Finite Element Analysis
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Lecture No. # 40

In the last class, we looked at derivation of finite element equations for four node tetrahedral elements is a part of three-dimensional elasticity, and as a part of that we looked at governing differential equations for three-dimensional elasticity problems. Also we looked at in detailed, the finite element equations for four node linear element for three-dimensional elasticity problems. In today's class, let us look at eight node solid element which is also known as brick element, and also twenty node solid element with curved sides for solving three-dimensional elasticity problems. At the end we will also look at thermal prestress prestrain effects, how to consider all these into account when we are solving three-dimensional elasticity problems, in fact any elasticity problem. And let us let me, briefly review what we have done in the last class, before we proceed with solving or before we proceed with formulating finite element equations for eight node solid element.

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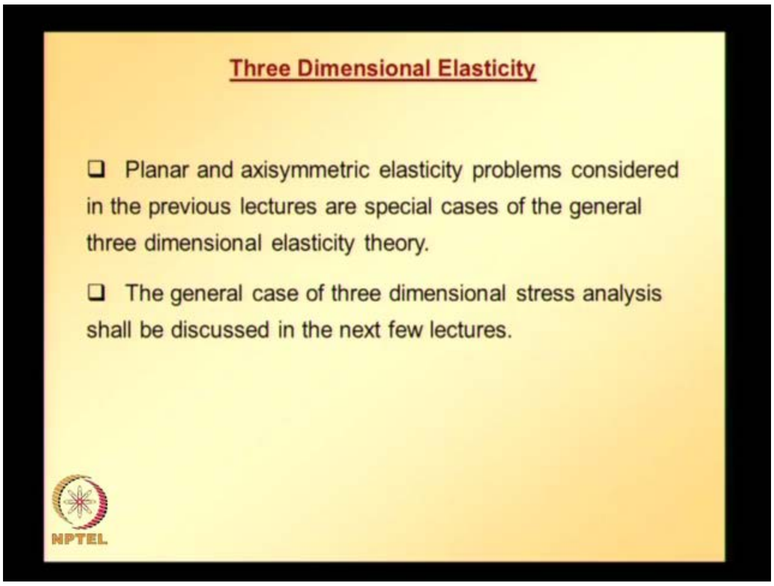

So, planar and axisymmetric elasticity problems that consider in the earlier lectures are special case of general three-dimensional elasticity theory. And general the general case of three-dimensional stress analysis shall be discussed in the next few lectures.

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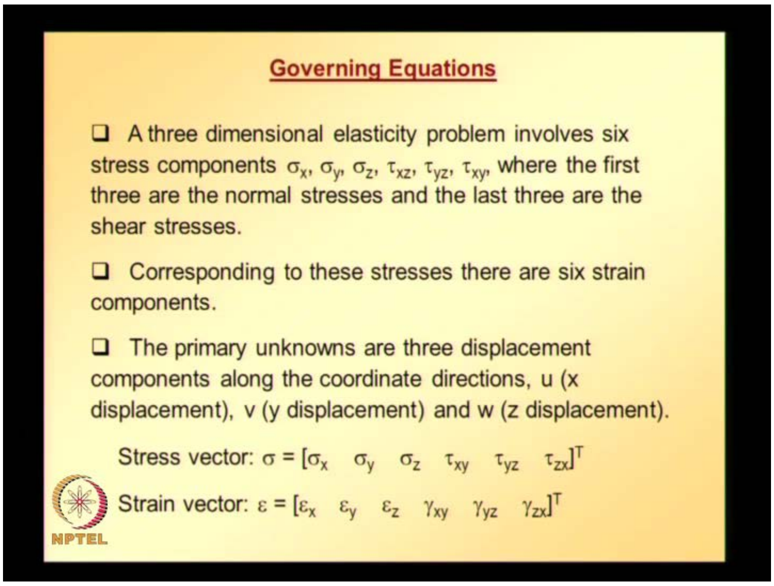

That is what we discussed in the last class. Three dimensional elasticity problems involve six stress components also six strain components. What are this six stress components? The first three components are normal stress components; the last three are the shear stress components. The primary unknowns are three displacements along x, y, z directions and the stress vector looks like this. The strain vector looks like this.

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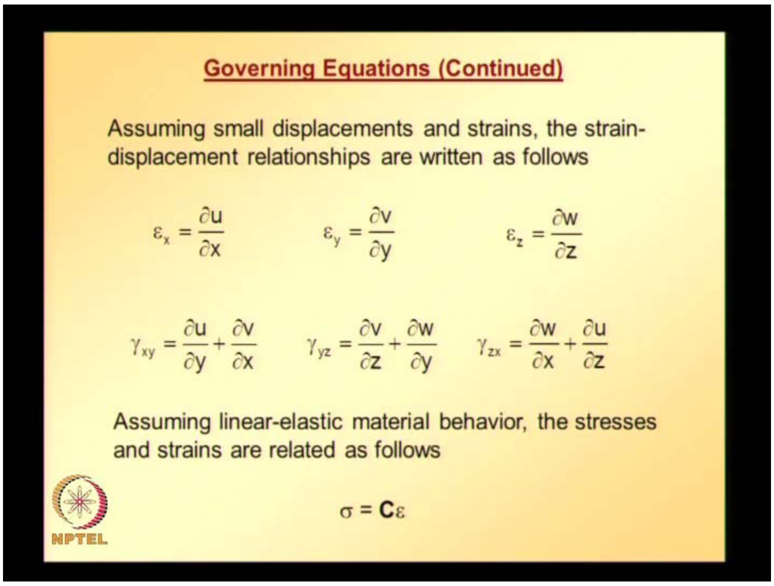

Assuming, small strain displacement relationship can be written as follows. Assuming linear elastic material behavior stresses and strains are related through this equation, where, C is the constitutive matrix for isotropic material.

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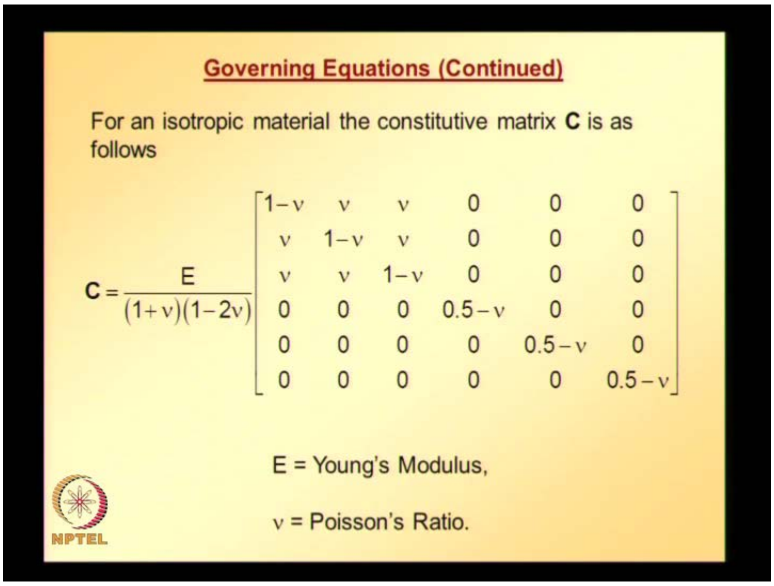

Isotropic material the constitutive matrix C is as follows, where E and v are Youngs modulus and Poissons ratio. Then, before we actually derive finite element equations for any kind of element we need to know what is potential energy functional.

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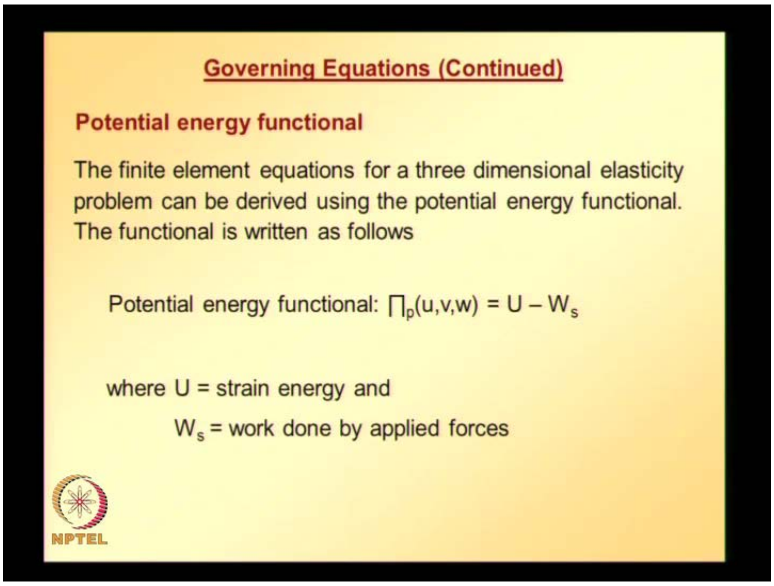

In the last class, we have seen potential energy functional for three dimensional elasticity problems looks like this, where u is strain energy w s is work done by the applied forces and also we looked in detail how to calculate the strain energy.

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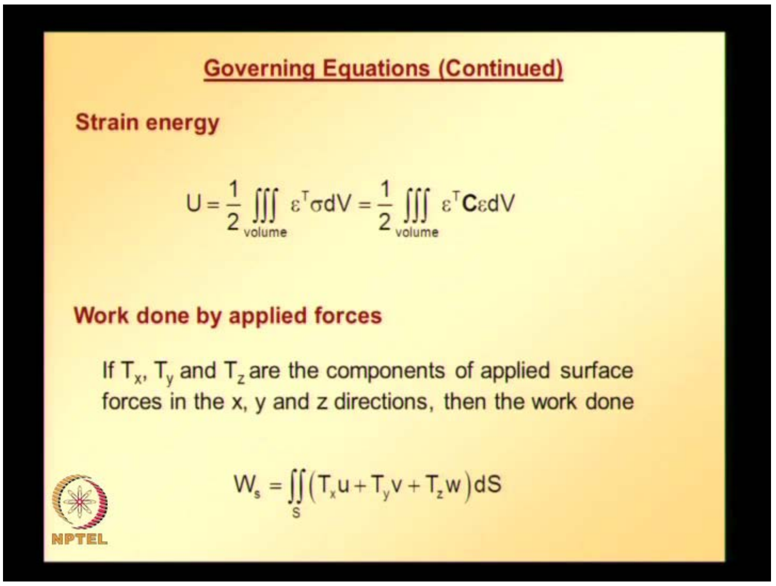

Evaluating, this integral which is going to be volume integral and work done by the applied forces is given by the traction components multiplied by a displacements, along that particular direction integrated over the surface on which traction is applied. If specified concentrated forces or body forces are present. Work done by the corresponding forces can also be computed in the similar manner. This is what, we have seen in the last class. Also last class we have seen how to calculate principle stresses once we solve for the displacements, find strains and stresses at the points that we are interested. Later using the six stress components, we can calculate principles stresses for subsequent use in the failure criteria.

