Finite Element Analysis

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Lecture No. # 33

Let us continue with what we left left behind in the last lecture. So, we will see some more examples on applications based on general two-dimensional boundary value problems. And as a part of that, in today's lecture, we are going to see two-dimensional ideal, irrotational, incompressible fluid flows around an object. So, basically if you recall, what we did in the last class, we looked at torsion problem.

Basically, once we get the governing differential equation and associated boundary conditions, we compare the differential equations, and the boundary conditions with the corresponding equations of general two-dimensional boundary value problems. And once we identify the corresponding coefficients, we can easily write the finite element equations. So, that is what basically we did. So, similar manner let us solve some problems in today's class using or the finite element equations; that we already developed for general two-dimensional boundary value problems.

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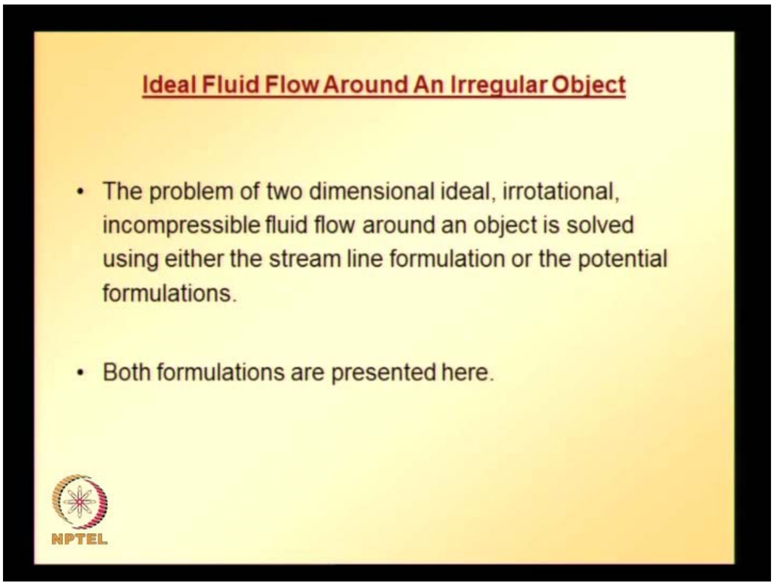

So, ideal fluid flows around that irregular object. The problem of two-dimensional ideal, irrotational, incompressible fluid flow around an object is solved using either stream line formulation or potential formulation. So, there basically there are two approaches through which, we can solve this problem, and both formulations are presented here that means, we are going to look at the both the formulations. So, first let us start with stream line formulation.

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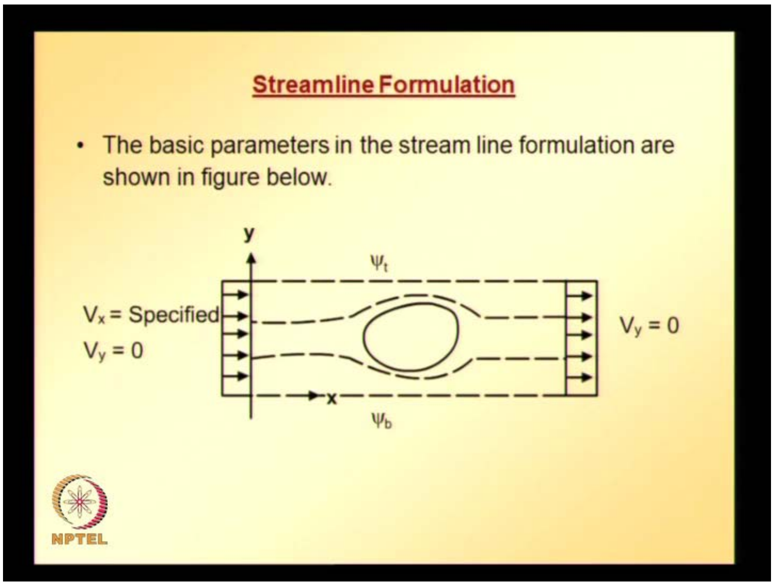

The basic parameters in the stream line formulations are shown in figure below. The field variable is here for stream line formulation. The field variable is the stream function, which is denoted with psi and it is related to the fluid velocities in x and y directions. And also, the solution domain is extended far enough from the obstruction. So, that there is no effect of obstruction on the flow characteristics characteristics.

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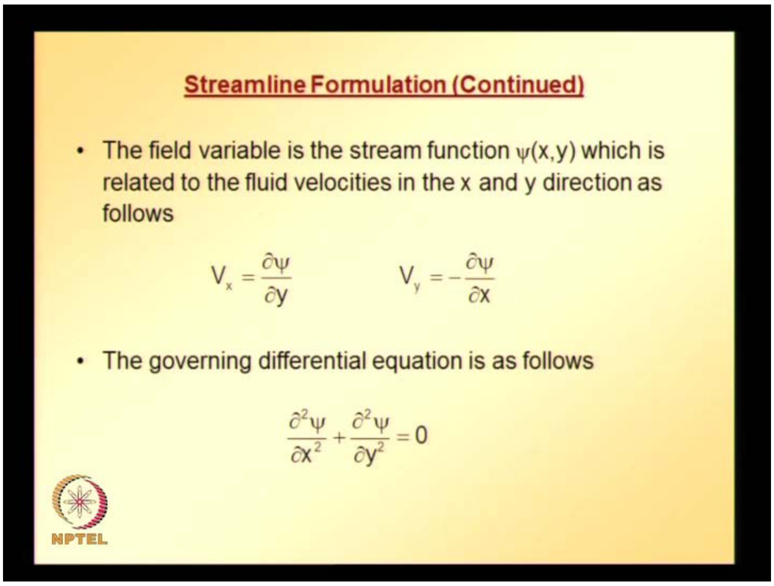

So, the fluid velocities in x and y directions are related to stream function psi, which is function of x and y through these equations. That is, velocity in the x direction is derivative of psi with partial derivative of psi with respect to y. Velocity fluid velocity in y direction is equal to minus of partial derivative of stream function with respect to x. The governing differential equation is as follows. So, basically we need to solve this differential equation subjected to some boundary conditions imposed on psi.

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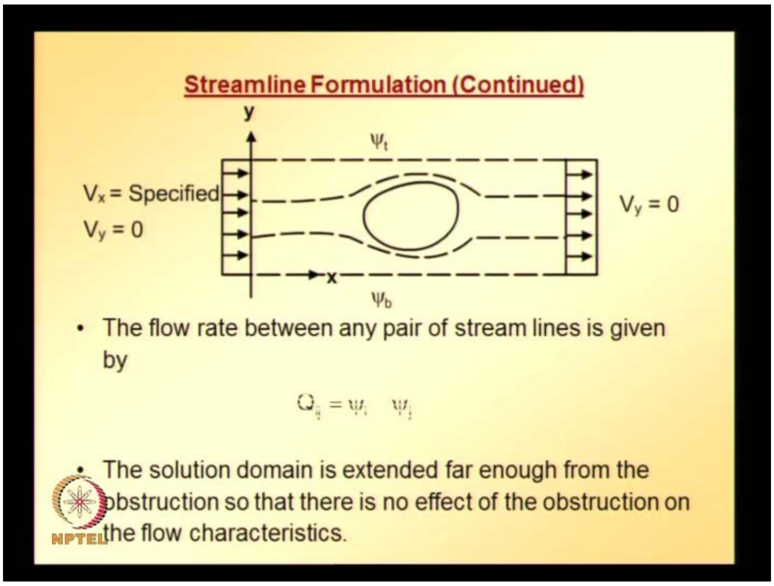

The fluid rate between any pair of stream lines is given by this equation Q i j is equal to psi i minus psi j. That is, i and j are any pair of stream lines and also the solution domain is extended far enough from the obstruction. So, there is no effect of obstruction on the flow characteristics. And for the figure given here, we can easily identify the boundary conditions.

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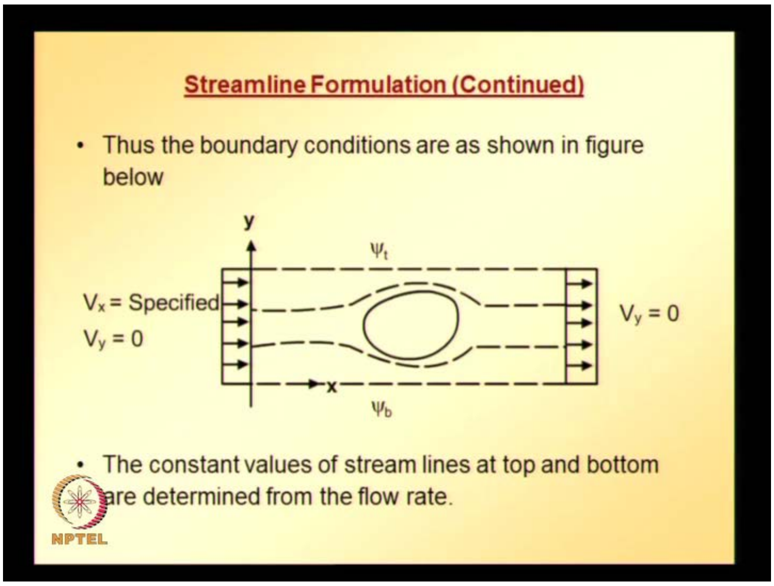

The boundary conditions are as shown. That is, at the far left end, V x is specified; V y is equal to 0 and at the far right end, V y is equal to 0 again. And stream function at the top is denoted with psi subscript t and stream function at the bottom is denoted with psi subscript b. The constant values of stream lines at the top and bottom are determined from the flow rate. So, what is the difference between psi t and psi b? That is given by the flow rate.

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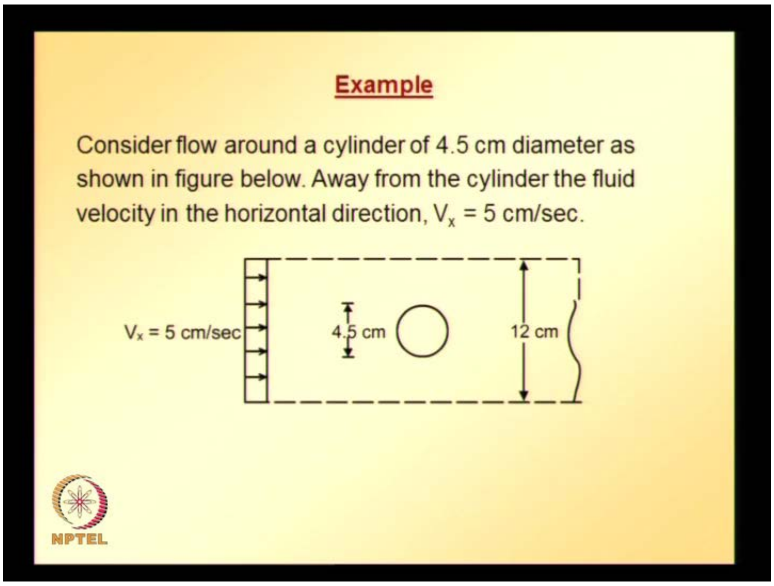

Now, let us consider an example. Consider flow around cylinder of diameter 4.5 centimeters. Away from the cylinder, the fluid velocity in the horizontal direction is V x is equal to 5 centimeter per second. So, this is the problem. V x is specified; V y is equal to 0. The diameter of a cylinder is given as 4.5 centimeters and total width of flow is given as 12 centimeters. So, the constant psi values; that is, psi t and psi b as I just mentioned, it can be determined from the flow rate per unit thickness. Here thickness is assumed to be 1 unit.

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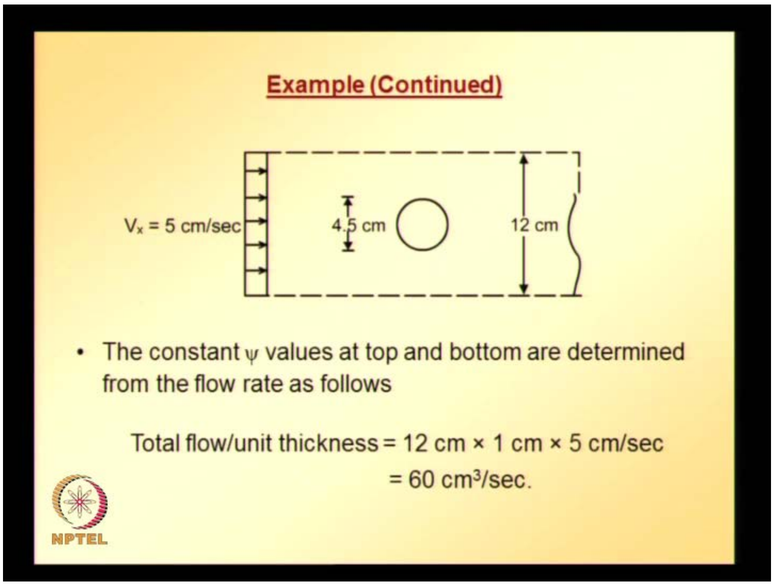

So, flow rate or unit thickness is given by 12 times 1 times V x; that is, 5 centimeter per second and it comes out to be 60 centimeter cube per second. So, the difference that is psi t minus psi b is 60 centimeter cube sorry 60 centimeter cube per second.

