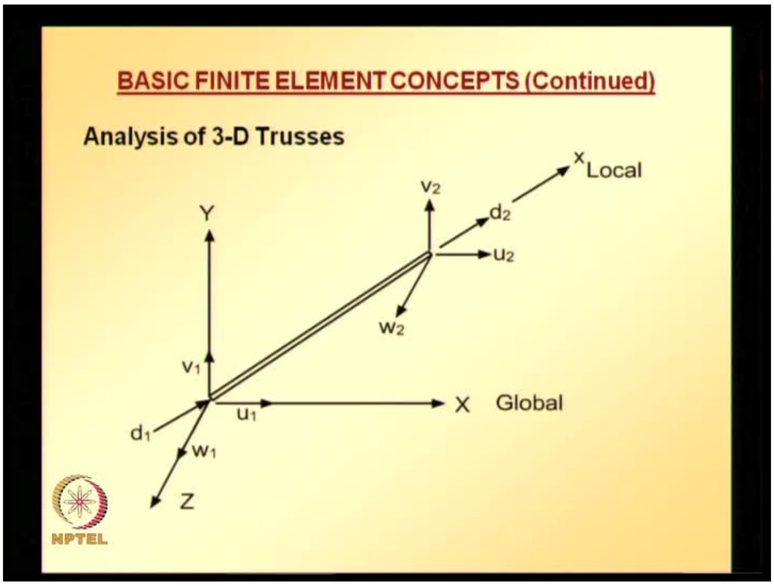
(Refer Slide Time: 30:31)



So, the local displacements can be obtained in this manner. Once we get local displacements, we can calculate strains, element strains, using this relation element stress and it turns out that element strain is negative quantity, that is, compressive and element stress also turns out to be negative and element axial force turns out to be negative. So, it is compressive force so, the member is in compression.

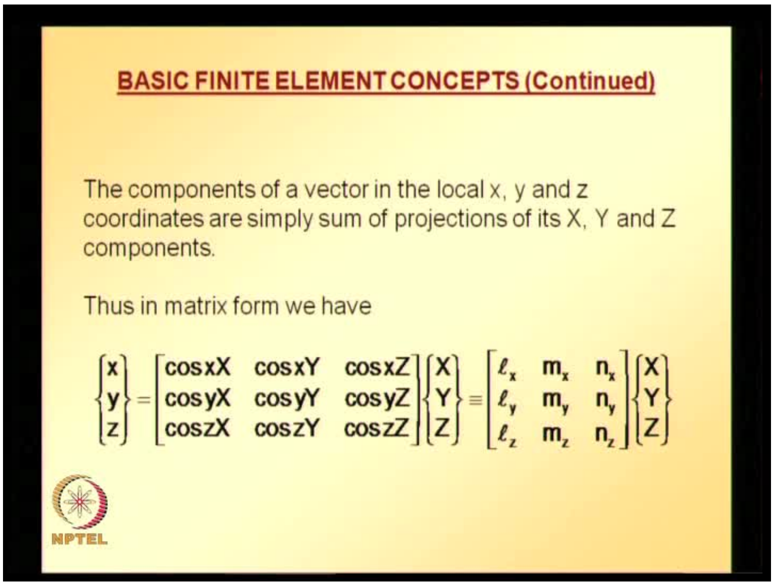
So, just now we have seen 2-D truss problems. How to solve 2-D truss problems? In a similar manner a truss having any number of elements can be analyzed. And now let us look an example, or first derive the equation system for a 3-D truss problem, 3-D space truss whereas, 2-D truss is a 2 plane truss; 3-D truss is going to be a space truss and 3 dimensional space truss element can be developed, using a coordinate transformation and using local elemental equation for axial deformation element in the manner, which we did for 2-D plane truss element.

(Refer Slide Time: 31:50)



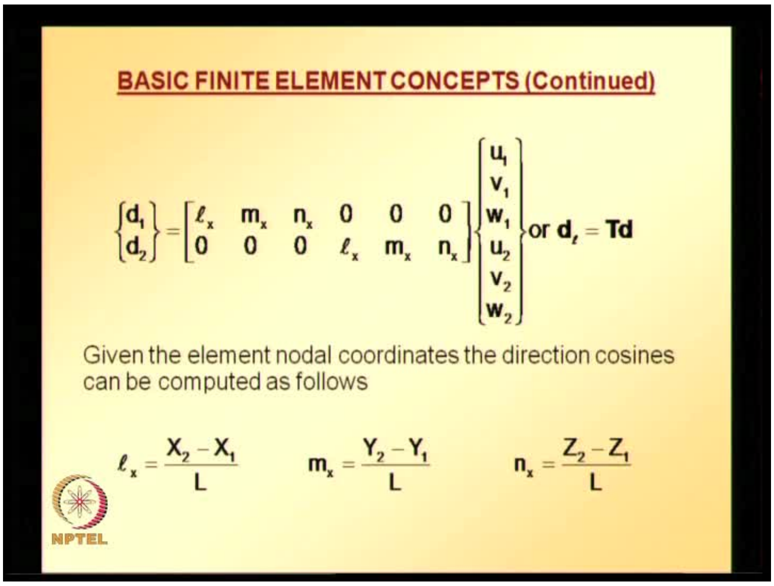
Here a 3-D space truss is shown and the local degrees of freedom are indicated or identified as u 1 v 1 u 2 v 2 and the global x axis, y axis, z axis are shown there and the local displacement at node 1, node 2, d 1, d 2 are also shown. Similarly, the global degrees of freedom at node 2, u 2 v 2z w 2 are shown there and the local x axis is oriented along the centroidal axis of the space element. So, the local displacement vector is local displacement vector consists of 2 components: d 1, d 2. And the global displacement vector consists of 6 components: u 1 v 1 w 1 u 2 v 2 w 2.