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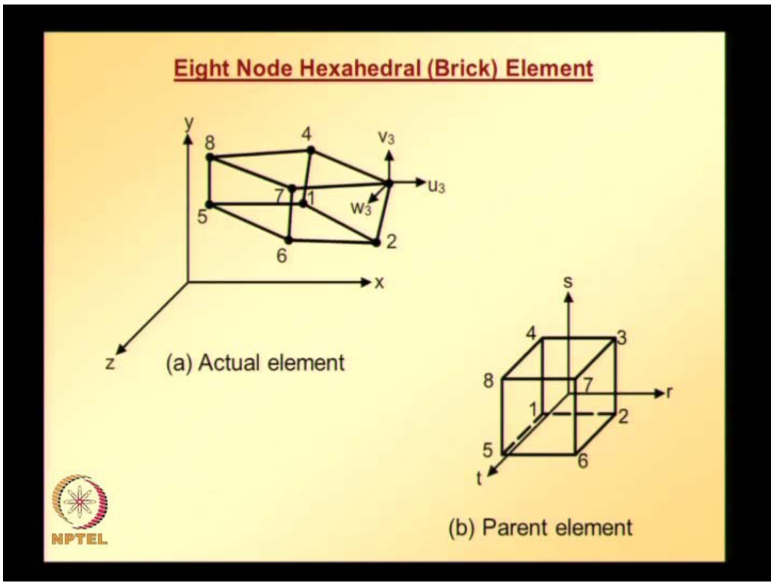

With that, let us start deriving finite element equations for eight node hexahedral or brick element. This is an eight node hexahedral element and similar to that we already did for four node tetrahedral element. The element equations can easily be derived by using isoparametric mapping concept and this actual element we are going to map it onto a parent element, which is going to be a cube having dimension 2 by 2 by 2. That is, x axis goes from minus 1 to 1, t axis goes from minus 1 to 1, r axis goes from minus 1 to 1. Parent element looks like this. Origin is located at the center, that is the center of this cube is located at s is equal to 0 or r is equal to 0, s is equal to 0, t is equal to 0. With that understanding three dimensions of this cube 2 by 2 by 2. With that understanding with the location of origin that is, at the center we can easily find what are the coordinates of various nodes.

For example, node 1 is located at r is equal to minus 1, s is equal to minus 1, t is equal to minus 1.Similarly, node 2 is located at r is equal to 1, s is equal to minus 1, t is equal to minus 1, node 3 is located at r is equal to 1, t is equal to 1, r is equal to 1, s is equal to 1, t is equal to 1 and node 4 is located at r is equal to minus 1, s is equal to 1, t is equal to minus 1. Node 5 is located at r is equal to minus 1, s is equal to minus 1, t is equal to 1. Node 6 is located at r is equal to 1, s is equal to minus 1, t is equal to 1. Node 7 is located at r, s and t is equal to 1. Node 8 is located at r is equal to minus 1, s and t equal to 1.

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Eight Node Hexahedral Element (Continued)

Node	r_i	s_i	t_i
1	–1	–1	–1
2	1	–1	–1
3	1	1	–1
4	–1	1	–1
5	–1	–1	1
6	1	–1	1
7	1	1	1
8	–1	1	1

With that understanding, we can make a note of the nodes in the parent element and their location as given here. Similar to four node tetrahedral elements are for that matter, the elements that we look for planar and axisymmetric elasticity problems. The trial solutions are written in terms of finite element shape functions for parent element.

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Eight Node Hexahedral Element (Continued)

Trial solution

The trial solutions are written in terms shape functions for the parent element. Thus

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots & N_8 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots & N_8 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots & N_8 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \vdots \\ u_8 \\ v_8 \\ w_8 \end{Bmatrix} = \mathbf{N}^T\mathbf{d}$$

Trial solutions u, v, w displacement component along x direction, displacement component along y direction, displacement component along z direction can be written in terms of finite element shape functions at all. The eight nodes of this eight node hexahedral element or eight node brick element the matrix consisting of finite element shape functions is denoted with letter N. All the displacement components are put together in a vector d. Trial solution can be written as N transpose d, the shape functions for this eight node hexahedral or eight node brick element can easily be obtained using Lagrange Interpolation formula. That we already looked at when we are deriving shape functions for two dimensional elements. Only difference is going to be, we need to apply Lagrange Interpolation formula in three dimensions. Using Lagrange Interpolation formula in three dimensions the shape functions for the parent element which are required can be obtained.

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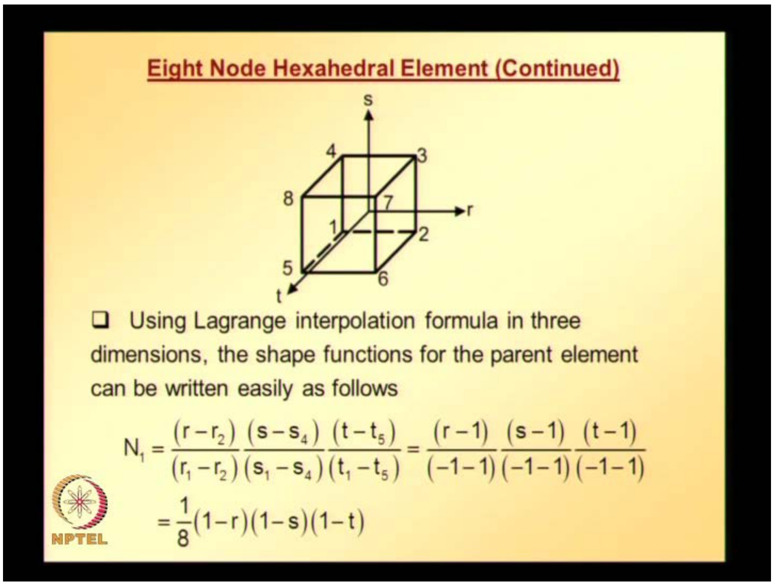

This is the parent element applying Lagrange Interpolation formula. We can write shape function corresponding to node 1. In this manner, N 1 is equal to r minus r 2, s minus s 4, t minus t 5 divided by r 1 minus r 2 times s 1 minus s 4 times t minus t 5. By substituting the nodal coordinates r 1, r 2, s 1, s 4, t 1, t 5, we get shape function corresponding to node 1 as 1 over eight, 1 minus r, 1 minus s, 1 minus t. It is just application of Lagrange Interpolation formula in three dimensions.

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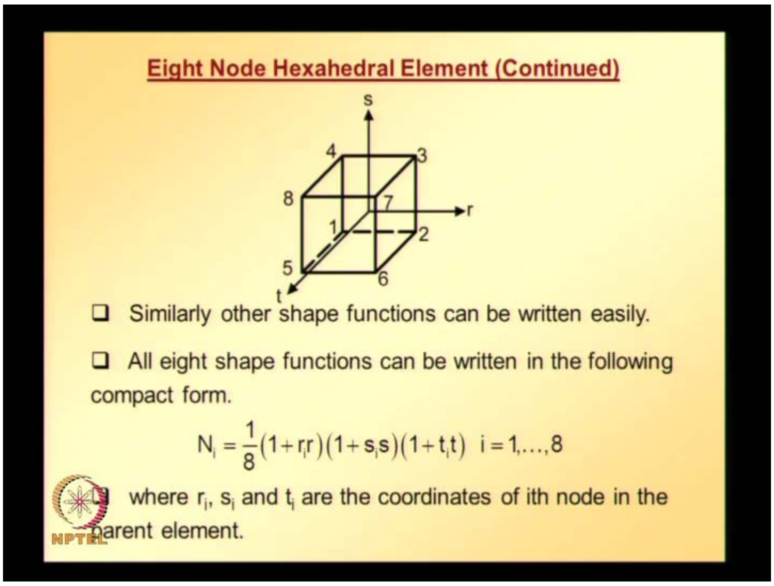

Similarly, shape functions for other nodes can be written, we can write all the eight shape functions to get that in a compact form in this manner, where, N i is shape function corresponding to eighth node, r i is the nodal coordinate r i, s i, t i are the nodal coordinates of that particular node in the parent element coordinate system, where, r i, s i, t i are the coordinates of eighth node in the parent element. Once, we know the shape function expressions for all the eight nodes, we can easily write the trial solutions in terms finite element shape functions.

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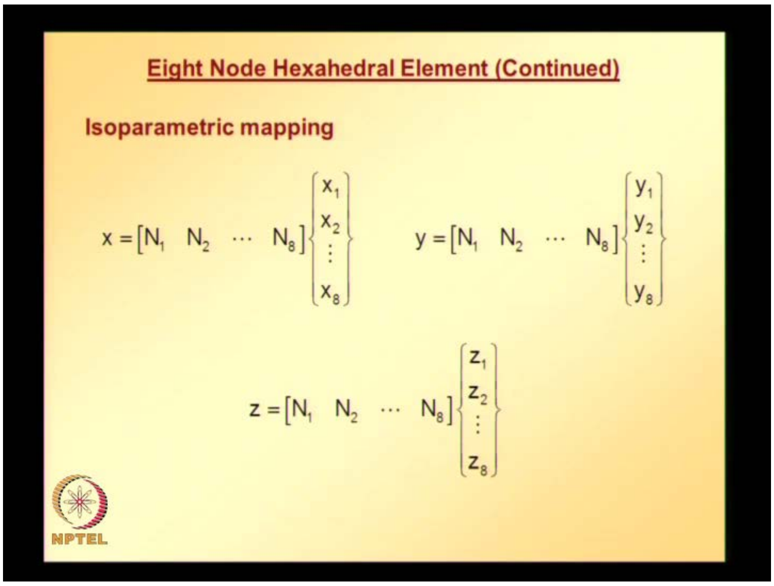

Next is Isoparametric mapping, this is similar to the earlier elements that we have seen. Except, there are eight nodes, eight shape functions in the corresponding x coordinates of all the eight nodes, x is equal to N 1 x 1 plus N 2 x 2 plus N 3 x 3 and so on. N 8, x 8 all this can put together in a matrix and vector form in this manner. Similarly, y can be written in terms of finite element shape functions and the nodal coordinates of all the nodal y coordinates of all eight nodes. Similar, z can be written as finite element shape functions and z coordinate of all the eight nodes. Once, we have this we can easily derive since, all the shape functions are functions of r s t, we can easily find what is partial derivative of x with respect to r s t, similarly y with respect to r s t and z with respect to r s t for subsequent calculations for strains.

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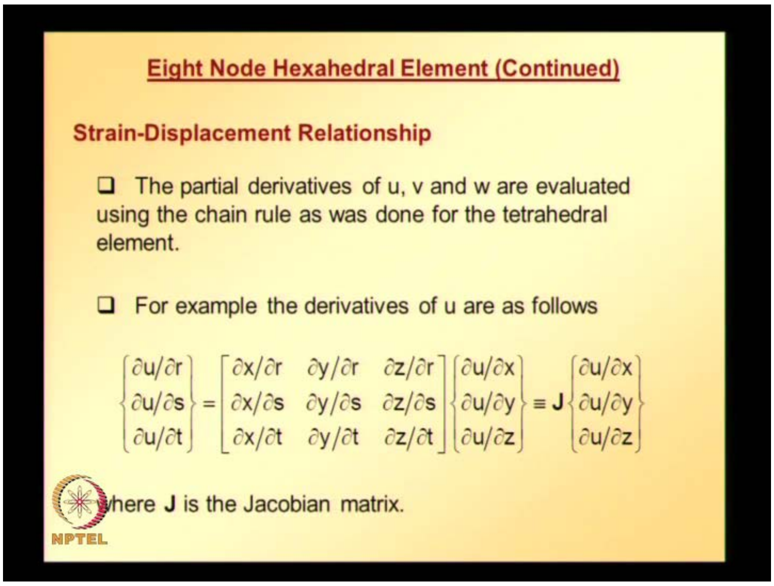

Strain displacement relationship, the partial derivatives of u, v, w are evaluated using chain rule, as was done for four node tetrahedral element. For example, derivatives of u are written as follows, this equation gives partial derivatives of displacement component, t along x direction with respect to r s t and partial derivatives of displacement along x direction with respect to x y z. This is where we required finding partial derivatives of x with respect to r s t, y with respect to r s t, z with respect to r s t. Once, we know partial derivatives of x with respect to r s t, y with respect to r s t and z with respect to r s t, we can easily find what is J, which is Jacobian matrix.

