Finite Element Analysis

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Lecture No. # 36

In last class, we have derived element equations for two d elasticity problems starting with governing potential energy functional. And using shape function corresponding to 3 node triangular element. And also the solution procedure is illustrated through an example, and at the end we found that, we have before that we have taken a cantilever plate, and using two elements, two triangular two triangular elements with 3 nodes each. We descritized, that cantilever plate and the solution that we got, when we compare it with exact solution.

We can actually compare the stresses along the the common side for the two elements, that is interface between the two elements. For exact solution stresses on two sides should be equal and opposite of each other; and the solution that we obtained using two linear triangular elements, for this cantilever plate problem. We observed that there is large discontinuity in stresses across element boundaries, so this solution is not good this is because, we used only two elements, the mesh is very coarse and obviously, we cannot expect very good results, but as we increase the number of elements, it discontinuity in stresses should reduce.

So, now in today's class, we will look at 4 node quadrilateral element that is derivation of finite element equations using 4 node quadrilateral elements for two d elasticity problem. And subsequently illustration to the same cantilever plate problem now is using only one 4 node quadrilateral element. Later introduce, let class we will be looking at 8 node isoparametric element as well.

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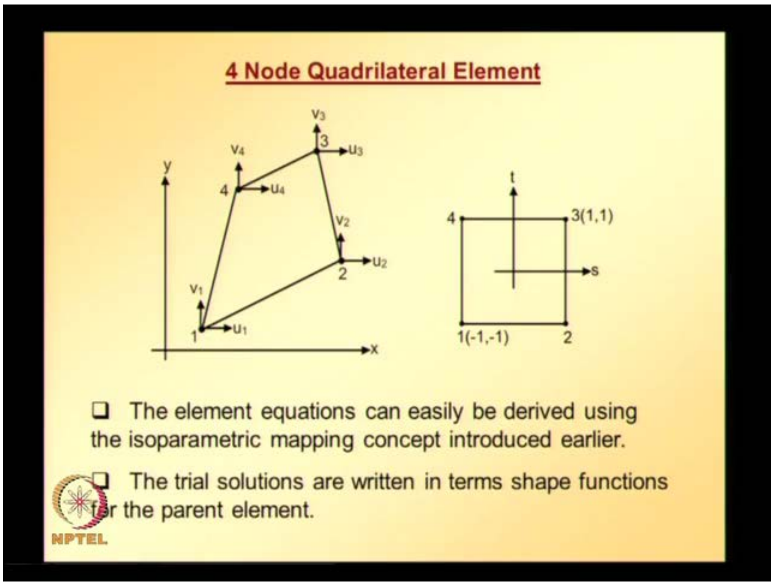

So, now, let us get started a 4 node quadrilateral element is shown on the left hand side, and for isometric mapping purpose, will be using the element that is shown, the parent element that is shown on the right hand side. The element equations can easily be derived using isoparametric mapping concept, introduced earlier, that we are familiar with.

The trial solutions are written in isoparametric mapping, the trial solutions are written in terms of shape function for parent element. So, we need to know what are the shape functions for the 4 nodes of this quadrilateral element in the parent element domain.

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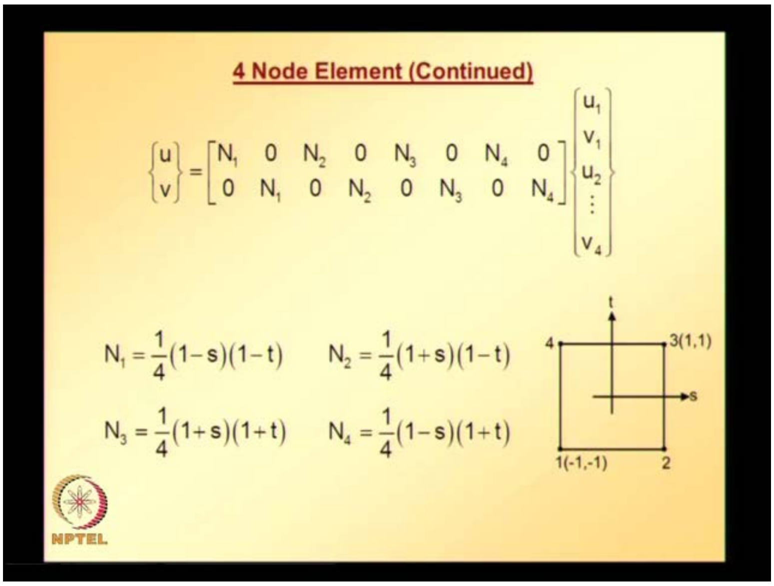

Once we have the shape functions of all 4 nodes of the parent element, we can write this equation which interpolates displacement, using the nodal values at the 4 nodes and the interpolation functions or the shape functions. Here, N 1, N 2, N 3 are nothing but, shape functions corresponding to the 4 node parent element.

And once, we know these shape function expressions we can as well write expressions for isoparametric mapping, that is how x y are related to s and t, how the axial element coordinate are related to the parent element coordinates, this is isoparametric mapping, written in matrix and vector form.

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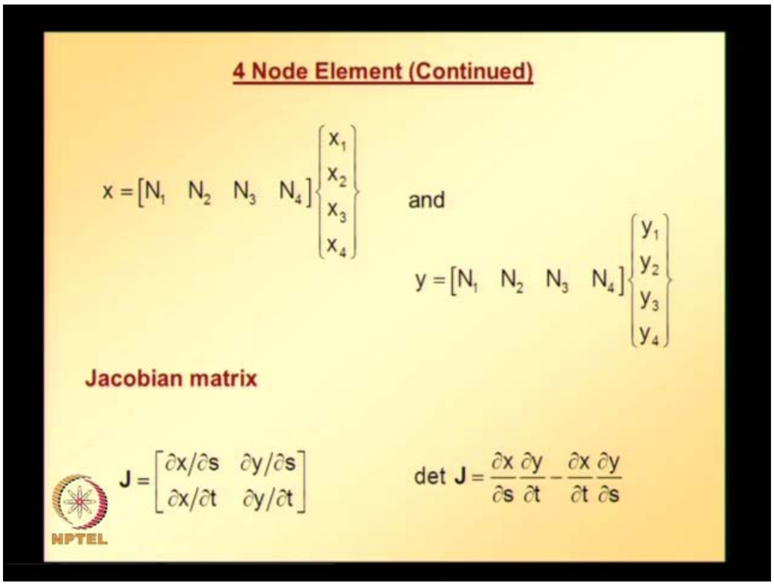

And once we have this information, we can easily find what is Jacobian matrix, and determinant of Jacobian Jacobian matrix compresses of all partial derivative of x with respect s, x with respect to t, y with respect s and y with respect to t. So, once we have these two equations, we can easily find those derivatives, so Jacobian matrix is defined like this, and from here we can find determinant of Jacobian.

There is nothing new in all this process, it is a same as for 3 node triangular element except that the number of nodes now became 4 instead of 3, so the matrix sizes becomes bigger. So, once we have x and y related to the nodal values like this, nodal coordinate values we can easily find the partial derivatives.

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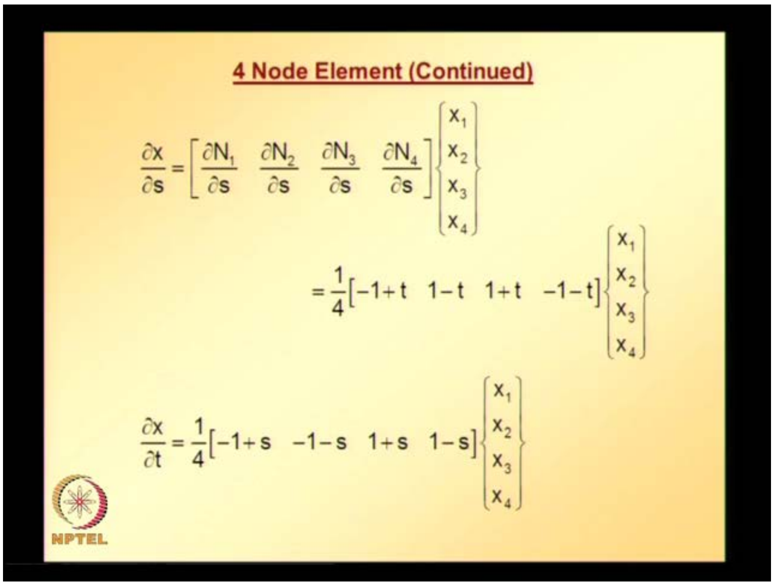

Using these relations, which can be simplified further as shown there, similarly derivative of x with respect to t.

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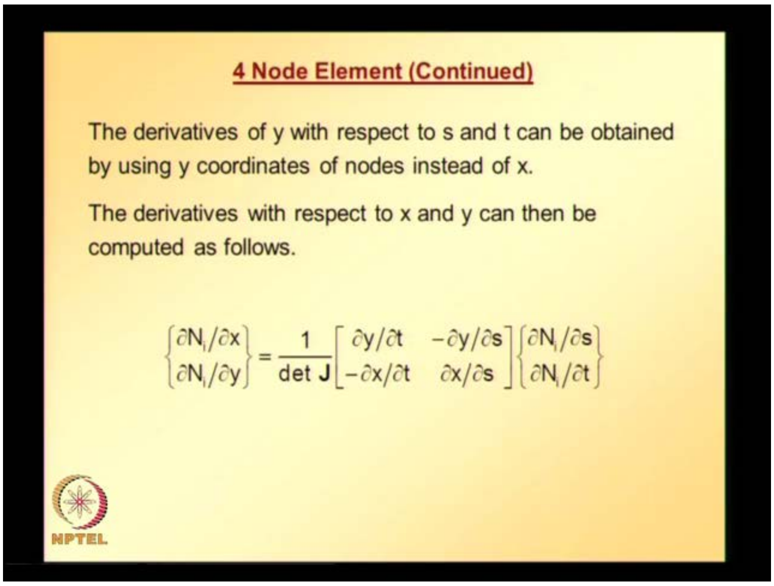

And we can also find the derivatives of y with respect to s and t, using y coordinates of node instead of x, in the previous two equations. And now, we also require knowing how the shape function derivatives in the axial element coordinate system or related on the parent element coordinate system that is given by this one. Derivatives with respect to x and y can be computed as follows, which is given there for which we require to calculate determinant of Jacobian.

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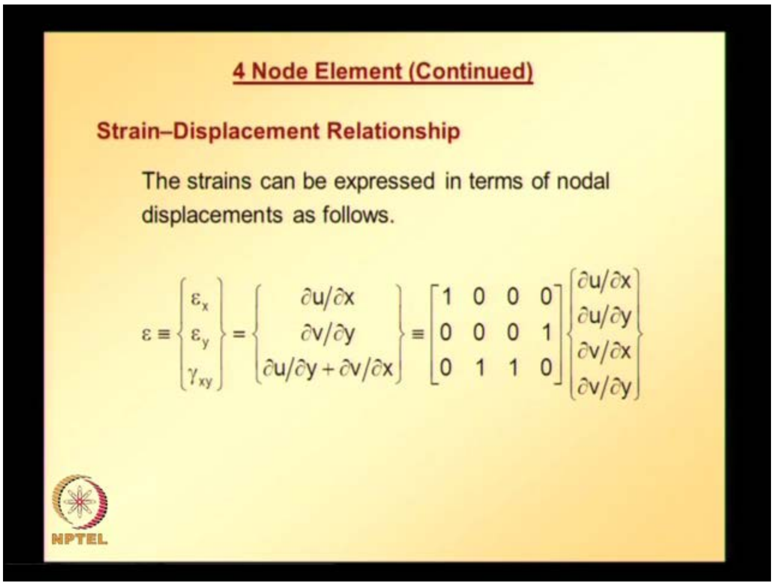

So, once we have all this information, we can easily write strain-displacement relationship, strains can be expressed in terms of nodal displacements; we are familiar with this definition of strain. So, now, substitute or this vector consisting of partial derivative of displacements can be rearranged, as shown in the later part of the equation, and partial derivatives of displacements with respect x and y can easily be calculated using this relation.

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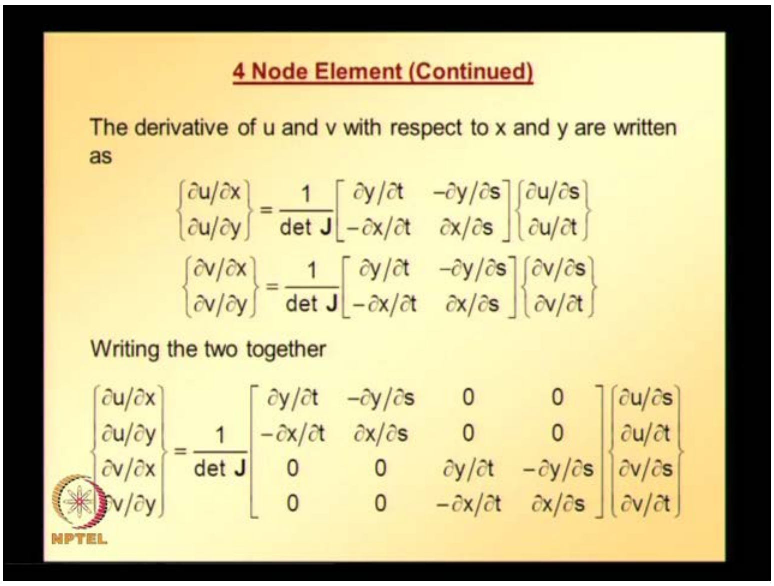

This is derivatives of displacement component in the x direction similarly, derivatives of displacement component in the y direction. And we can put these two equations together, and get this one and here in the last equation, we have derivatives of displacement components both in x and y directions with respect to x and y.

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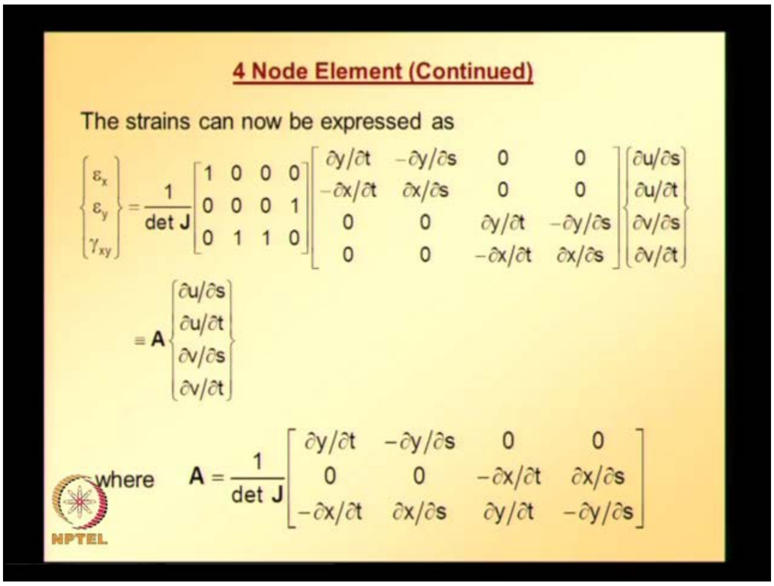

So, now once we have that information strains, we know definition of strain and we just found what are the derivatives of displacement components with respect x and y, so we can plug in that and rearrange the equations and finally we get this. Where A is a matrix which is defined like this, please note that this kind of definition of A and other intermediate variables is choice of the teacher. Sometimes, if you refer some other books you may see different definition, are all together they may avoid this kind of intermediate definition or intermediate variable definition. Now, derivatives of trial solution with respect to s and t are easy to compute, here if you see A matrix you have derivatives of trial solution with respect to s and t sorry, the vector beside A matrix it consists of derivatives of trial solution with respect to s and t, which can be easily calculated.