(Refer Slide Time: 33:17)



Similarly, global local force vector consists of 2 components: capital P 1 capital P 2 and global force vector consists of 6 components: F 1x F 1y F 1z F 2x F 2y F 2z. So, the notation that we are following is similar to what we used for 2-D plane truss and what we require the important thing is transformation matrix. The components of the vector in the local x, y, z coordinate system are simply projections of its capital X, capital Y, capital Z components that is the global components. So, the relation between the local spatial coordinate system to the global spatial coordinate system is given by this equation, which is just extension of what we have seen for 2-D plane truss problems. You have one more additional dimension. So, the corresponding components appear in the transformation matrix and the coordinate system vectors. The cos cosine of small x capital X is denoted using l x.

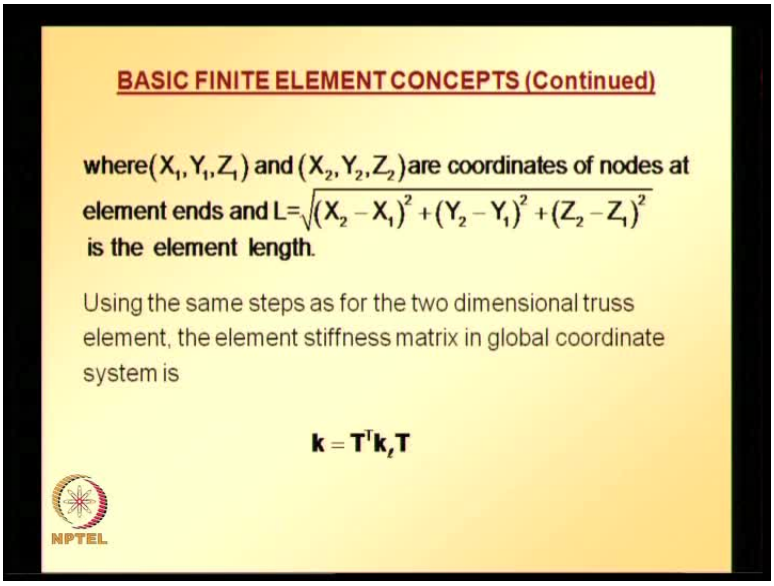
(Refer Slide Time: 34:32)



Similarly, other components are denoted using l x m x n x; l y m y n y; l z m z n z, these are nothing but direction cosines and similar to 2-D plane truss element, the displacement components or displacement in the x direction transforms in the same manner as a spatial coordinate in the x direction. So, d 1 is given by l x times u 1 plus m x times; v 1 plus n x times w 1. Similarly, d 2 is given by l x times u 2 plus m x times v 2 plus n x times w 2 these two equations can be written in a matrix and vector form in these manner, d 1, d 2 is equal to l x m x n x zero 0 0, 0 0 0 l x m x n x, u 1 v 1 w 1 u 2 v 2 w 2 and this can be compactly written as d l is equal to T times d where, d l is local displacement vector T is transformation matrix.

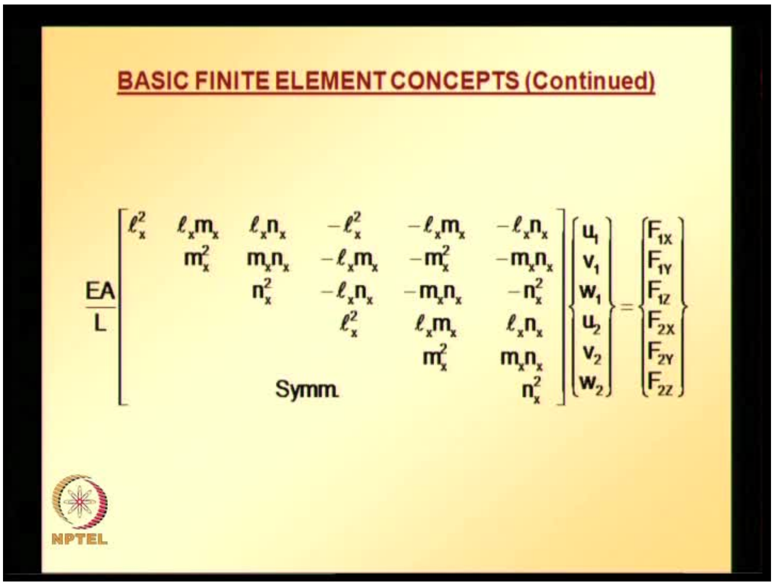
And d is global displacement vector and transformation matrix as you can see here, it is different from what we have seen for 2-D plane truss element and this transformation matrix can also be verified to be orthogonal matrix that is, T transpose T turns out to be identity matrix or T transpose is same as T inverse. So, the inverse relation that is d is equal to T transpose d l.