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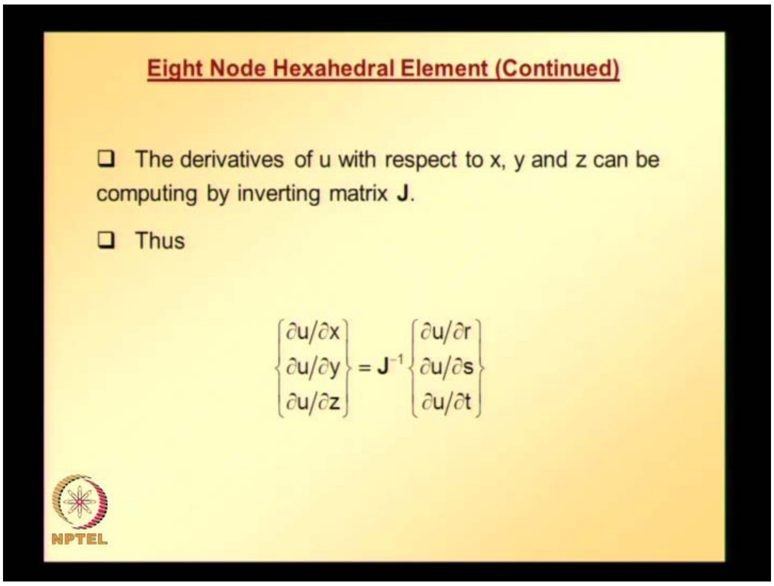

This is the inverse relation derivatives of u that is, displacement component along x direction with respect to x, y, z can be computed by inverting matrix J. This gives the inverse relationship of the previous equation. Here, substituting partial derivatives of u with respect to r s t, we know u in terms of finite element shape functions. Since, finite element shape functions are in terms of r s t, we can easily take derivatives of finite element shape functions with respect to r s t. We can easily find what is partial derivative of u with respect to r s t in terms of finite element shape functions.

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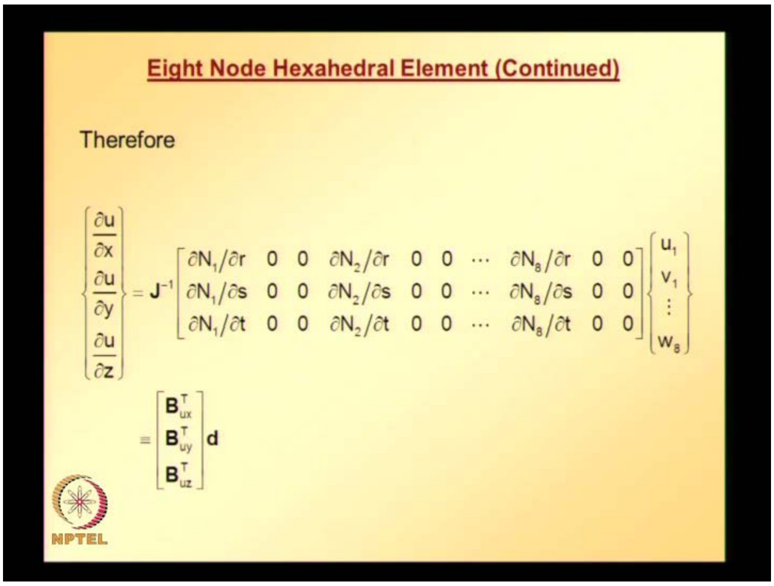

Substituting all that information we get this, which can be compactly written using B u x, B u y, B u z. This gives partial derivatives of displacement along x direction with respect to x, y, z. Similar relations can also be developed for partial derivatives of displacement component along y direction, z direction with respect to x,y,z.

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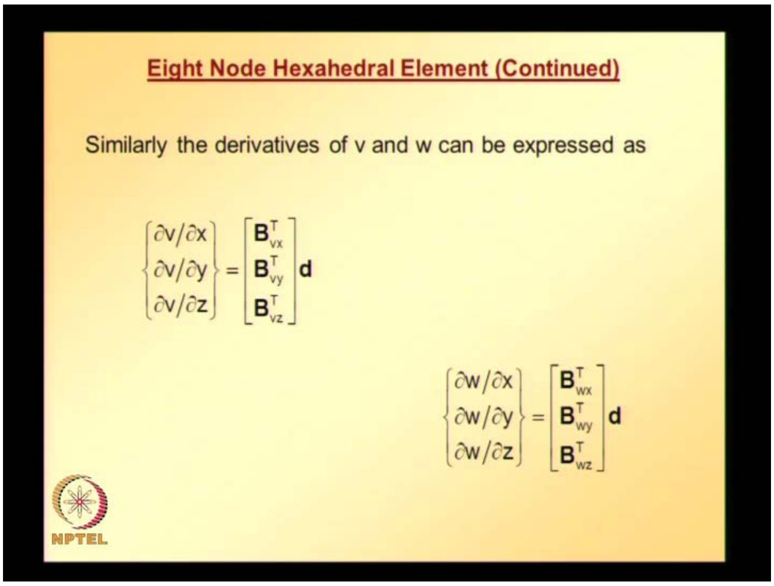

These two equations gives derivatives of v with respect to x,y,z, w with respect to x,y,z, which is compactly written in terms of B v x, B v y, B v z, B w x, B w y, B w z. Now obtained all the quantities that are required for calculation of strains. Strains requires derivatives of displacement components with respect to x, y, z. Using the definition of strain vector, the strains can now be expressed in terms of nodal displacements by choosing appropriate rows from the above matrices.

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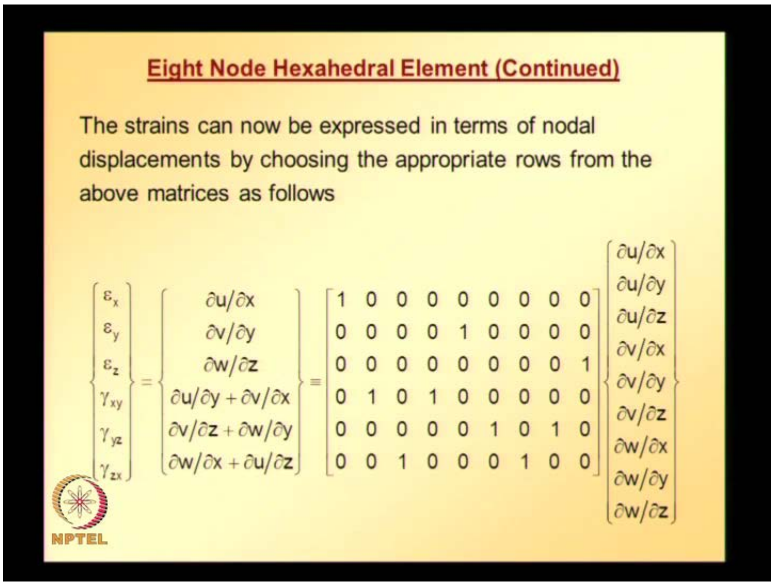

First, definition of strain epsilon x is equal to partial derivative of u with respect x, epsilon y is equal to partial derivative of v with respect to y, epsilon z is equal to partial derivative of w with respect to z. Similarly, gamma x y partial derivative of u with respect to y plus partial derivative of v with respect x, gamma y z partial derivative of v with respect to z, partial derivative of w with respect to y, gamma z x partial derivative of w with respect x, partial derivative of u with respect to z, this entire vector can be written or rearranged in the manner that is shown on the right hand side of the equation.

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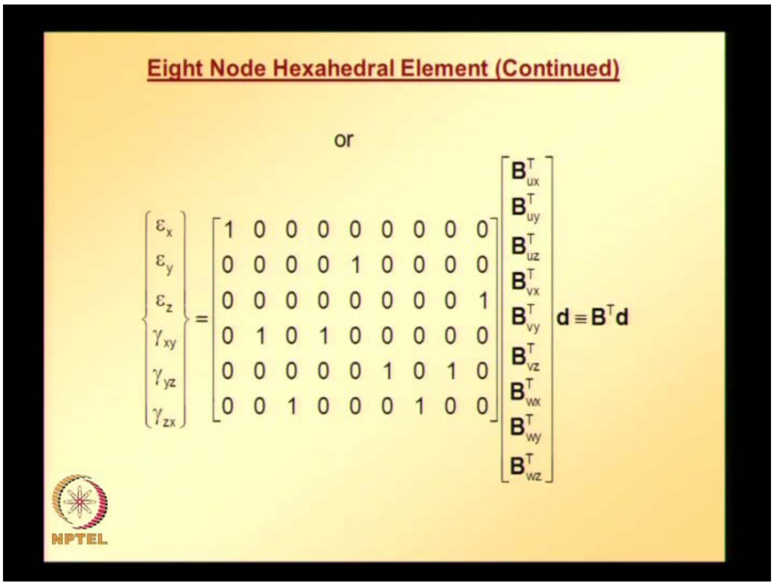

Once we have the right hand side, we can now replace that with appropriate rows in the equations that we have already derived in terms of Bs, B u x, B u y, B u z, B v x, B v y, B v z, B w x, B w y, B w z in this manner, which can be compactly written as B transpose d where, B is strain displacement matrix. This is how we can calculate these strains for this eight node hexahedral element. Now, we are actually ready to get the elements stiffness matrix by substituting strains into strain energy expression, we can get this, where k is the element stiffness matrix.

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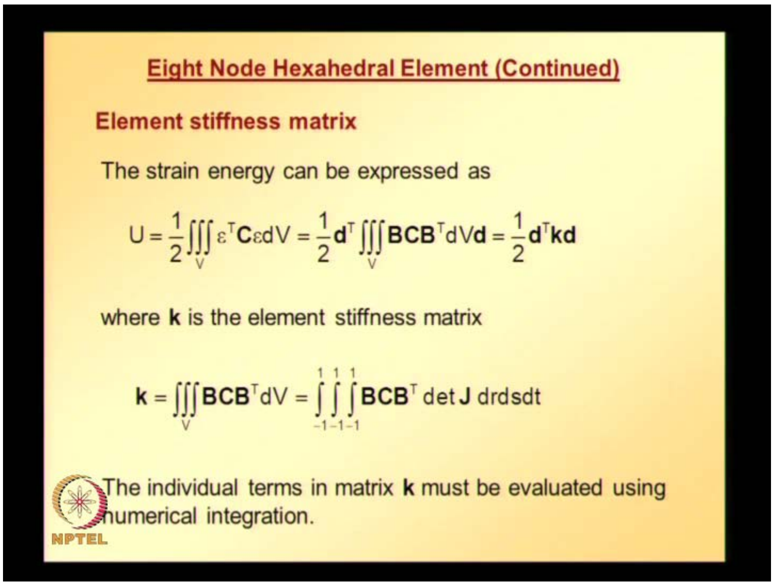

We can see here for this eight node hexahedral element strain displacement matrix B is not a constant. We need to adopt numerical integrations came to evaluate this element stiffness matrix. The individual terms in k matrix must be evaluated using numerical integration like Gaussian quadrature. Now, let us briefly look at numerical integration in three dimensions, which helps us to evaluate these kinds of integrals.

