Finite Element Analysis

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Lecture No. # 23

Let us continue with two-dimensional boundary value problems that we started looking in the last class and basically let me summarize, what we heard in the last class. We started out with the problem statement of general two-dimensional boundary value problem. We also looked at equivalent functional of that problem. This is the problem that we have taken is a second order differential equation defined over a two-dimensional domain surrounded by a boundary. We also looked at how to derive the element shape functions for a 3 node triangular element and also we looked at how to get the element equations using Galerkin Criteria in conjunction with the three node element shape functions. That we derived and finally we got element equations.

Now, let me quickly go through what we have done in the last class. This is the second order differential equation for a two-dimensional boundary value problem. This need to be solved over arbitrarily defined area A, bounded by surface S and a part of surface on which a natural boundary condition are specified is denoted with S 1. Part of the boundary on which essential boundary natural boundary conditions are specified is denoted with S 2. These are the two boundary conditions either of this can be specified and subjected to these two boundary conditions. We need to solve this two-dimensional boundary value problem that is the problem statement.

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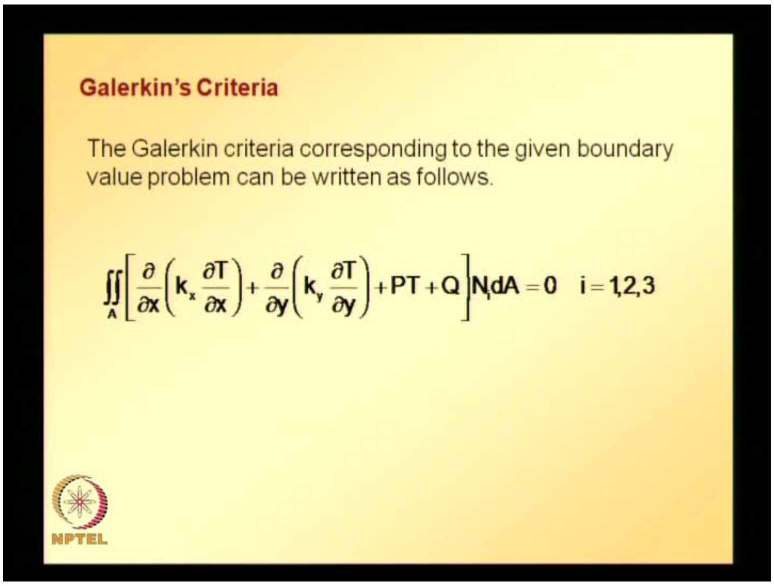

Using Galerkin Criteria we get this equation, where I takes values 1, 2, 3, because there are three shape functions for a three node triangular element using integration by parts, that is Green's Theorem.

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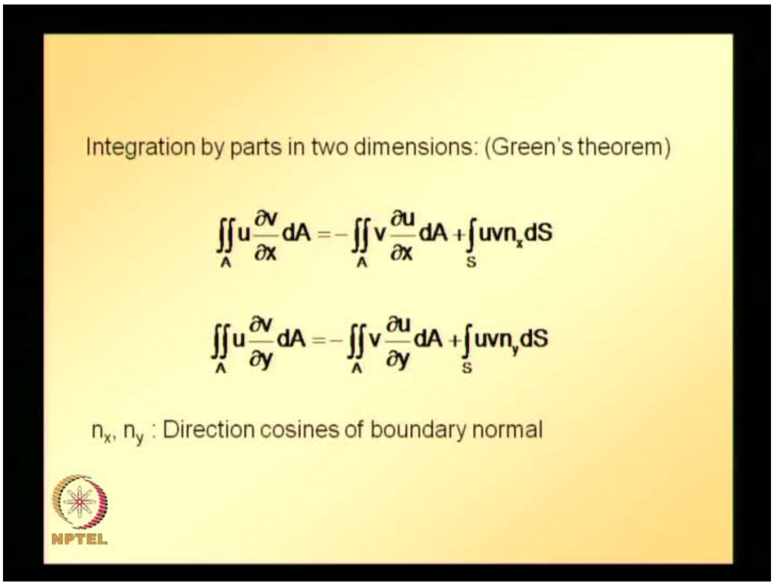

We get these two identities or two formulas; we use these two formulas to simplify the earlier equation. Here, n x, n y are the direction cosines of the outer normal to the boundary.

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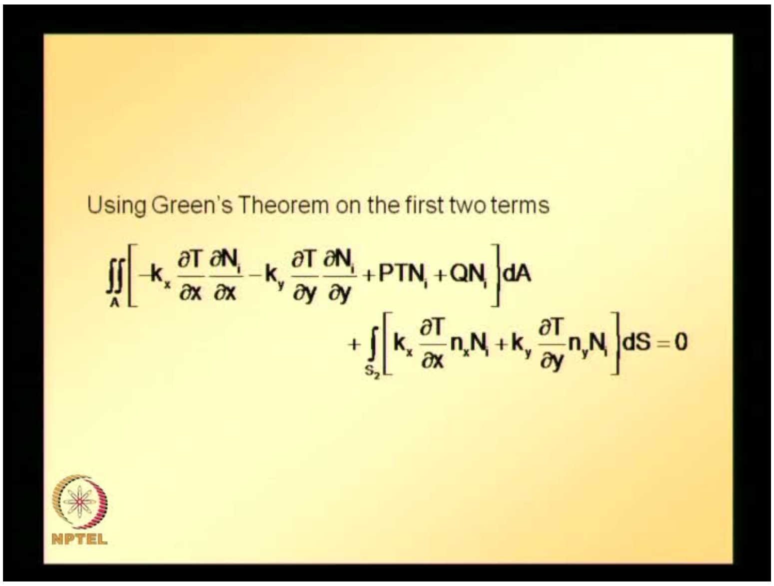

Using Green's Theorem on the first two terms of the equation that we obtained using Galerkin Criteria we get this equation.

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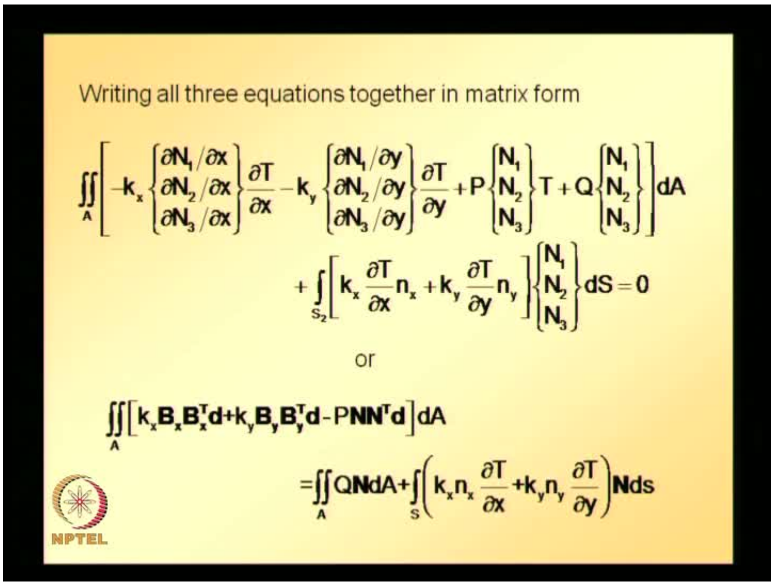

This can be further simplified by writing equations corresponding to N 1, N 2, and N 3. Writing all three equations together in matrix form, we get this equation. Here, this boundary integral over S 2 actually gets changed in the second equation, as boundary integral over the total boundary S. I gave detailed explanation in the last class, integral we can write like this because integral over S 1 is unknown, and there is no need to evaluate it.