(Refer Slide Time: 36:23)



Given the element nodal coordinates, the direction cosines can be computed. Once, a special element is given once we know, the spatial coordinates of ends of that element we can calculate l x m x n x using these relations where, l is nothing but length of that element. Length of the element can be obtained, using the coordinates of the ends of the element using the relation given here. And the relation between the element stiffness matrix in the global coordinate system to the local coordinate system, the equation look similar to what we derived for 2-D plane truss element. Except that transformation matrix T is different. Here now, it is l x m x n x 0 0 0 0 0 0 l x m x n x and k l is same as what we used for 2-D plane truss element, because local stiffness matrix is same, that is EA over L 1 minus 1 minus 1 1.

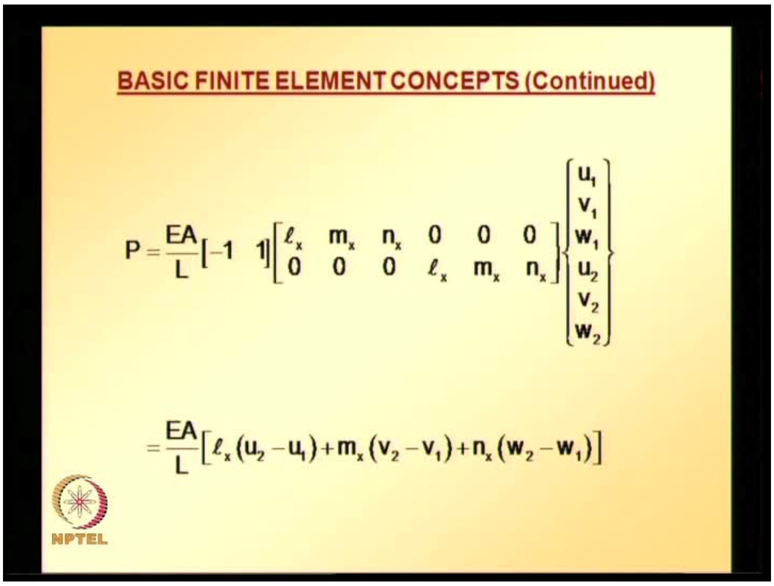
(Refer Slide Time: 37:25)



So, using these transformation matrix and local element stiffness matrix, we can get the global equation element equations for a 3-D truss element in this manner and it is easy to remember or memorize this equation system without much effort. If we remember, 1 quadrant of this, the components in 1 quadrant of this stiffness matrix, we can write the other the components in the other coordinates quadrants, that is whatever elements you have in the first quadrant, the components in the fourth quadrant of this stiffness matrix are same. First quadrant and first coordinate quadrant are same and whatever, components are there in the second quadrant and third quadrant, they are nothing but with a negative sign appended to whatever components are there in the first quadrant.

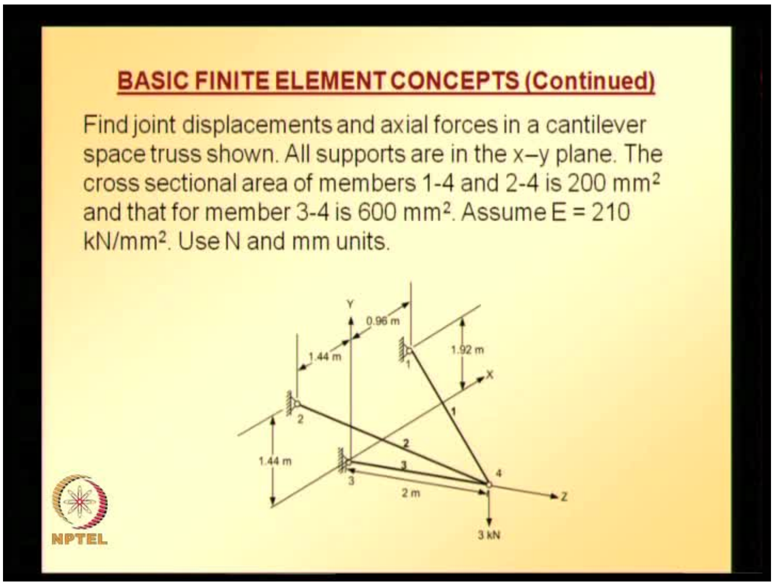
So, in that manner we can easily memorize this stiffness matrix for 3-D truss element and EA over L is just they are material in geometric parameters and the displacement vector consists of u 1 v 1 w 1; u 2 v 2 w 2 and similarly, force vector consists of 6 components. So, once element equations are written in terms of global coordinates, the assembly and solution process proceeds in a same manner as, we have seen for 2-D truss element. Axial forces can be computed by transforming displacements, back to the local element coordinate system.