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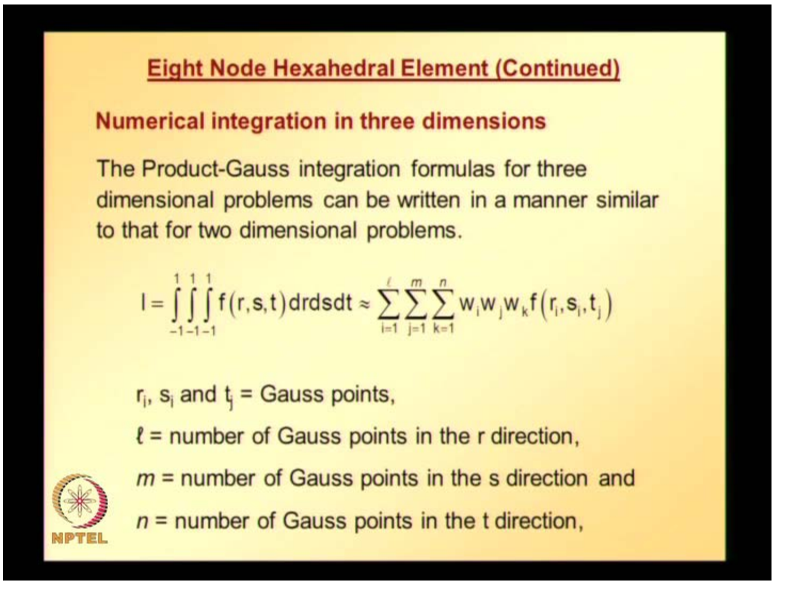

The product Gauss integration formulas for three dimensional problems can be written in a manner similar to that for two dimensions. Except that, we need to do whatever we have done for two dimensions, we need to extend by one more dimension to get integration formulas for three dimensional case. Integral something like this I is equal to minus 1 to 1, minus 1 to 1, minus 1 to 1, integral f d r, d s, d t. Basically, stiffness matrix element stiffness matrix the components of element stiffness matrix usually will be in this form. This can be evaluated or numerically approximated using Gaussian quadrature, the way it is shown on the right hand side of the equation, where r i, s i, t i are gauss points and w i, w j, w k are the weights. r i, s i, t i gauss points. l, m, n are number of integration points along r direction, s direction, t direction. These need not necessarily be same depending on the order of polynomial along r, s and t directions. We can select different values for l, m and n.

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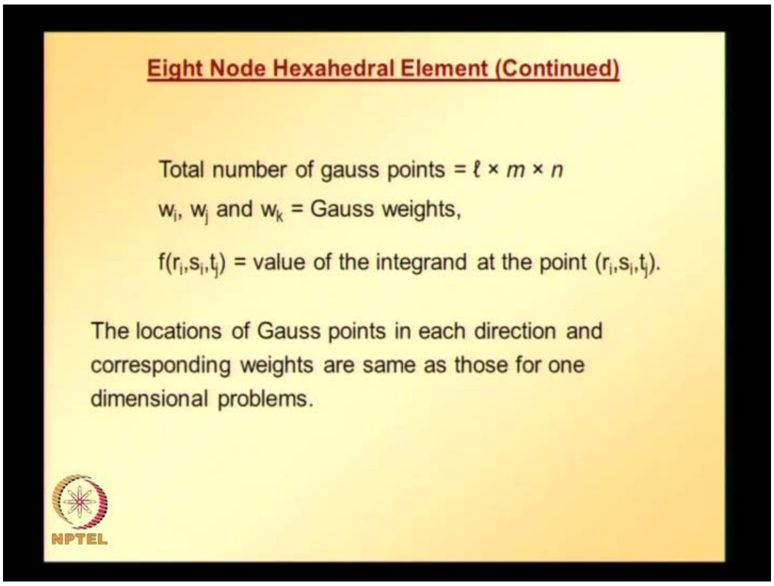

The total number of integration points are going to be l times, m times, n and w i, w j, w k are the Gauss weights and f as a function of r i, s i, t i, s value of integrand, at point r i, s i and t i. Now the important thing is how to know the locations and weights. The locations of gauss points in each direction and corresponding weights are same as those for one dimensional problems that, we are already familiar with. Let us look at a Gauss weights at each Gauss point and corresponding locations of the Gauss points. If we try to adopt 2 by 2 by 2 integration, if we select 2 number of integration points along r direction, 2 number of integration points along s direction, 2 integration points along t direction.

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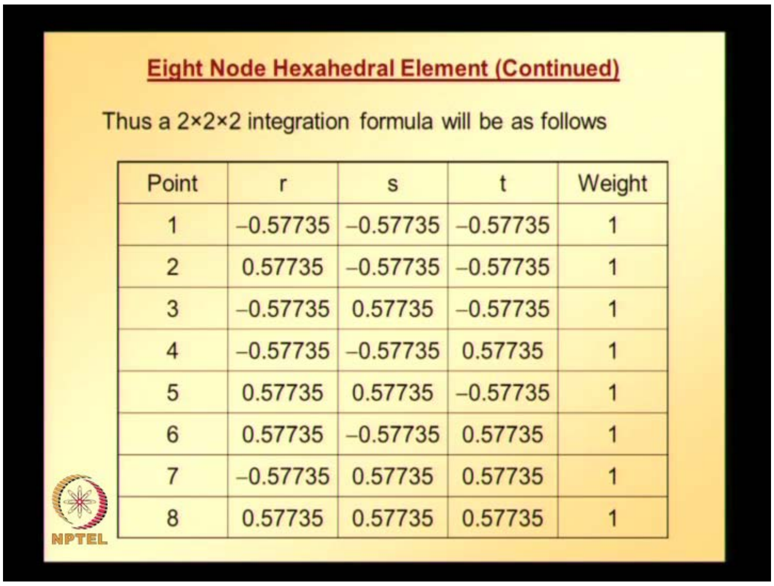

That is 2 by 2 by 2 integration formula, 2 by 2 by 2 integration formula point locations integration. They are going to be eight integration points 2 times, 2 times, 2 is eight. So, they are going to be eight integration points. The corresponding locations of these integration points, what is the r coordinate, s coordinate, t coordinate? The weight at each of these integration points is indicated in the table. This is just arrived at using the information that we have already for one dimensional problems, which we looked at in the earlier lectures. This is how we can evaluate numerical integration for three dimensional cases and using this kind of formula, we can evaluate individual terms in the element stiffness matrix.

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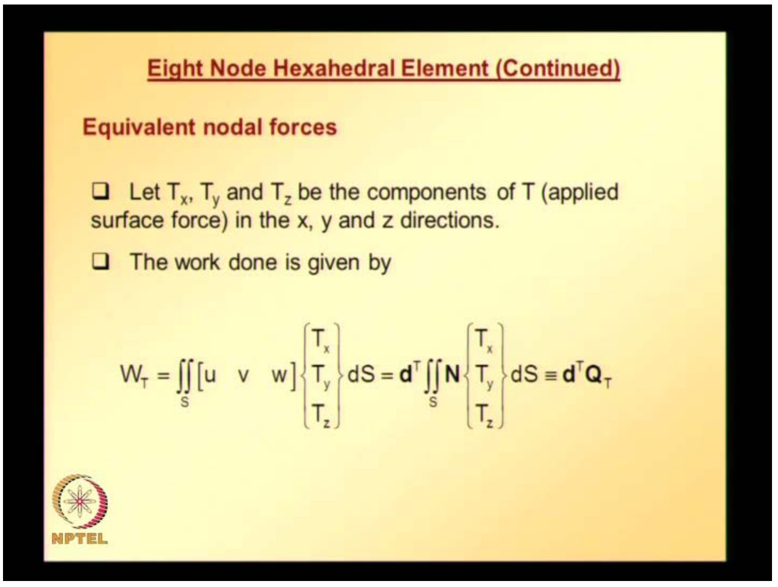

Equivalent nodal forces, this is similar to four node tetrahedral element that T x, T y, T z. With the components of traction surface forces that are applied along x, y, z directions. Work done by these forces is given by the traction components multiplied by the corresponding displacement components integrated over the surface on which the forces are applied. This can be further written compactly in the manner that is shown on the right hand side of the equation d transpose Q T where, Q T is equivalent nodal load vector.

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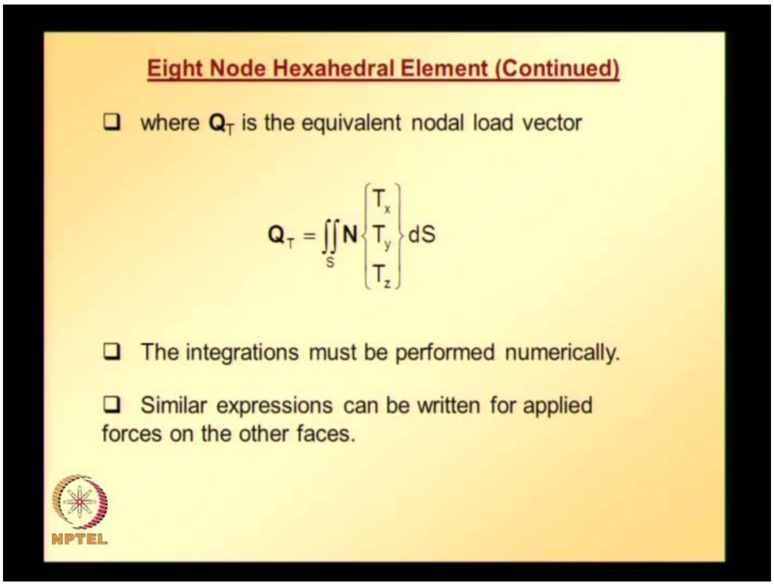

This is going to be a surface integral for eight node hexahedral element. The integrations can be performed numerically similar to that; we already looked at in the earlier classes for evaluating integrals in two dimensions. Similar kind of expressions can be written for other kinds of applied forces body forces and other forces. We looked at how to get element stiffness matrix for eight node hexahedral element also, how to evaluate equivalent nodal force vectors. With this we can actually assemble element stiffness matrix for each of the elements in the finite element discretization for the particular three dimensional elasticity problems. We can also assemble the equivalent nodal load vectors for the particular loading. Then we can assemble using these element equations we can assemble global equations and applying appropriate boundary conditions.

We can get the reduced equation systems solve for the unknown displacements. Subsequently, we can use strain displacement matrix. We can solve for strains and stresses and then we can do all kinds of post processing of stresses using principle stresses that we discussed in the last class, all that procedure is similar to that we have seen for two dimensional problems. Let us look at the other element that is twenty node isoparametric solid elements. Before, we proceed let me summarize this four node tetrahedral element is counter part of three node linear triangular element. The eight node hexahedral element is counter part of four node elements, for two dimensional problems and twenty node isoparametric element is going to be counter part of eight node serendipity element.

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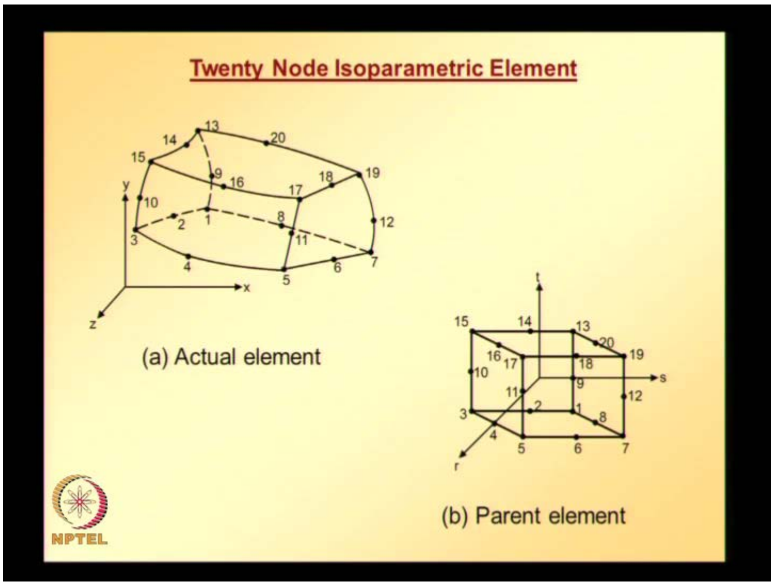

This is a twenty node isoparametric element with curved edges. Similar to that we have seen so far element equations are derived using isoparametric mapping concept. For that we require a parent element. Parent elements for this twenty node element looks like this where, center of element are located at the center of the q, which is having dimensions 2 by 2 by 2. With that understanding we can easily write the locations of all the nodes with respect to the parent coordinate system and once we have that information we can write the trial solution.