Since, corresponding nodal parameters are known. Essential boundary condition is specified, equation containing these terms can be removed from the system before solving for unknown nodal parameters. Thus the integral over S 1 can be ignored from element equations integral over S 2 can be evaluated using the specified natural boundary conditions. That is the reason the first equation can be re written as given in the second equation, where the boundary integral over S 2 is replaced with integral over entire boundary.

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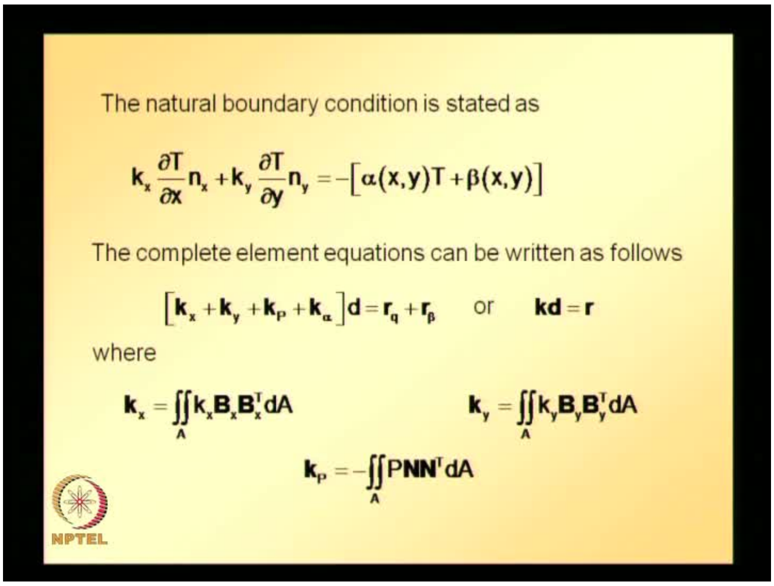

Now, using the specified natural boundary condition the boundary integral can be simplified further using this natural boundary condition. When we plug in this boundary integral term in to the equations. The complete element equations can be written like this, k x plus, k y plus, k p plus, k alpha times d is equal to r q plus, r beta, or it can be compactly written as k d equal to r, where k is defined as k x plus, k y plus, k p plus, k alpha and r is defined as r q plus, r beta. If you see each of this k x, k y, k p, k alpha they are all area integrals or integrals over the entire domain. We need to adopt numerical integration scheme, then the integrant becomes complicated. This is how k x, k y, k p are defined and k alpha and r beta corresponds to the natural boundary conditions.

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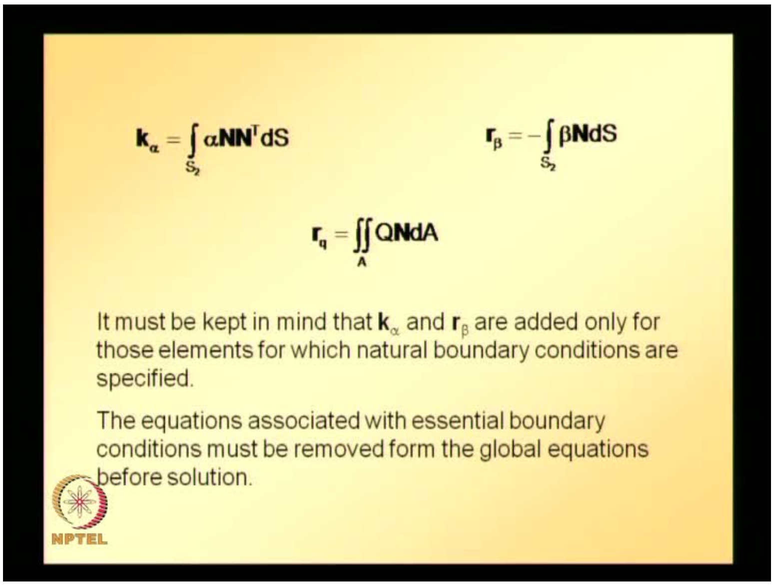

k alpha, r beta and r q is defined like this. Since, k alpha and r beta corresponds to the boundary on which natural boundary conditions are specified. It must be kept in mind that k alpha, r betas are added only for those elements, for which natural boundary conditions are specified. For a particular problem, no natural boundary condition is specified then k alpha and r beta are going to be 0. Please note that each of these surface integrals are going to be a vector having a column vector, which is going to be having three components.

If there are three nodes for a triangular element that we are considering, if we take each node there is 1 degree of freedom than these boundary integrals. That is k alpha, r beta, r q are going to be a vector having three components. k x, k y, k p and k alpha are going to be a matrix having dimension 3 by 3. Since, this corresponds to a triangular element having 3 nodes. Assuming 1 degree of freedom at each node we get 3 by 3 matrix. This k alpha, r beta are added only for those elements for which natural boundary conditions are specified. Equations associated with essential boundary condition must be removed from the global equations before solving. Once we assemble all the element equations, once we come up with the global equations from the information of element connectivity, we need to remove the equations associated with essential boundary conditions, before we solve the global equations.

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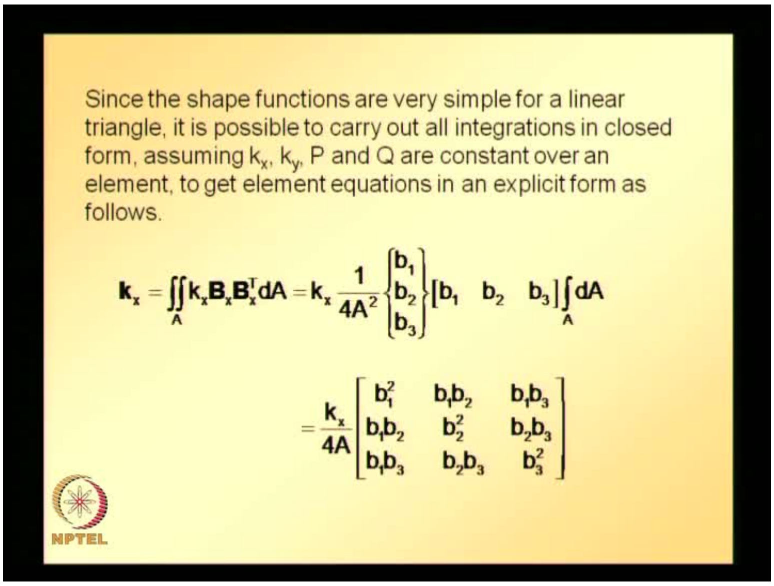

Let us see how each of these matrices and vectors looks like, for a 3 node triangular element. The shape functions for 3 node triangular element are very simple. Since, 3 node triangular element is a linear element or linear triangle; it is possible to carry out all the integrations in a closed form.