Because, trial solution is function of s and t, because the shape functions or functions of s and t. And this can be done in this manner that is calculation of derivatives of trial solution with respect s and t can be obtained through this equation, which can be compactly written like this.

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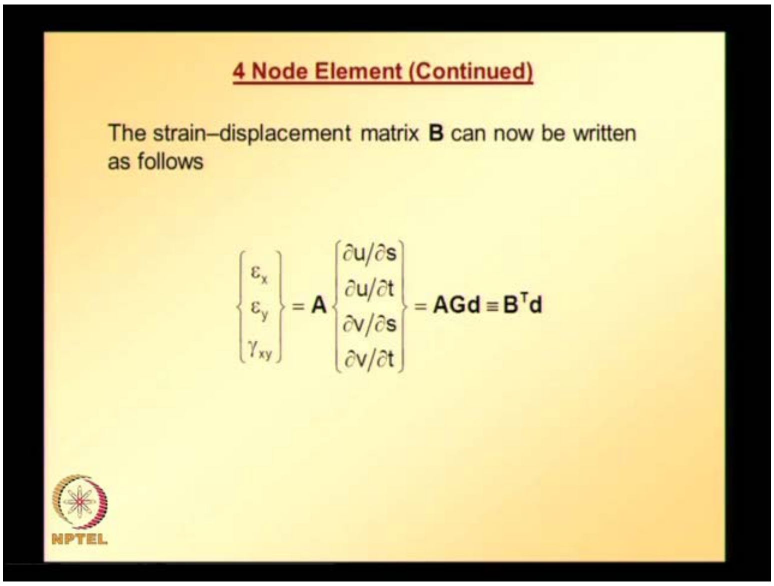

So, finally, strain-displacement matrix can be written like this, so the key thing is we need to get the nodal coordinate information, and then we can easily follow these steps. And finally get the required quantities to calculate strains, once we knew the nodal values.

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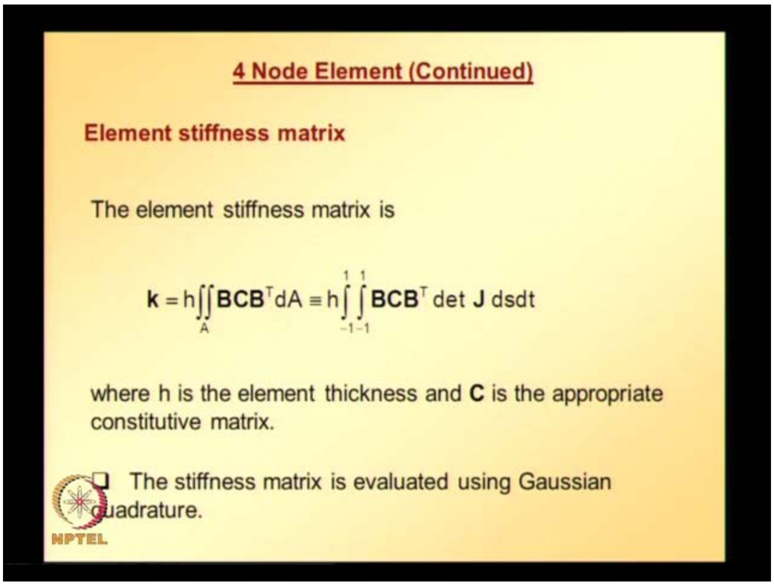

So, now let us look at element stiffness matrix, element stiffness matrix is defined like this, please note here for 3 node linear triangular element B matrix is a constant whereas, for 4 node quadrilateral element B matrix is not a constant, so we need to perform numerical integration. So, for that purpose we need to change the limits of integration from to minus 1 to 1, and the details of how we do that we already discussed several times in the earlier lectures.

And here in this equation h is the element thickness; C is the appropriate constitutive matrix depending on plane stress or plane strain. The stiffness matrix is obtained using Gaussian quadrature can be evaluated using Gaussian quadrature, which is a numerical integration procedure.

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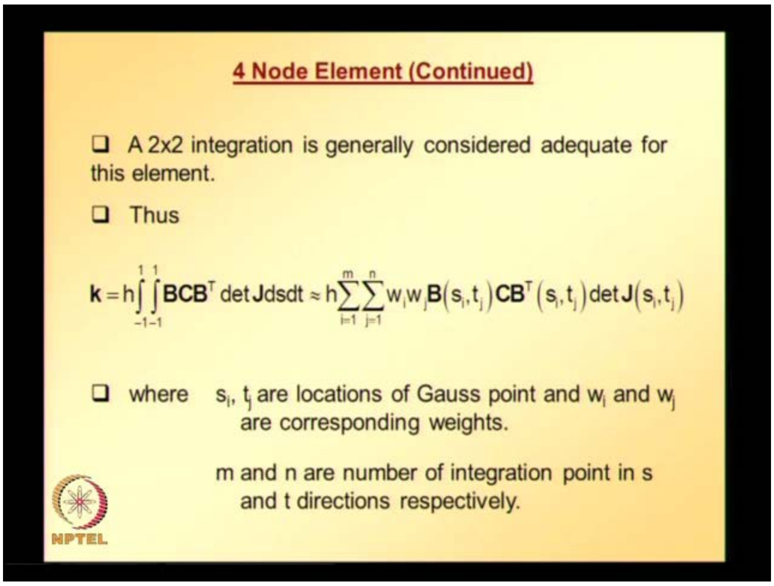

2 by 2 integration is generally considered adequate for this element, and so element stiffness matrix can be approximated by taking two integration points along s direction and two integration points along y direction sorry along t direction; and weight at each integration point is weight in the s direction times weight in the t direction. And evaluate all the quantities in the integrand, at these integration points and sum up and multiply the integrand value at each integration point with the corresponding weight, and sum up the contribution from all the integration points.

So, that is basically what numerical integration schemes will do, and that is here expressed in the form of an equation. Where s and t are the locations of Gauss point and w's are the corresponding weights, m and n are the number of integration points in s and t directions respectively. These integration points in s and t direction need not be same, which we already looked at; it depends on the order of the integrand in the s direction, and order of integrand in the t direction, so this is how element stiffness matrix can be evaluated.

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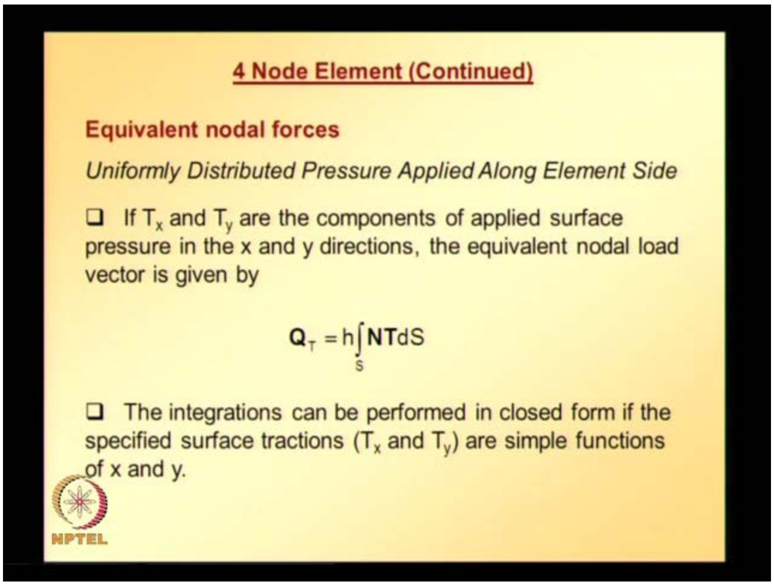

So, now, let us look at how to assemble or how to obtain equivalent nodal forces, and for illustration purpose, we consider uniformly distributed pressure along element side. If T x, T y are the components of applied surface pressure in x and surface pressure is same as surface traction are the components of applied surface pressure in the x and y directions, the equivalent node nodal load vector is given by this one.

So, here we need to substitute N, N is nothing but, a shape function matrix of all non of all the shape functions along that particular side, along which the pressure is applied. Integrations can be performed in a in the in a closed form, if the specified surface tractions T x, T y are simple functions of x and y. This also we discussed in the last class, if T x, T y are simple functions we can easily perform the integrations, in a can be we can perform integrations in a closed form, but if T x, T y are complicated functions; then we need to adopt numerical integration. And the simplest case that you can have is T x, T y being constant or uniform.

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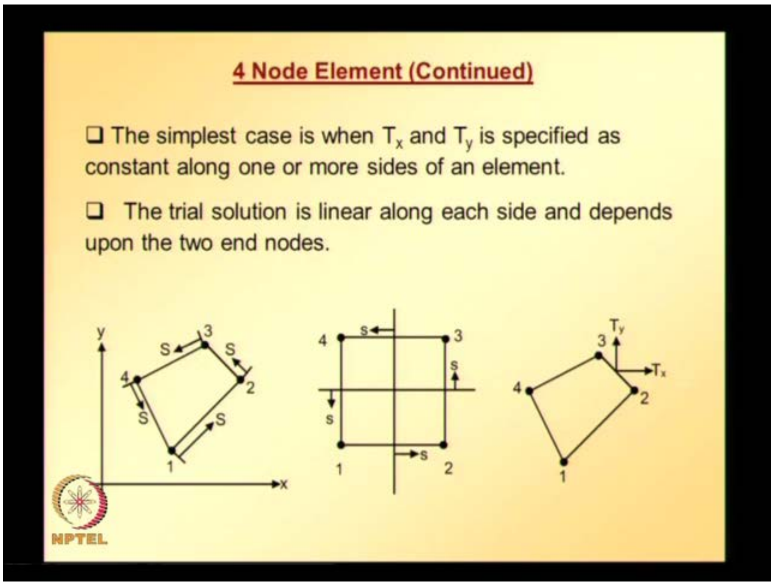

Simplest case is when T x, T y is specified as constant along one or more sides of an element. And also please note that, we are dealing with 4 node elements, so the trial solution is linear along each side and depends upon the two end nodes, so we can easily write the shape function of the nodes using Lagrange interpolation formula, which we discussed earlier. Here, both actual element and the parent element are shown and also the traction is applied on side 2-3.

For each side the coordinate's capital S for actual element and the corresponding small s for the parent element are shown the shape functions can for the shape functions, for each side can be expressed as follows, alongside 1-2. If you take parent element along side 1-2, shape functions of node nodes 3 and 4 are going to be 0 alongside 1-2.

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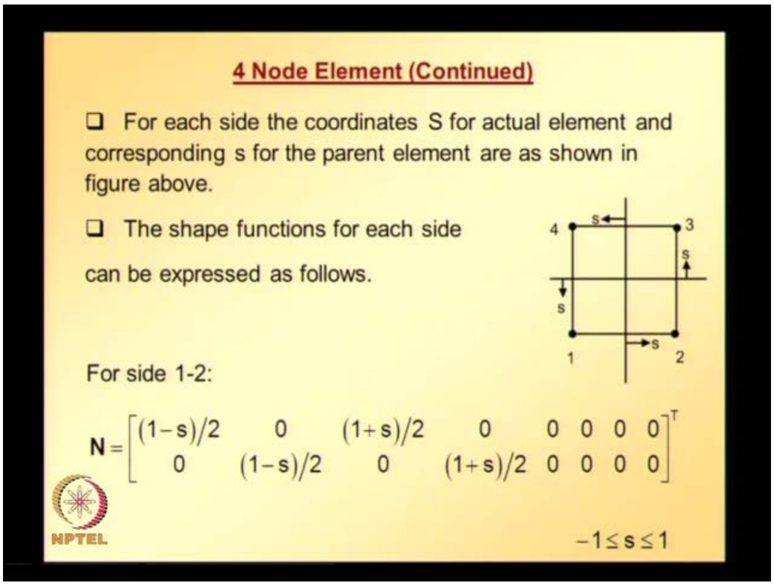

So, the shape function vector, shape function sorry shape function matrix is going to be like this, because shape function shape functions of nodes 3 and 4 alongside 1-2 are going to be 0.

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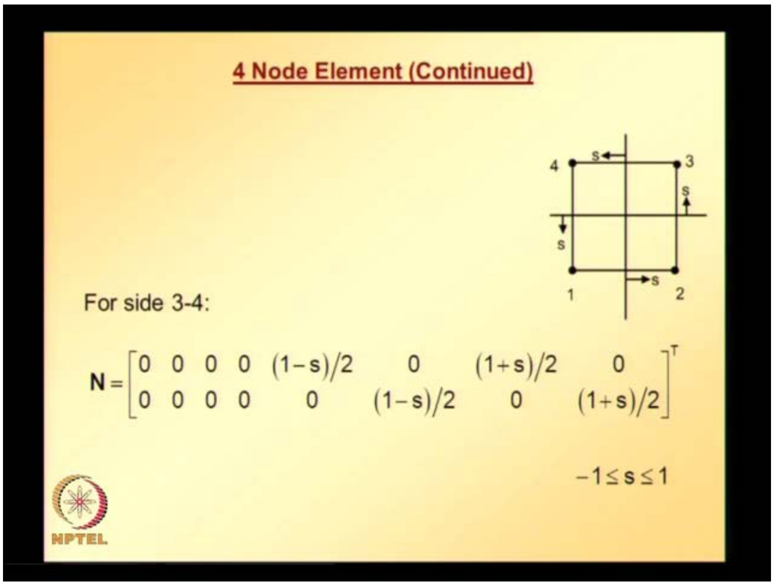

And similarly, side 2-3, shape functions of nodes 1 and 4 are going to be 0 alongside 2-3. Anyway we discussed all these several times in the earlier classes, but for completeness all those are repeated here, for side 3-4, shape functions of nodes 1, 2 are going to be 0. And the shape function matrix for evaluating the equivalent nodal vector.

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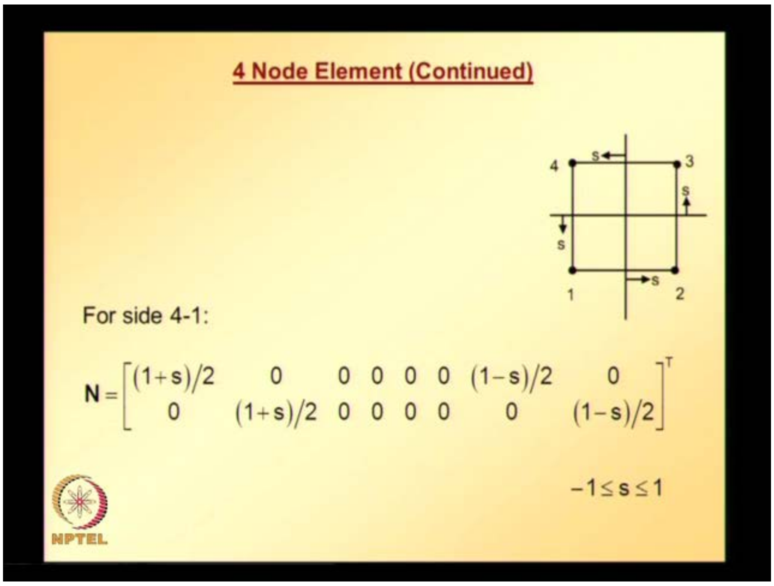

The shape function matrix that is required, if the traction is applied along side 1-2 looks like this, similarly 4-1, side 4-1. So, once we obtain this shape function matrix, we can easily evaluate equivalent nodal load vector; for illustration consider the case, when uniform pressure is applied along side 2-3.