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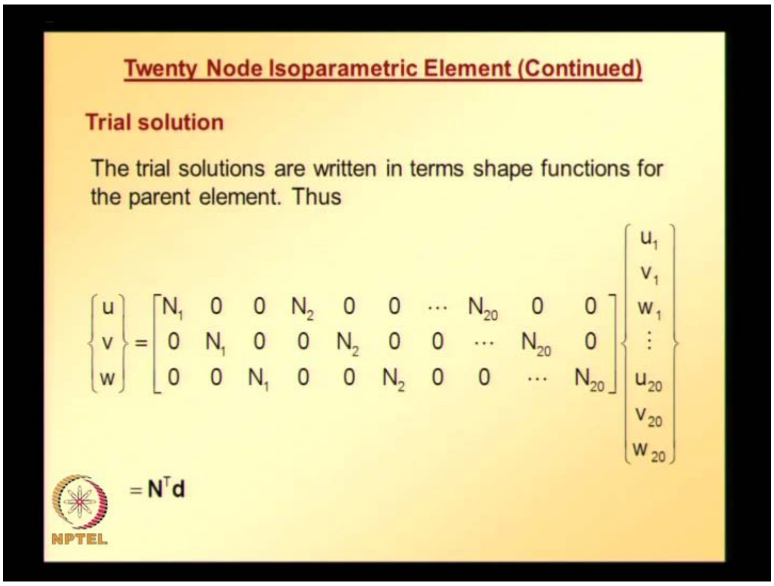

Trial solution written in terms of finite element shape functions .This is this equation is similar to that, we have seen earlier except that number of nodes increased. Matrix consisting of finite element shape functions is denoted with letter N. The displacement component at all the nodes is denoted with letter d. It is compactly written as N transpose d, the shape functions for the parent element can be obtained using similar kinds of procedures that we adopted for obtaining shape functions for eight node serendipity element for two dimensional problems. Here, directly the expressions are given.

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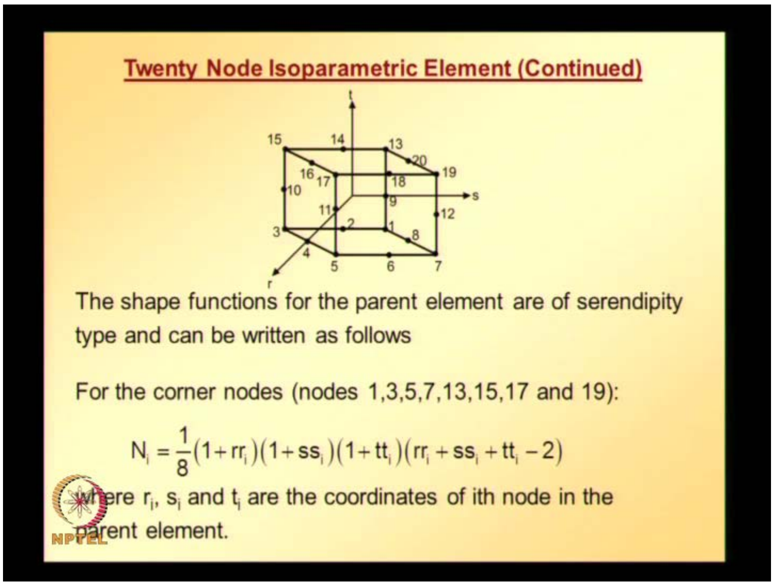

Shape functions for parent element are of serendipity type and can be written as follows. For corner nodes we cannot write one general expression, similar to that we have done for eight node hexahedral element. The shape functions expressions are going to be different or they look different for corner nodes mid side nodes. We need to write those separately for corner nodes. That is nodes 1, 3, 5, 7, 13, 15, 17, 19 for the node numbering that are given or that is shown in the figure there. For corner nodes the shape function expression looks like this, where r i, s i, t i are the coordinates of node in the parent element.

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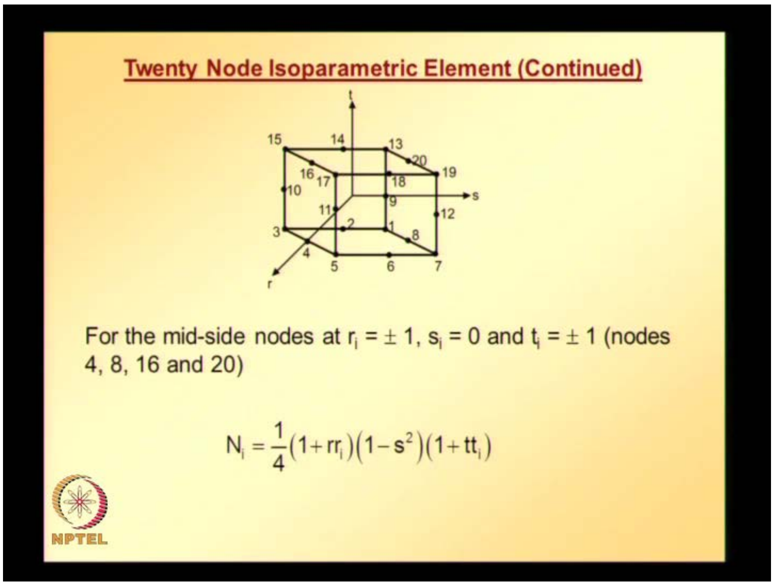

For mid side nodes, that is nodes which are located at or nodes whose s coordinate is 0. r i the nodal coordinate r coordinate of that particular node or mid side nodes, whose r coordinate is plus or minus 1, t coordinate is plus or minus 1, s coordinate is 0, such nodes that is node 4,8,16, 20. For these nodes the shape function expression is given by this.

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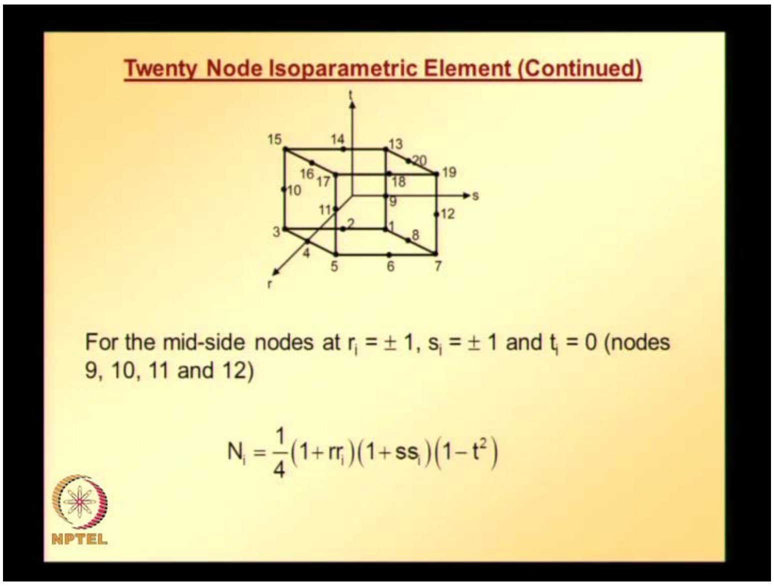

Similarly, for mid side nodes whose t coordinate is 0 and r coordinate is plus or minus 1, s coordinate is plus or minus 1. That is nodes 9, 10, 11, 12. Shape function expression is given by this.

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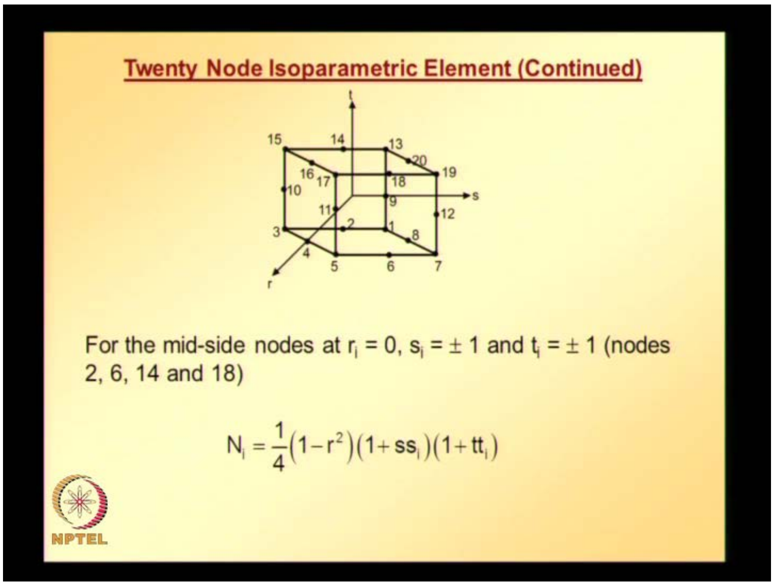

For mid side nodes whose r coordinate is 0, s coordinate, t coordinate are plus or minus 1. That is nodes 2, 6, 14, 18 is given by this. Please note that, these expressions are valid only for the node numbering that is shown in the figure here. If you adopt a different node numbering then these expressions are going to be different and again these expressions are developed using the serendipity. That, we adopted similar to that when we are deriving shape functions for serendipity element, eight node serendipity element for two dimensional problems.

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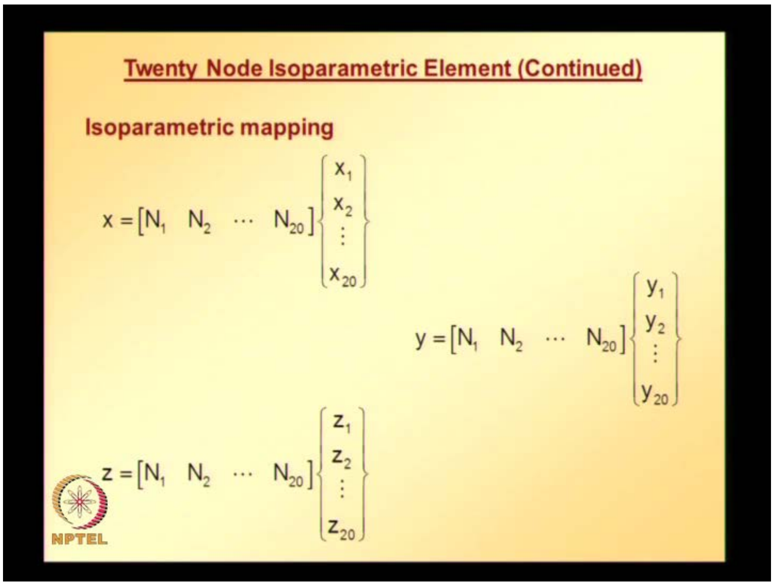

Once we got all the finite element shape function expressions, we can write isoparametric mapping for twenty nodded element. Similar to that we did for earlier elements except the dimensions of this vector are going to be increased. Because, there are 20 nodes. Once, we have isoparametric mapping expressions we can easily find partial derivative of x with respect to r s t, y with respect to r s t, z with respect to r s t for subsequent use in strain displacement relations.