Assuming k x, k y, P Q are constant over element to get element equations in an explicit form. This is possible only if k x, k y, P Q are constant. If they are not constant the integrant becomes complicated. We need to adopt numerical integration scheme which will be looking at in the later lectures. If k x, k y, P and Q are not constant, now let us assume these are constant. Let us see how to simplify further the matrix k x, matrix k x is defined like integral k x coefficient k x B x times B x transpose. Integrated over entire element area. Substituting what is B x, B x is nothing but derivative of shape functions a vector consisting of derivative of shape functions with respect to x. Substituting B x, B x transpose you can see or if you recall the expressions for shape functions for 3 node triangular element these N1, N 2, N 3 are all linear in x and y.

When you take derivative with respect to x or y, you are going to get a constant vector a vector, which is going to be constant over the entire element. B x is going to be a constant consisting of coefficients b 1, b 2, b 3, which are functions of the spatial coordinates of the nodes. Substituting B x and B x transpose, this integral can be simplified in the form given in the slide. An integral over d A is going to be A, this can be further simplified in this manner, this kind of expressing k x in a closed form is possible only if the coefficient k x is constant. If it is not constant the integrant becomes complicated.

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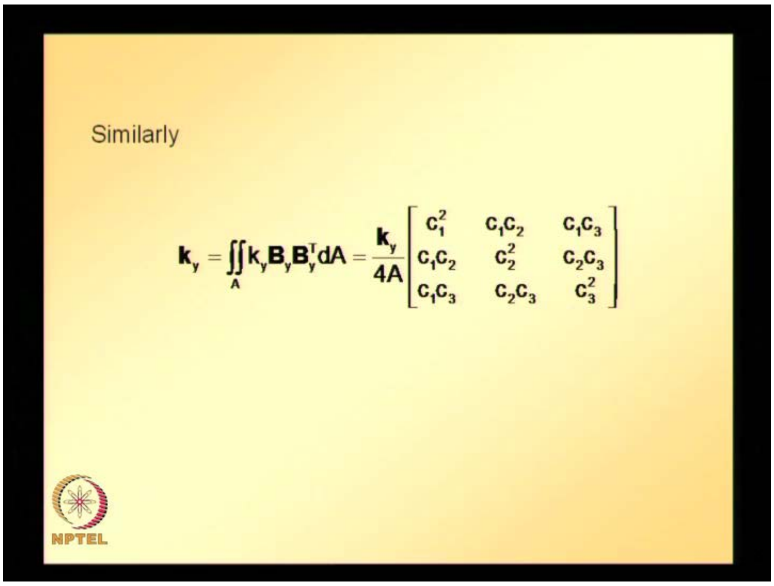

Similarly, we can write explicit expression for matrix k y except that we need to substitute B y here, which is nothing but a vector consisting of derivative of shape functions with respect to y, which is nothing but c 1, c 2, and c 3, where c 1, c 2, c 3 are functions of spatial coordinates of the nodes. Substituting all that information and simplifying this integral similar to what we have done. Adopting similar procedure as for the previous case of k x, we get k y in this manner in a closed form.

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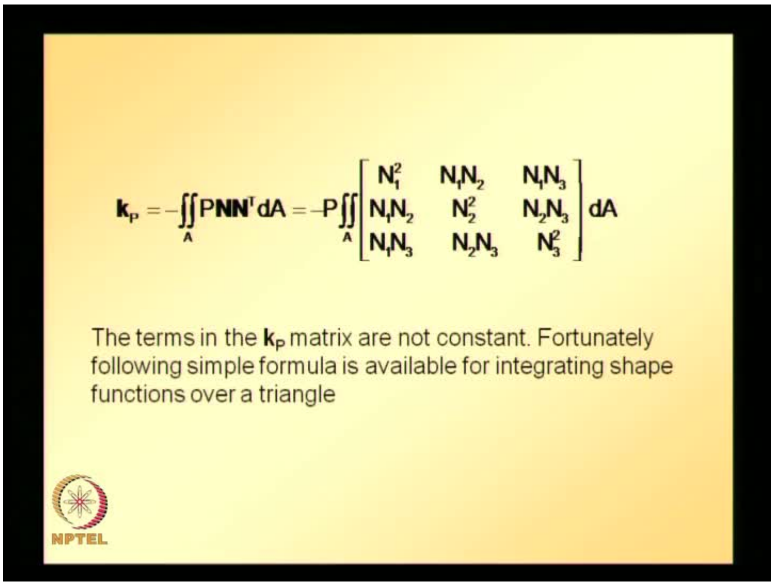

Let us look at the other term k p, k p is defined like this, assuming P to be the constant we can take P out of the integral and multiplying N. Which is nothing but a vector consisting of shape functions N 1, N 2, N 3. Carrying out multiplication of N, N transpose, this can be further simplified in the manner which is shown in the slide. Please note that this N 1 square, N 1 N 2, N 1 N 3 and all these quantities are not going to be constant. They are going to be functions of spatial coordinates. To carry out these integrations, we need to adopt some kind of numerical integration scheme or fortunately there is a formula to evaluate this in a closed form. The terms in the k p matrix are not constant. Fortunately following simple formula is available for integrating shape functions over a triangular element. Whatever formula that is going to be given is valid only for integration over triangular region.

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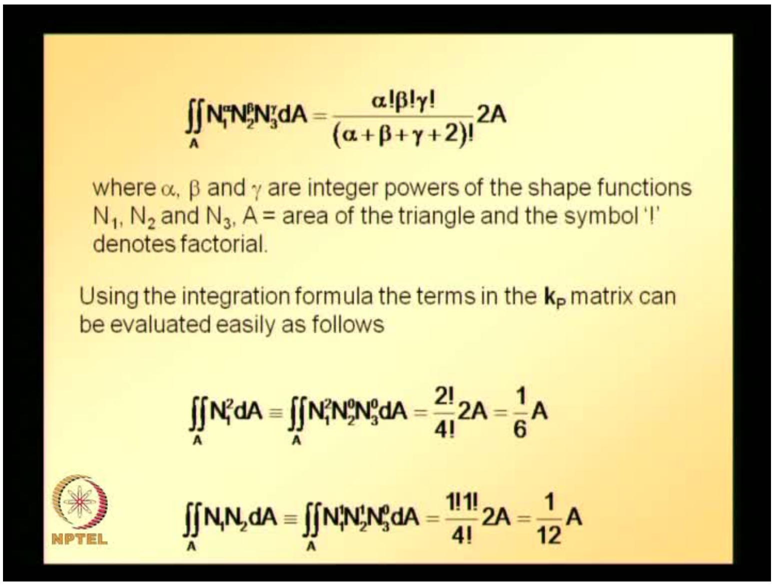

This formula looks like N 1 raise to power alpha, N 2 raise to power beta, N 3 raise to power gamma integrated over a triangular area, which is equal to alpha factorial multiplied by beta factorial, multiplied by gamma factorial divided by alpha plus, beta plus gamma plus 2 factorial, entire thing multiplied with 2 A, where A is areas of the triangular region and these alpha, beta, gamma are all integers, where alpha, beta, gammas are integer powers of shape functions N 1, N 2, N 3. A is area of triangle. This exclamation symbol denotes factorial operation.