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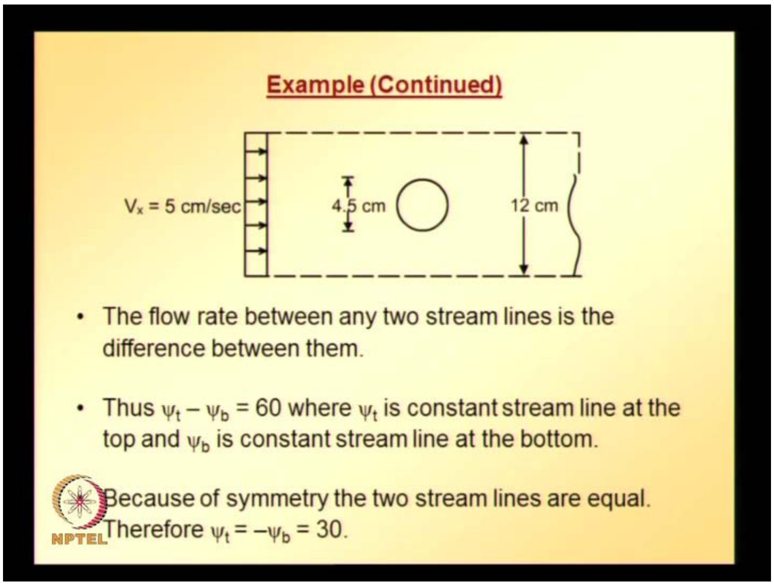

So, the flow rate between any two streams stream lines is the different between them. So, and because of symmetry, the two stream lines are equal and actually because of symmetry, we can model only a quarter of this domain. In that case, psi t minus or psi t is going to be minus of psi b is equal to 30, which is half of 60.

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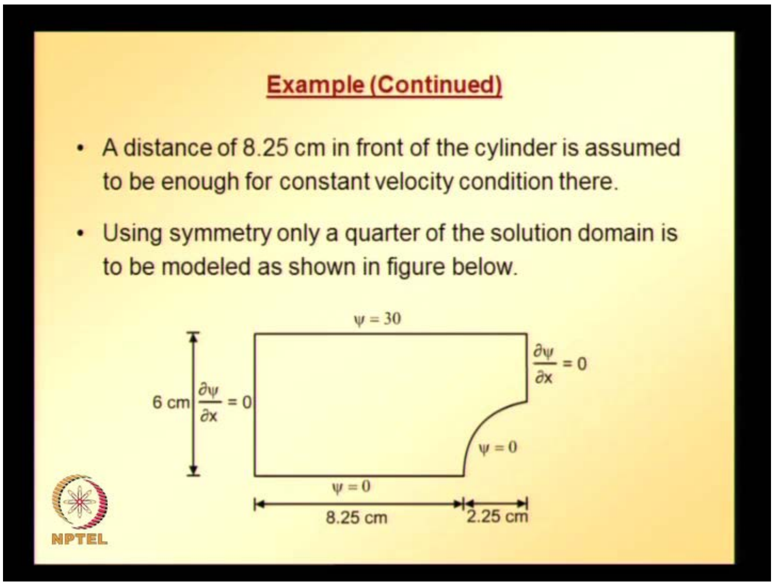

A distance of ah 8.5 8.25 in front of cylinder is assumed to be enough for constant velocity condition; because we need to select solution domain far enough from the obstruction. So, that no effect of obstruction on the flow characteristics. Using symmetry, only a quarter of solution domain is to be modeled and the quarter model is shown are along with all the boundary conditions. Since V y is equal to 0, partial derivative of stream function with respect to x is equal to 0 on the left extreme left edge.

And because of symmetry, partial derivative of stream function with respect to y, which is indirectly related to to the V y; velocity fluid velocity along y direction is also 0 along the line of symmetry on the right hand side. And again, since the model is symmetry; because of symmetry even psi is equal to 0 on the bottom edge of this quarter model. Similarly, the edge which is coinciding with the cylinder psi is equal to 0 and psi is equal to 30 and that is coming from flow rate, which we just calculated. So, this is the quarter model of the solution domain. So, we need to solve the differential equation a second order differential equation; that we have seen earlier in terms of psi subjected to these boundary conditions.

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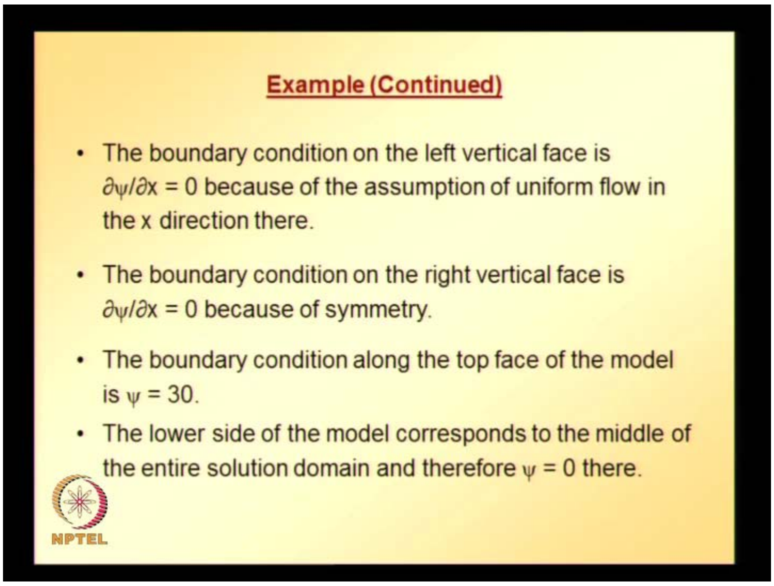

So, the boundary condition on the left vertical face is partial derivative of psi with respect to x is equal to 0; because of assumption of uniform flow in the x direction there. The boundary condition on the right right vertical face is partial derivative of stream function with respect to s is equal to 0, because of symmetry. And the boundary condition along the top face of the model is psi is equal to 30. Lower side of the model corresponds to the middle of entire solution domain. So, stream function is equal to 0 there. So, these are the boundary conditions.

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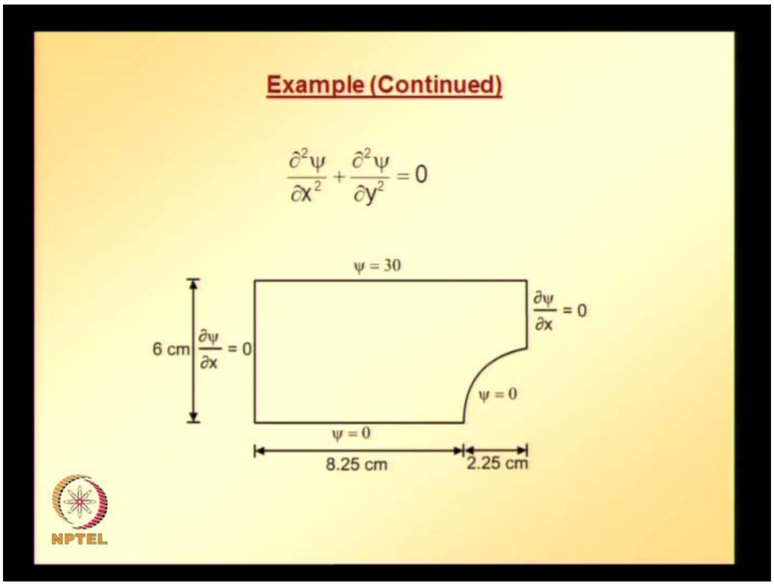

So, we need to solve this second order differential equations subjected to these boundary conditions. So, that is the problem statement. Now, everything is given to us. Now, our job is to compare this governing differential equation with general two dimensional boundary value problems; the differential equation, that we have taken for general two dimensional boundary value problems. And the corresponding boundary conditions, we need to compare with the boundary conditions of this particular problem. Identify the coefficients and we can actually write the finite element equations.

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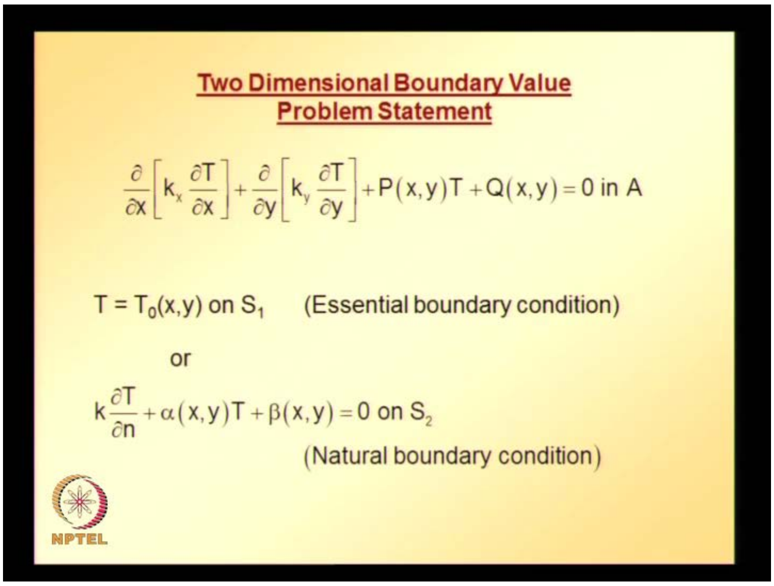

So, now let us look once again general two dimensional boundary value problems. This is a statement, that we started out with and this needs to be satisfied over the domain A subjected to any of these boundary conditions; essential boundary condition and natural boundary condition.

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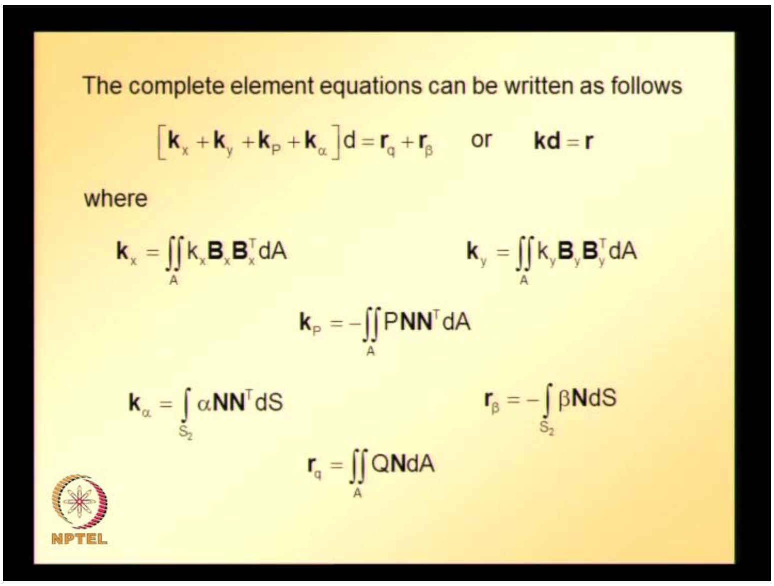

And using Galerkin criteria, what we did is, after substituting finite element approximations, a complete equations for general two dimensional boundary value problems reduces to this; where, each of these k x, k y, k p, k alpha, r beta, and r q are defined here.

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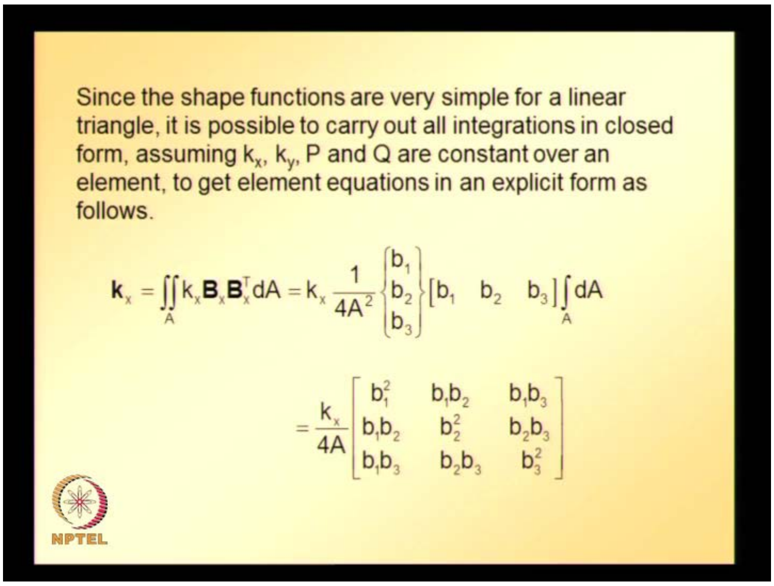

And for linear three node triangle element, since the shear functions are very simple for linear triangle, it is possible to carry out all integrations in closed form. Assuming k x, k y, P, Q are constant over element to get element equations in explicit form. So, for three node triangle element, k x turns out to be this; after multiplication of the two matrix two vectors, we get this.

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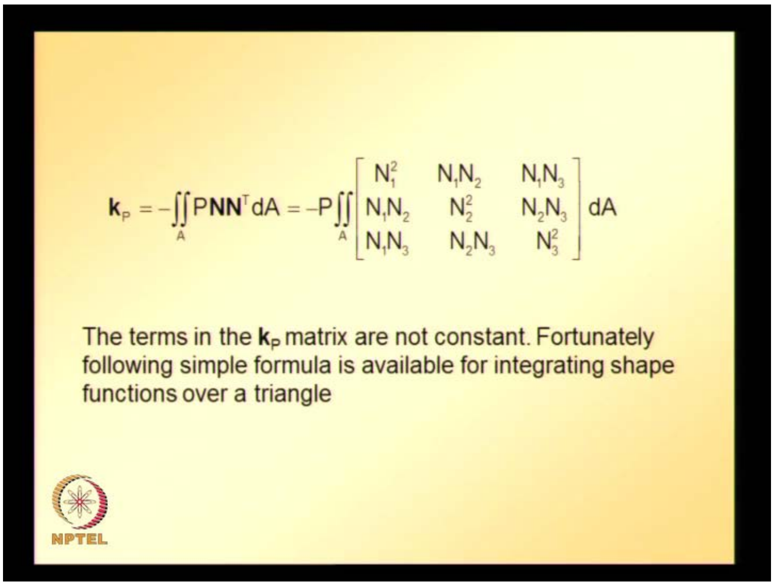

Similarly, k y and k p and here, since the components of P are not going to be constant, we need to perform some numerical integration. Terms in k p matrix are not constant. Fortunately, simple formula is available for integrating shape functions over triangle.