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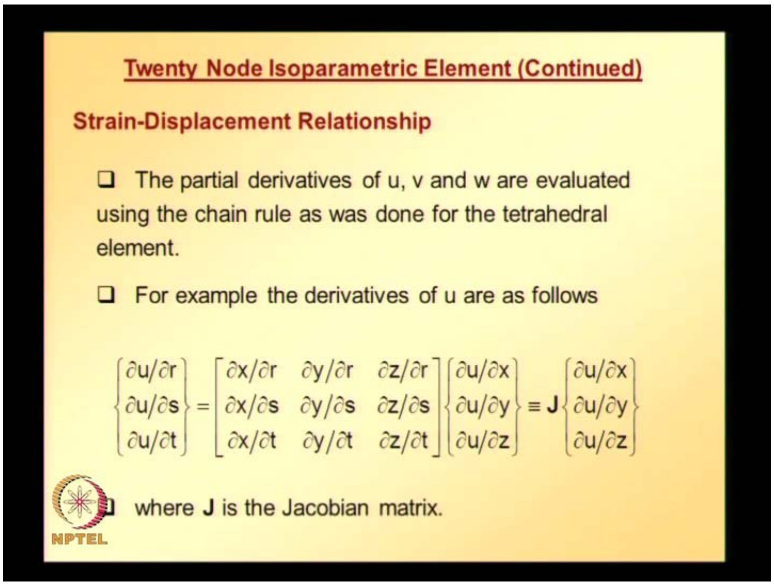

Strain displacement relationship partial derivatives u, v, w are evaluated using chain rule as was done for tetrahedral or hexahedral element. These equations looks or these equations are similar to that we have seen for other elements earlier. Except that some of the dimensions may be different, where J is Jacobian matrix.

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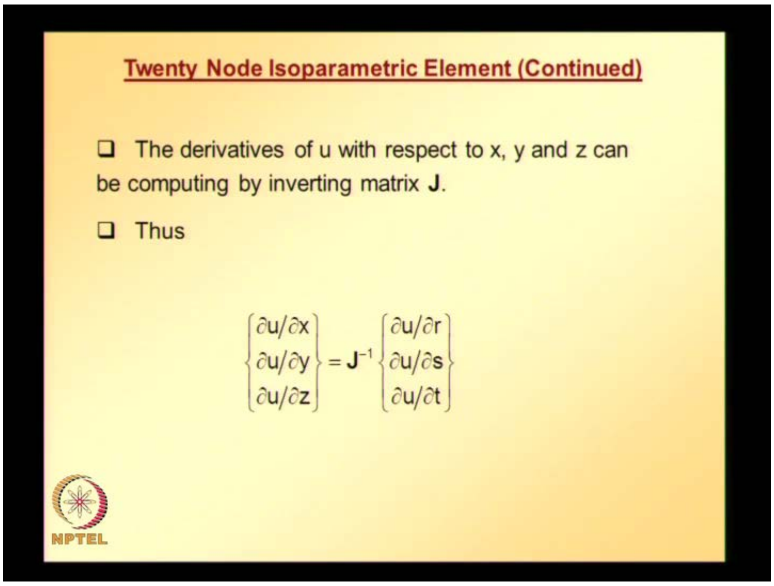

The inverse relation is given by this.

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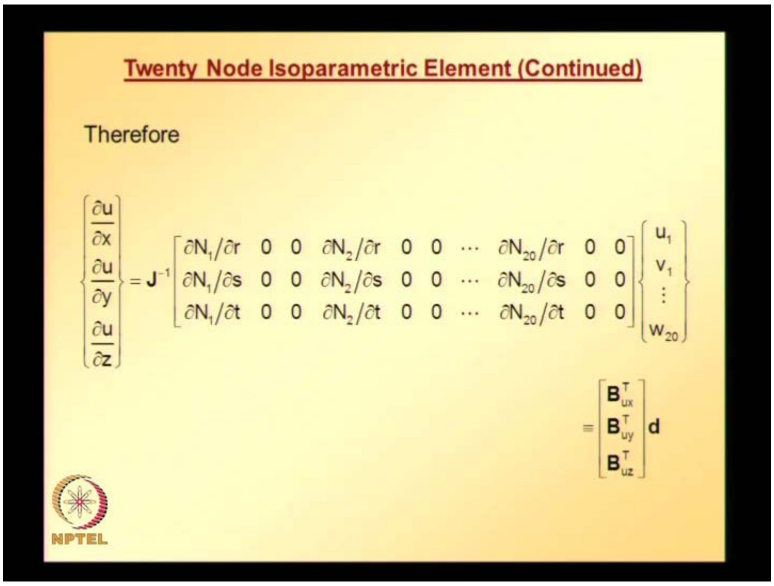

This equation gives derivatives of u with respect x, y, z, which can be compactly written similar to that what we did for eight node hexahedral element, except that the dimensions of matrices are going to be more.

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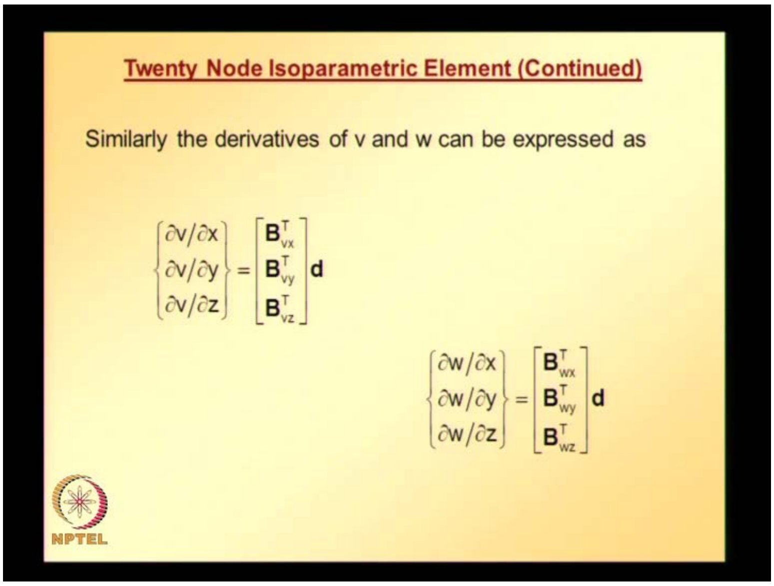

Similar derivatives for v and w can be written or expressed as follows. Once we have all this information using the definitions of strains in terms of displacements they can easily substitute these quantities. Get this strain displacement matrix or strain displacement relationship for twenty node isoparametric element.

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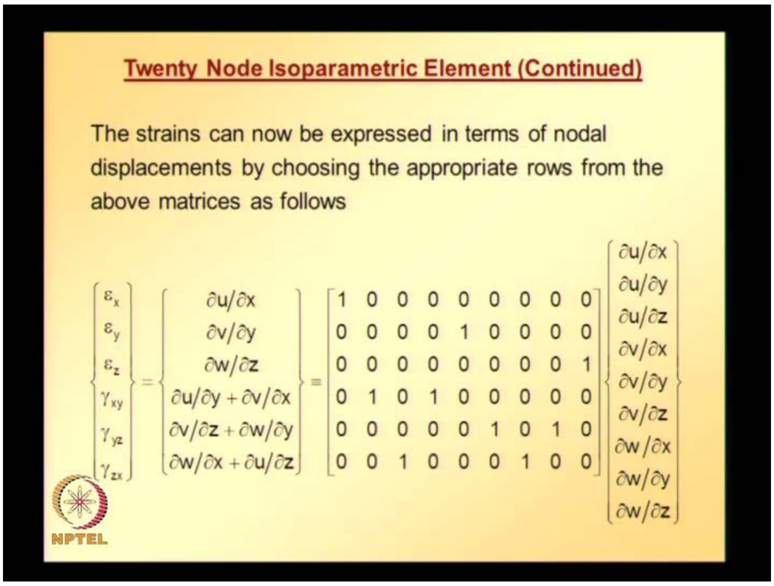

Strains now can be expressed in terms of nodal displacements by choosing appropriate rows from the above matrices. Strain definition, in terms of displacement components which can be rearranged as shown on the right hand side. Now plugging in or selecting the appropriate rows from the previous equations we can get this.

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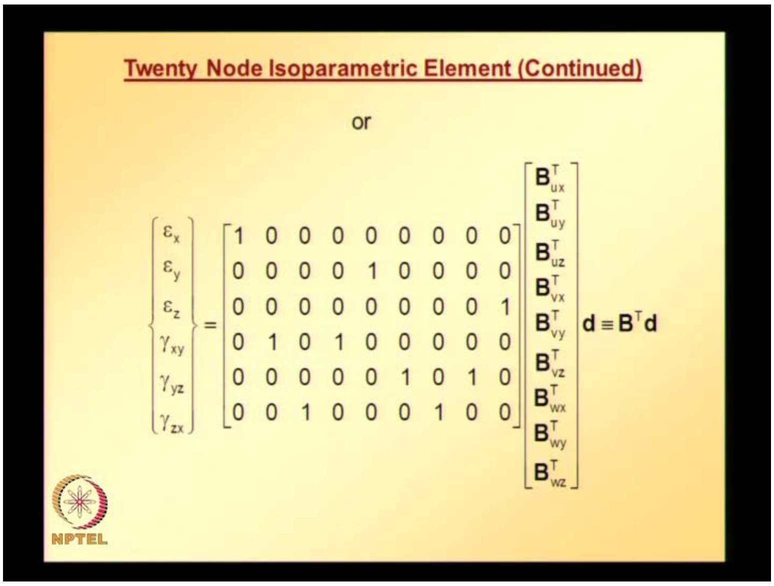

Basically, all these details are similar to that we already looked for eight node hexahedral element. Except that dimensions of matrices and vectors are going to be longer. Strain can be compactly written as B transpose d.

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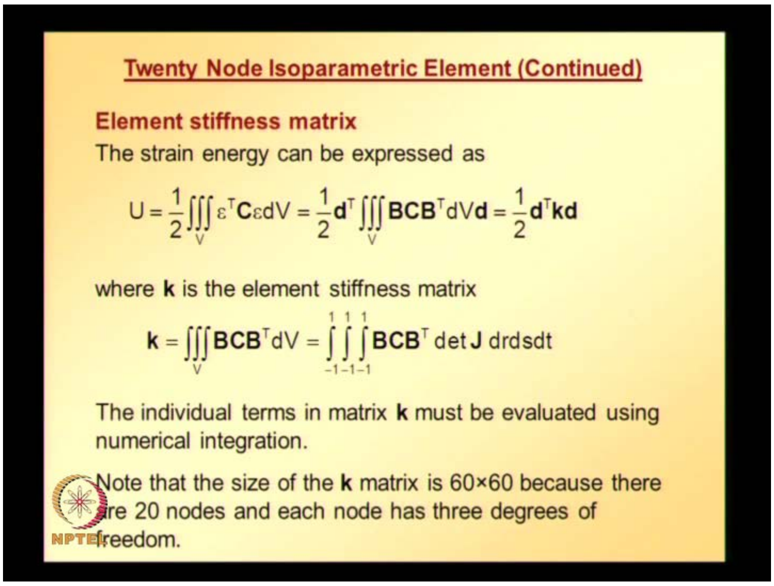

Substituting this definition and the strain energy expression, we get element stiffness matrix, where k is element stiffness matrix defined like this. The individual terms in k matrix must be evaluated using numerical integration like Gaussian quadrature. Because, B matrix is not a constant and only thing is size of the k matrix you need to keep in mind here. There are three degrees of freedom, at each node there are twenty nodes. Size of stiffness matrix is going to be 60 by 60 because, there are twenty nodes and at each at each node there are three degrees of freedom. So, we got element stiffness matrix.