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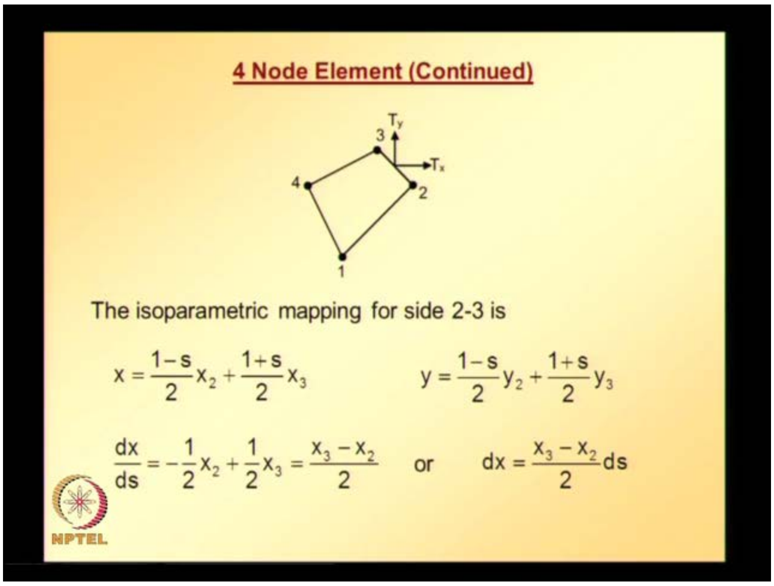

So, to proceed further we need to write, what is isoparametric mapping for side 2-3 and only non-zero shape functions are going to be shape function corresponding to node 2 and 3. Other shape functions are going to be 0 and shape functions of node 2 and 3 for side 2-3 can easily be obtained using Lagrange interpolation formula. And finally, we can write the relationship between x coordinate or local coordinate along that side 2-3 for the actual element, the relationship between the local coordinate system for a side 2-3 of actual element, and local coordinate system of side 2-3 of the parent element. How they are related, we can write through this isoparametric mapping. So, this is how x and y are related to S the local coordinate system of the parent element.

So, once we have this information of x and y, we can easily take partial derivative of x with respect s partial derivative of y with respect to s, this is partial derivative of x with respect s, which can be rearranged as the weight is shown there d x is equal to x 3 minus x 2 divided by 2 times d s.

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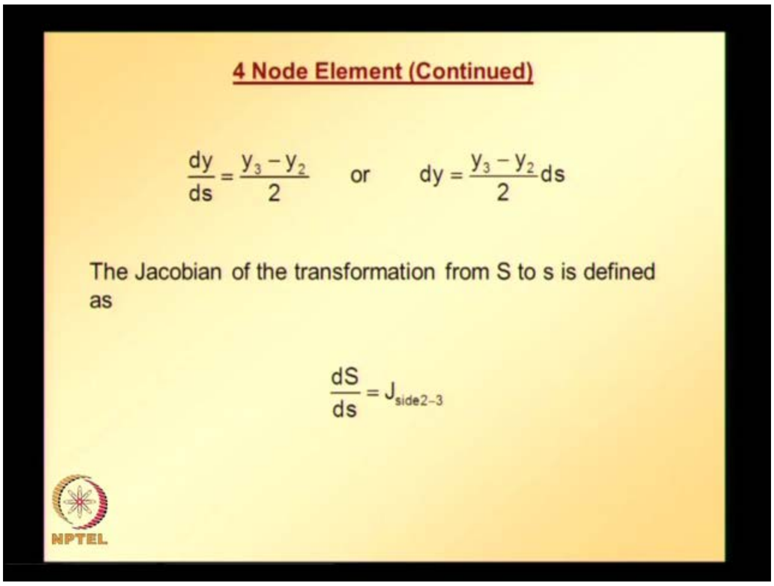

Similarly, partial derivative of y with respect to s, and once we have this we can easily write Jacobian transformation of local coordinate system, along side 2-3 of the actual actual element. And local coordinate system along side 2-3 of the parent element, so the Jacobian transformation from capital S to small s is defined as, this one and we just calculated what is partial or derivative of x with respect to s, derivative of y with respect to s.

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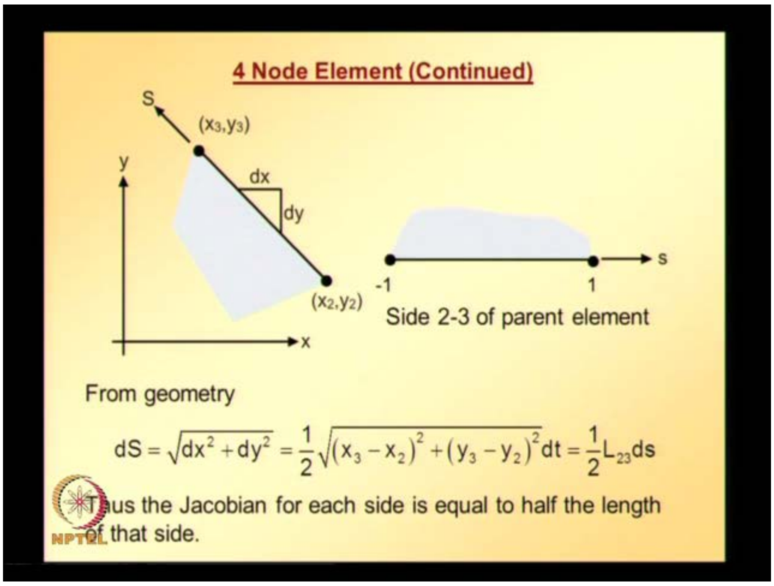

So, using that information and with reference to this figure, which map side 2-3 of the actual element to side 2-3 of the parent element. Differential element in the actual element capital D capital S is from the geometry, we can easily see that, it is going to be square root of d x square plus d y square. And substituting what is d x, d y we get this and here, there is a small type typographical error, it should be d s instead of that d t is typed.

So, finally, we get d capital S is equal to L 2 3 over 2 times d small s, where L 2 3 is nothing but, length of side 2-3. So, the Jacobian is d capital S over d small s is nothing but, length of side 2-3 over 2 half the length of that side, Jacobian for each side is equal to half the length of that side.

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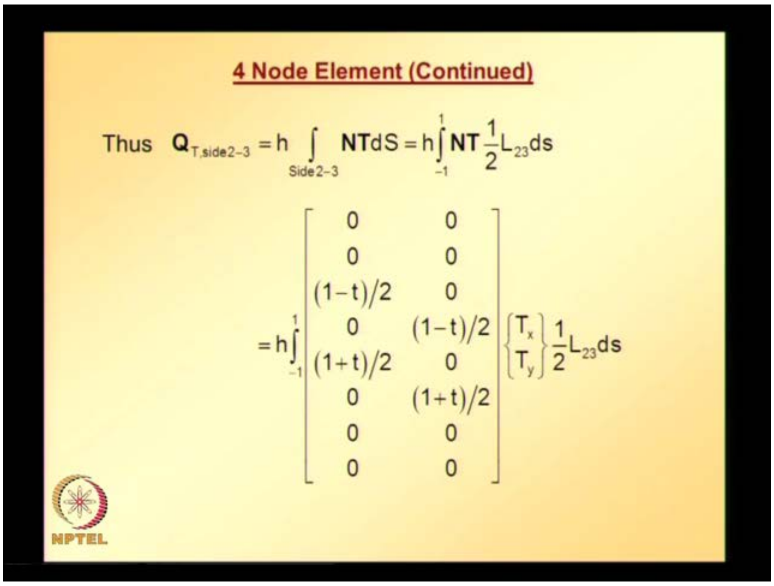

So, we can easily now evaluate numerically by changing limits of integration, once we got this Jacobian, once we obtained the relationship between d capital S and d small s we can change the limits of integration; and adopt numerical integration if required, if the tractions applied are constant, then we do not need to adopt numerical integration, we can do integrations in a closed form. So, since here we assume T x, T y are uniform or in other words they are constant.

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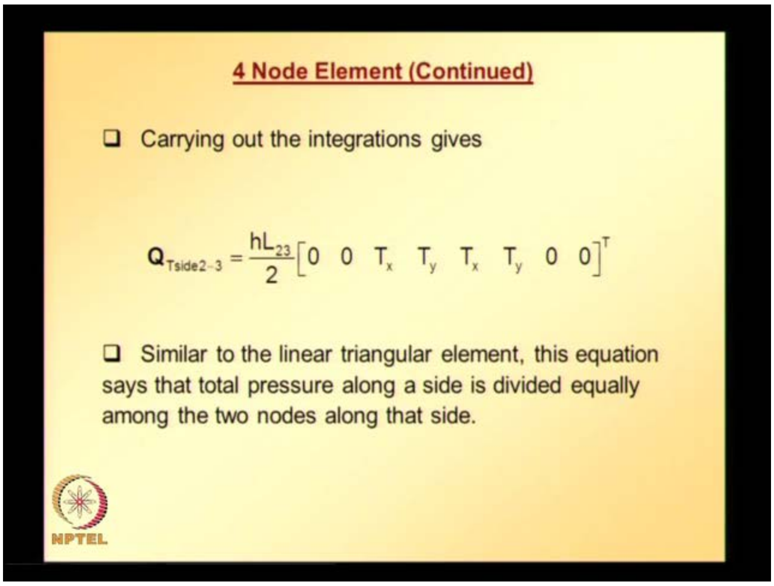

So, we can easily obtain this Q vector in a closed form, so carrying out integrations we get Q in this manner. Similar to linear triangular element this equations this equation says total pressure along side along the side is divided equally among 2 nodes along that side.

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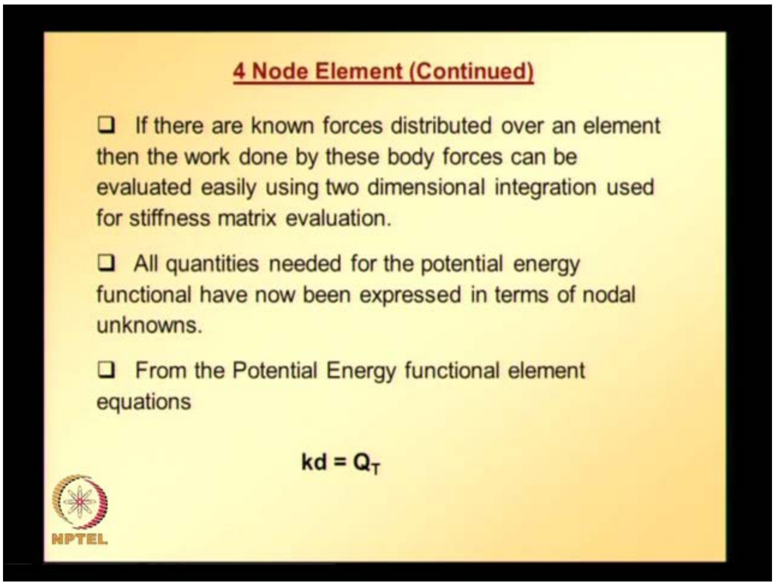

If there are known forces distributed over an element, than the work done by these body forces can be evaluated easily using two-dimensional integration used for stiffness matrix evaluation. All quantities needed for potential energy functional have now been expressed in terms of nodal unknowns, so finally, from potential energy functional, we get k d equal to Q t.

So, to illustrate all these details, let us take an example, example is same as what we used for illustrating 3 node triangular elements, except that there we adopted two elements 2, 3 node triangular elements. Whereas, here we will be using only one 4 node quadrilateral element for solving the same problem, cantilever plate problem.

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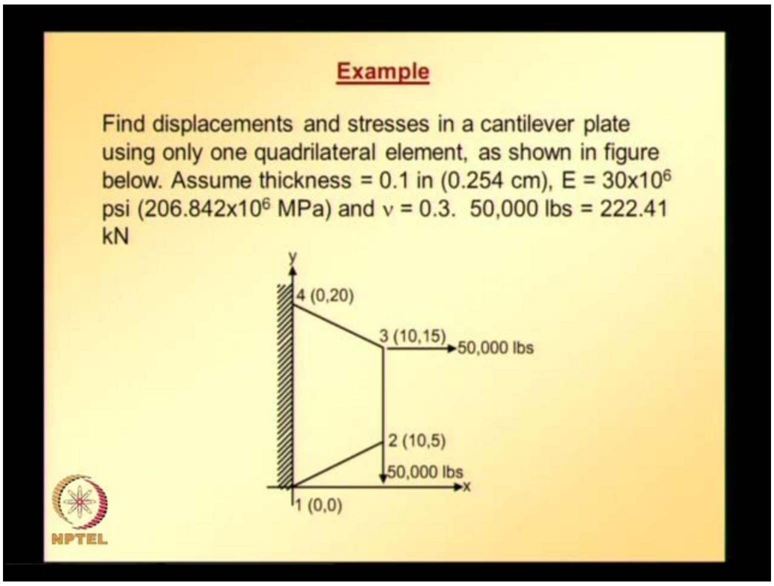

Again all the quantities are given both in FPS units and SI units and since, the quantities that are given in FPS units are round numbers, we will proceed with FPS units, but it is not going to make much difference, because as long as we are consistent with the units we get solution.

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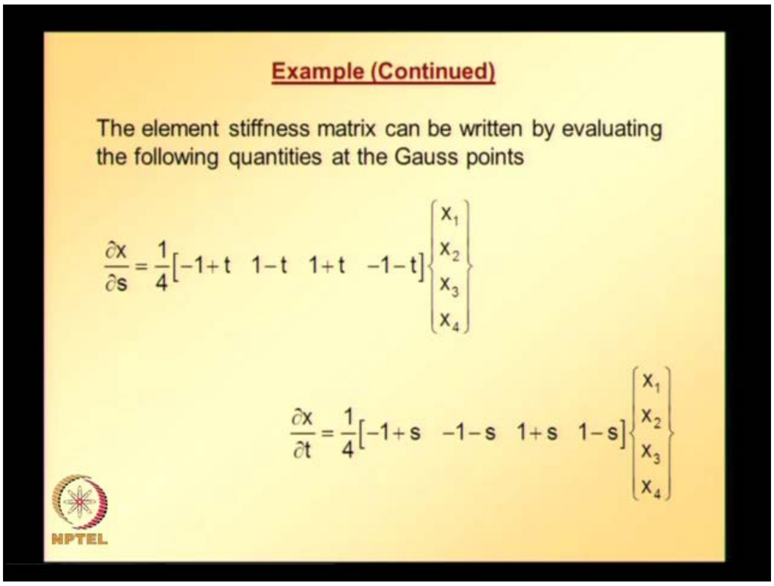

Element stiffness matrix can be written by evaluating following quantities at Gauss points, so we require to evaluate, we are now adopting 4 node quadrilateral element. So, we need to use 4 node element, 4 node parent element shape functions and try to find what is partial derivative of x with respect s, partial derivative of x with respect to t, partial derivative of y with respect s, partial derivate of y with respect to t, and after that we can easily find determinant of J.