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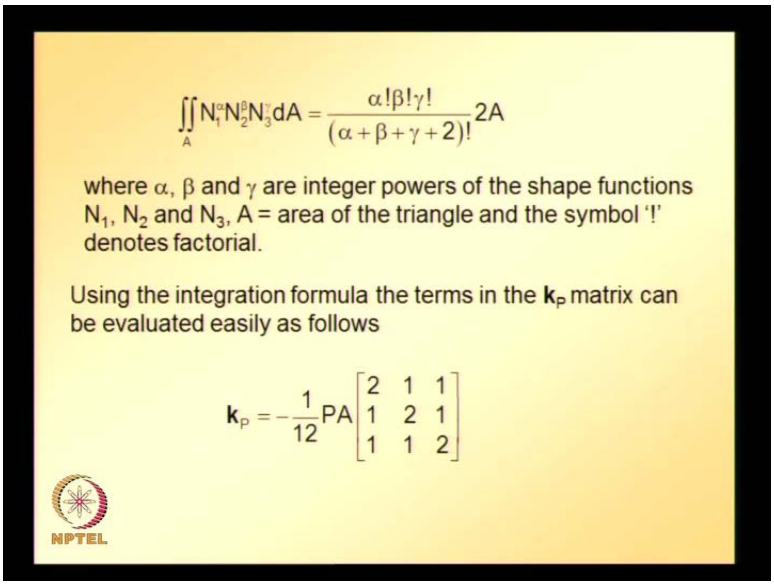

This is the formula. So, applying this formula, all the parameters in that formula are defined; alpha, beta, gamma. N 1, N 2, N 3 are the three shape functions of the linear triangular element. A is the area of triangle and symbol exclamation is for factorial and using integration formula, the terms in k p matrix can be evaluated like this.

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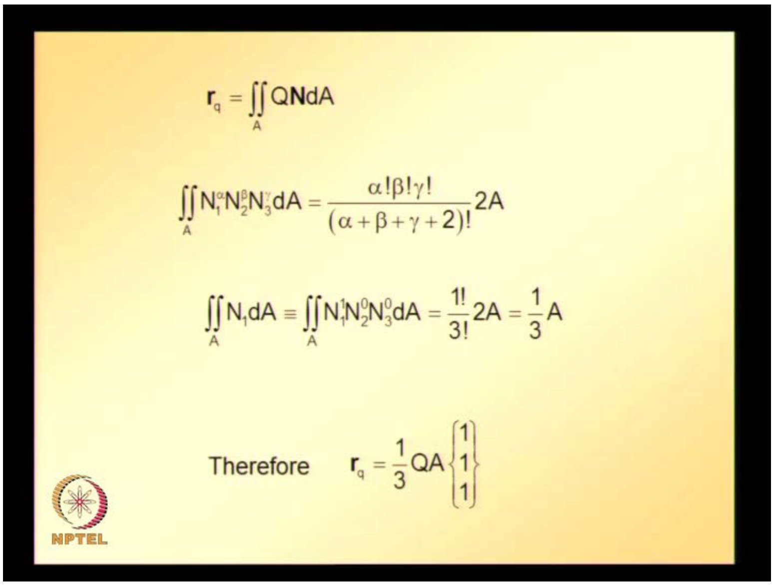

And then, we required to evaluate r q. Once we identify q, what is q? We can evaluate r q. For evaluating r q also, we can use this formula and one component of that r q is simplified and shown the details are shown. And carrying out similar kind of integrations, we can get all components of r q. So, therefore r q is given by one third Q times A and 3 by 1 column vector consisting of all 1's.

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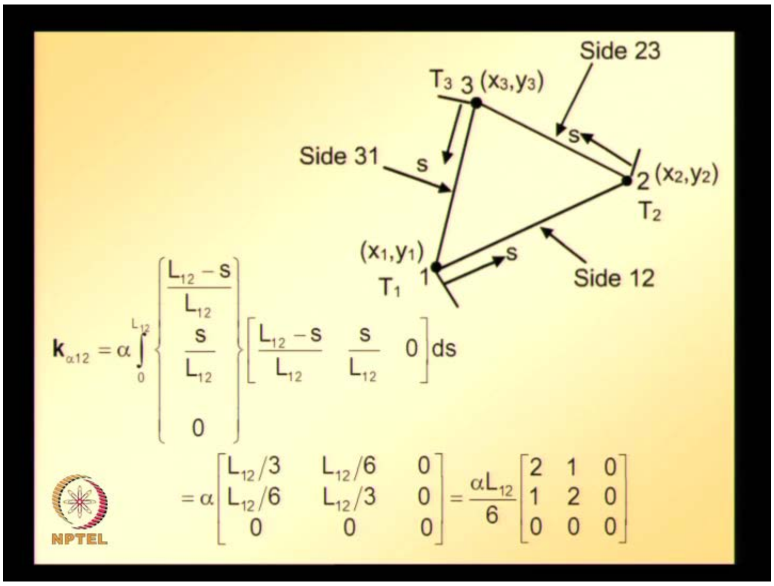

And now coming to the line integrals, once we identify once we carefully compare the given boundary conditions with the corresponding boundary conditions of general two dimensional boundary value problems, we can identify what are the corresponding parameters? And we can assemble the boundary integrals, k alpha. For illustration, k alpha is given here; it is k alpha evaluated along side 1 2 details are given here. The shape functions of nodes 1 and 2 can be obtained using one dimensional Lagrange interpolation formula. And simplification of this leads to the similar integrations can be carried out along the other two edges.

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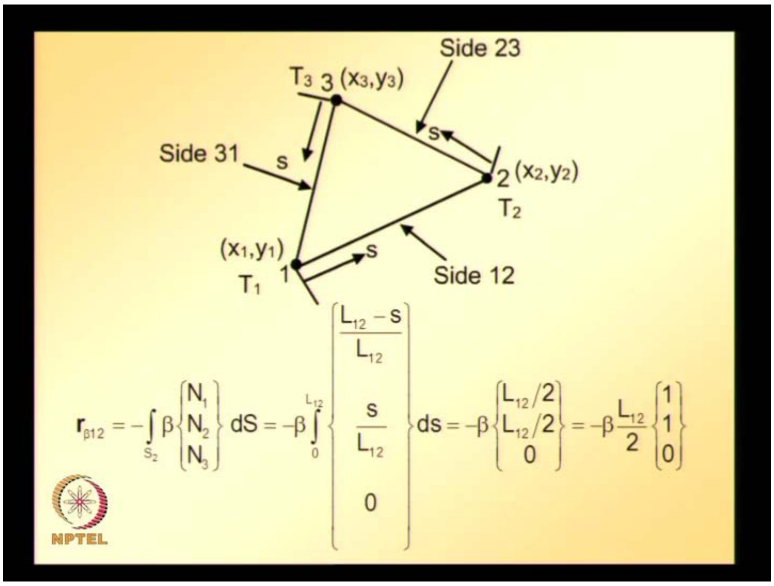

Similarly, r beta along side 1 2: So, all these procedures, which we have seen earlier needs to be repeated.

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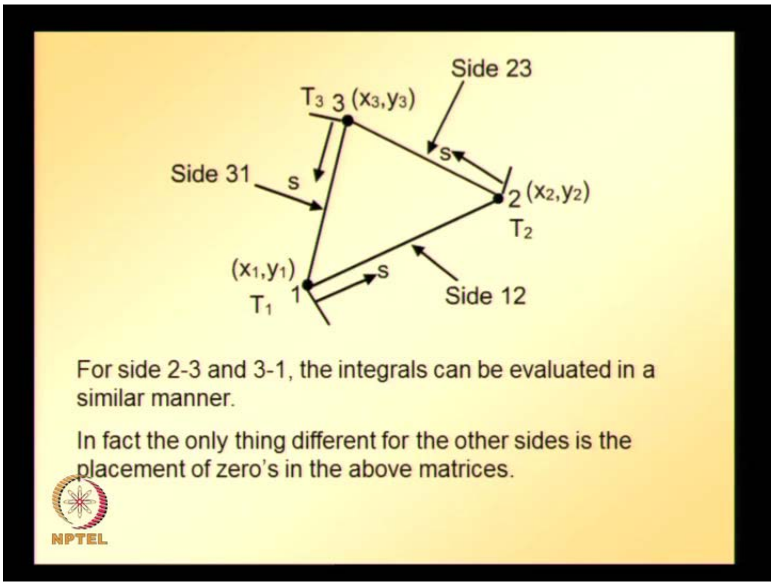

Once we identify the corresponding parameters for this ideal fluid flow around an irregular object. So, for side 2-3, 3-1 integrals can be evaluated in similar manner and only difference is going to be the placement of zero's in the matrices consisting of shape functions.

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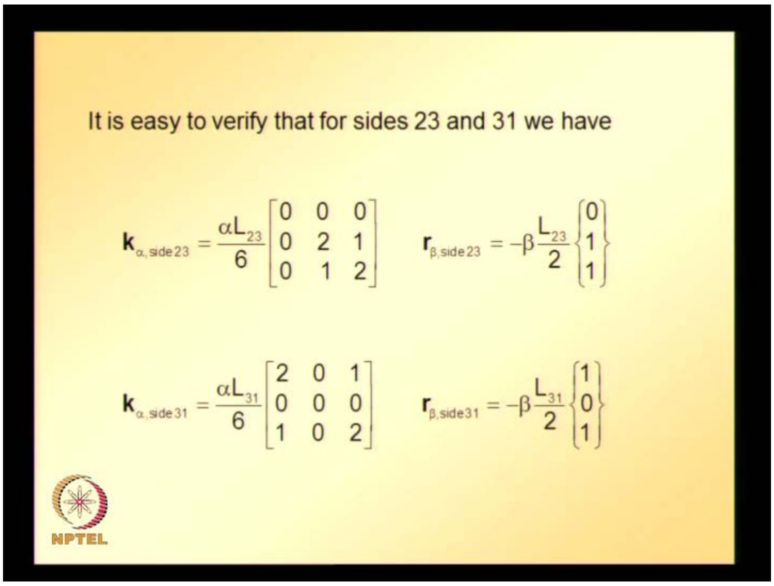

And for side 23 and 31, k alpha and r beta are looks like this. So, once we identify the corresponding coefficients, we can get the element equations. And once we get element equations, we can get the global equation system based on nodal connectivity. And after imposing the essential boundary conditions, we can solve for the nodal values.

And once we solve for the nodal values here, for this particular problem of fluid flow around an irregular object, around an irregular object the nodal parameters are going to be the stream function values. Once we obtain stream function values, we can find partial derivatives of stream functions; stream function with respect to x and with respect to y and which are related to the velocity components or fluid velocities along x and y directions.

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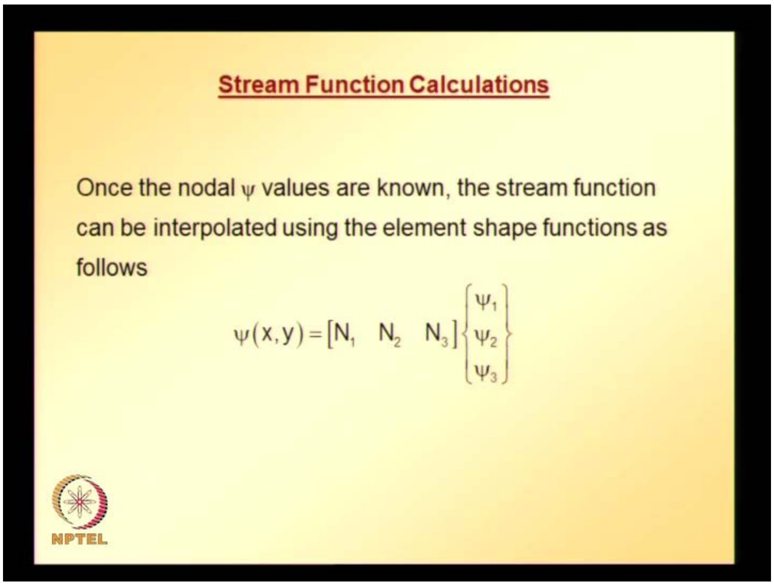

So, stream function calculations: Once the nodal values psi are known, stream function can be interpolated using element shape functions like this.

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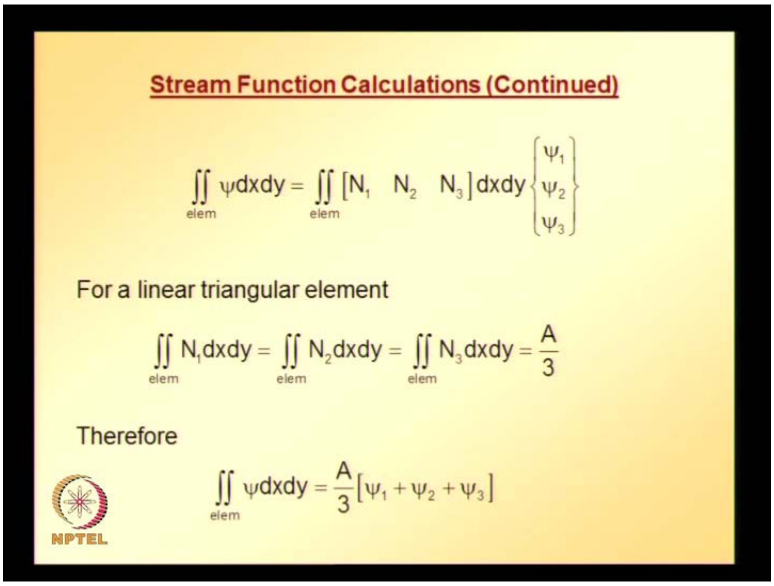

And also, we require sometimes we require this. What is the integral psi dx dy over the element or entire domain? Here, the formula is written for one particular element. So, we know psi is equal to N 1 psi 1 plus N 2 psi 2 plus N 3 psi 3, which can be written in matrix and vector form. As it is shown on the right hand side of the equation and substituting, since psi 1, psi 2, and psi 3, the nodal the stream function nodal corresponding to the nodes. Nodal stream function values are constant. They can be taken out of the integral.

And once we simplify this integral with N 1, N 2, and N 3 integrated over dx dy or the entire area of the element for linear triangular element, these integrals turns out to be area of triangle divided by 3. So, substituting this this integral, that is psi integrated over the entire element can be approximated as area of the triangle element divided by 3 or area of triangle element multiplied by average value of a stream function at all the nodes. That is, A over 3 times in are in brackets psi 1 plus psi 2 plus 3 as shown in the right hand side of the equation last equation.