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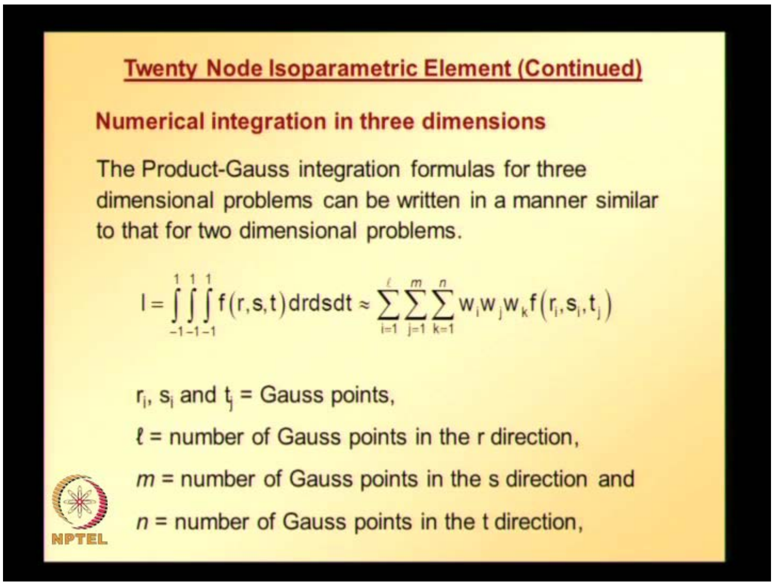

Next is equivalent nodal force vector or before we proceed there numerical integrations. The previous element stiffness matrix, we need to evaluate using numerical integration. Quickly, let us go through numerical integration in three dimensions similar to that we already looked for eight node hexahedral element. Product Gauss integration formulas for three dimensions can be written in a manner similar to that for two dimensions. Each component of element stiffness matrix can be evaluated using this formula, where definitions of various quantities are given.

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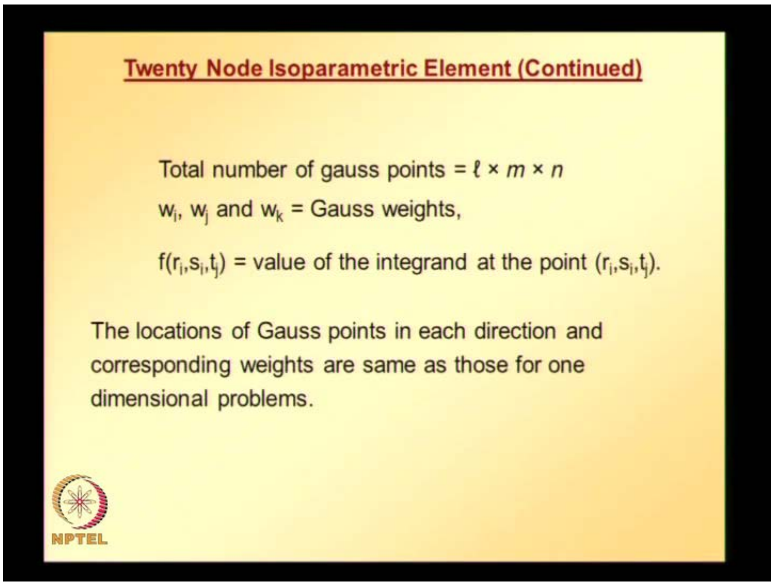

This is just for completeness; I am showing you numerical integration three dimensions again.

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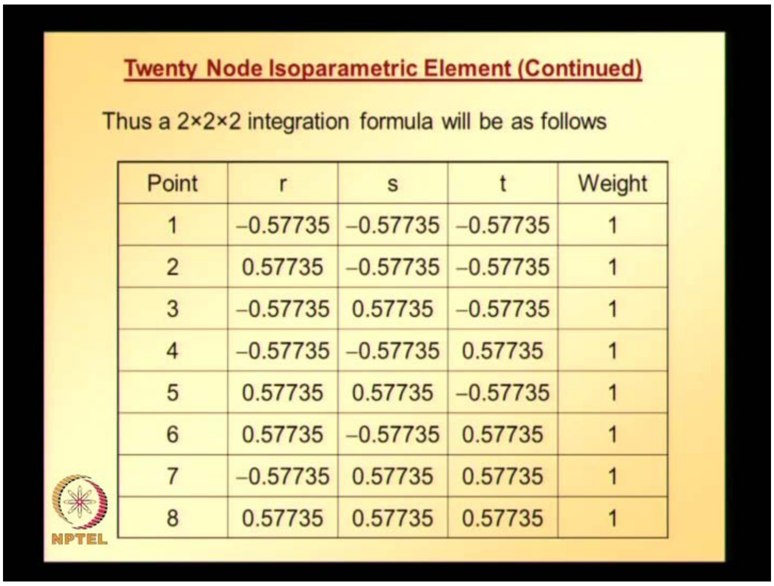

For 2 by 2 by 2 integration, the coordinates and weights of various integration points are given. Here, adopting this kind of formula we can evaluate each component of element stiffness matrix.

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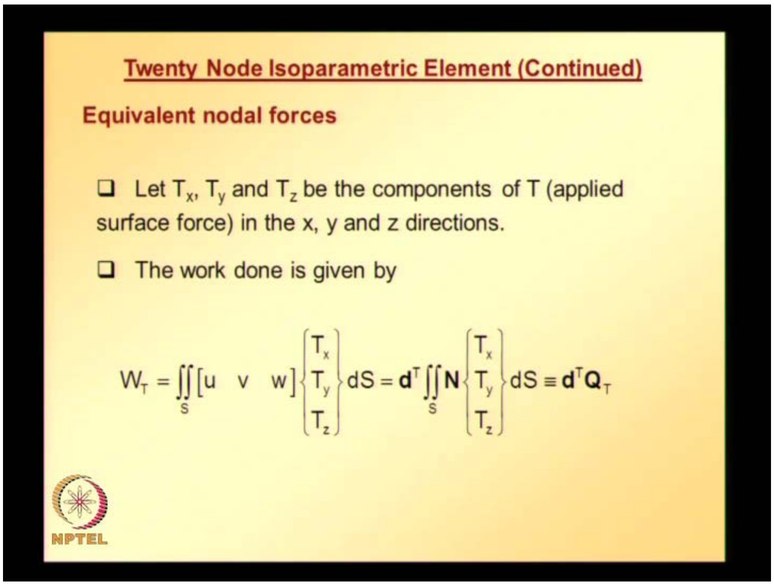

Next is equivalent nodal force vector if T x, T y, T z are the components of tractions along x, y, z directions. Work done is given by this integral which can be compactly written as d transpose Q T, where Q is the equivalent nodal load vector.

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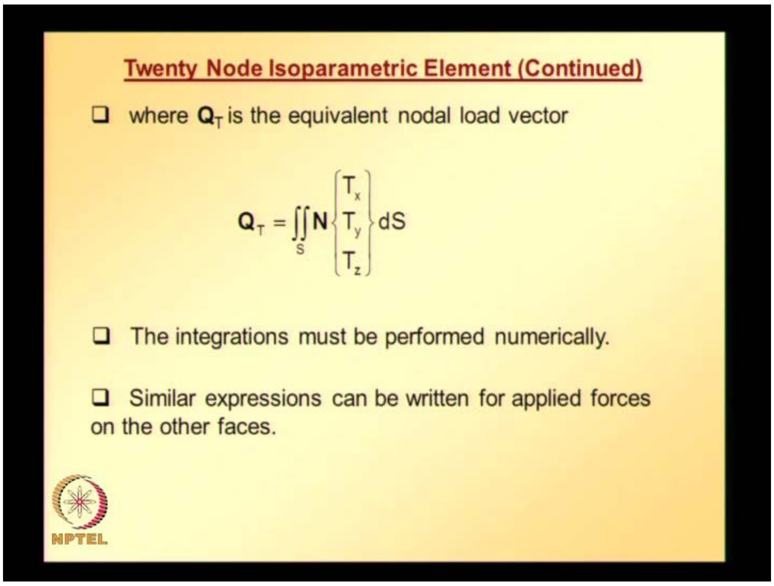

Integration must be performed to evaluate this numerically and similar expressions can be written for other kinds of forces acting on other faces. Once again element stiffness matrix for twenty node isoparametric element is going to be 60 by 60 from each element. And also equivalent nodal force vector length 20. That is vector is going to have 20 components. Procedure wise, the dimensions of matrices and vectors are going to be longer, other than that conceptually the procedure for assembling the element matrices and vectors. Getting the global equation system and applying the essential boundary conditions and getting the reduced equation system. Solving for the unknown nodal displacements and calculations of strains stresses all those details are similar to that for the problems that we looked at earlier.

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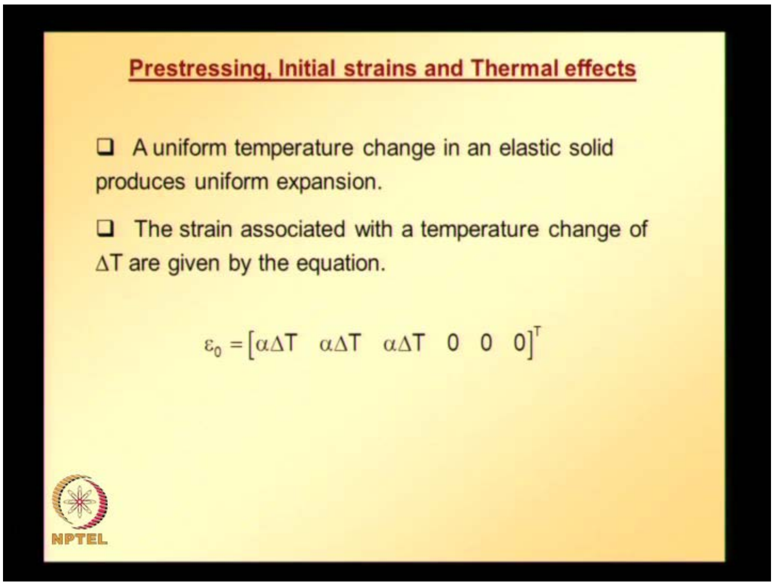

Let us look at prestressing initial strains and thermal effects. How to handle this? A uniform temperature change in an elastic solid produces uniform expansion that we already know. Strain associated with temperature change delta T is given by this. Only normal components will be non zero shear components are going to be 0. This is a strain associated with temperature change of delta T in an elastic solid. For plane stress and plane strain case is given by this.

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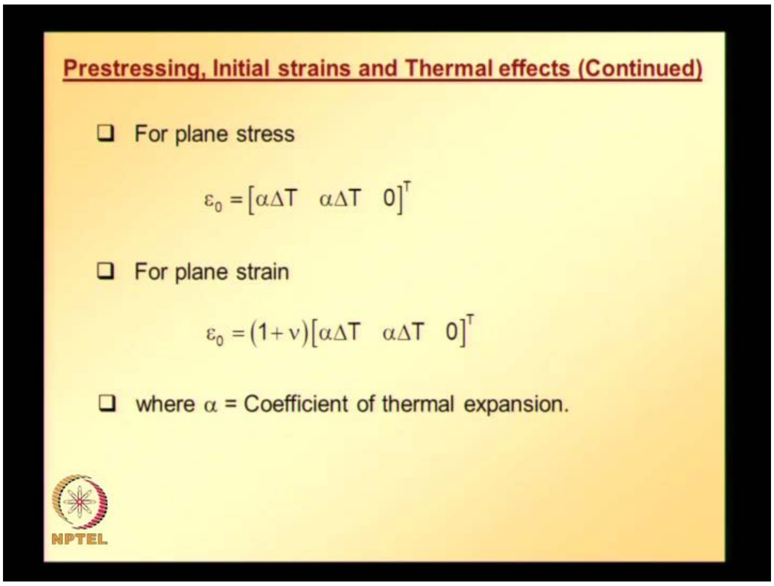

Plane stress and for plane strain case, in all these equations alpha is coefficient of thermal expansion. Delta T is the change in temperature, n u is the Poissons ratio. This is how we can calculate strain associated with temperature change. Prestressing implies presence of some unknown initial stress in the body. Basically that is, what we are going to do, when we are actually analyzing prestress concrete. Prestressing implies presence of some unknown initial stress in the body, the stresses the corresponding stresses or prestressing.