(Refer Slide Time: 39:19)



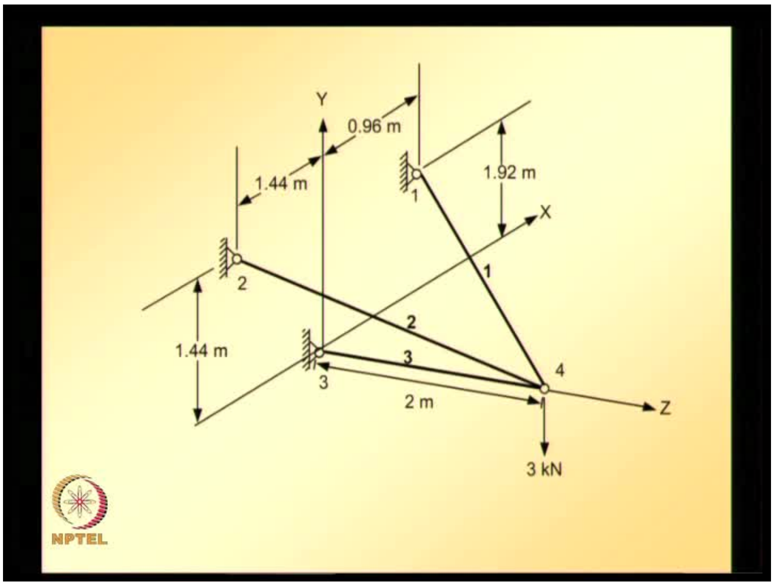
So, once we get the global displacements, we can calculate local displacements; once we get local displacements, we can calculate strains and then stresses and using those values, we can calculate the forces in a truss element 3-D truss element. The equation is given here and this equation in terms of direction cosines after manipulating the first equation is shown there.

(Refer Slide Time: 39:58)



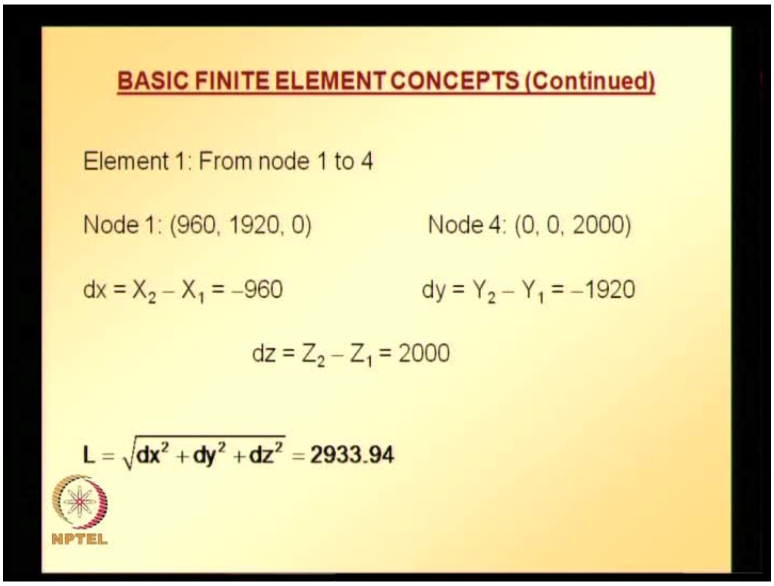
So, to understand these concepts, let us solve a 3-D truss problem. Find joint displacement and axial forces in a cantilever space truss shown. All supports are in x, y plane. The coordinate system global x, global y, global z coordinate system is also indicated in the figure. The cross sectional areas of member 1-4 2-4 are 200 millimeter square and that for member 3- 4 is 600 millimeter square. Young's modulus is 210 GPa or kilo Newton per mm square and because Newton per mm square is MPa, kilo newton per mm square is GPa all units are in SI units. So, looking at this the various dimensions that are given here for this 3-D, a space truss we can easily identify, what are the x, y, z coordinates, for each of the nodes 1, 2, 3, 4.