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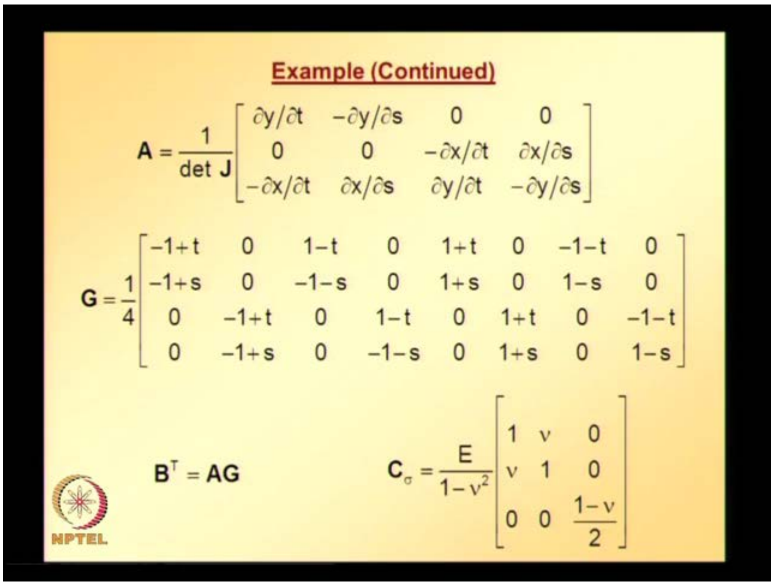

And once, we have determinant of J, we can even calculate what is that intermediate matrix A required for getting strain displacement relation, and also one more intermediate matrix is required; and once we have these two, we can get strain displacement matrix. And constitutive matrix for plane stress, and once we have all these evaluate all these quantities at each integration point, and then depending on the number of integration points.

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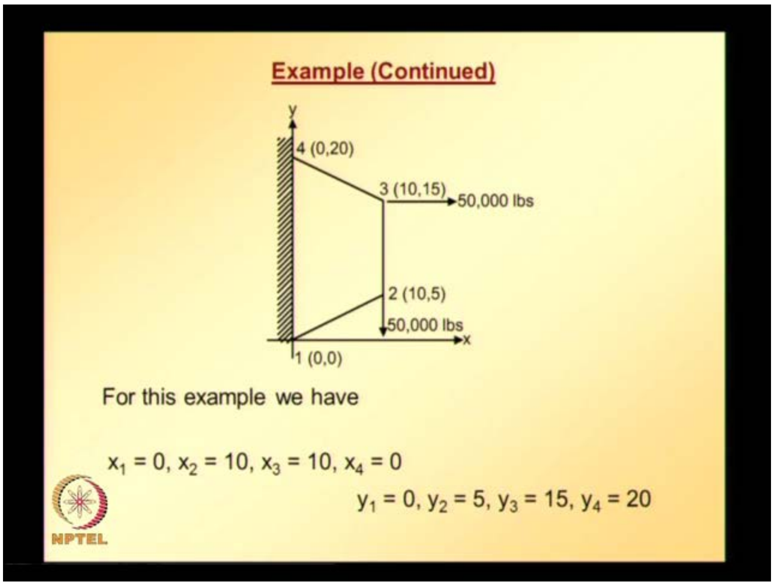

Depending on the number of integration points, evaluate the integrand at each of the integration point multiply with corresponding weight, and sum it up we are going to entire approximate value of entire stiffness matrix.

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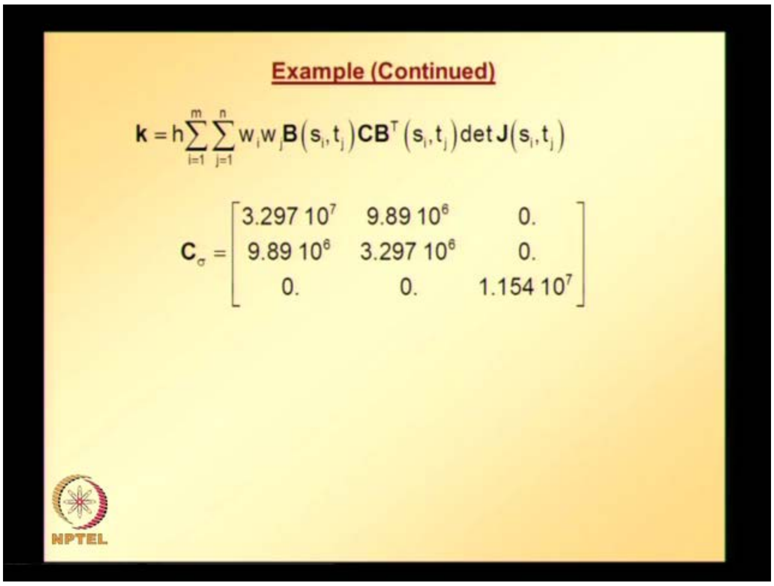

So, to proceed further, we need to make a note of all the coordinates coordinates of all nodes, x coordinates and y coordinates.

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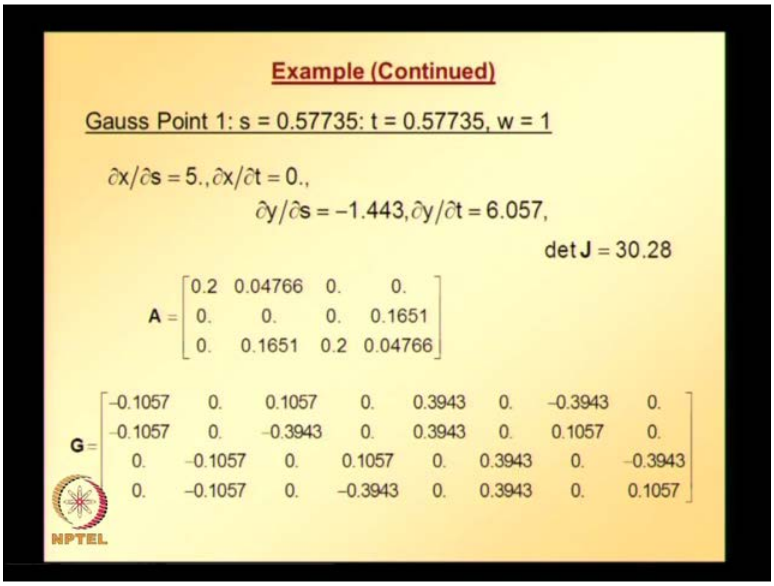

And once we make a note of it, they are ready to evaluate stiffness matrix using Gauss integration, so here we are adopting 2 by 2 integration, so there there are total in total there are going to be 4 integration points, how to obtain coordinates and weights of the corresponding integration points, we already discussed several times earlier. So, now let us take 1 integration point, and see the details and similar procedure can be repeated in the other integration points. So, Gauss point that is selected is given the details are given coordinates and weights.

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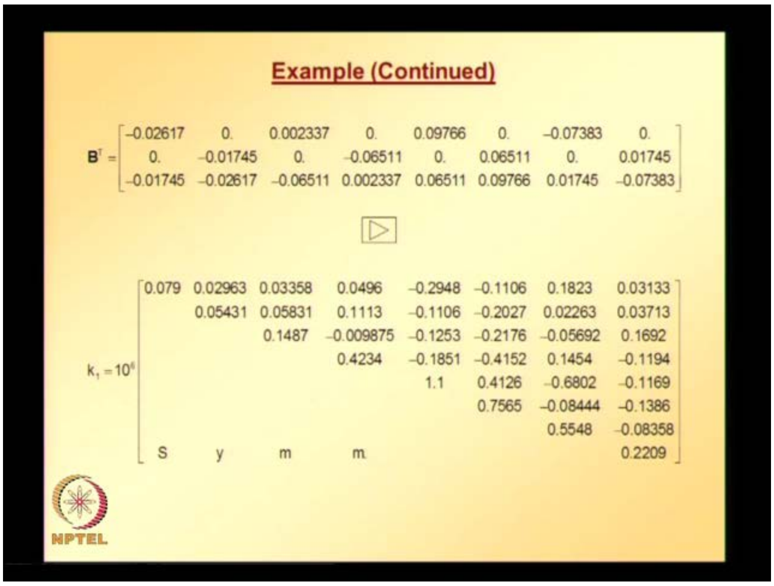

So, evaluate all the required quantities at this integration point and finally, we can evaluate at this integration point, what is strain displacement matrix, and once we have that we can easily evaluate what is stiffness matrix at this integration point. Performing similar calculations for other integration points, and adding the resulting matrices, we get the following or we get the element stiffness matrix.

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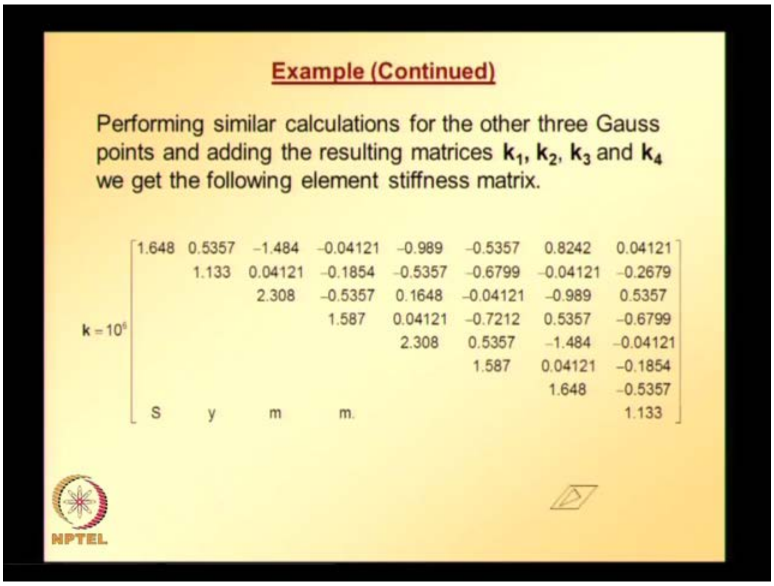

So, we obtained the element stiffness matrix, and there is not much inward as far as assembly of equivalent load vector is concerned, because all the point loads are given, so the all the loads that are applied are concentrated loads. So, only we need to just plug in the the corresponding value at the appropriate location in the load vector. And if you see this problem, there are 4 nodes and nodes 1 and 4 are fixed, so all degrees of freedom corresponding to these nodes are going to be 0.

So, in the final global equation system, we are going to anyway eliminate the rows and columns corresponding to degrees of freedom which are 0. So, finally, we can directly write the reduced equation system, which consists of degrees of freedom corresponding to nodes 2 and 3, which are u 2, v 2, u 3, v 3 because, degrees of freedom corresponding to nodes 1 and 4 are going to be 0.

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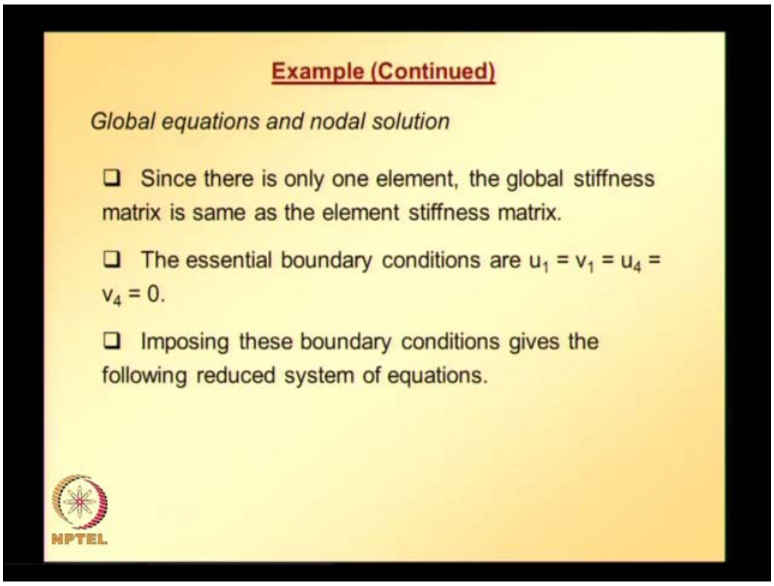

Global equations and nodal solution, since there is only one element the global stiffness matrix is same as element stiffness matrix, essential boundary conditions. These are the essential boundary conditions; imposing these boundary conditions gives the following reduced system of equations.

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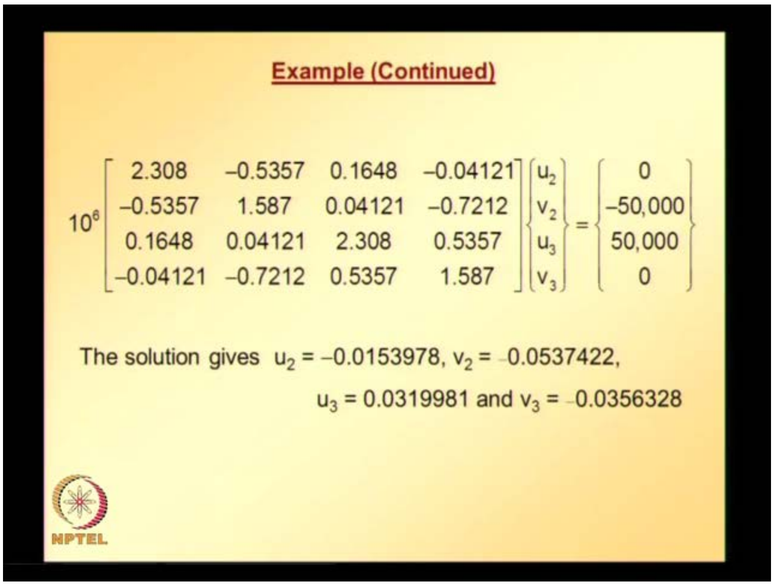

And solving this we get the displacements at nodes 2 and 3, once we obtained the displacements at nodes 2 and 3, we can easily do post processing like calculation of stresses.

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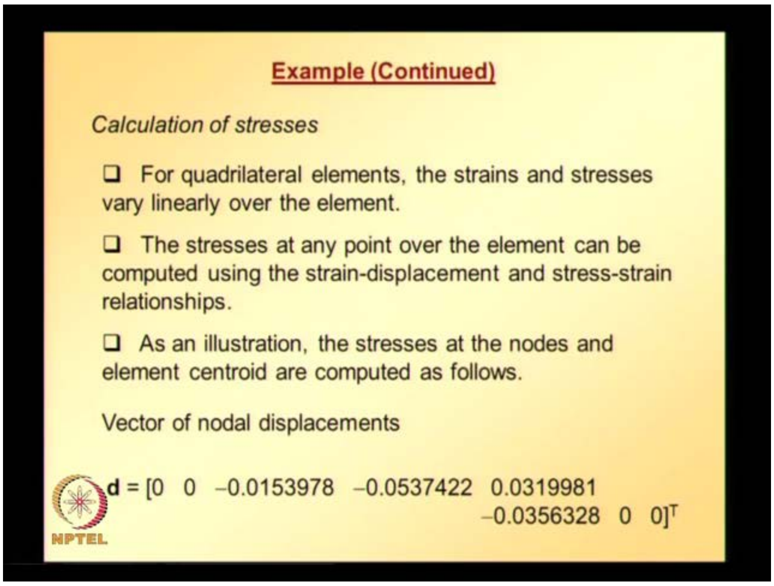

So, calculation of stresses for quadrilateral elements, the strains and stresses vary linearly over the element because, if you see the derivatives or if you see the B matrix, B matrix is going to be linear over the element. So, for quadrilateral element strains and stresses vary linearly over element; stresses at any point over element can be computed using strain-displacement and stress-strain relations, as an illustrations stresses at the nodes and element centroid are computed as follows.

First to do that, we require vector of nodal displacement since, we calculated displacements at node nodes 2 and 3, and the other displacements are 0, we can easily write the vector of nodal displacements. And now, we are interested in finding stresses at nodes, so we need to know what is the corresponding s and t coordinates.