Using this formula we can evaluate each of the components of k p matrix, using the integration formula terms in the k p matrix can be evaluated. Here, for two terms the details are shown integral N 1 square over triangular region. We can rewrite as N 1 power 2, N 2 power 0, N 3 power 0, because any number raise to power 0 is equal to one. alpha is equal to, 2 beta is equal to 0, gamma is equal to 0. With this information substituting this into the formula, we get two factorial divided by 4 factorial times 2 A. This can be simplified as A over 6, which is one component of k p matrix integral N 1 square over the triangular region. Similarly, product of N 1 N 2 integrated over triangular region. It can be written as N 1 power, N 2 power 1, N 3 power 0. That means alpha is equal to 1, beta is equal to 1, and gamma is equal to 0. Substituting all this information into the formula we get 1 factorial times 1 factorial, divided by 4 factorial times 2 A. which can be simplified as A over 12. In this manner, we can carry out integration of all the components and k p matrix can be obtained by plugging in all these integrals.

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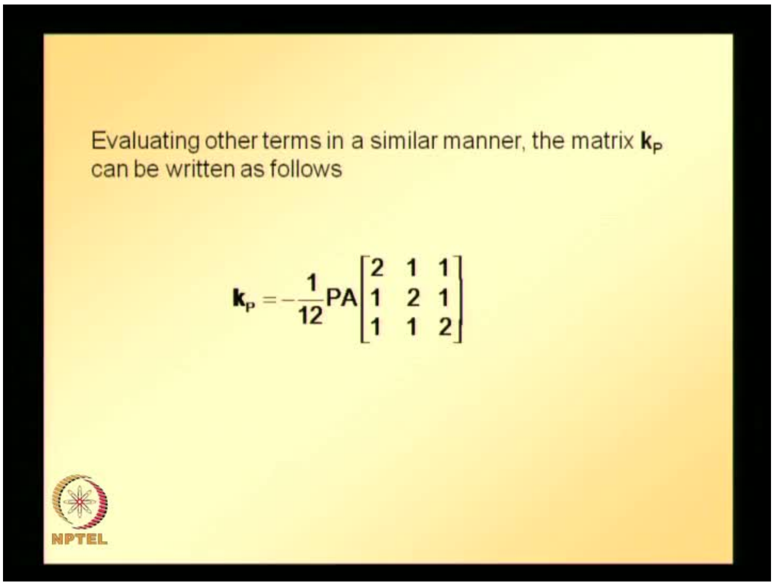

Evaluating other terms in a similar manner, the matrix k p can be written like this. Here, once again application of this formula is possible only if we are integrating the shape functions over triangular region, P is constant. Here, P is assumed to be constant, if P is not constant the integrant becomes complicated. We need to adopt numerical integration scheme. We have seen how to evaluate k x, k y and k p for a 3 node triangular element.

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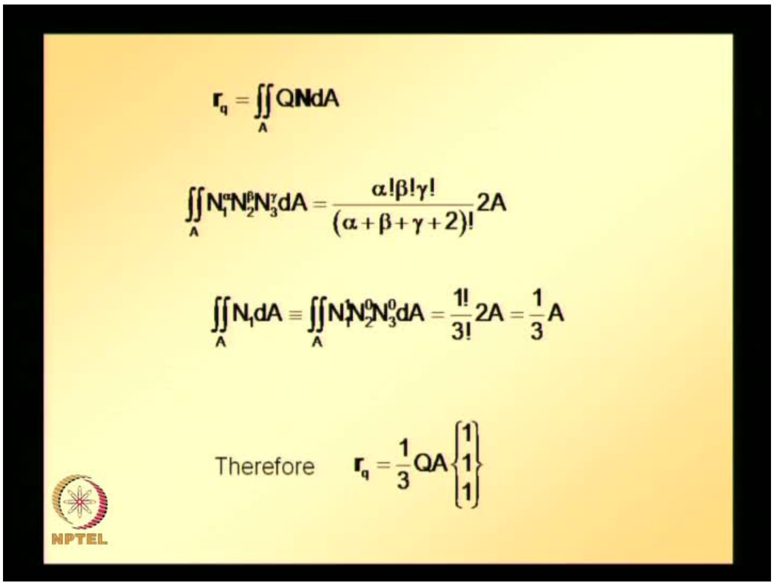

Now let us look at the other quantities, r q is defined like this, integral or Q times N integrated over triangular region. If Q is constant we can take Q out of the integral. For evaluating rest of the quantity that is, integral of any shape function N 1, N 2, N 3. We can adopt the formula that we have seen earlier, the formula we have seen N 1 power alpha times N 2 power beta, N 3 power gamma integrated over triangular a region is alpha factorial times, beta factorial times gamma factorial, divided by alpha plus, beta plus, gamma plus, 2 factorial times. Entire thing multiplied by 2 A. This formula can be applied to simplify each of the components of r q.

Example integral N 1 over triangular region can be written as N 1 power 1, N 2 power 0 and N 3 power 0. This can be simplified to 1 factorial divided by 3 factorial times 2 A, which is equal to A over 3. Similarly, evaluating other components that are integral N 2 over triangular region and integral N 3 over triangular region, we get all the components of r q, r q can be written like this. Here, once again Q is constant then only this is valid, this is how we can evaluate area integrals.

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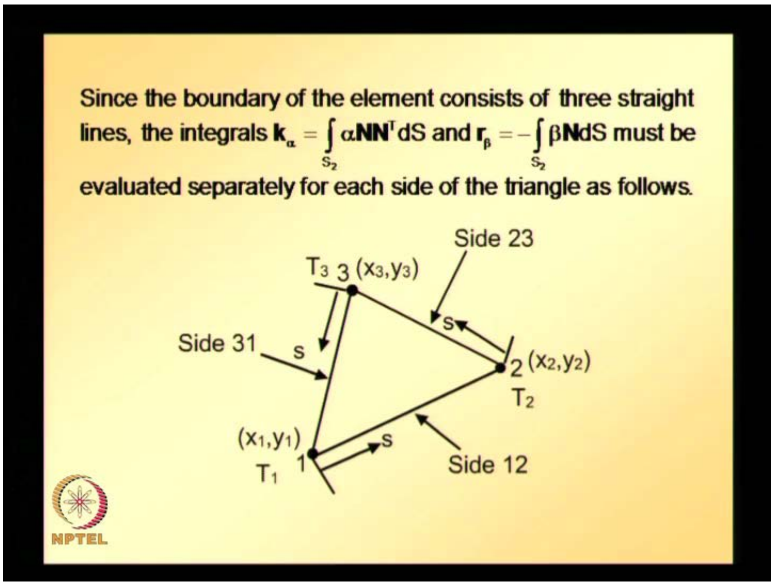

We are left with two more integrals which are boundary integrals k alpha and r beta. These integrals we need to evaluate along the element edges. Since, boundary integrals or since the boundary of element consists of three straight lines like this. This is a triangular element that we are looking at or we are trying to develop element equations for this kind of triangular element. Since, the boundary of element consists of 3 straight lines integrals k alpha and r beta must be evaluated separately for each side of triangle. We need to evaluate these integrals for each of the sides 1 2, 2 3, 3 1 separately. If you see the schematic there, for each of these edges local coordinate system are defined. s for side 1 2 starting at node 1, s is equal to 0 corresponds to node 1 and s is equal to length of side 1 2 corresponds to node 2. Similarly, for side 2 3 local coordinate system is defined, side 3 1 local coordinate system is defined.

Now, we need to evaluate these integrals for each of the sides taken each of them separately. The details for one of the sides will be illustrated here, one more thing to note is alongside 1 2, the shape function of node 3 is going to be 0, alongside 2 3 shape function of node 1 is going to be 0, alongside 3 1 shape function of node 2 is going to be 0.