(Refer Slide Time: 41:11)



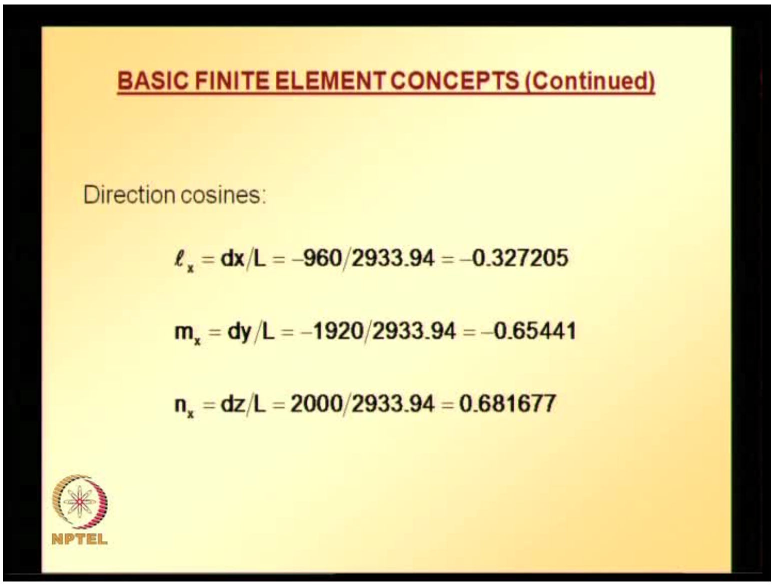
And now again here for simplicity, the element equations are directly written in terms of global degrees of freedom while, taking into consideration, the boundary conditions the unknown reactions at ith node corresponding to 0 displacements are identified as R xi, R yi, R zi i takes values 1 2 3.

(Refer Slide Time: 41:50)



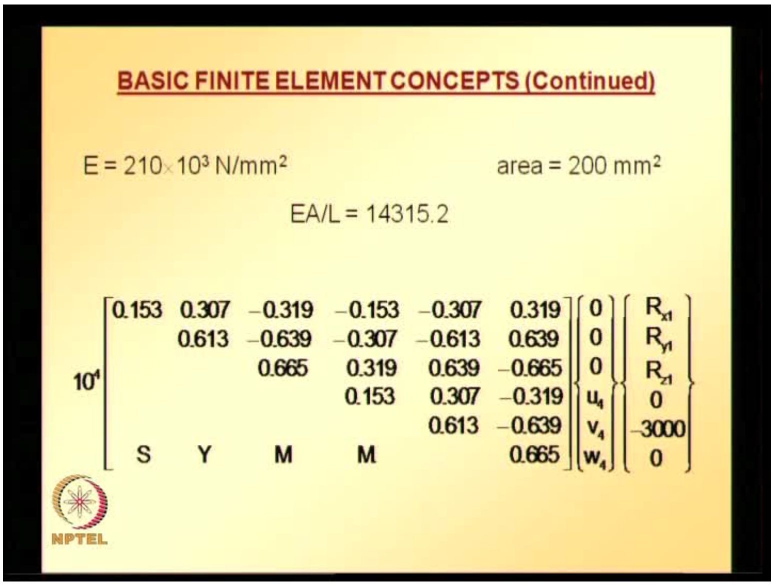
It is similar, what we did for 2-D plane truss elements. Now for element 1: element 1 goes from node 1 to node 4 that is, local node for this local node 1; node 1 for this element is global node 1; local node 2 for this element is global node 4. So, using the nodal values all are given all the units are here in millimeters.

(Refer Slide Time: 42:44)

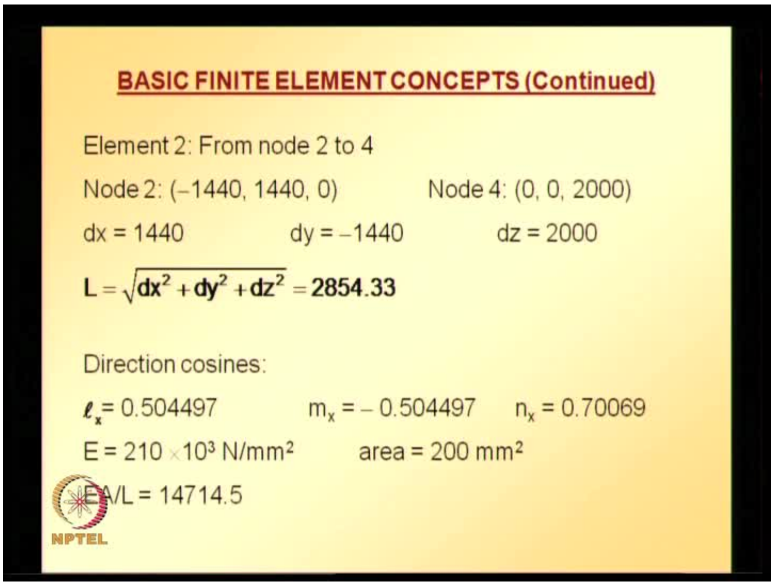


So, the coordinates are given there for node 1, node 2 for element 1 and using these values we can easily calculate, what are the direction cosines, using the values difference between the spatial coordinates dx dy dz and since we know, the coordinates of end points. We can easily calculate, what is the length of this element? And we can calculate, what are the direction cosines, l x m x n x and material properties cross sectional areas and a length we just calculated.