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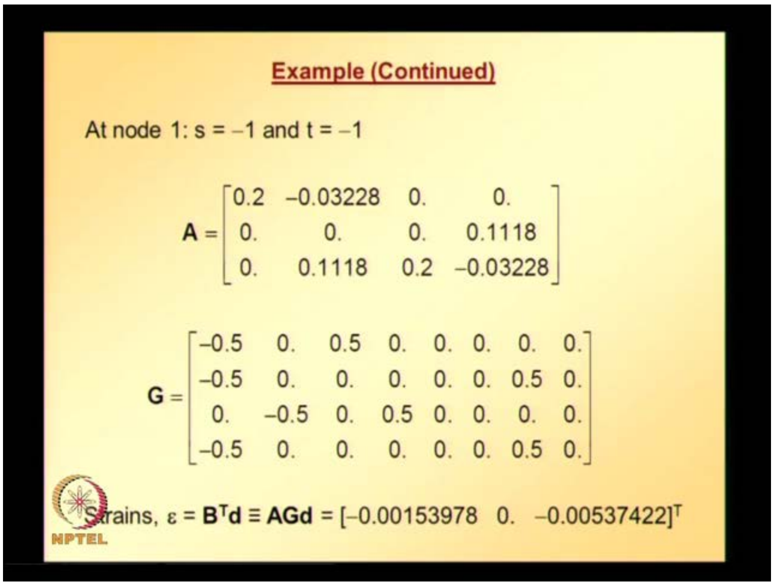

So, at node 1 s is equal to minus 1, t is equal to minus 1, so A matrix, G matrix once we have these two matrices, we can easily get what is B matrix. And once we know B matrix we can get this strain, because strain is related to nodal displacements y of this B matrix.

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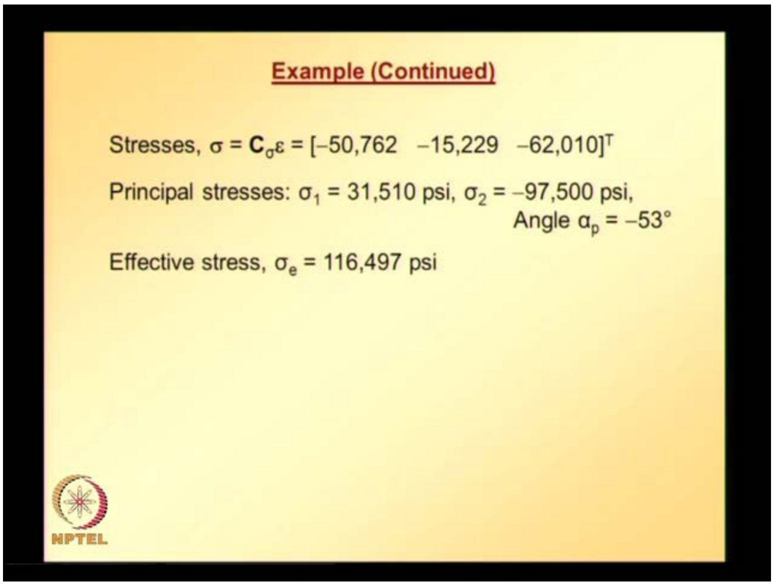

Hence, once we have these strains, we can calculate stresses and then do rest of the post processing similar to what we did in the last class, like calculating principle stresses and other details.

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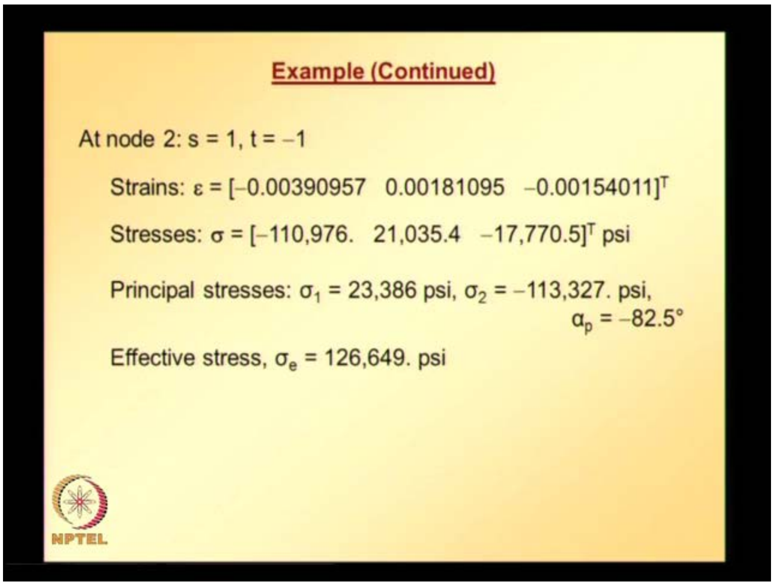

Similar, calculations can be repeated at other nodes by taking appropriate s and t values, this is at node 2, node 3 (Refer Slide Time: 33:38).

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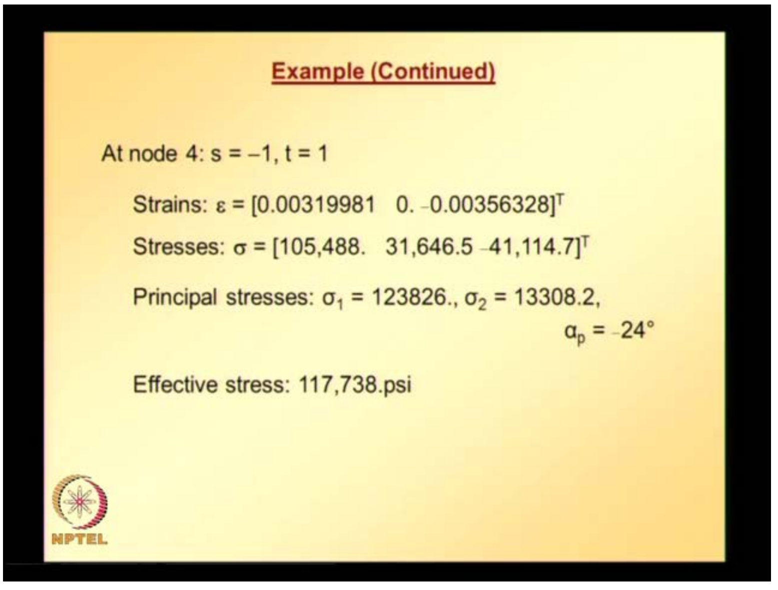

And that node 4, and please note that element centroid corresponds to s is equal to 0, t is equal to 0.

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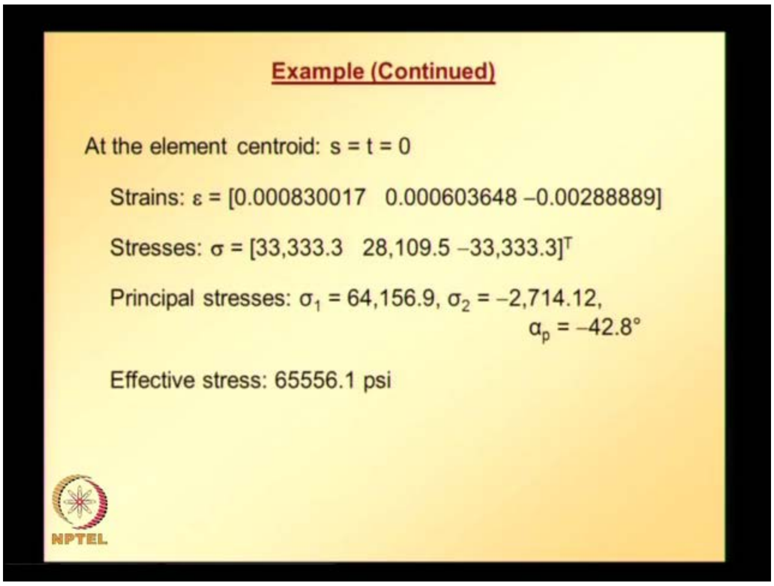

So, if somebody is interested in finding the stresses and strains, and the corresponding principle stresses and equivalent stress quantities are effective stress quantities at the centroid, we need to calculate do the perform the calculations by taking s is equal to 0, t is equal to 0. So, strain, stresses and all those details at this point are given.

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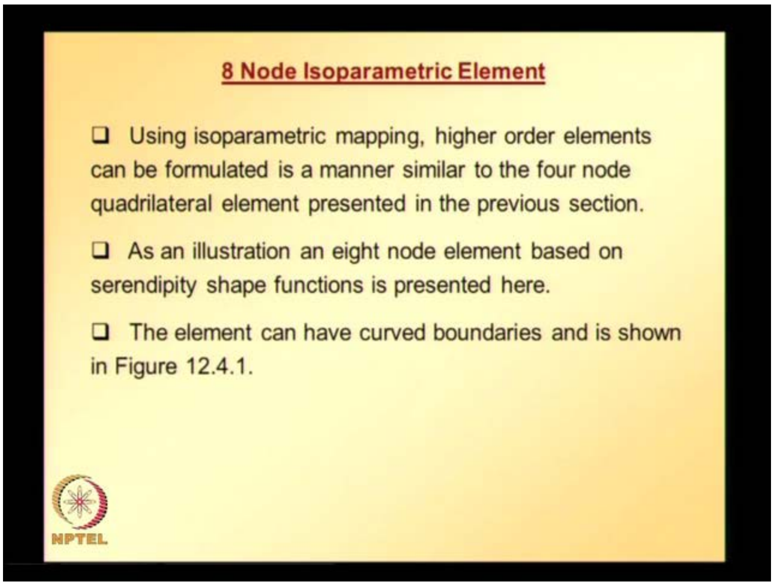

So, the next element that we are going to look at is 8 node isoparametric element, using isoparametric mapping higher order elements can be formulated in a manners, similar to 4 node quadrilateral element.

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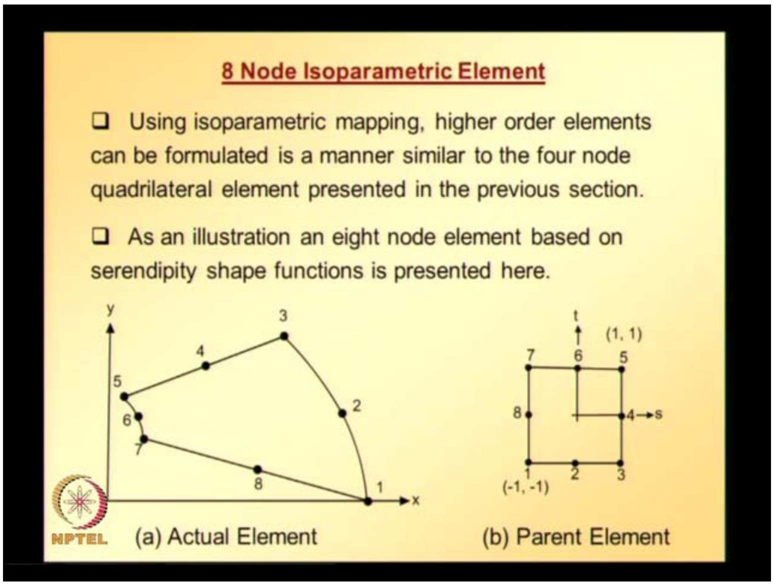

As an illustration an 8 node element based on serendipity shape functions is presented, element can have curved boundaries, like this actual 8 node element is shown and also 8 node parent element is also shown. So, for by this time you are familiar for writing isoparametric mapping relations, we require to know, what are the shape functions of all the 8 nodes of 8 node parent elements.

So, first thing is we require to write or note down what are the shape functions of all the 8 nodes of 8 node serendipity element, which is shown serendipity element, parent element which is shown on the right hand side, and the actual element in the figure is shown on the left hand side.

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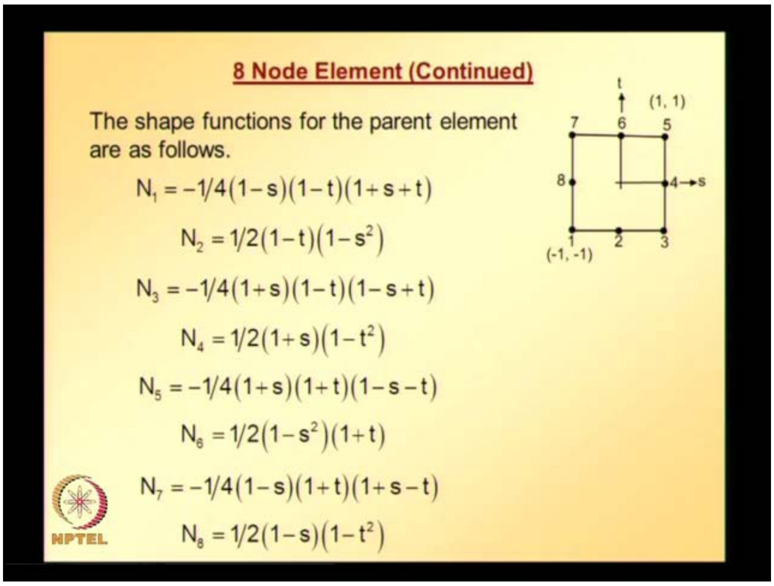

This shape functions for the parent element are as follows, all the shape function expressions we have seen earlier, and also we discussed how to derive this.

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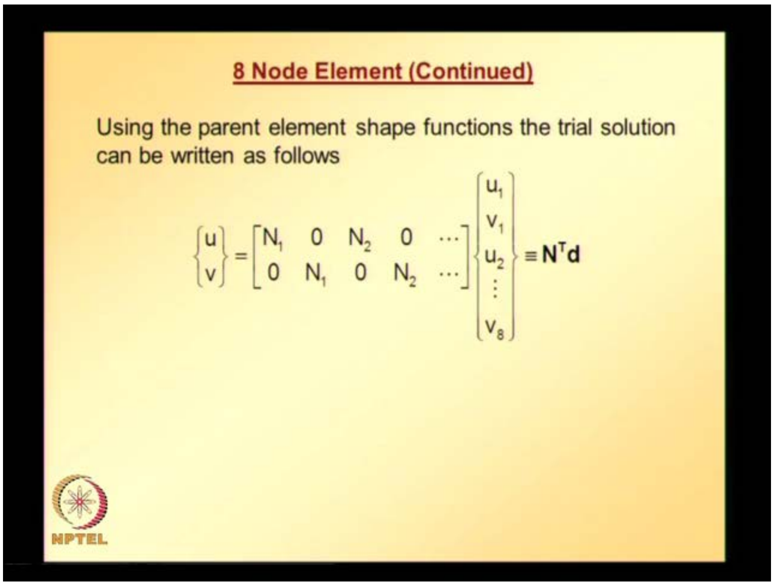

Using the parent element shape functions, the trial solution can be written as follows displacement component in the x direction is given by, is can be obtained using all the 8 node shape functions, and the corresponding nodal values at the 8 nodes. So, suppose if you if you want to displacement in the x direction, then it can be interpolated using displacement component at all the 8 nodes in the x direction, and the corresponding shape functions. Similarly, in the y direction, so this is how trial solution can be written and isoparametric mapping, once we have these two relations, we can write what is Jacobian matrix, determinant of Jacobian and also shape function derivatives an x with respect to x and y; all this is again same as for 4 node quadrilateral element.