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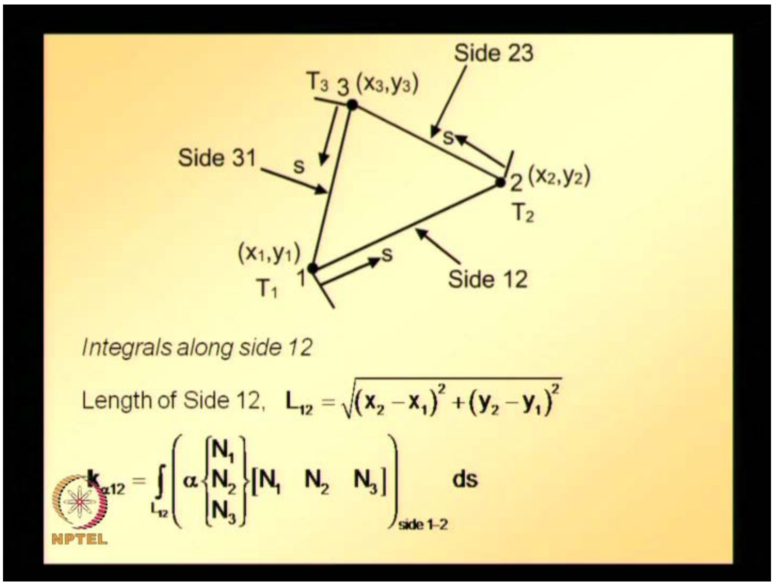

To evaluate integral alongside 1 to 2, we require what is the length of side 1 2. Side 1 2 can be calculated, once we know the coordinates of each of the nodes. With the information of coordinates of node 1 and 2, we can calculate length of side 1 2. Then integral k alpha along side 1 2 becomes like this. Integral evaluated along side 1 2 the value of alpha is going to come from natural boundary condition alpha times N 1, N 2, N 3 vector. We are going to get a 3 by 3 matrix, we need to evaluate this integral over side 1 2. We can adopt numerical integration scheme, because we cannot use the formula that we have seen earlier, because that is only applicable for integration over triangular region that is for area integral.

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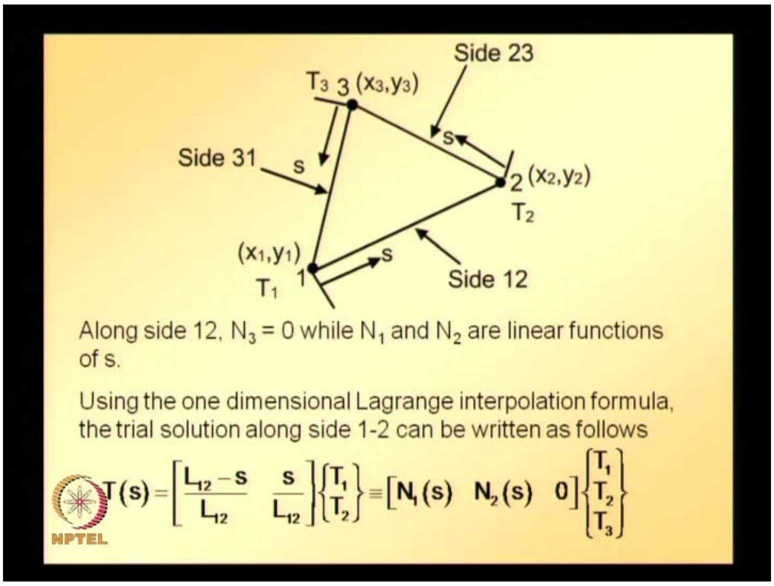

Now, alongside 1 2 shape function of node 3 are going to be 0. While N 1 and N 2 are linear functions of s. How to get the shape functions of N 1 N 2 along side 1 2? We can use Lagrange Interpolation formula that we already learned for one-dimensional, when we are looking at one-dimensional elements, we already looked at Lagrange Interpolation formula for 2 node linear element. We can use that using the one-dimensional Lagrange Interpolation formula.

The trial solution along side 1 2 can be written like this, N 1 times T 1, T 1 is the field variable value at node 1, N 2 times T 2 T 2 is field variable value at node 2, and the shape function of N 1 can be obtained using Lagrange Interpolation formula taken side 1 2 alone. Shape function of node 1 along side 1 2 is given by L l 2. Where, L l 2 is length of side 1 2 minus s divided by length of side 1 2. That is shape function of node 1 alongside 1 2, shape function of node 2 along side 1 2 is given by s divided by length of side 1 2. This formula can easily be obtained using Lagrange interpolation formula. Since, shape function of node 3 is 0 along side 1 2 we can rewrite this equation as N 1 N 2 0 times T 1, T 2, T 3 as given in the equation. This is how trial solution can be written in terms of nodal values T 1, T 2, T 3 along side 1 2.

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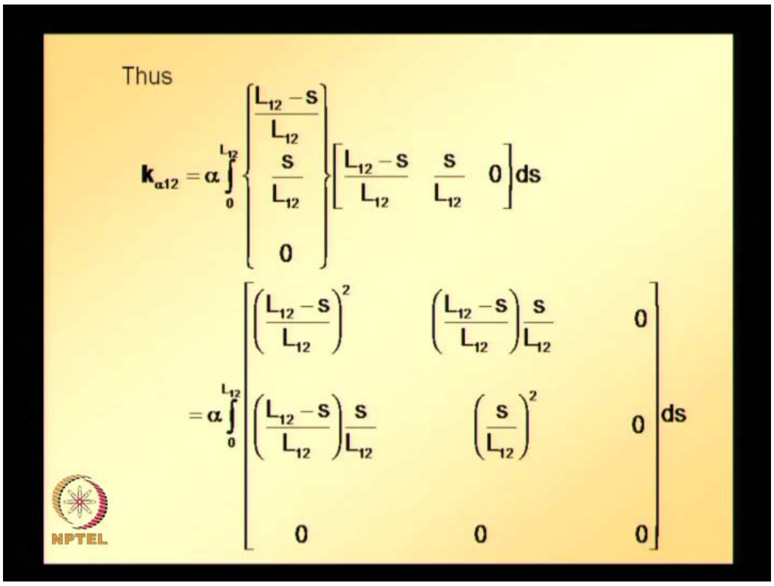

Substituting the shape function values of N 1, N 2, N 3 along side 1 2 into k alpha we get this. Carrying out the vector multiplications, we are going to get this matrix and still this needs to be evaluated over side 1 2. That is small s going from 0 to length of side 1 2, we can adopt numerical integration scheme if the integrant is complicated or if integrant is simple, we can evaluate this in a closed form. Since, here integrant is simple, we can evaluate in a closed form by integrating each of these components with respect to S, substituting the limits of integration 0 and L l 2, we get the closed form.