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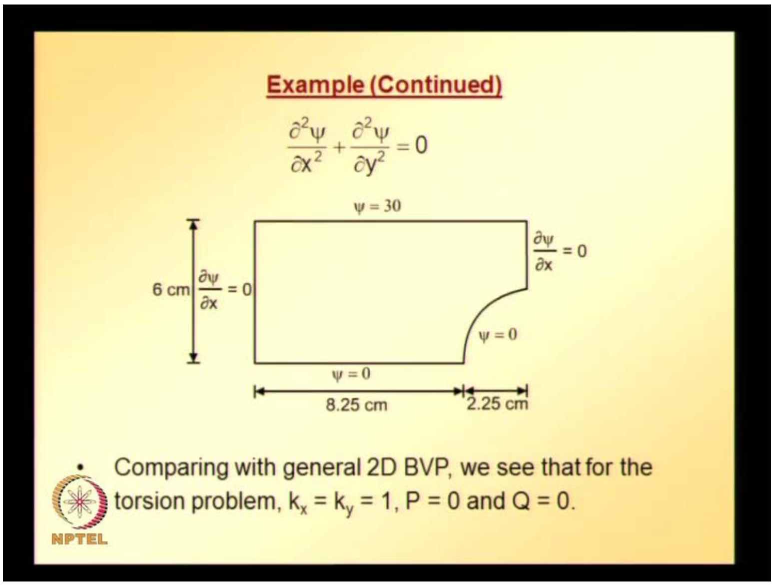

Now, we require to solve this differential equations subjected to these boundary conditions. So, proceeding in the way that is explained, we can get the element equations. And once the element equations are obtained, we can assemble the global equation system; impose the essential boundary conditions. Here, essential boundary conditions are psi is equal to 30 on the top phase; psi is equal to 0 on the bottom face. So, these are the essential boundary conditions. After imposing these essential boundary conditions, we can or we need to impose these essential boundary conditions; the nodes which are along these boundaries.

Once we impose these essential boundary conditions, we can solve for the nodal values and do all kinds of post processing. So, now comparing with general two dimensional boundary value problems, see that for this for this ah sorry it is mentioned as torsion problem; it should be ideal fluid flow problem. So, for this particular problem, when we compare with general two dimensional boundary value problem, it turns out that k x is equal to k y is equal to 1; P is equal to 0 and Q is equal to 0. So, once again there is a mistake. It should be should be fluid flow problem, instead of torsion problem.

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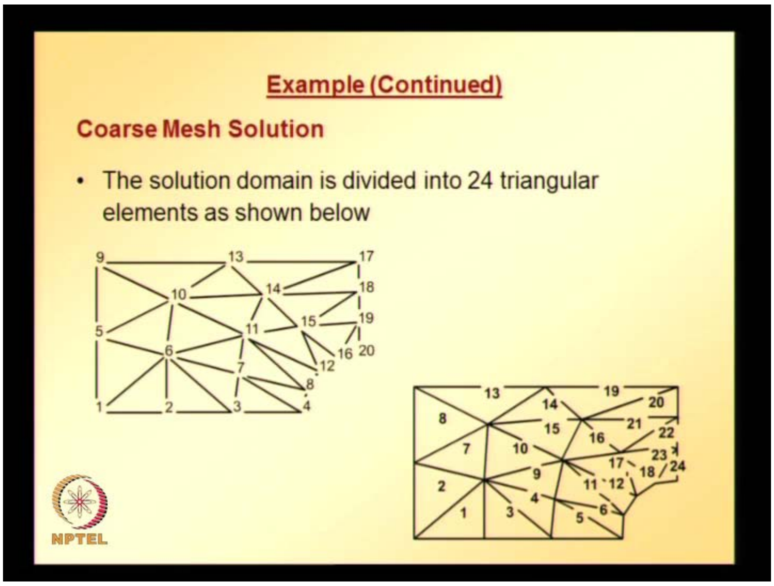

So, now let us see the solution, how it looks or how it converges, when we take different meshes? Here coarse mesh solution, by coarse mesh I mean the mesh; the quarter model consists of 24 triangle elements triangular elements. So, this is the quarter model and it consists of 20 nodes. The corresponding elements are here 24 elements 24 triangle elements triangular elements. So, using this discretization, we can solve this problem by imposing essential boundary conditions at nodes 9, 30, 70, 1, 2, 3, 4, 8, 12, 16, and 20. By imposing essential boundary conditions over these nodes and we can solve for the stream function value at other nodes. And here at each node, there is only one degree of freedom, which is going to be the stream function.

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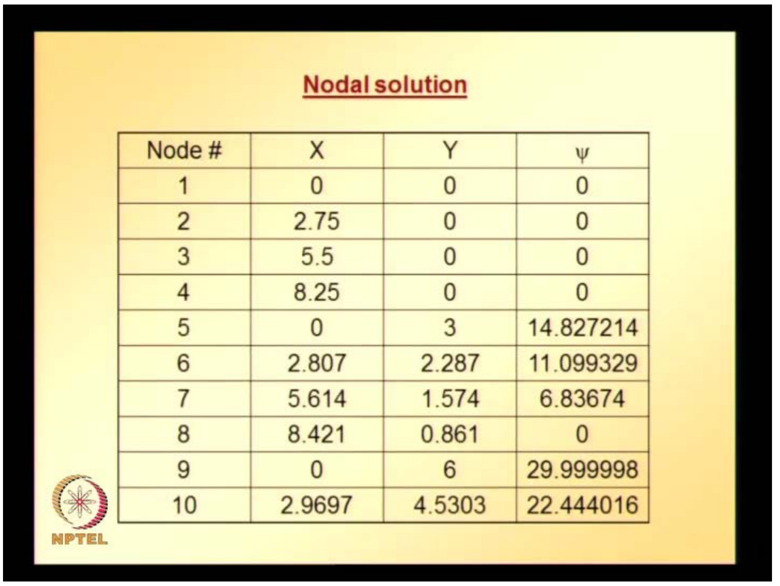

Nodal solution

Node #	X	Y	ψ
1	0	0	0
2	2.75	0	0
3	5.5	0	0
4	8.25	0	0
5	0	3	14.827214
6	2.807	2.287	11.099329
7	5.614	1.574	6.83674
8	8.421	0.861	0
9	0	6	29.999998
10	2.9697	4.5303	22.444016

So, the nodal solution, the details are given here at all nodes 1 to 10 in this table and next table shows for the rest of the nodes.

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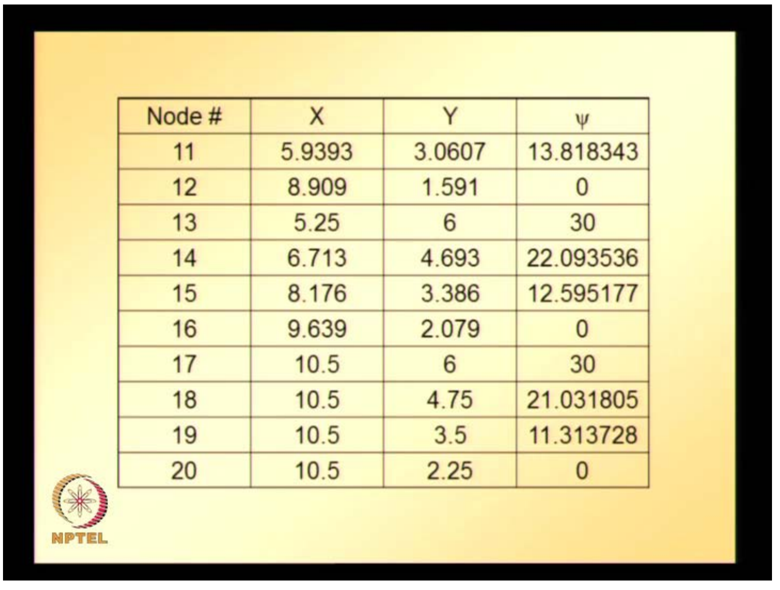

Node #	X	Y	ψ
11	5.9393	3.0607	13.818343
12	8.909	1.591	0
13	5.25	6	30
14	6.713	4.693	22.093536
15	8.176	3.386	12.595177
16	9.639	2.079	0
17	10.5	6	30
18	10.5	4.75	21.031805
19	10.5	3.5	11.313728
20	10.5	2.25	0

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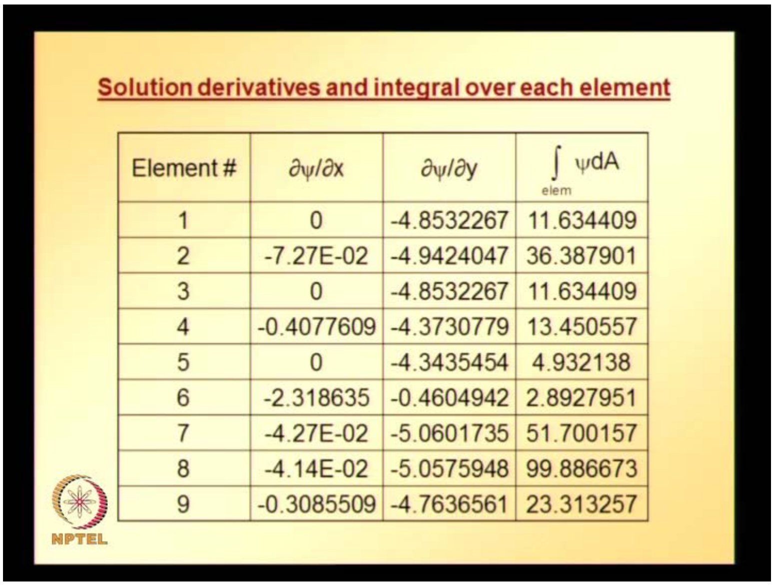

Element #	$\partial\psi/\partial x$	$\partial\psi/\partial y$	$\int_{elem} \psi dA$
1	0	-4.8532267	11.634409
2	-7.27E-02	-4.9424047	36.387901
3	0	-4.8532267	11.634409
4	-0.4077609	-4.3730779	13.450557
5	0	-4.3435454	4.932138
6	-2.318635	-0.4604942	2.8927951
7	-4.27E-02	-5.0601735	51.700157
8	-4.14E-02	-5.0575948	99.886673
9	-0.3085509	-4.7636561	23.313257

And solution derivatives and integral over each element, they are shown in this table. Partial derivative of stream function with respect to x, partial derivative of stream function with respect to y and integral of stream function over the element domain 1 to 9 elements are shown here.

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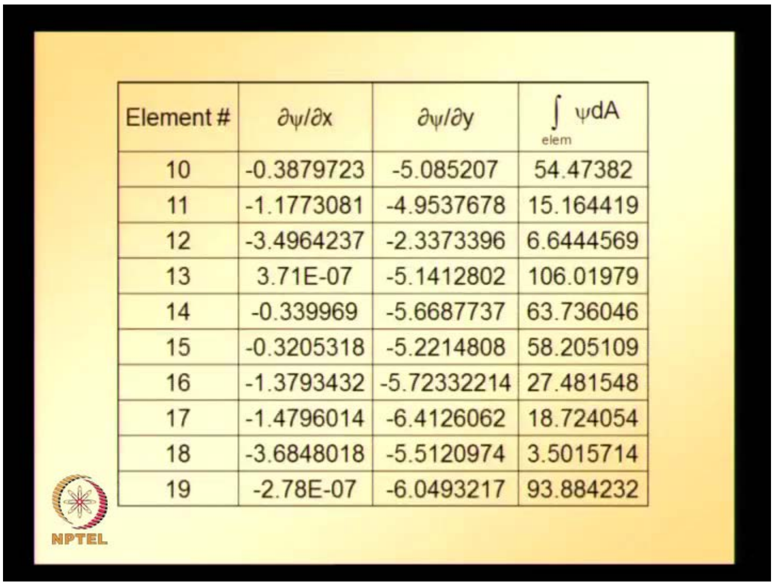

Element #	$\partial\psi/\partial x$	$\partial\psi/\partial y$	$\int_{elem} \psi dA$
10	-0.3879723	-5.085207	54.47382
11	-1.1773081	-4.9537678	15.164419
12	-3.4964237	-2.3373396	6.6444569
13	3.71E-07	-5.1412802	106.01979
14	-0.339969	-5.6687737	63.736046
15	-0.3205318	-5.2214808	58.205109
16	-1.3793432	-5.72332214	27.481548
17	-1.4796014	-6.4126062	18.724054
18	-3.6848018	-5.5120974	3.5015714
19	-2.78E-07	-6.0493217	93.884232

And rests of the elements are shown. The other slide 10 to 19, details are given here.

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Element #	$\partial\psi/\partial x$	$\partial\psi/\partial y$	$\int_{elem} \psi dA$
20	-0.38835	-7.1745563	57.692852
21	-0.3832884	-6.838263	46.740227
22	-0.9327613	-7.7744613	21.758797
23	-0.9708058	-8.5500298	12.768353
24	-1.7975818	-9.0509825	2.0294001

20 to 24 details are given here. We can also plot, what are called stream lines. Corresponding to the lines along which, stream function value is constant.

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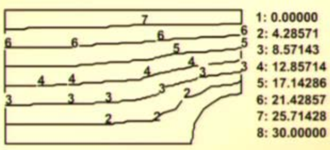
Example (Continued)

- The stream lines (ψ = constant) are shown in figure below.



Stream lines

So, stream lines are shown in this figure. Also we can plot, derivatives of stream functions stream function with respect to x and y.