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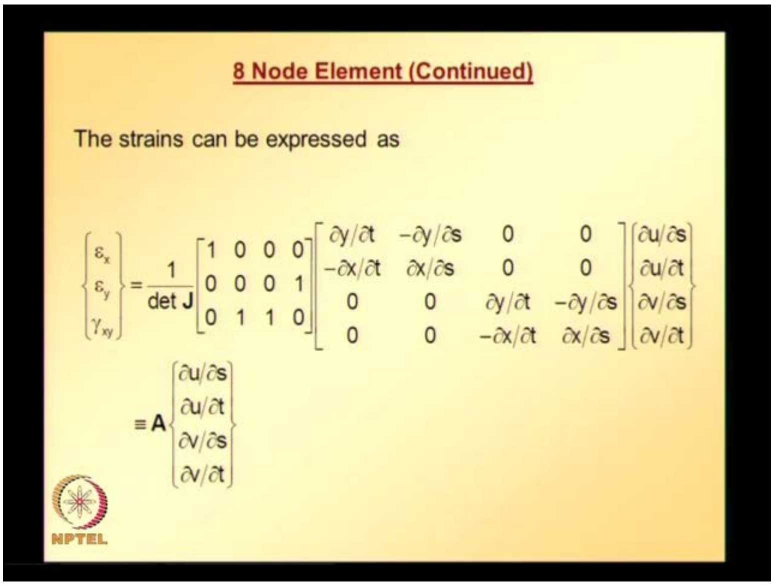

So, strains except the dimension of the matrices get increased, because we have 8 nodes, so we have more degrees of freedom. And derivatives of trial solution with respect to s and t.

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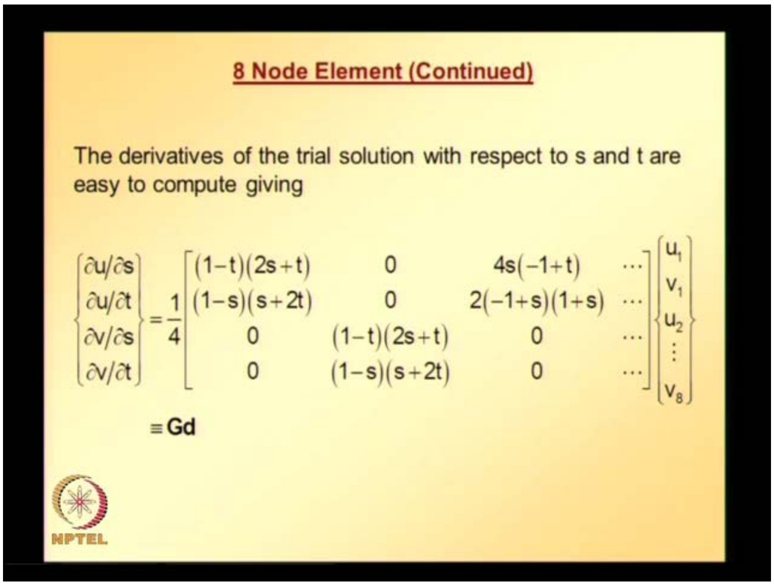

Again the dimension of G matrix increases, because there are 8 nodes, so once we have this we can write strain displacement matrix.

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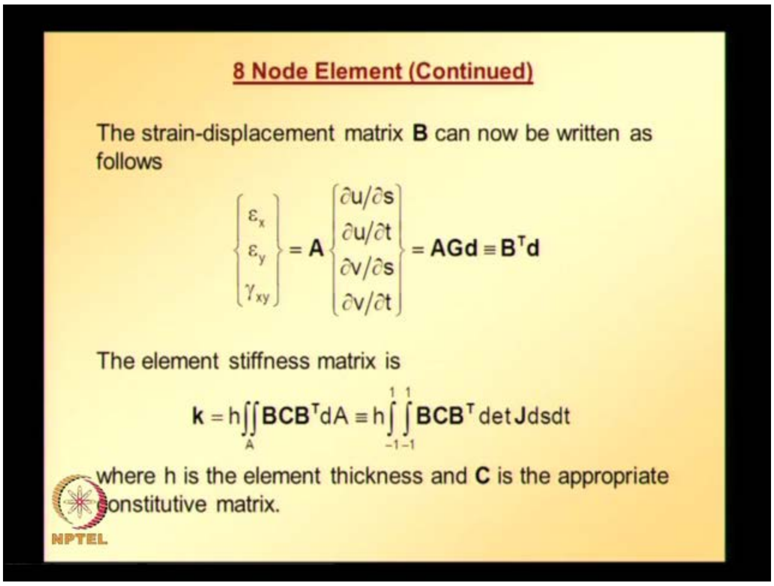

And also which can be written as epsilon is equal to B transpose d in compactly, so then we can write the element stiffness matrix, again now B matrix becomes more complicate, a B matrix is more complicated for 8 node case. So, the integrand becomes more complicated, so better we adopt numerical integration, because it is not possible to obtain or evaluate the integral in a closed form, where h is thickness element thickness, c is appropriate constitutive matrix.

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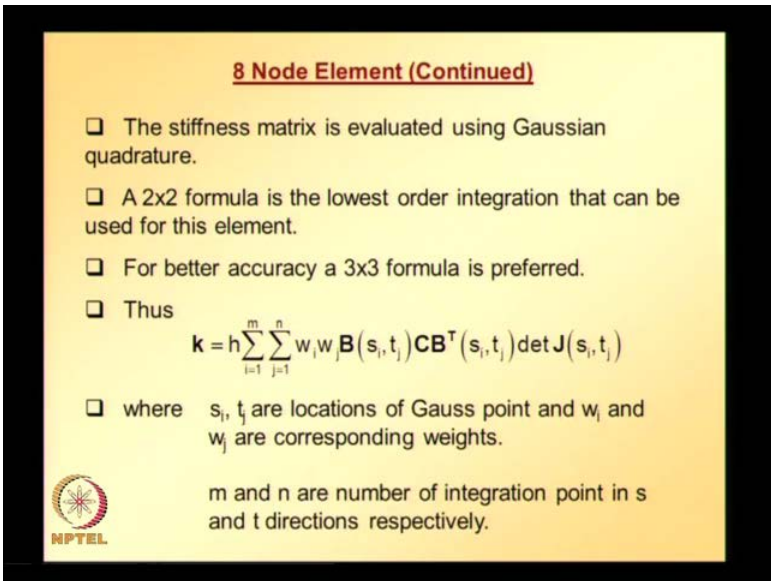

Stiffness matrix is evaluated using Gaussian quadrature 2 by 2 formulas is the lowest order integration that can be used for this element; you can easily guess why it is the lowest order of integration that can be used for this element, because we are 8 node element is the quadratic element. So, if you go back and apply the formula 2 n minus 1 criterion for deciding the number of integration points that are required to exactly evaluate an integral that we discussed earlier.

You can easily check that, the lowest order integration that can be used for this element is 2 by 2, for better accuracy 3 by 3 formulas, is 3 by 3 integration is preferred. So, the k matrix stiffness matrix can be numerical evaluated like this, were rest of the details are similar to 4 node quadrilateral element, so this is about element stiffness matrix.

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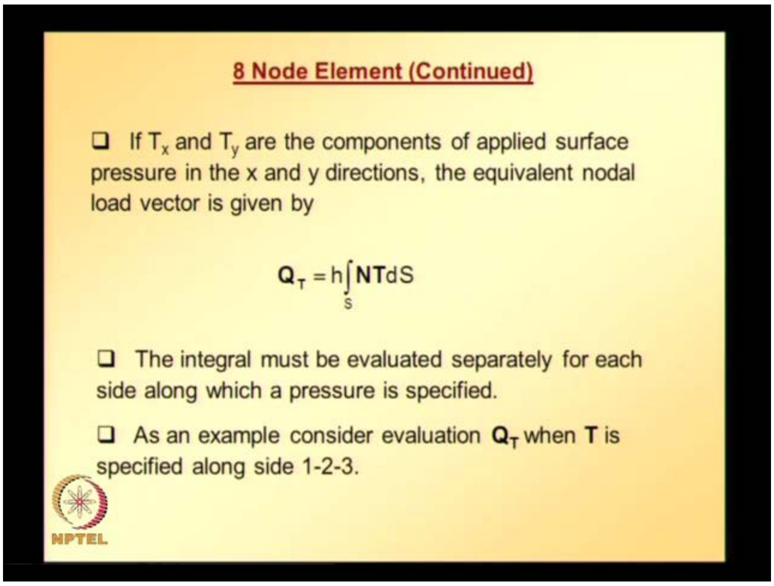

So, what about equivalent load vector, if T x, T y are the components of applied surface pressure in x and y directions, the equivalent nodal load vector is given by this one. So, to evaluate equivalent nodal load vector again we require knowing, what is the shape function matrix along that particular side, along which fractions are specified. Integral must be evaluated separately for each side along which a pressure is specified, as an example considers evaluation of Q T, when T is specified alongside 1-2-3.

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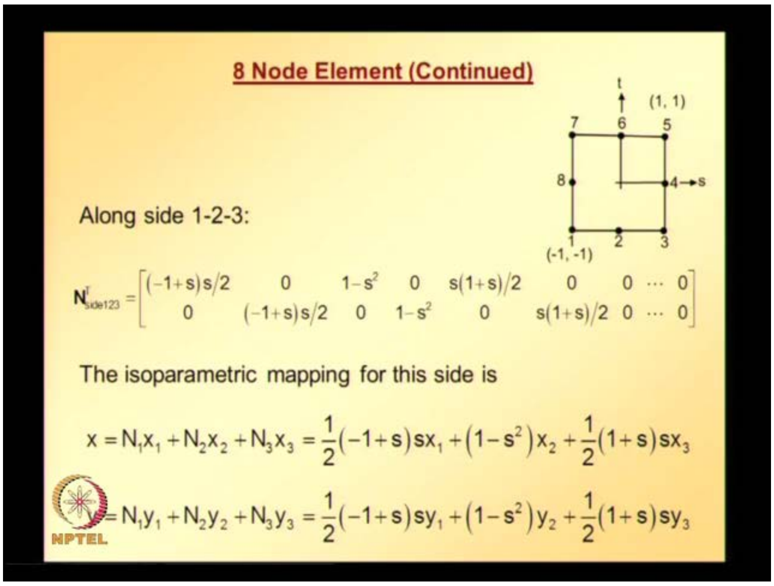

So, T is specified along side 1-2-3, so we need to write shape function matrix along side 1-2-3, and then we need to write isoparametric equations and because, we need to know what is the Jacobian, how differential element in the parent element along that particular side is related to differential element, in the axial element along that particular side. So, for that we require isoparametric mapping relation for this side, so that is given by the shape functions, which are non-zero along that side 1-2-3.

And since, shape functions of other nodes alongside 1-2-3 except nodes 1, 2, 3 are 0, we can easily obtain shape functions of nodes 1, 2, 3 along side 1-2-3 using Lagrange interpolation formula. And then subsequently we can write the relationship between x and local s coordinates system, and y and local s coordinate system.

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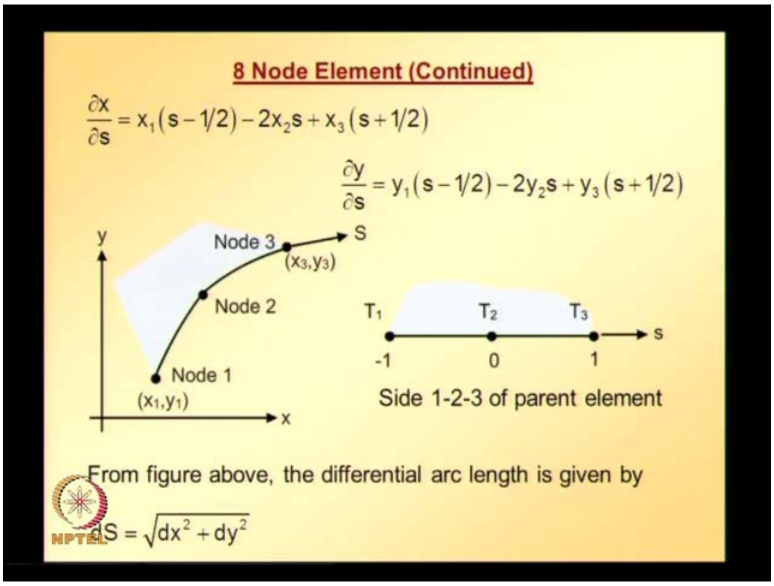

And once we have these two relations, we can easily calculate what is partial derivative of x with respect s, partial derivative of y with respect s, and this schematic shows the differential element, how the differential element in the axial how differential element in the axial element is related to differential element in the parent element. So, differential arc length is given by in the axial element d capital S is given by square root of d x square plus d y square, this is similar to 4 node quadrilateral element except that now, we need to substitute d x, d y corresponding to 8 node elements.

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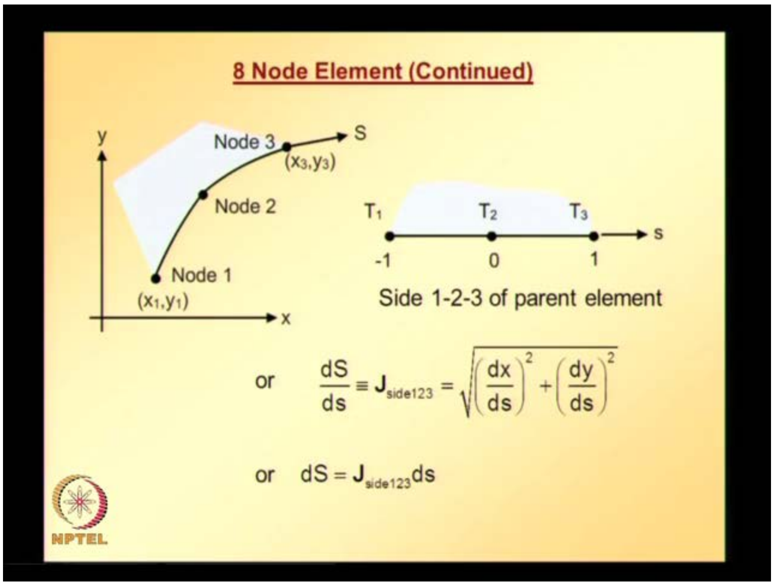

And dividing on both sides with d small s, we get this and now substituting what is d derivative of x with respect s, derivative of y with respect s in this, we can easily find what is the relationship between d capital S and d small s are the Jacobian.

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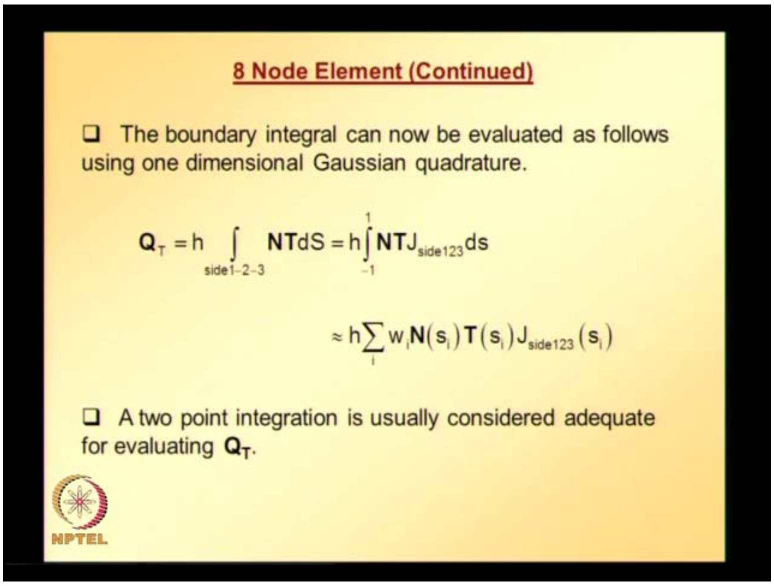

Once we have that relation, we can change the limits of integration and evaluate the integral using numerical integration, one-dimensional numerical integration; two point integration is usually considered adequate for evaluating Q T.