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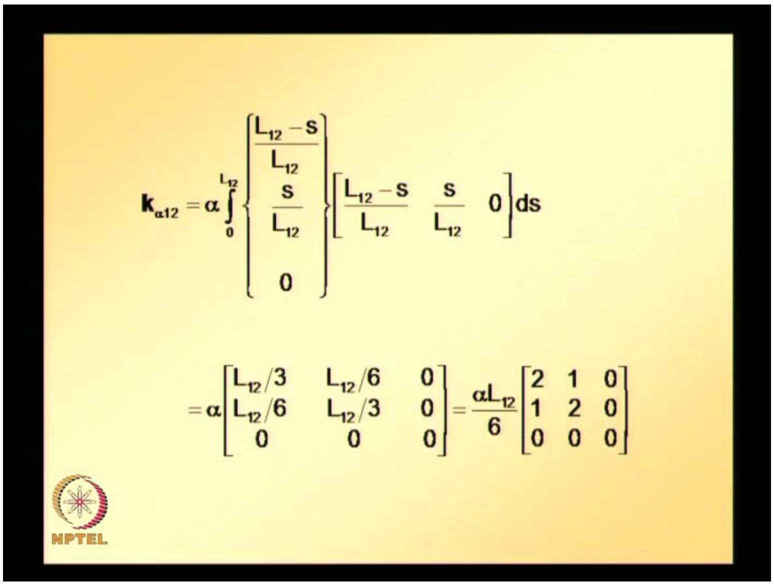

Solution of this k alpha, assuming alpha to be constant this formula or this equation is valid only when alpha is assumed to be constant. If alpha is not constant, then we need to integrate keeping alpha inside the integral. This is how k alpha can be evaluated.

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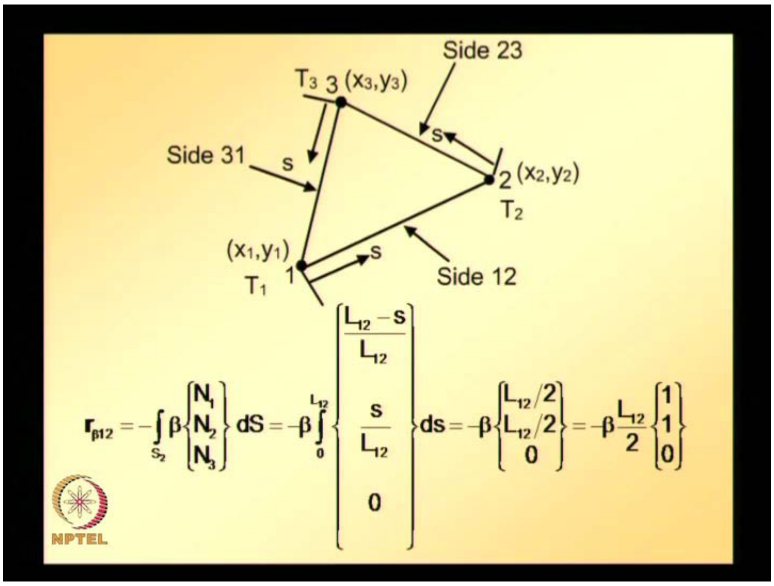

Similarly, r beta is defined like this, again we need to evaluate r beta separately along each of the sides 1 2, 2 3 and 3 1. Here, evaluation of r beta along side 1 2 is illustrated. Substituting N which comprises of component of shape functions or shape function value N 1, N 2, N 3 put in a vector form. Substituting that r beta can be written as beta times vector consisting of N 1, N 2, N 3 integrated over side 1 2. That can be re written by substituting the shape function of N 1, N 2, N 3. Since, N 3 is 0, we can simplify this further and assuming beta to be constant we can pull beta out of the integrant, we can get the closed form solution of this integrals. r beta is given by the value which is given in the slide. That is beta times length of side 1 2 divided by 2, multiplied by a vector consisting of 1 1 and 0. Similar operations can be repeated if required for side 2 3 and 3 1. If natural boundary condition is not specified along 2 3 or if natural boundary condition are not specified along any of sides along that side we do not need to evaluate these integrals.

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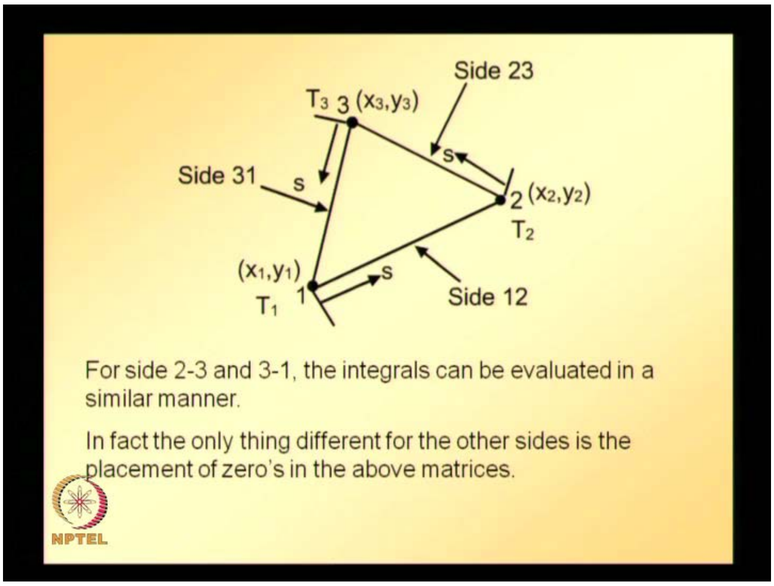

For side 2 3 and 3 1 integrals can be evaluated in a similar manner, except that local coordinate system should be defined as shown in the schematic, noting that a shape function of node which is not part of a side is going to be 0 along that side. In fact the only thing different from other sides is the placement of zeros in the above matrices, so that is what I mentioned. So, carefully noting which node is not part of the edge the shape function of that node is going to be 0 along that edge. Taking care of that, we can get the integrals for side 2 3 and 3 1.

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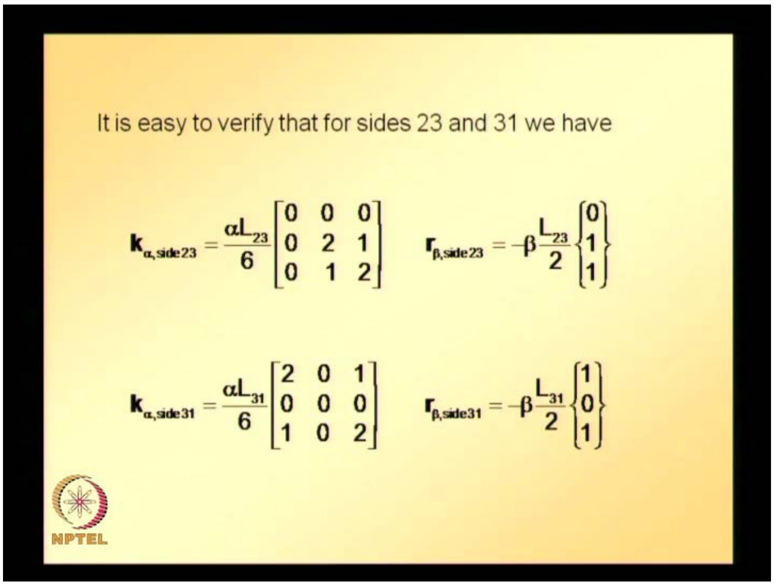

Now, to see application of this, whatever we have developed element equations, we will take up a numerical example, also it is easy to verify that for sides 2 3 and 3 1, the simplified form of k alpha and r beta are this.