(Refer Slide Time: 43:02)

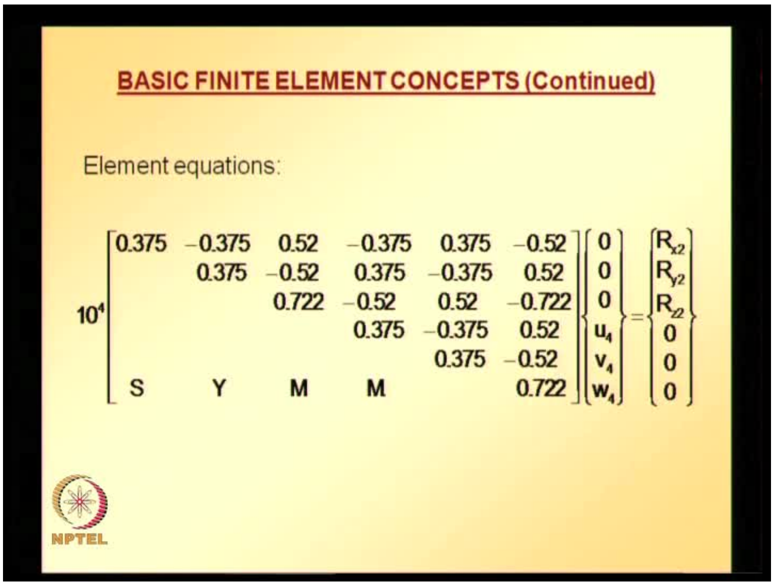


(Refer Slide Time: 43:30)



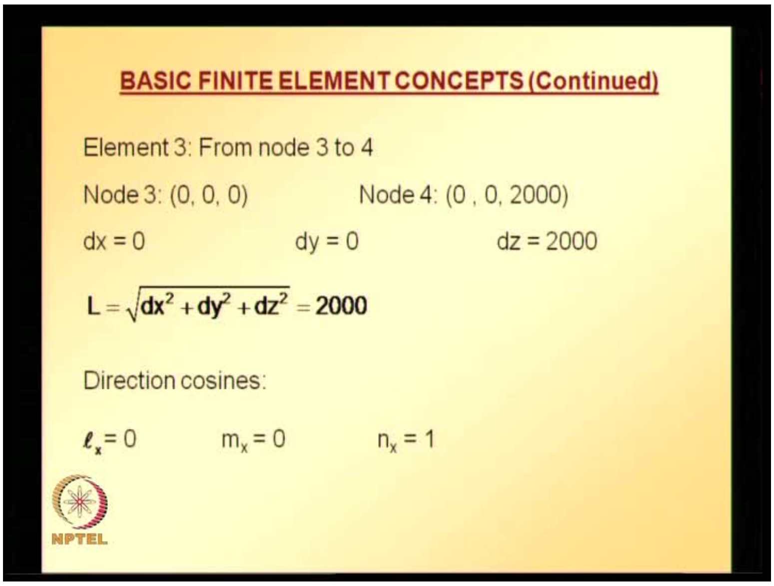
So, using those values we can easily write, the element equations for this particular element, using the formula that we just developed for 3-D space truss element, similar process can be repeated for element 2 and element 3; element 2 local node 1 is nothing, but global node 2; local node 2 is global node 4; so, using this information again all units are in millimeters. We can calculate, using the difference between the spatial coordinates we can calculate dx dy dz and length of the element can also be calculated, direction cosines and using material properties and geometrical properties, we can calculate, elemental equations for element 2.

(Refer Slide Time: 44:08)



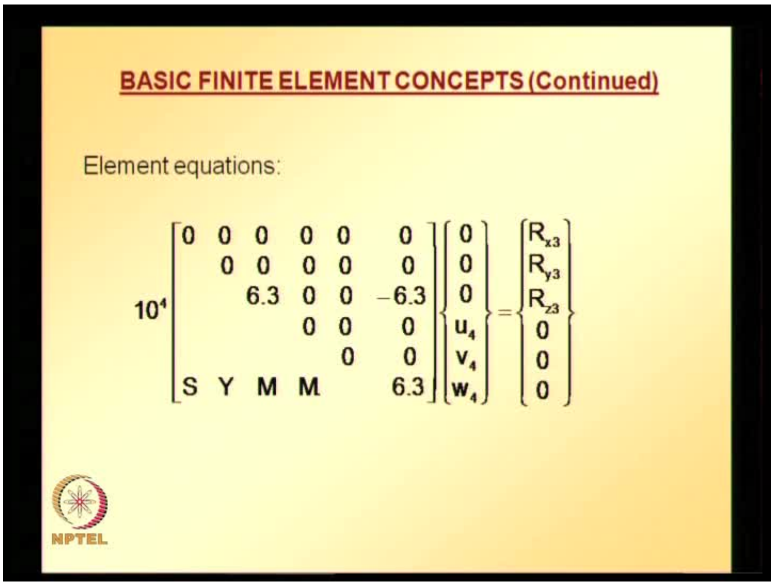
As already mention, since node 1, 2 and 3 are fixed the corresponding degrees of freedom are 0 and the corresponding locations in the force vector reactions will be at the corresponding locations reactions will be developed. So, those are indicated using R x2 R y2 R z2; similarly, in the earlier equation we have, R x1 R y1 R z 1.