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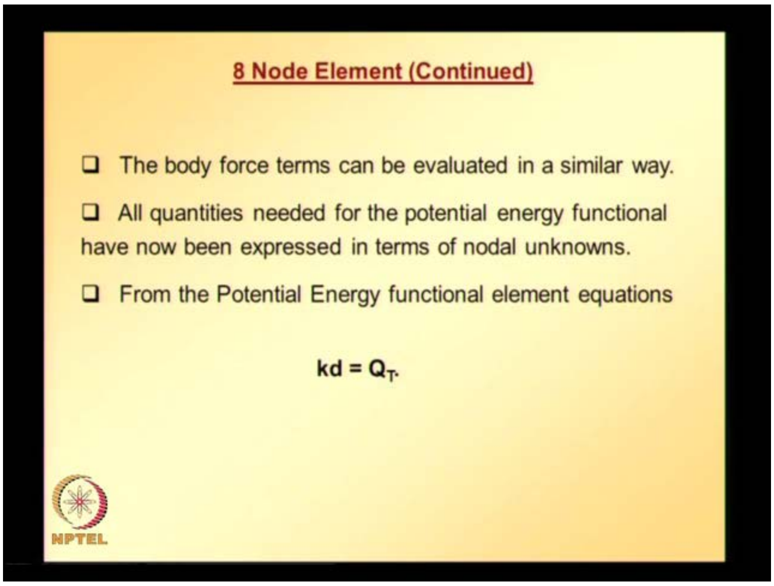

Body force terms can be evaluated in a similar way, all quantities needed for potential energy functional have now been expressed in terms of nodal unknowns, so finally, we get k d equal to Q T. So, all these details will illustrate using example 8 node element.

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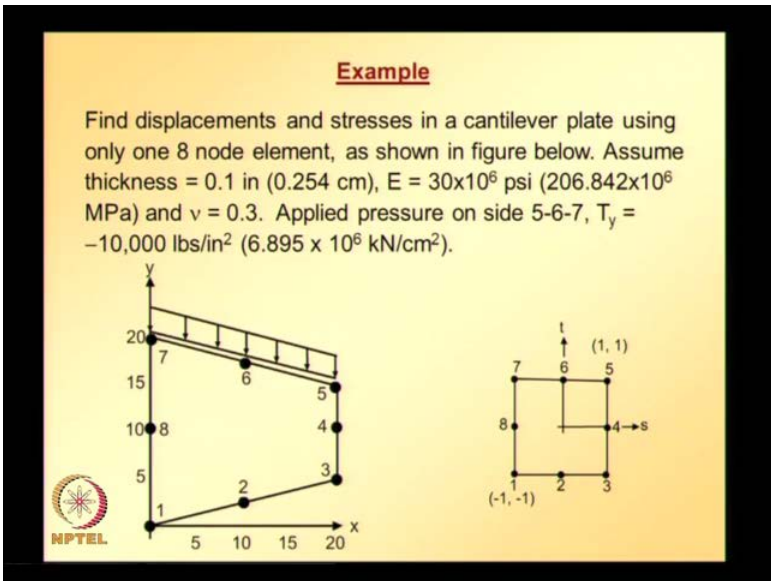

Here, also we will be taking cantilever plate, all the geometrical details, material details are given both in FPS units and SI units. And also pressure here, the difference between the earlier examples, which we have seen, when we are looking at 3 node triangular element and 4 node quadrilateral element is there we have concentrated forces, where we where it is not required for us to evaluate the equivalent nodal load vector by doing integration, we can directly obtain that from the values of the concentrated forces that are applied. Whereas here, uniform are applied pressure is applied, so we need to perform integration, and we will see or we will find an interesting observation when we when we do that.

So, now let us proceed this is the problem, so we need to find displacements and stresses in this cantilever plate using 1, 8 node element and pressure is applied on side 5-6-7, and its applied in the y direction and the value is given both in FPS units and SI units. This 8 node element, this entire cantilever plate is discretized using only one 8 node element this, and 8 node element, actual element we are going to mapped on to a a parent element which is shown on the right hand side.

And we will be using 3 by 3 Gaussian integration, so total you can guess total 9 integration points are there and I guess, you know how to obtain the corresponding weights and coordinates for all this points, so now the calculations will be illustrated for one integration point. So, similar process can be repeated for the other integration points, and contribution from all integration points can be added up to get the final element stiffness matrix.

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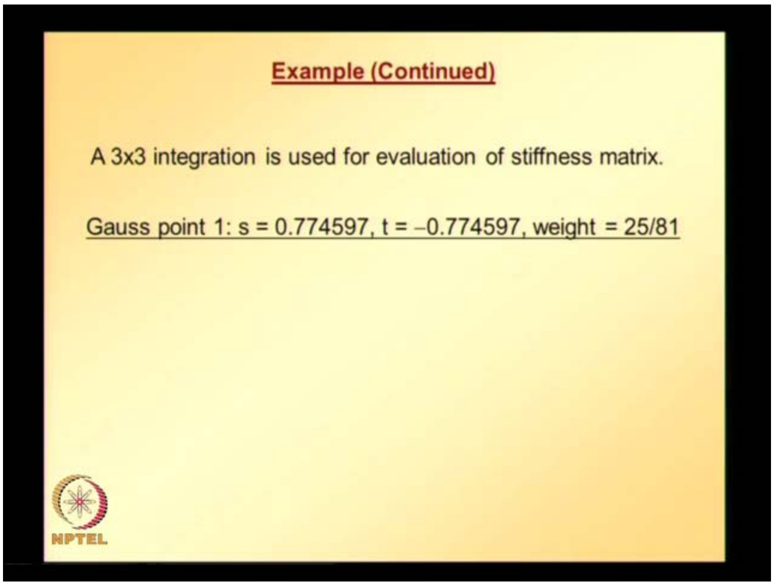

So, now let us take 1 integration point, coordinates and weights are given.

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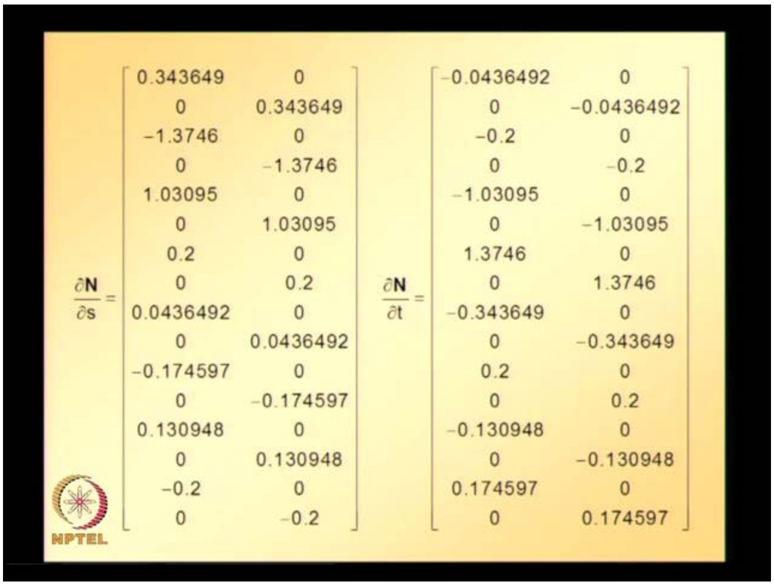

And we require, at that point we require to find what is partial derivative of shape function vector with respect to shape function derivative; shape function vector derivative with respect to s and t those details are given here.

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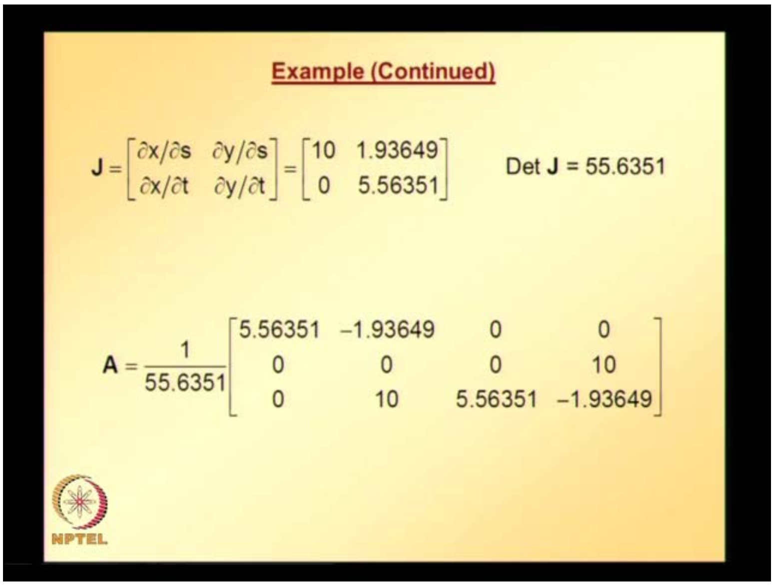

And once we have that, we can easily calculate, what are these quantities.

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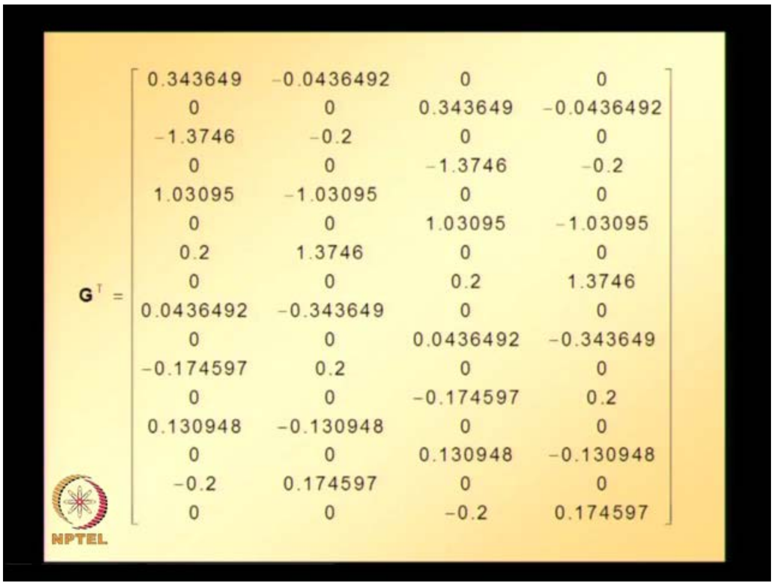

And once, we have this we can also calculate what is G, and once we have A and G we can calculate what is B matrix, and plane stress conditions are assumed here.

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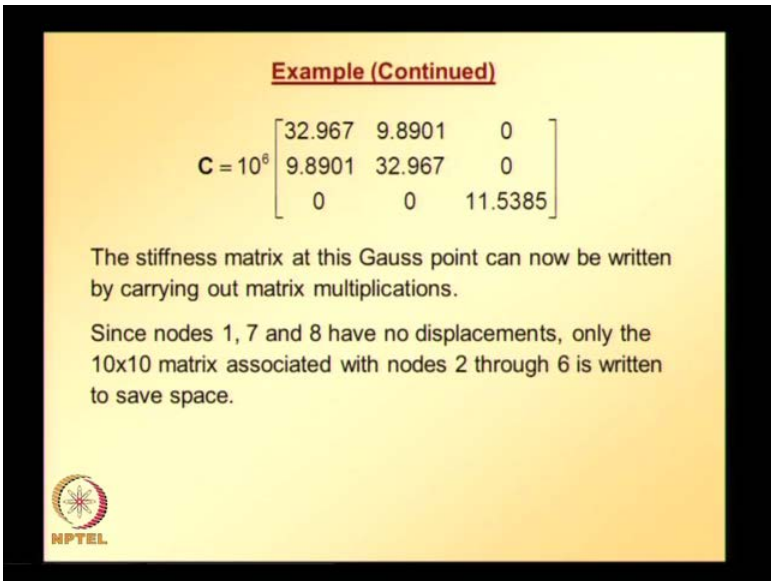

So, using that information we can easily using the material property details, we can easily write constitutive matrix for this particular problem. So, stiffness matrix at this Gauss point can now be written by carrying out matrix multiplications; since, nodes 7 1, 7, 8 have no displacements, only the 10 by 10 matrix associated with nodes 2 through 6 is written to save space. So, basically the nodes along which are fixed since both degrees of freedom are constraint. So, finally, when if you go to the final reduced equation system the equations are the the rows and columns corresponding to the degrees of freedom of which are 0 are going to be eliminated, so they are eliminated at this stage itself to save this space. So, the stiffness matrix contribution, the global reduced equation system from integration point can be obtained by carrying out matrix multiplications.

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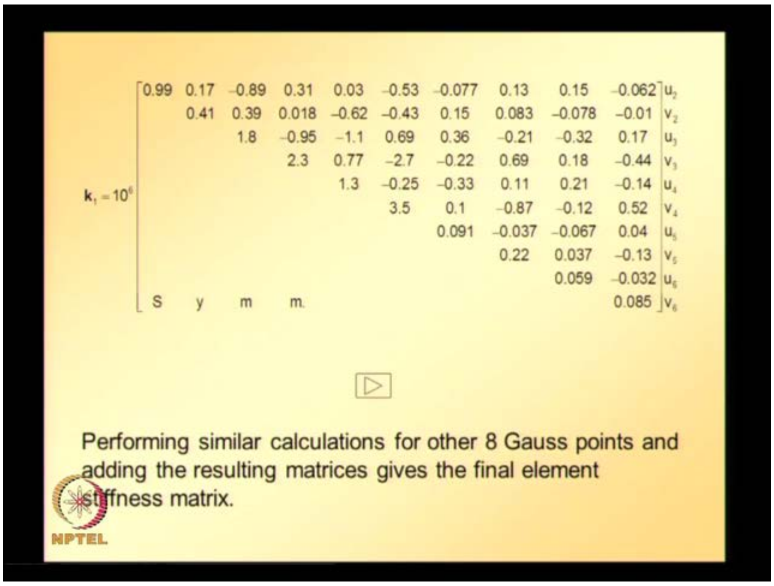

And performing similar calculations for other 8 Gauss points, and adding the resulting matrices gives final element element stiffness matrix.

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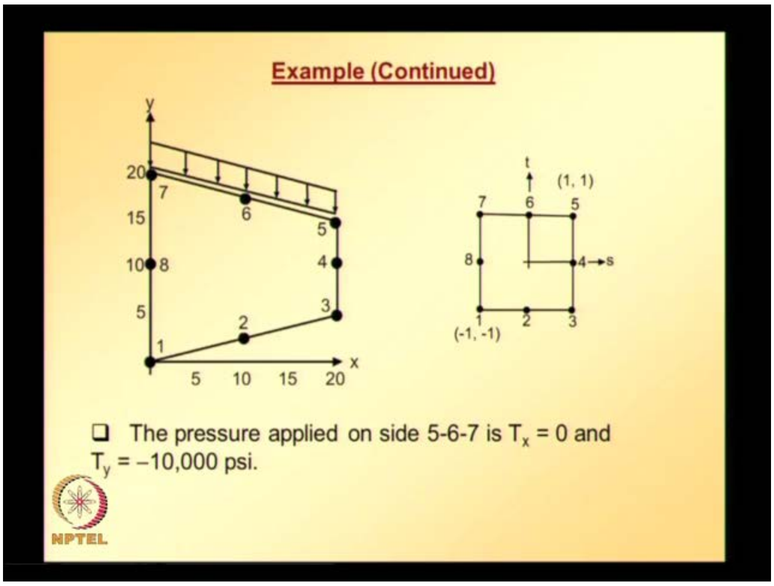

And now, since pressure lode is applied along side 5-6-7, let us discuss how to assemble the equivalent load vector for this applied pressure, so the value of applied pressure is given here, T x is equal to 0, T y is equal to minus 10000 psi. And also because, this pressure load is applied on side 5-6-7, it will result in nodal loads in y direction only at nodes 5-6-7.