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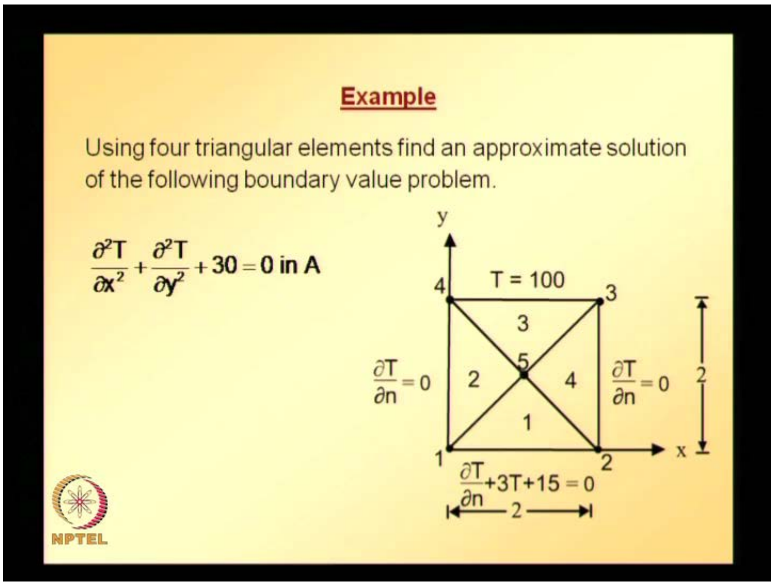

Let us take a numerical example here to illustrate how to use the above element equations and to illustrate the assembly process for two-dimensional problems. Now let us take the problem statement using four triangular elements find an approximate solution of following boundary value problem. This is the boundary value problem or the differential equation for which we need to find solution. The field variable is T and this differential equation is valid over domain A, where A consists of 4, 3 node triangular elements all the nodes are numbered 1, 2, 3, 4 and 5. Fifth node is at the centre of A. Along each of the edges the boundary conditions are also shown. Also the geometrical details are given the problem dimensions and from the details that are geometrical details that are given in the schematic. We can easily use the coordinate system that is defined, we can easily figure out what are the nodal coordinates of each of the nodes 1, 2, 3, 4, 5, and also elements are numbered as 1, 2, 3, and 4.

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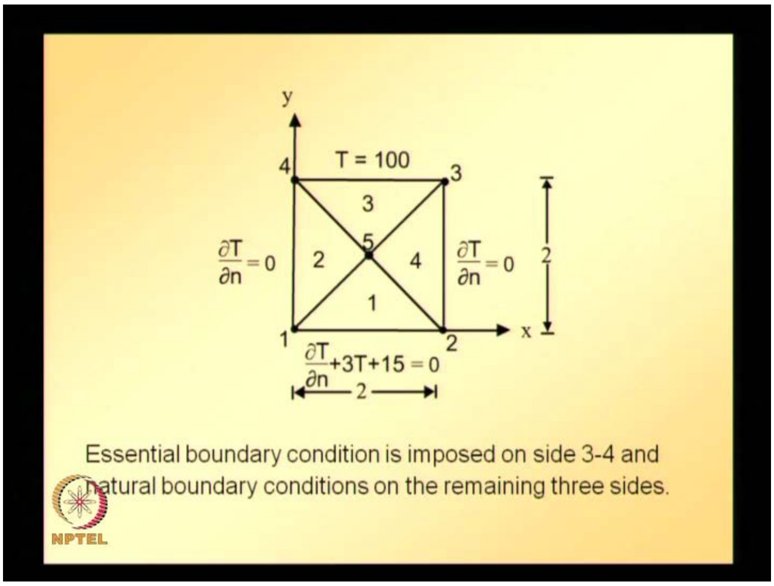

Essential boundary condition is imposed looking at the boundary condition, we can easily figure out which is essential boundary condition, which is natural boundary condition. Essential boundary condition is imposed on side 3 4 along the edge which is connecting nodes 3 and 4; natural boundary conditions are specified along edges 1 2, 2 3 and 4 1 along the remaining 3 sides. Now we have all the information like boundary conditions are given along all the edges, we can easily identify what is alpha and beta when we compare this with general two-dimensional boundary value problem. Also given the differential equation we can also find what is k x, k y, P Q. If we compare with general two dimensional boundary value problem.

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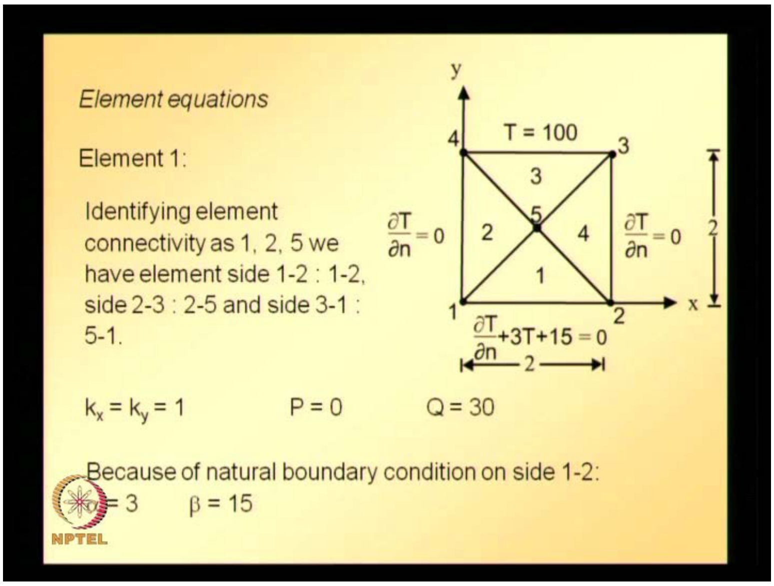

Let us start assembling the element equations starting with element 1. First we need to identify what are the nodes identifying element connectivity as 1, 2, 5. This is how element 1 connectivity is identified 1, 2, 5. Local node 1 is same as global node 1, local node 2 is same as global node 2, local node 3 is global node 5. Identifying element connectivity as 1, 2, 5 we have element sides. Local side 1 2 is same as global side 1 2, local side 2 3 is going to be side 2 5, and side 3 1 is going to 5 1. With this understanding, also identifying from the given differential equation, what are the coefficients k x, k y, P Q comparing with general two-dimensional boundary value problem. We can easily identify k x, k y, as P is equal to 0 and Q is equal to 30. Alongside 1 2, if you see this element 1 only along side 1 2 natural boundary condition is specified. Side 2 5 and 5 1 no natural boundary condition is specified. Alongside 1 2 the given natural boundary condition compare that with the boundary condition. For general two-dimensional boundary value problem and identify what is alpha and beta, alpha is 3 and beta is 15, it can be easily verified.