(Refer Slide Time: 44:51)



Similarly, we can assemble for element 3, the equation system it goes from node 3 to 4. So, local node 1 is 3; global node 3; local node 2 is global node 4 and we can calculate, dx dy dz and length and we can calculate direction cosines and we have the material properties again geometrical properties. Using these we can calculate, what are the element equations for this particular element and now, we have the elemental equation for element 1 2 3.

(Refer Slide Time: 45:28)

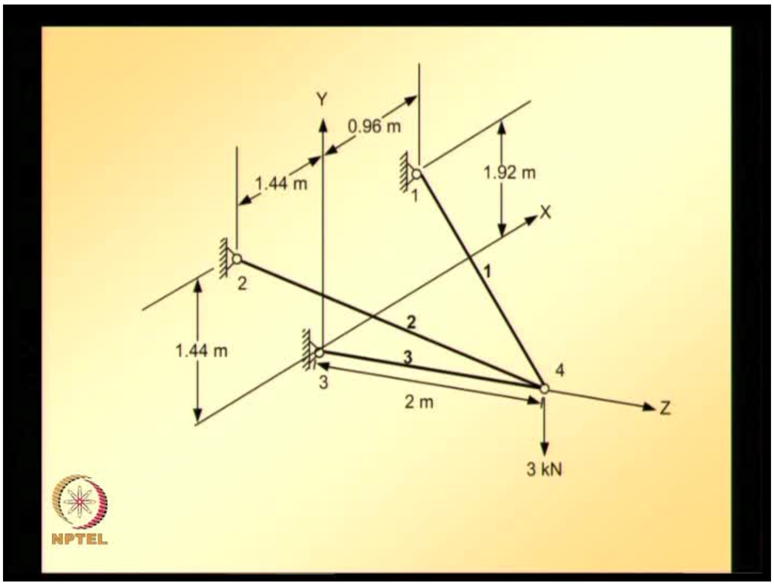


And now we are ready to assemble the global equation system, but since at each node, we have 3 degrees of freedom and there are 4 nodes. So, global equation system will be 3 times 4 that is, 12 by 12 and it is very cumbersome to write a big 12 by 12 equation system instead of that with a reasoning that since, node 1, 2, 3 are fixed and the corresponding degrees of freedom are 0 and we have seen from the 2-D truss problem.

In the global equation system and the finally, when we are solving for the unknown degrees of freedom, what we will be doing is; we will be eliminating the rows and columns corresponding to the degrees of freedom at which 0 value is applied or 0 value is specified. So finally, in the global equation system to solve for the unknown degrees of freedom, we need to eliminate those rows and columns corresponding to the degrees of freedom of which the value is 0.

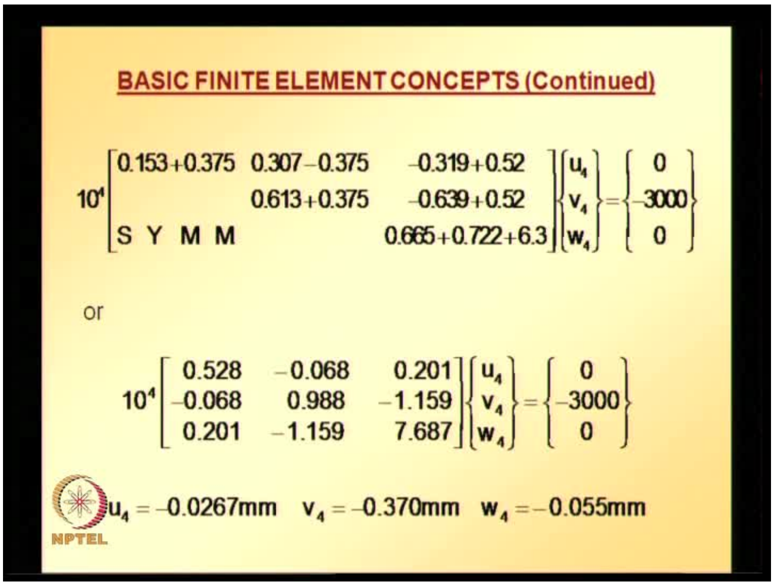
So anyway, in the global equation system, we will be eliminating the rows and columns corresponding to nodes 1, 2, 3 which are nothing but for the node numbering that we have given these corresponds to the location 1 to 9, that is 1 to 9 rows and columns will be eliminated in the 12 by 12 equation system. So, whatever we will be left with in the reduced equation system, we will be at the locations 13, 14, 15.