A two point integration is used for evaluating the line integral and also you can see, when we are trying to write the shape function matrix along side 5-6-7, the shape functions of all other 5 nodes are going to be 0 along side 5-6-7. So, to simplify writing only non-zero entries are written.

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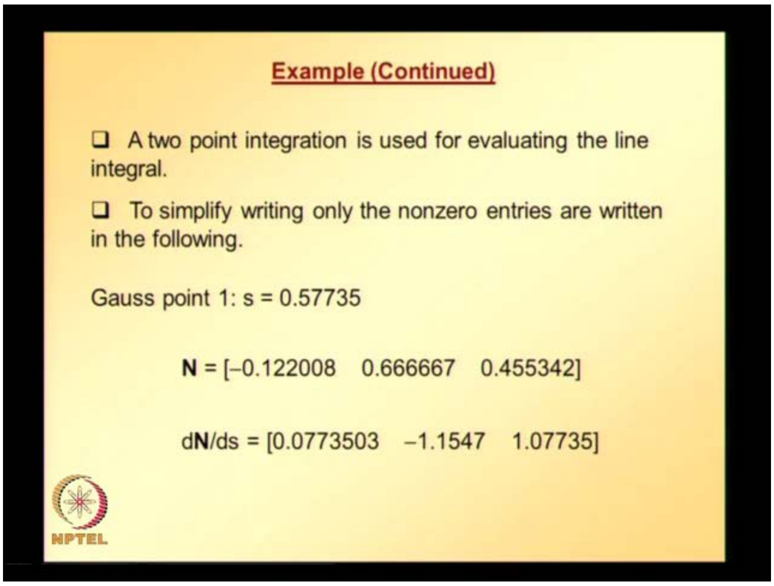

So, at integrate two two point integration is used to evaluate this line integral to get the equivalent nodal load vector. So, the first integration point shape function vector, which comprises of shape function value of node 5, 6, 7 and the derivative of that and Jacobian.

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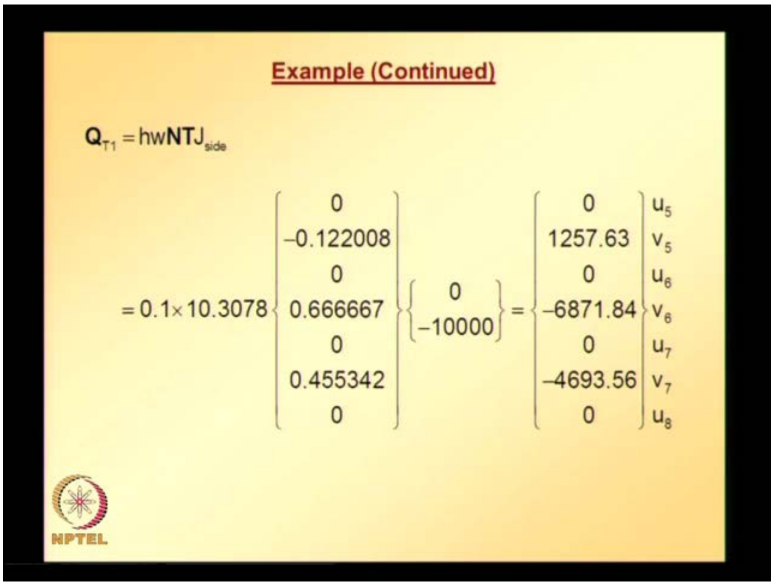

And finally, evaluating equivalent nodal load vector at integration point one we get this.

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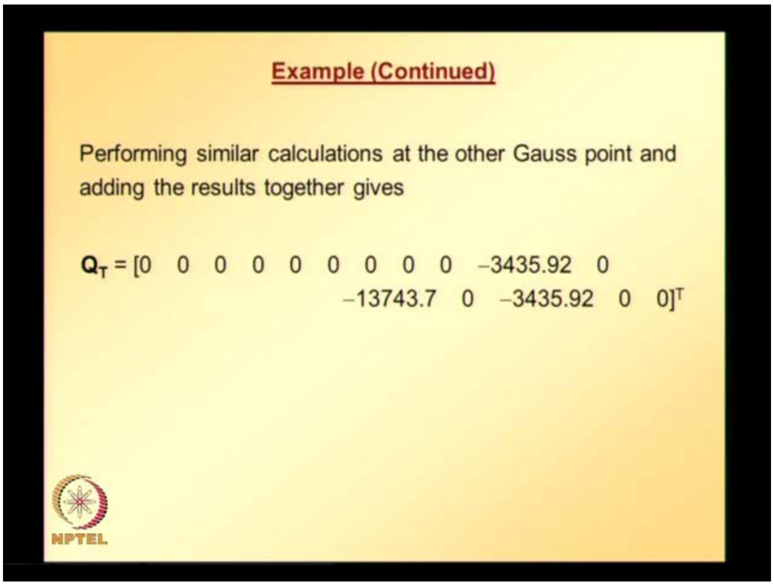

Performing similar calculations at the other integration points and adding the results together we get, and here the entire entire vector is written, entire load vector is written.

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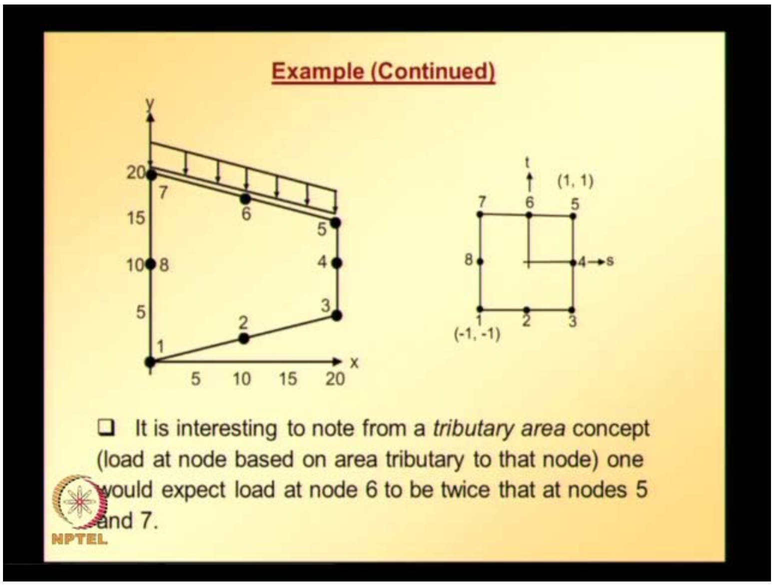

And if you look at this one, it is interesting to note that from tributary area concept, lower at the node based on area tributary to that node; one would expect load at node 6 to be twice that at nodes 5 and 7.

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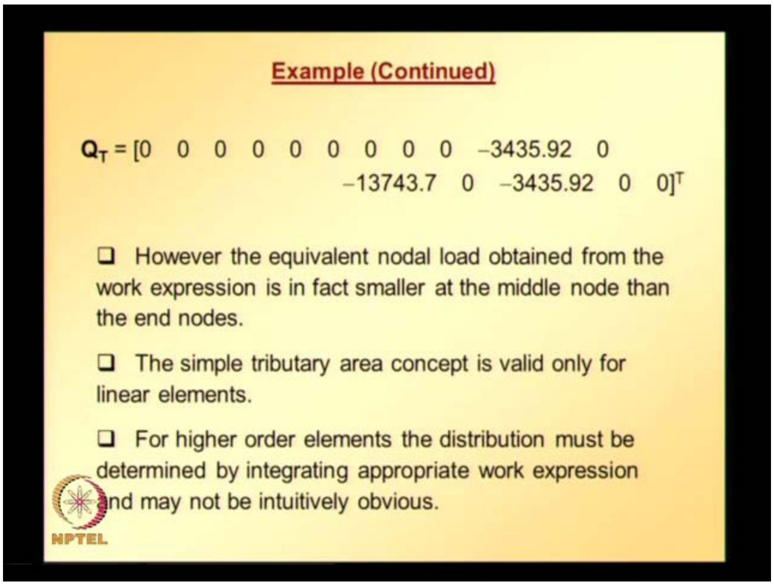

But, that is not happening here; however, equivalent nodal load obtained from the work expression is in fact, smaller at middle node than at the end nodes. The simple tributary area concept is valid only for linear elements, for higher order elements the distribution must be determined by integration integrating appropriate work expressions, may not be intuitively obvious, so keep this in mind.

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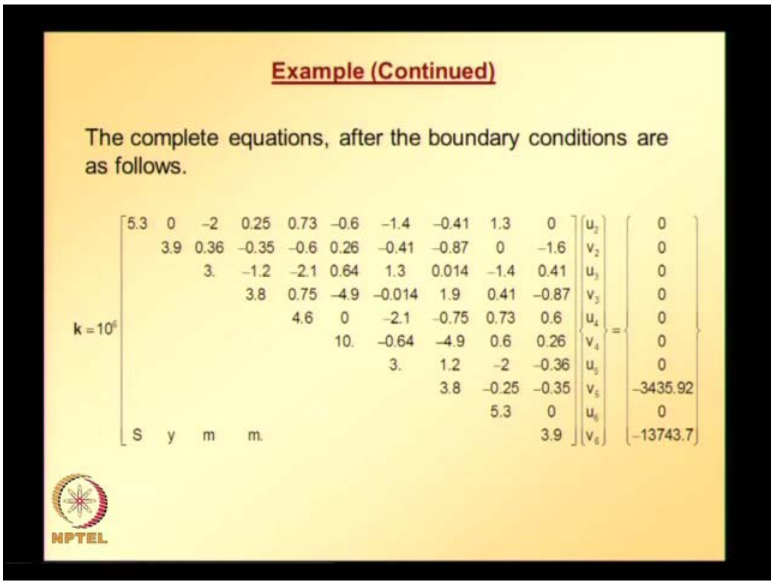

So, we need to perform, so the basic thing is simply by using tributary area concept, if we do the equivalent nodal load vector is going to be different from that that we get by performing integration of the the work expression; complete equations after applying boundary conditions are given here, complete reduced equation system.

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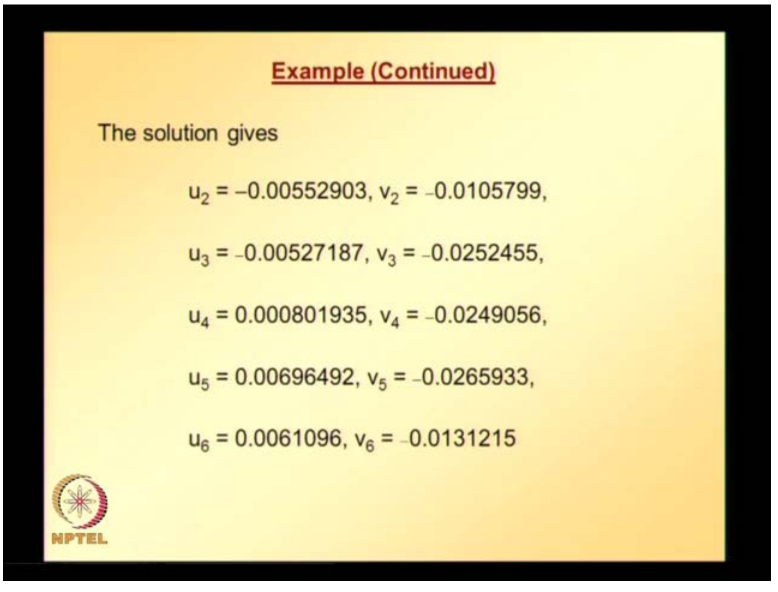

So, solving this we can find solution at all the 5 nodes, and once we have the displacements we can calculate stresses, similar to what we did for 3 node linear triangular element, and 4 node quadrilateral elements.

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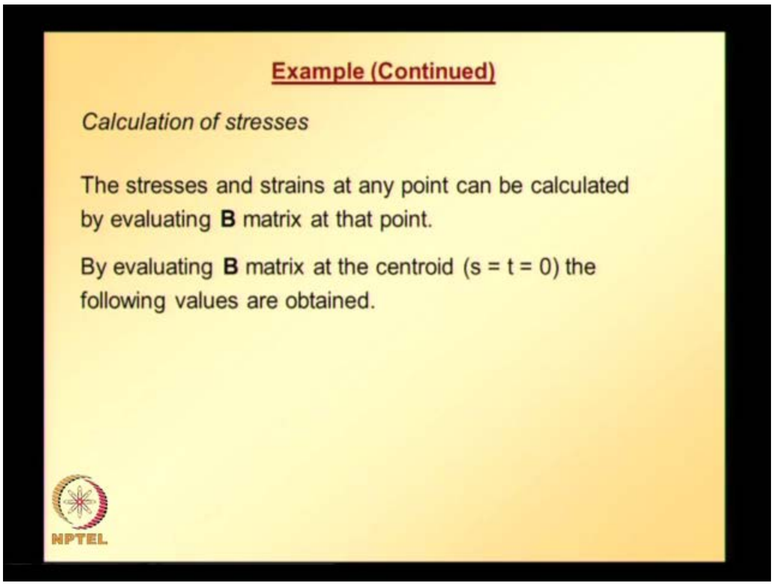

Stresses and strains at any point can be calculated by evaluating B matrix at that point by evaluating B matrix at the centroid, which corresponds to s is equal to 0, t is equal to 0, this is only for illustration.

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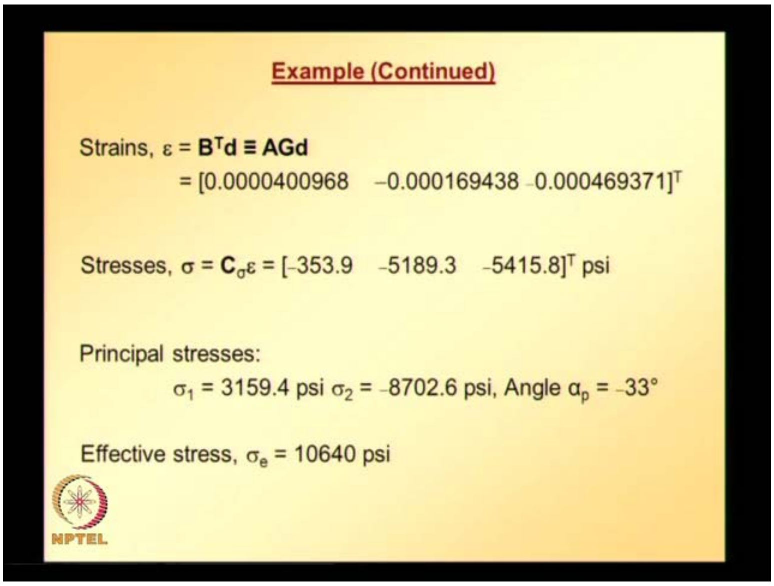

So, this in essence is what plane stress, plane strain problems are or two d elasticity problems using finite element method, so we discussed about the finite element formulation for two d elasticity problems. And afterwards we looked at finite element equations for triangular element, and 4 node quadrilateral element, and 8 node isoparametric element; so in next class, we look at the case of axisymmetric problems.