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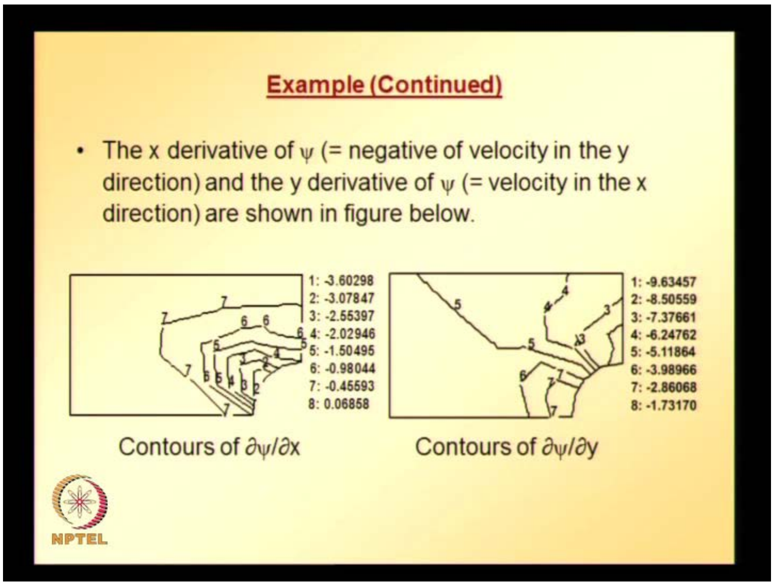

Derivative of stream function, x derivative of stream function, y derivative of stream function; because x derivative of stream function is related to velocity in the y direction and y derivative of stream function is related to velocity in the x direction. And they are shown here, contours of derivatives of stream function with respect to x and y. So, whatever solution finite element solution that is shown is using 24 triangular elements. The problem can be repeated by using finer mesh.

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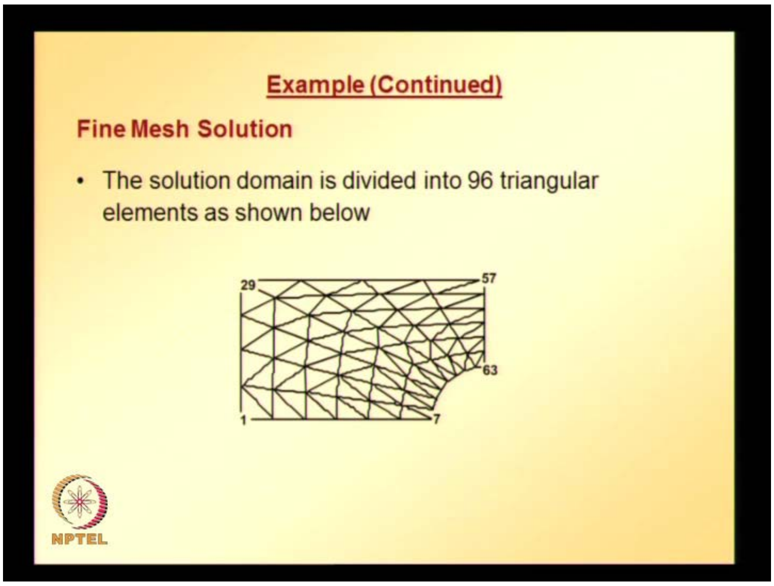

Solution domain is divided into 96 triangular elements. So, the discretization is shown here and everything remains same, except that size of the problem increases; because we require to solve for more number of nodal values.

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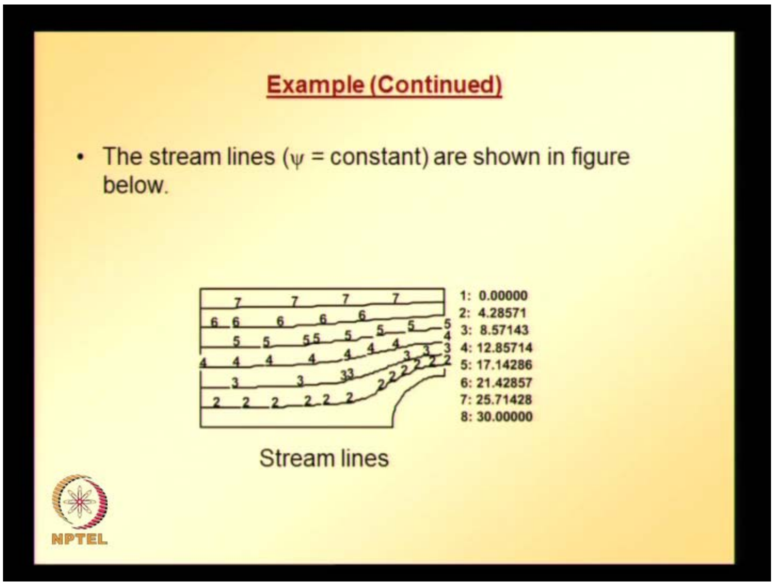

So, let us look at, how the solution looks? Stream (( )) stream lines, psi is equal to constant; those lines are these.

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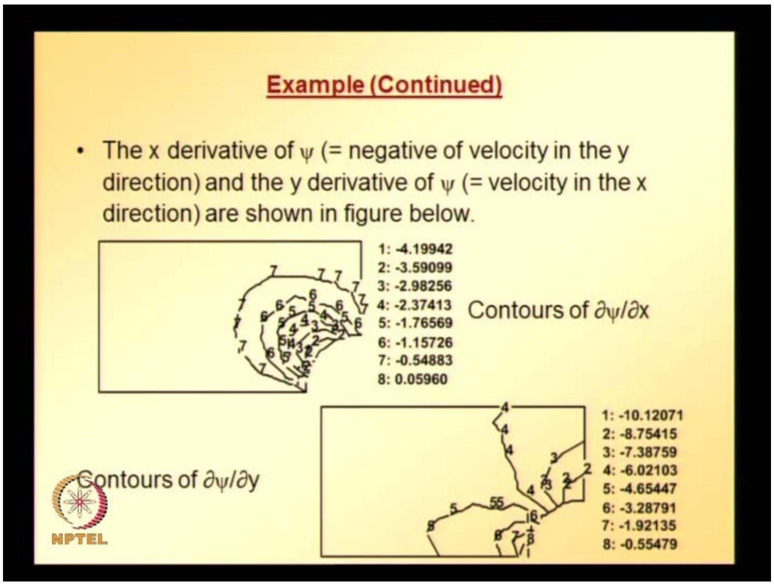

And the derivatives of stream function, psi with respect to x and with respect to y. So, we solve this problem using 24 elements and 96 elements.

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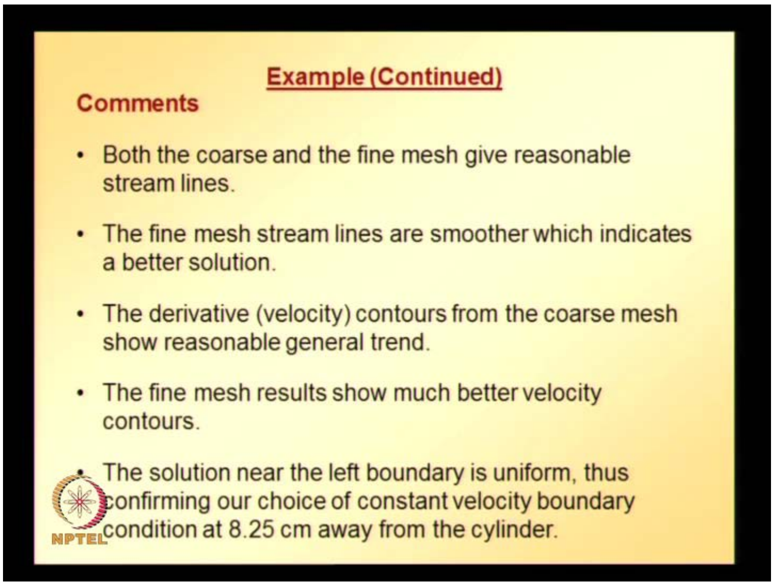

Let us see, how the solution matches? Both coarse and fine mesh gives a reasonable stream line that is expected. Because the solution itself may converge very fast; whereas, derivatives of solution require finer mesh. Fine mesh stream lines are smoother, which indicates a better solution. Derivative contours for coarse mesh show reasonable agree reasonable general trend. Fine mesh solution results show better velocity contours. And solution near left boundary is uniform; thus confirming our choice of constant velocity boundary at a distance of 8.25 centimeters away from cylinder.

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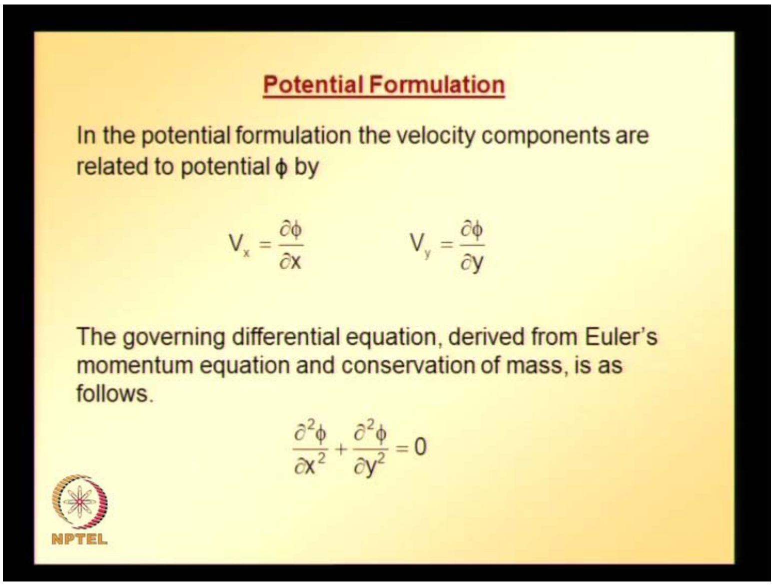

So, now let us look at, solving this ideal fluid flow around an irregular object through the different formulation; that is potential formulation, potential formulation potential formulation. The velocity components are related to a potential phi by these relations. velocity fluid fluid velocity along x direction, y direction are related to the potential phi, where this equations. The governing differential equation, derived from Euler's momentum equation and conservation of mass is this one. So, basically we require to solve this second order differential equation; solve for phi subjected to some boundary conditions.

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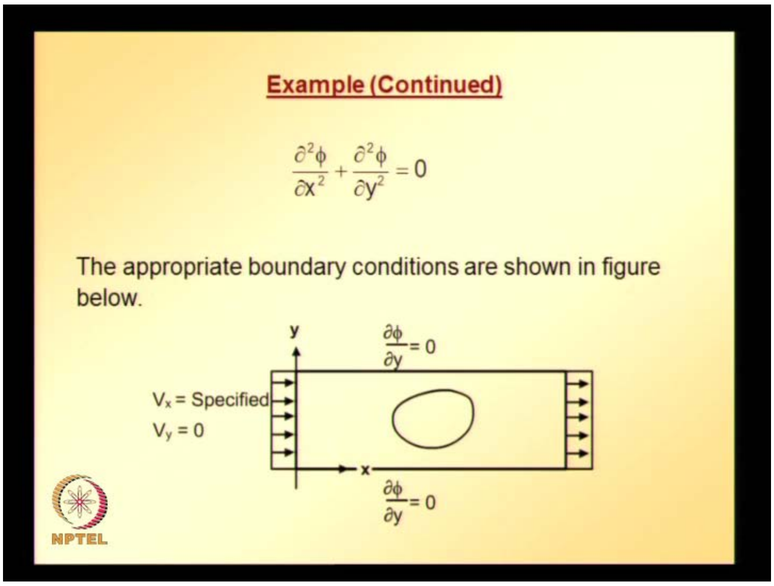

Appropriate boundary conditions are as given here in the are are as shown in the figure. That is, essential boundary condition at the top and bottom are partial derivative of potential with respect to y is equal to 0. And V x is specified on the left vertical side and V y is equal to 0. So, we can solve or we can redo the problem that we already did, using potential function for potential formulation.

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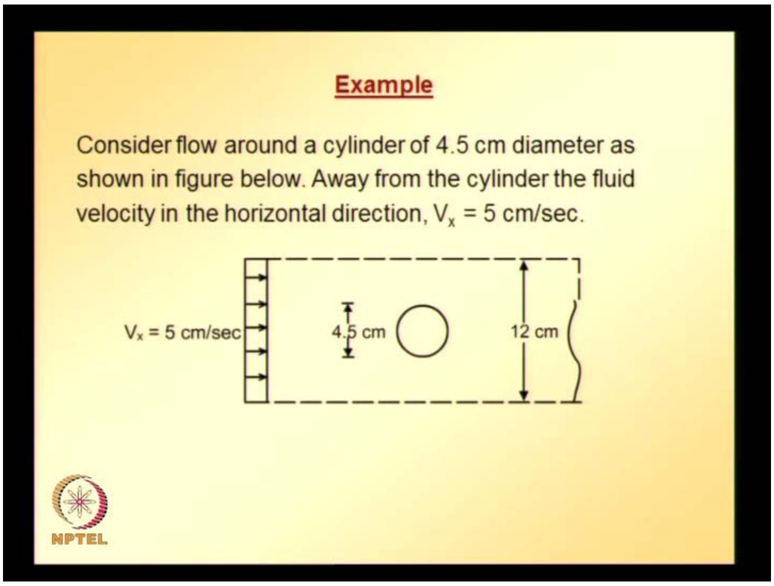

And we let us look at the problem statement. For the problem that we already looked at, consider fluid flow around cylinder of 4.5 centimeters and V x is 5 centimeters per second. Again, only a quarter of solution domain needs to modeled, because of symmetry.