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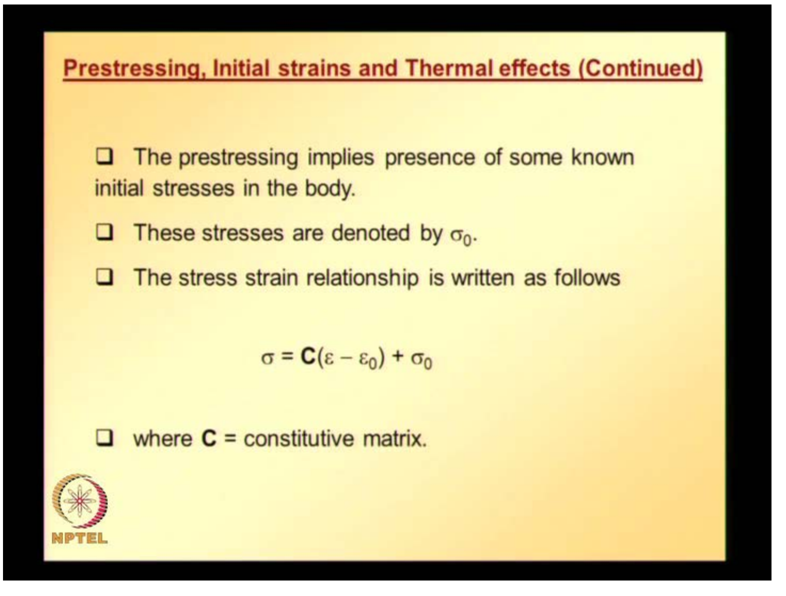

The stresses corresponding to this prestress are denoted with sigma naught and taking care of these prestressing prestresses. The strain stress strain relationship can be written like this. The actual stress that is developed in the body is going to be dependent only on the strains excluding the strains. That is associated with change in the temperature or initial strains. Stresses are going to be given by C times epsilon minus epsilon naught plus initial stresses sigma naught, where C is the constitutive matrix. So, this is how stresses in a body subjected to temperature change or initial strains can be calculated.

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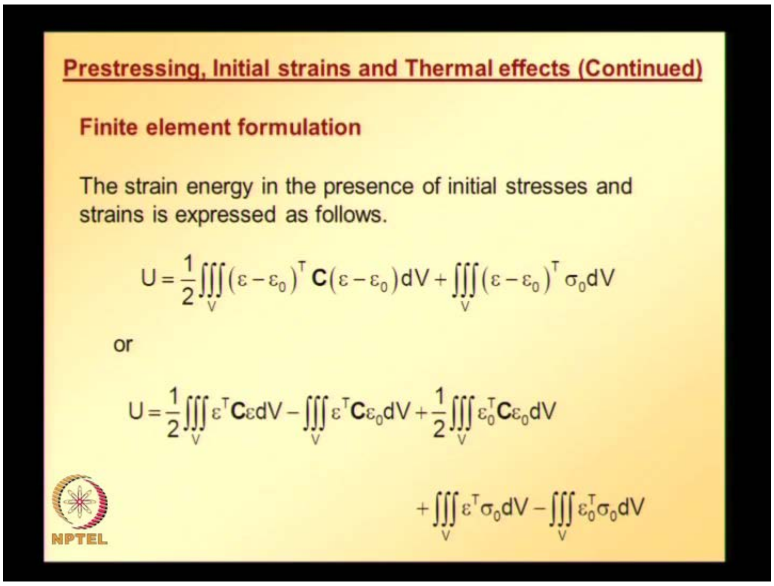

Using this definition strain energy in the presence of initial stresses and strains can be written like this. Only difference here is instead of epsilon, epsilon minus epsilon naught is used in the first expression, second one is coming from the work done by the strain energy, because of developed strains, in addition to initial strains and initial stresses. This can be further simplified and we get this all terms and we know that by differentiating strain energy we get element equations.

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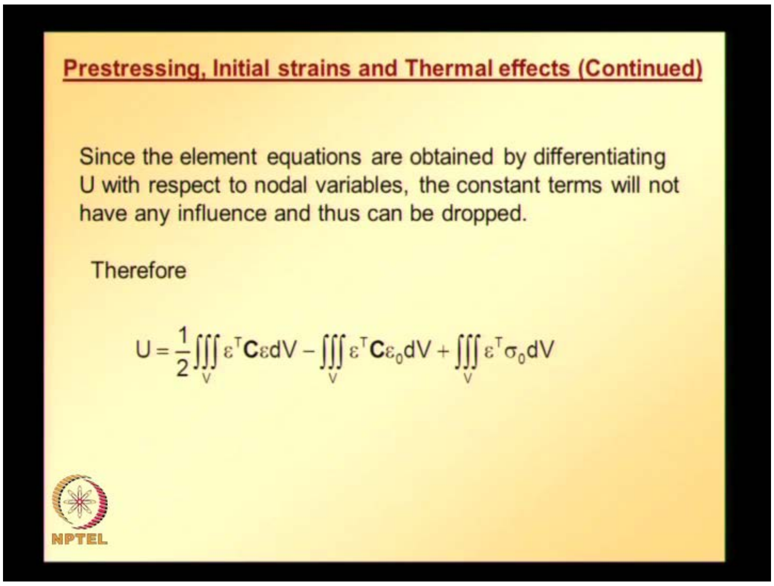

Since, element equations are obtained by differentiating strain energy with respect to the nodal variables, which are going to be nodal displacements. The constant terms will not have any influence and can be dropped. Whatever constant terms are there in the previous equation they will all drop off, when we take derivatives with respect to the nodal displacements or nodal variables. Finally, we are going to get this one before we do differentiation; we can remove the constant terms or we can take derivatives of constant terms automatically they are going to be 0.

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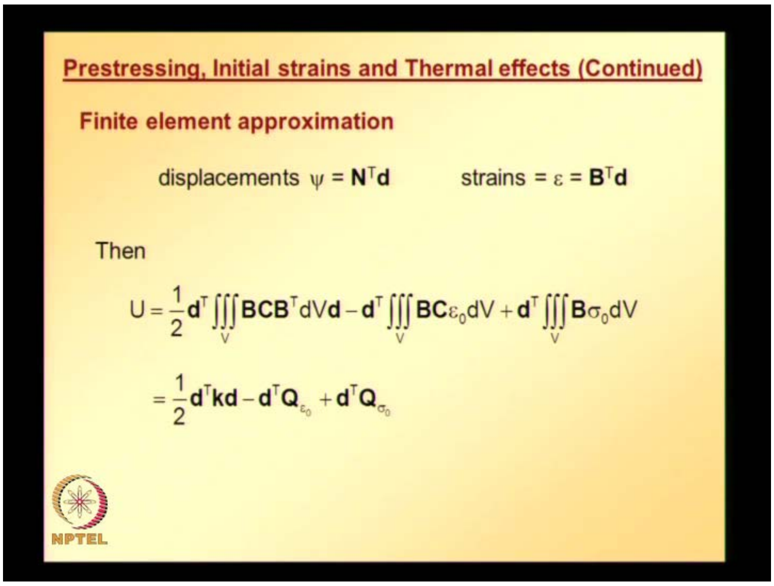

By taking the strain energy expression without constant terms, before we proceed actually we need to substitute the definitions of strain or strain in terms we need to express strain in terms of nodal displacements. Finite element approximation of nodal displacements N transpose d strains, B transpose d. Substituting this into the previous equation in which constant terms are avoided, we get this which can be written in a matrix and vector form in this manner, where each of the terms are defined like this.

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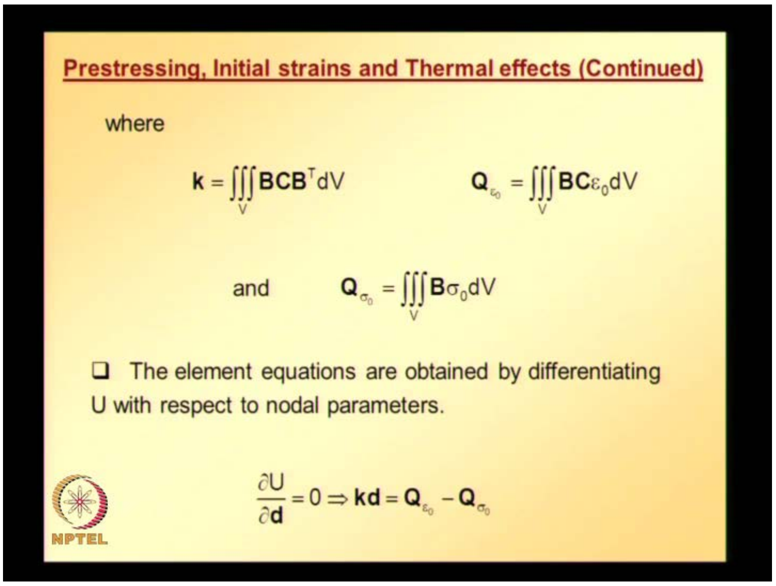

Q epsilon naught, Q sigma naught are due to initial strains, due to the presence of initial strains or stresses. The element equations are obtained by differentiating the previous equation strain energy u with respect to the nodal displacements or nodal parameters, we get this equation and you can see this equation.

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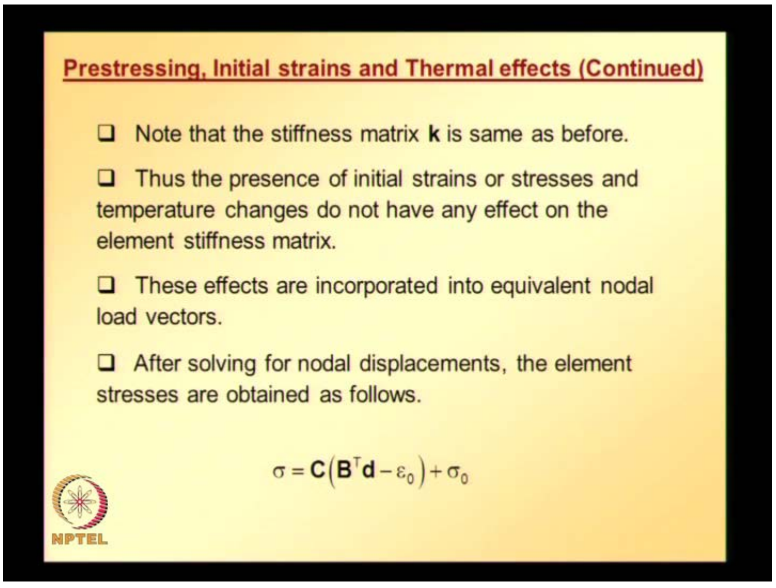

It can be notice that stiffness matrix, k is similar to that we already looked at in the earlier cases or stiffness matrix is same as before, except that the presence of initial strains and stresses or temperature changes do not have any effect on the element stiffness matrix. That is what we can notice, also all these effects that is initial strains or stresses and temperature changes are incorporated into the nodal force vectors or nodal load vectors. We can proceed similar to that we already discussed for other kinds of elements for various kinds of elements that we discussed during this lecture. Similar to that what we can do is, with this definition of element stiffness matrix and force vectors, nodal load vectors we can assemble.

In case, initial strains or stresses are present or if there is change in temperature we can assemble the element equations as usual, and also the equivalent nodal load vectors can be assembled using the equations that we just saw, that we have just seen the equation using the Q epsilon naught, Q sigma naught equations. And get the element equations and using nodal connectivity we can assemble the global equations, and apply appropriate essential boundary conditions, and solve for the nodal displacements, and do all kinds of post processing. So, this completes three-dimensional elasticity problems after solving for nodal displacements, element stresses can be calculated using the last equation that is shown.