(Refer Slide Time: 41:11)



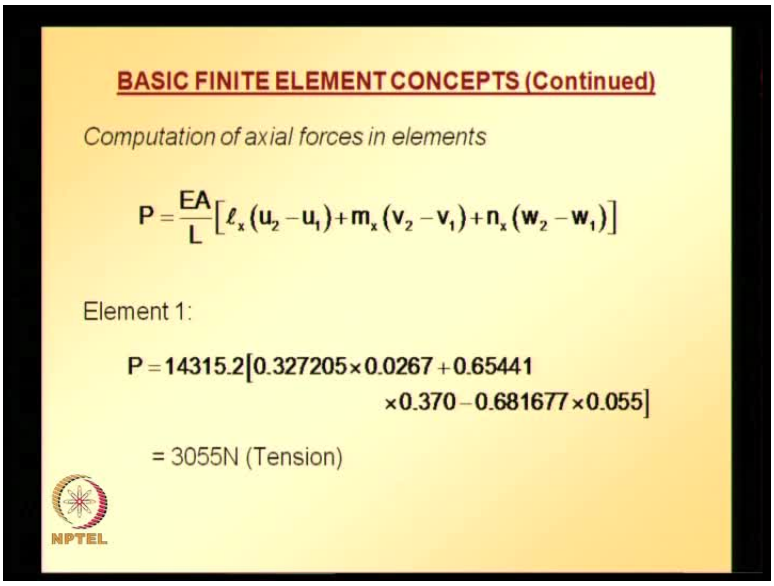
So, we can directly write the global equations corresponding, these degrees of freedom. So, this is what I just mentioned nodes 1, 2, 3 are fixed so, the corresponding degrees of freedom was 0. So, we can directly write the reduced equation system, which corresponds to the global equation system locations 30 10 11 12 locations.

(Refer Slide Time: 47:55)



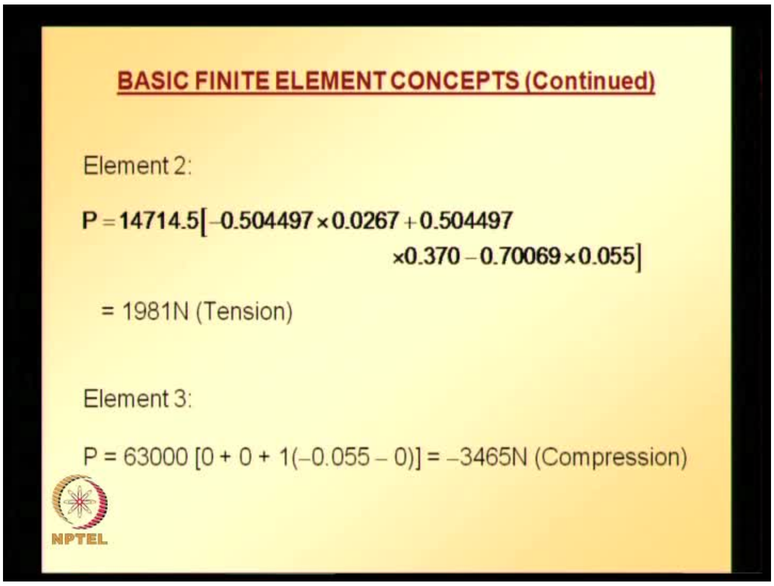
So, this is the reduced equation system, which is obtained by summing up the contribution from element 1, 2, 3 at the locations in their lower quadrant of these element equations. The contribution is taken from there and once we add up, we get this reduced equation system. We can solve this equation system for the unknown degrees of freedom which are u 4 v 4 w 4, this is a 3 by 3 equation system. So, those unknown u 4 v 4 w 4 can be easily solved and once we solved for u 4 v 4 w 4, we are ready to calculate the elemental forces by calculating strains stresses.

(Refer Slide Time: 49:02)



So, computation of axial forces can be calculated using this equation. For element 1: applying this equation and noting that for element 1, local node 1 is same as global node 1 and local node 2 is same as global node 4 and with that understanding, the axial force in element 1 is given by this and it turns out this element is in tension.

(Refer Slide Time: 49:37)



Similarly, element 2: also it turns out to be in tension. Element 3: it turns out that this element is in compression. So, in this lecture, we have seen how to solve 2-D truss problems and 3-D truss problems and one of the important thing is, assembling of global equation system or we can also obtain directly the reduced equation system by calculating or doing all the calculations in brain, that is, directly writing the reduced equation system by eliminating the rows and columns corresponding to the degrees of freedom. In the global equation system at which the degrees of freedom are specifies to be 0.

So, that is what we adopted in the second example, which is a 3-D truss problem. So, in the next lecture, we will be seeing, how to solve the truss problems for temperature changes stresses due to temperature changes and lack of it.

Thank you.