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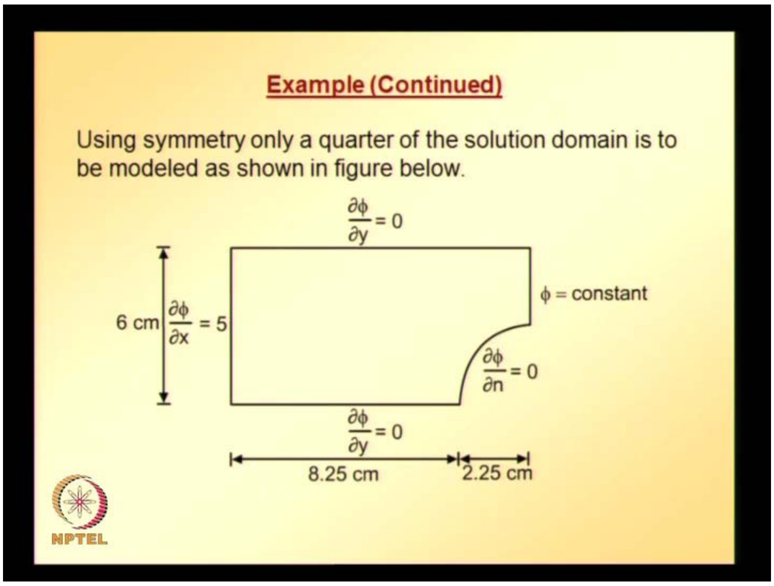

And the boundary condition on the left vertical phase is partial derivative of potential with respect x is equal to phi; because V x, fluid velocity in the x direction at that location is already given in the problem statement. Because and this is because of the assumption of uniform flow in the x direction. Boundary condition on the right vertical phase is potential is equal to 0 or potential sorry potential is equal to constant; because of symmetry. For the numerical solutions a constant a value of 50 is assigned to psi sorry 50 is assigned to the potential phi along this phase.

This is arbitrarily assigned. Arbitrarily, since phi is equal to potential is equal to constant and that constant value is taken as 50 just ah just arbitrarily. The boundary condition along the top and bottom phase of this model are partial derivative of potential with respect to y is equal to 0, which are indicated in the figure and all the details of geometry are shown in the figure. So, now with this information, we can solve the second order differential equation with respect to the potential; for this particular problem, over this domain subjected to these boundary conditions.

By comparing this with the problem statement of general two dimensional boundary value problem and then, we can actually get the finite element equations. And then, as usual the procedure is same; assembling and applying the essential boundary conditions solving for the for the nodal values. Here the nodal values, turns out to the potential value. And again this in this problem, at each node you have only one degree of freedom, which is going to be the potential phi value.

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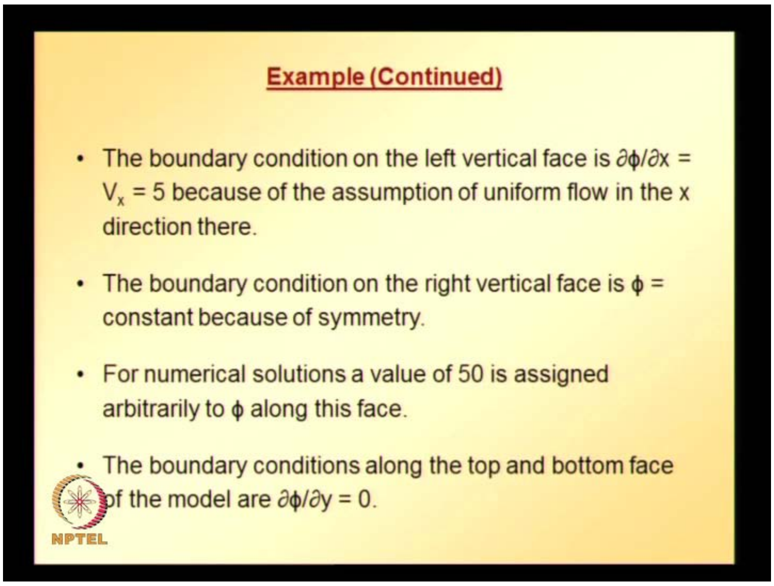

So, these are the boundary conditions that we already looked at.

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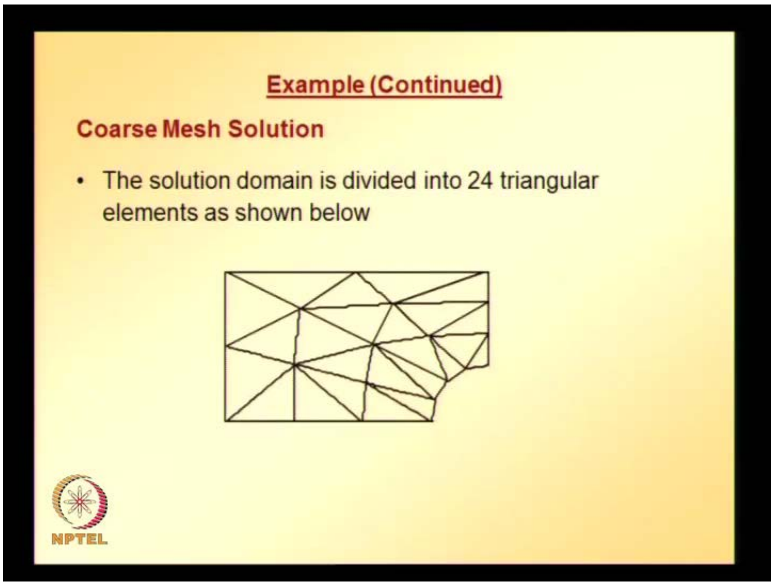

So, now let us look at, again for different mesh solution, how it looks coarse mesh solution? Solution domain is divided into same number of elements as we did for the previous formulation; that is 24 triangle elements. The discretization is shown here.

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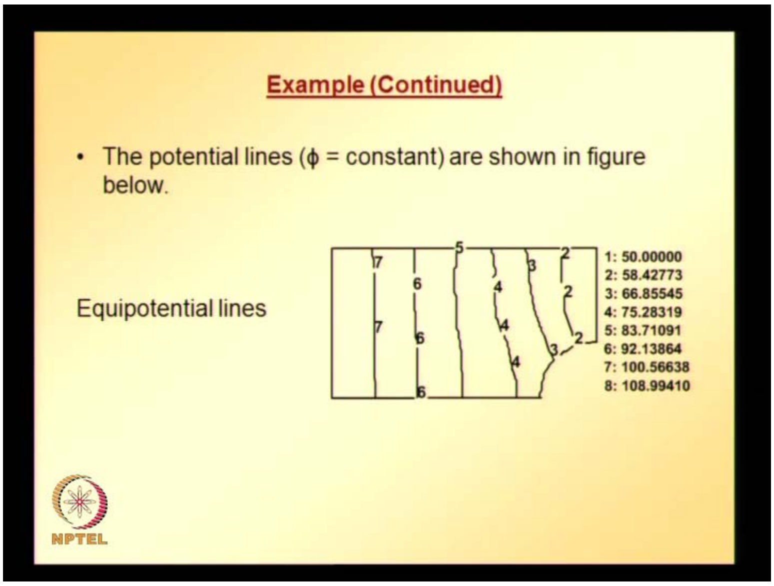

And the potential potential lines that is, phi is equal to constant are shown in this figure. Equipotential lines; potential line with phi is equal to constant are called equipotential lines.

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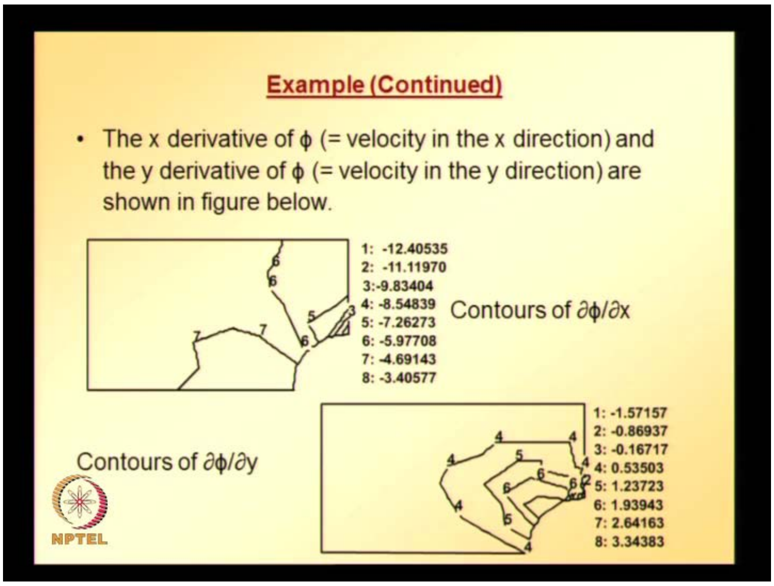

And x derivative of potential and y y derivative of potential, which are related to the velocity in the x and y directions respectively are shown here. The contours and these solutions correspond to 24 triangle elements, as we did earlier; we can actually go for finer mesh.

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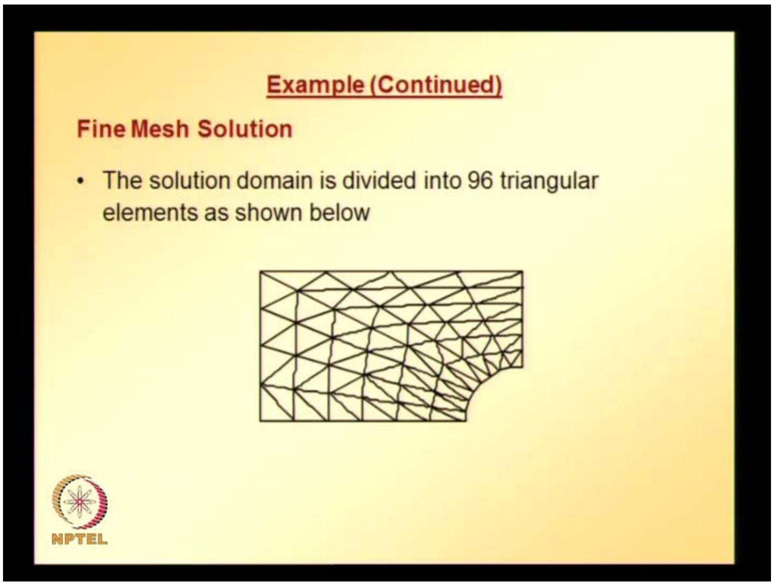

Solution domain is divided into 96 triangle elements. The mesh is shown here.

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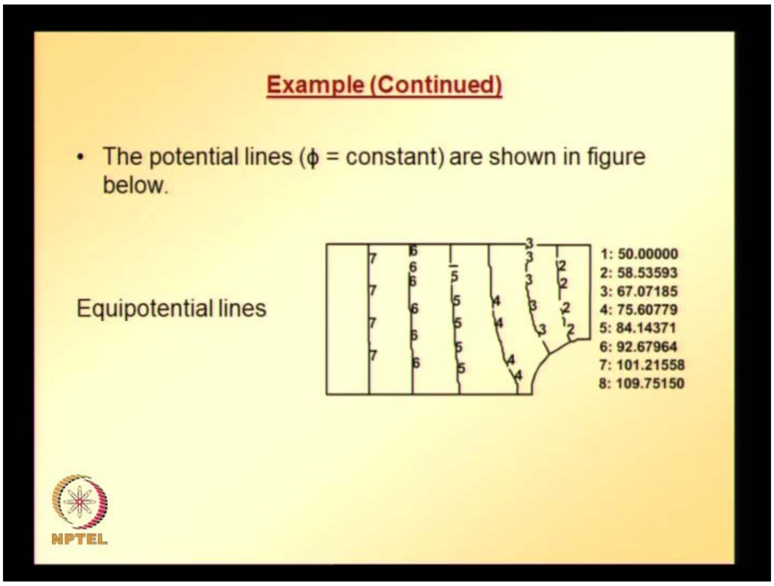

And the potential lines or equipotential lines obtained, using 94 or 96 sorry 96 triangle elements. The solution is shown here.

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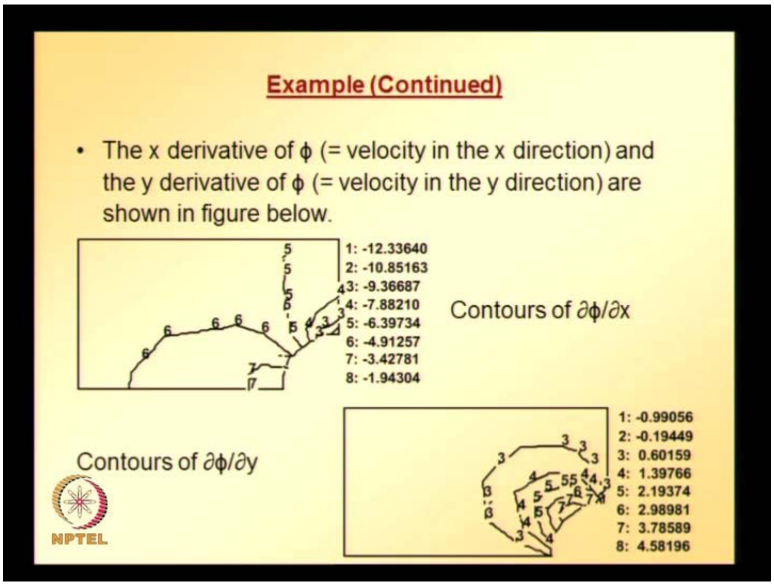

And contours of derivative x derivative of potential, y derivative of potential; they are shown here.

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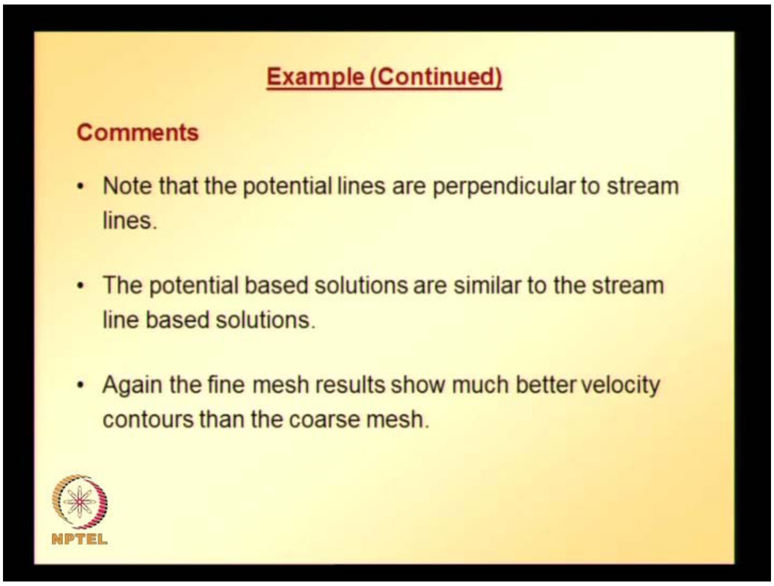

And let us see, let us make some observations. The potential lines are perpendicular to the stream lines. The stream lines are the lines that we obtained using the previous formulation. Stream function formulation and the potential lines are the lines that we just obtained using potential. Potential based solutions are similar to the stream line based solutions. Fine mesh results show better velocity contours than the coarse mesh.

(No audio from 36:15 to 36:25)

So, this is one application of general two dimensional boundary value problems. So, now let us look at other application, that is two dimensional steady state heat flow.

As usual only thing is, we require to know, what is the governing differential equation and what are the boundary conditions? So that, we can make a comparison with general two dimensional boundary value problems; identify the coefficients and get the finite element equations. So, consider the problem of finding temperature distribution through long chimney like structures. A slice of t units thick of such body is shown in the figure. Assuming no temperature gradient through thickness, the problem can be modeled as two dimensional problems. So, we are assuming, no gradient of temperature through the thickness.

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The governing differential equations can easily be derived using conservation of energy. And to do that, consider a differential element as shown here. So, the following notation is used. Temperature at any point is denoted with dx y; k x, k y are the coefficients of thermal conductivity along x and y directions. The corresponding units are given and q x is the heat flux in the x direction and q y is the heat flux in the y direction and Q is heat generated for unit volume. So, using this notation and taking a differential element, we can derive the governing differential equation. So, let us consider this differential element, in which all quantities are indicated.

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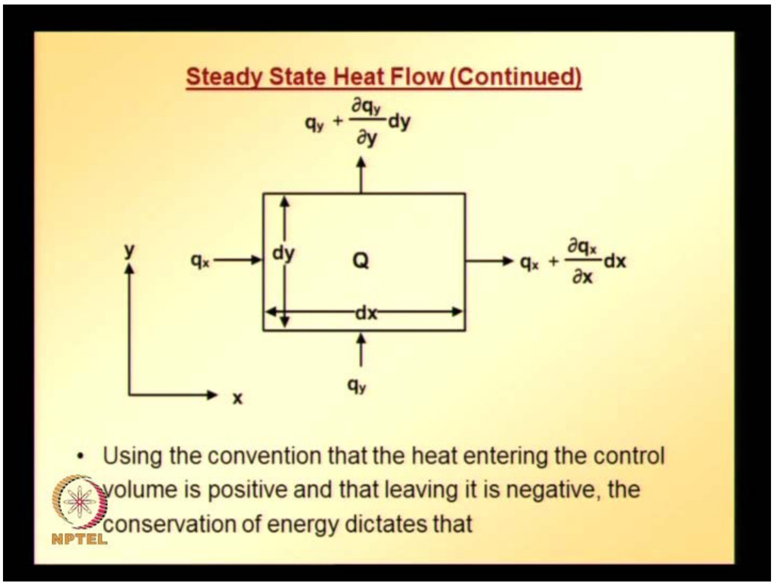

Heat flux along x direction, y direction and the heat generated and the dimensions of this differential elements are dx, dy and the coordinate system is also defined x y in the figure. So, using the convention that heat entering the volume control volume is positive and leaving and that is leaving is negative.

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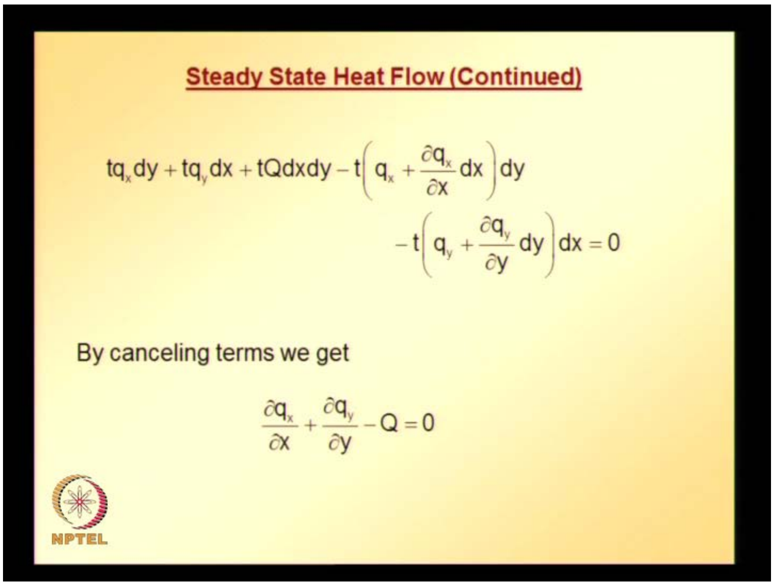

The conservation of energy dictates that, writing the equilibrium equation, we get this. And cancelling terms, which are higher order or cancelling terms, which are same in values, we get this. And substituting the heat flux, that is q x, q y in terms of thermal conductivity coefficients along x and y directions.

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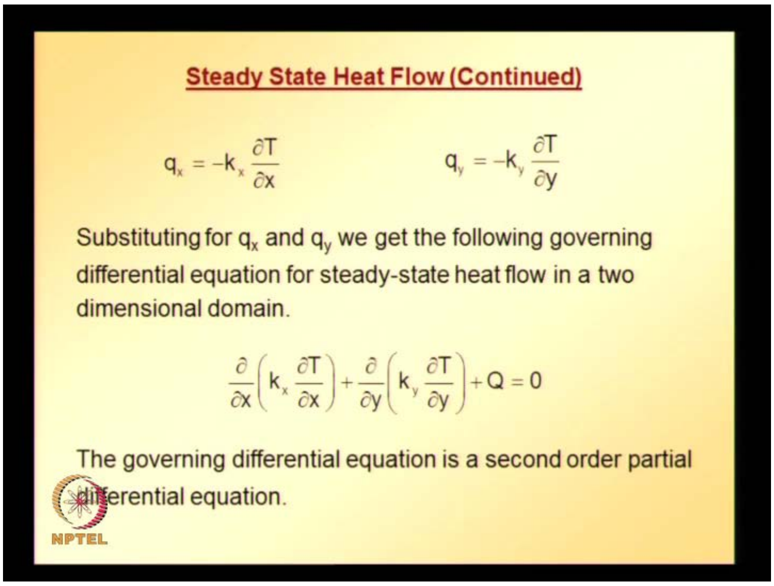

We know that, q x and q y are related to thermal conductivity coefficients through these equations. So, substituting q x, q y we get the following governing differential equations for differential equations for steady state heat flow in a two dimensional domain. And clearly, this is a second order differential equation and it is similar to what we already looked for general two dimensional boundary value problem. So, the governing differential equation is a second order partial differential equation.

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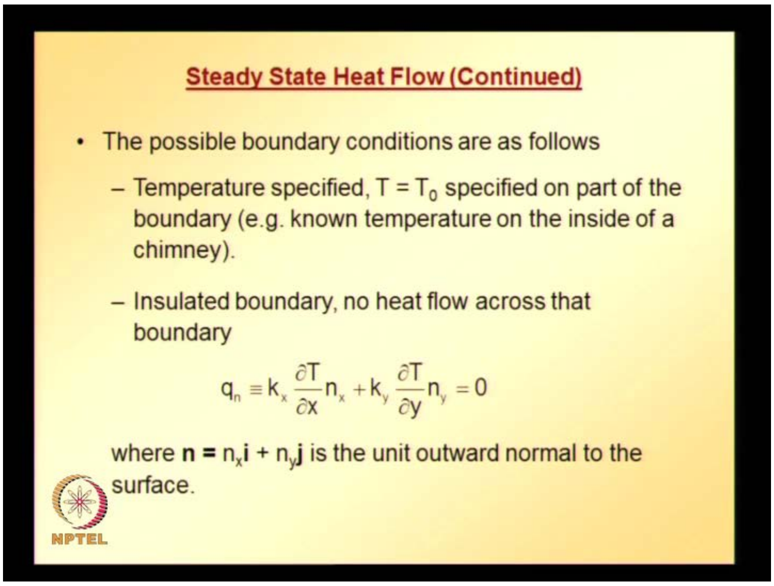

And the boundary conditions are as follows. Temperature specified on part of boundary, insulated boundary, no heat flow across the boundary. In that case q n, n is nothing but outward normal to the surface; n x, n y are the components of the out word normal to the surface. Since this is insulated, so q n is equal to 0. And we can also consider the other case, heat loss due to convection on the boundary. In that case, q n is non going to be a nonzero value, which is going to be the function of convection coefficient and temperature of the ambience.

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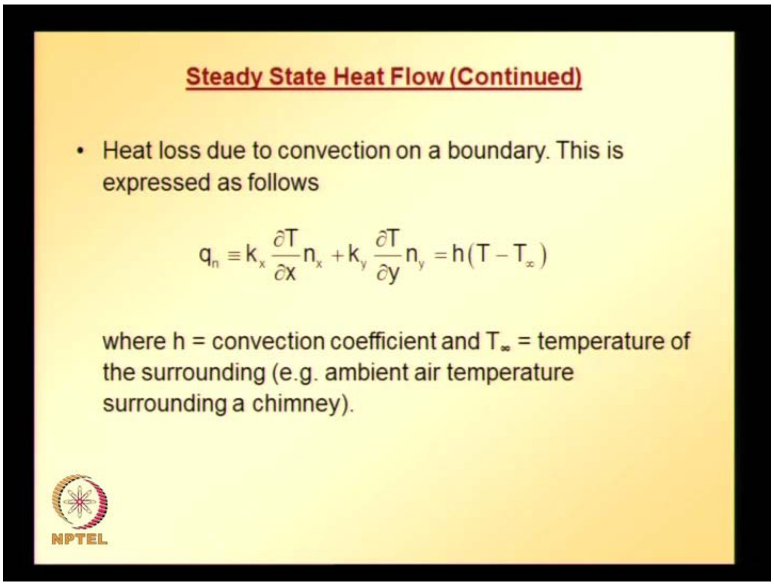

Heat loss due to convection on a boundary, in that case q n is given by this. So, we have two kinds of boundary conditions. Irrespective the boundary condition, what we need to do is, we need to compare the corresponding equation with the equation of general two dimensional boundary value problem and identify, what are corresponding coefficients, or corresponding parameters?

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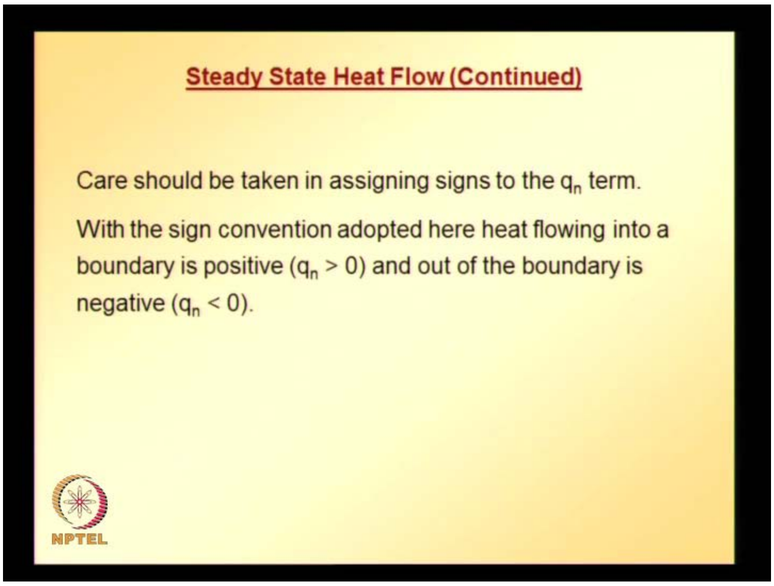

And here one more thing, care should be taken in assigning signs to q n term. And the sign convention adopted is heat flowing into the boundary is positive and heat flowing out of the boundary is negative.

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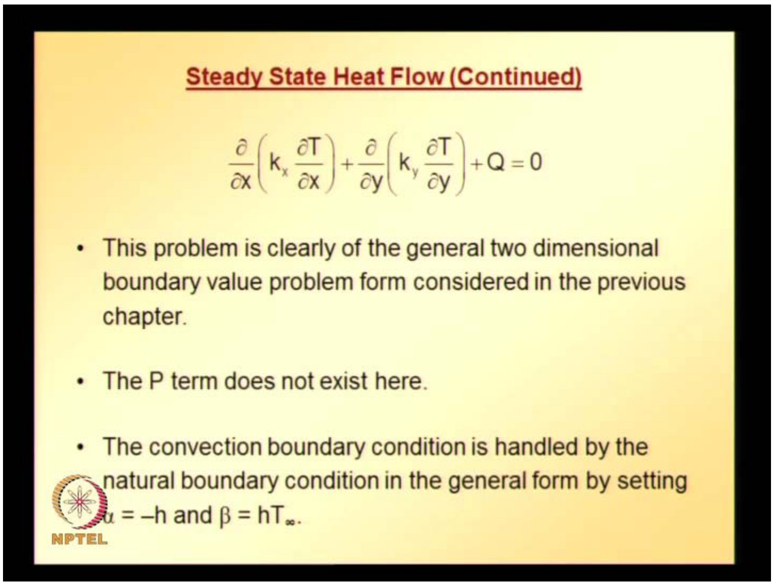

This is the governing differential equations. So, this problem is clearly of general two dimensional boundary value problem form considered in the previous chapter or previous lectures. And if we compare, only thing that we observe is, when compare with the general two dimensional boundary value problem, P term is missing. So, P does not exist here or P is equal to 0; so K P is going to 0. Convection boundary condition, if it is insulated boundary, alpha and beta are going to be 0. If it is convection boundary condition, then it is handled by taking alpha is equal to minus h; beta is equal to h times T infinity. T infinity is nothing but, temperature of the surround. So, with this understanding, we can solve a problem.

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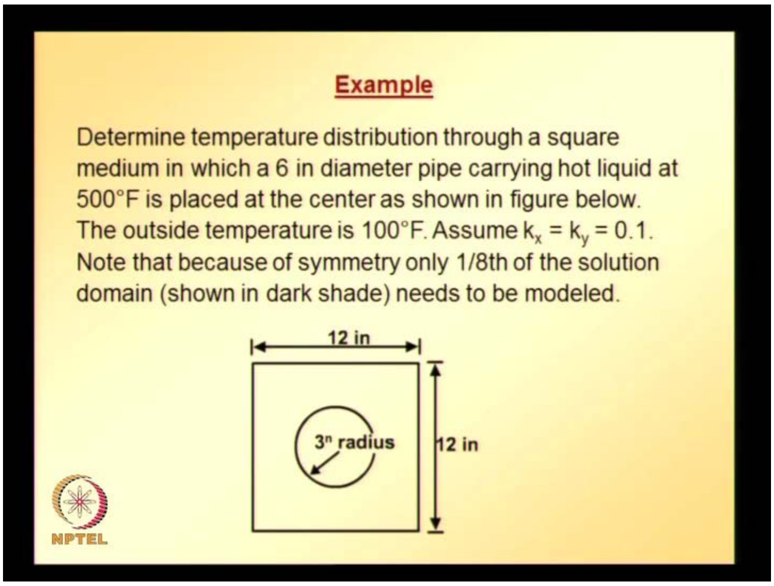

So, now let us take an example. Determine temperature distribution through a square medium with which is a 6 inches in diameter. And here the problem is given in f p s units but does not matter as long as we stick with the consistent units. And pipe carrying hot liquid at 500 Fahrenheit is placed at the center as shown in the figure below. Outside temperature is at 100 Fahrenheit and k x, k y thermal thermal conductivity coefficients along x and y directions are given here. And and you can see here, this problem we can take symmetry into account and because of symmetry, only one eighth of the solution domain needs to be modeled.

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So, now let us look at, one eighth model.

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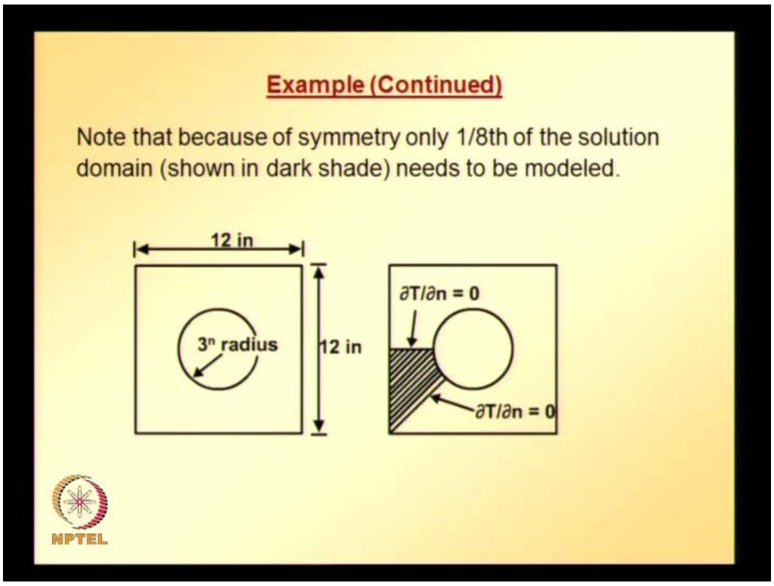

Note because of symmetry only, one eighth of the solution domain needs to be modeled. So, this is the one eighth model. And once we take symmetry into account, the associated boundary conditions along the line of symmetry are shown there, that is partial derivative of T. T here is temperature; partial derivative of T with respect to normal is equal to 0 along their lines of symmetry.

So, as usual we need to solve the second order differential equation, partial differential equations subjected to these boundary conditions, that is T. T value at the inner diameter is 500 degrees Fahrenheit or sorry T value at the center is 500 degrees Fahrenheit or T value at the inner diameter and T value at the outer diameter outside is 100 degrees Fahrenheit. Subjected to those conditions and subjected to the natural boundary conditions along the line of symmetry, we need to solve this problem.

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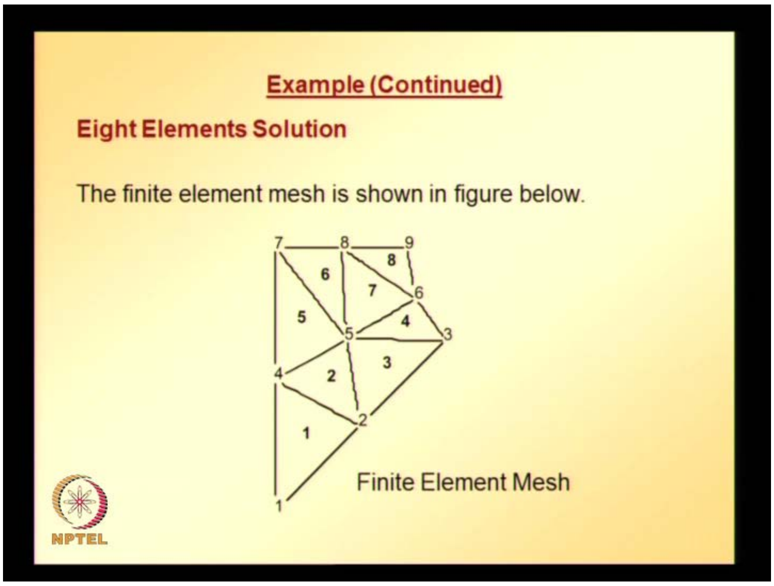

And as usual we can proceed with finite element discretization and here eight element solution is shown. So, finite element mesh with eight element eight triangle elements is shown here.

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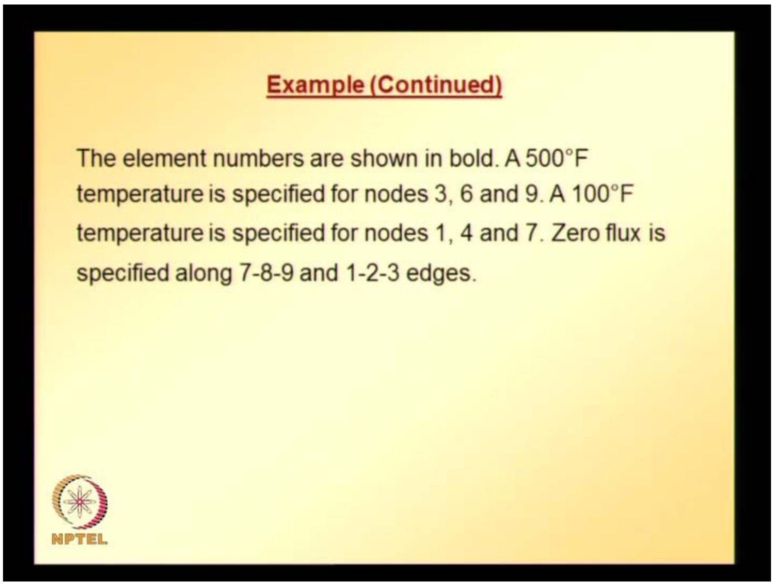

Element numbers are shown in bold. A 500 degree Fahrenheit temperature is specified at nodes 3, 6, 9. A 100 degree Fahrenheit temperature is specified at nodes 1, 4, 7. And zero flux is specified along 7-8-9 and 1-2-3 edges. So, let us see. So, 500 degrees Fahrenheit temperature is specified on on the nodes 3, 6, 9; because they those are the nodes, which are along the inner diameter of the inner diameter of the model. And temperature of 100 degrees Fahrenheit is specified along 1, 4, 7; because that is the edge, which is in contact with outside of the model. And zero flux is specified along 7, 8, 9; which is one of the line of symmetry and 1, 2, 3 which is the other line of symmetry and the nodal so with that boundary conditions.

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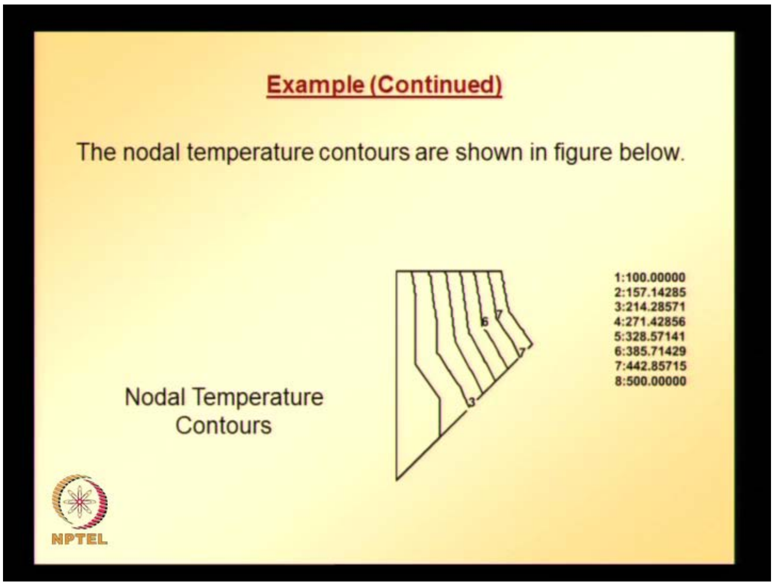

Nodal temperature values are computed, the model is solved. After applying the essential boundary conditions and the nodal temperature contours are shown here.

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Nodal coordinates and computed temperatures

Node #	X-Coord	Y-Coord	Temperature
1	-0.6000e+01	-0.6000e+01	0.1000e+03
2	-0.4061e+01	-0.4061e+01	0.1887e+03
3	-0.2121e+01	-0.2121e+01	0.5000e+03
4	-0.6000e+01	-0.3000e+01	0.1000e+03
5	-0.4386e+01	-0.2074e+01	0.2605e+03
6	-0.2772e+01	-0.1148e+01	0.5000e+03
7	-0.6000e+01	0.0000e+00	0.1000e+03
8	-0.4500e+01	0.0000e+00	0.2803e+03
9	-0.3000e+01	0.0000e+00	0.5000e+03

NPTEL

And numerical values at all the nodes are given in this table. Nodes 1 to 9.

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Conduction Heat Flow Per Unit Area In Each Element

Elem. #	$q_x = -k_x \partial T/\partial x$	$q_y = -k_y \partial T/\partial y$
1	0.4573e+01	-0.4578e-05
2	-0.7193e+01	-0.4790e+01
3	-0.1069e+02	-0.5362e+01
4	-0.1073e+02	-0.7168e+01
5	-0.9940e+01	0.2035e-05
6	-0.1202e+02	-0.1617e+01
7	-0.1385e+02	-0.1718e+01
8	-0.1465e+02	-0.2914e+01

NPTEL

And elements 1 to 8, heat flux along x direction, y direction; they are given here.

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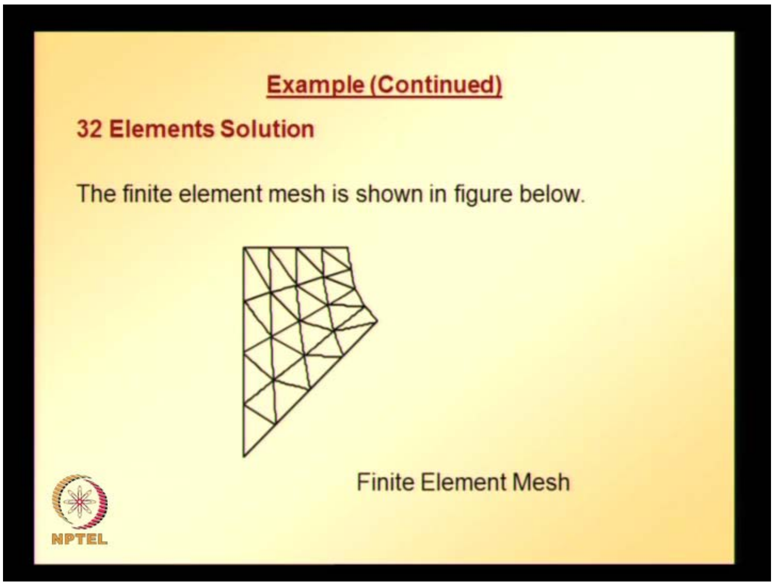

And now, the model is as we did for fluid flow case, a finite discretization is taken. So, 32 element solution is shown here. For that, the finite element mesh looks like this.

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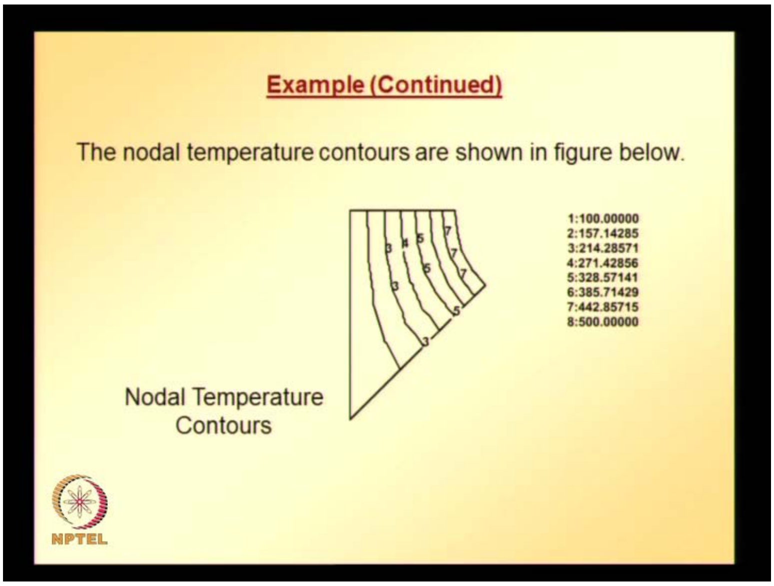

And the nodal temperature contours and still model is refined further. Using 72 elements sorry it should be 72; instead of that, it is typed as 32; and 72 element mesh is shown here. And the nodal temperature contours, for 72 element solution is given in this figure. If we compare this solution with 8 elements, 32 elements and 72 elements, we can clearly see convergence of solution both for temperature and heat flux values.