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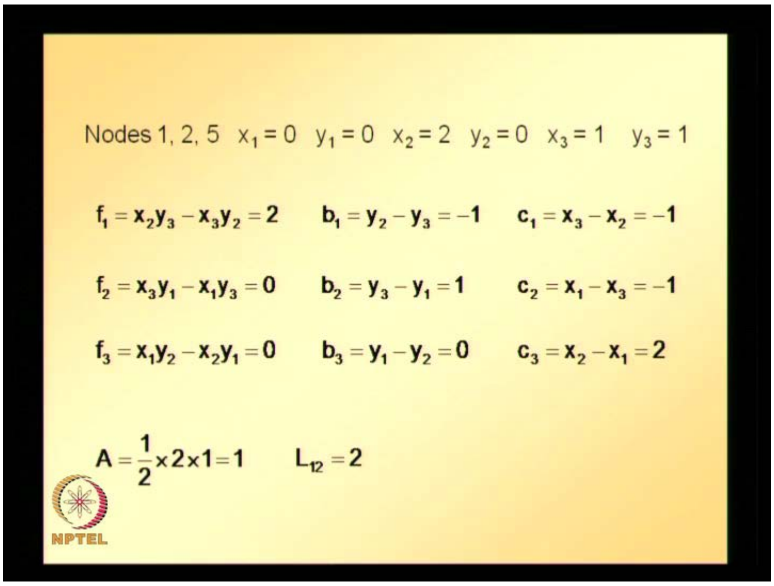

We have all the information, now our job is to plug in all these quantities into the element equations that we developed for a 3 node triangular element. Nodes are the element connectivity is like this, nodes 1, 2, 5. Noting down the coordinates of these nodes from the geometrical details that are given in the problem, x coordinates and y coordinates of nodes 1 2 5. This information is required, because we need to calculate the shape functions, for which we need to calculate f 1, f 2, f 3, b 1, b 2, b 3, c 1, c 2, c 3, which are all functions of spatial coordinates, we can calculate f 1, b 1, c 1. Similarly, f 2, b 2, c 2, f 3, b 3, c 3. Finally, we can calculate the shape function expressions using these coefficients f, b and c. Also we require what is the area of triangle A, what is the length of the side 1 2. All that information can also be computed, once we note down this nodal coordinates.

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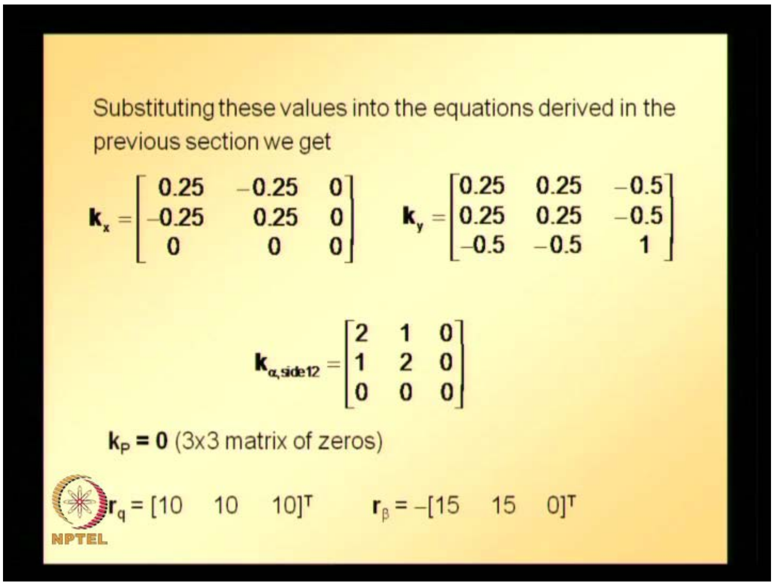

With this information, we can easily find the shape function. Shape function expressions N1, N2, N3 corresponding to the element 1 connecting nodes 1, 2, 5. Substituting these values into equations derived in the previous section. We already derived element equations for 3 node triangular element. Substituting the bs value b 1, b 2, b 3 into k x matrix that we already have and c 1, c 2, c 3 values into k y matrix that we have we get this 2 matrices k x and k y. Corresponding to element 1 and also k alpha identifying what is alpha. Plugging in into the element equation that for k alpha that we already developed. We get this k alpha for side 1 2. Since, P is 0 k p is going to be 0. It is going to be a 3 by 3 matrix of zeros also r q and r beta can be assembled, r q is going to be 10. A vector consisting of three components each of which is equal to 10 and r beta is a vector consisting of minus 15, 15, 0.

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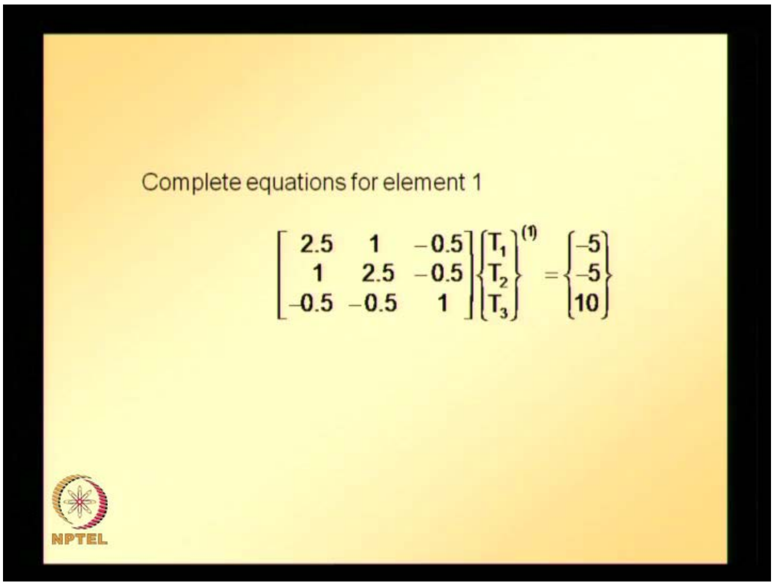

This is for element 1, these are the quantities for element 1. Using all these quantities complete equations for element 1 can be written like this, where T 1, T 2, T 3 are the local nodal parameters for element 1; that is why it is written as T 1, T 2, T 3. Super script 1 that means it corresponds to element 1. Similar operation we can also repeat for other elements like element 2, element 3 and element 4. Except that we need to keep a note carefully on the element connectivity and also the coordinate of coordinates of nodes.

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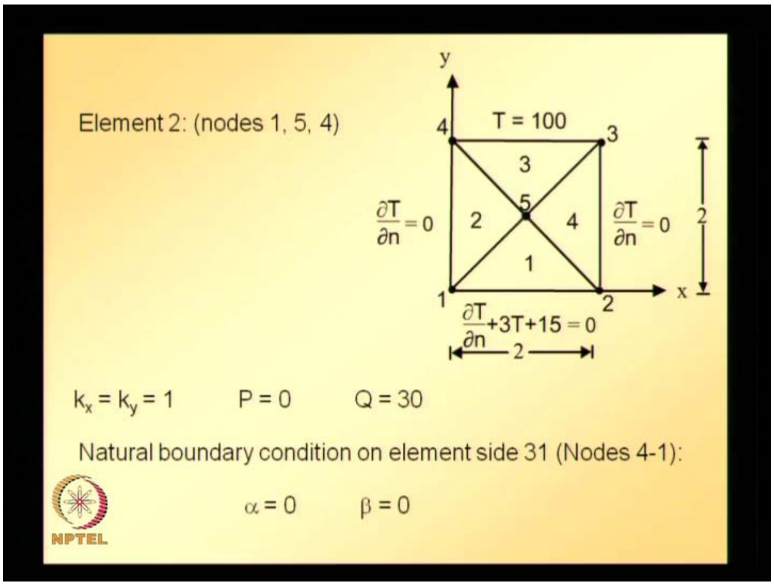

For element 2, the details of workout is shown here, element 2 the connectivity is defined like this 1, 5, 4. That means local node 1 is same as global node 1, local node 2 is global node 5, local node 3 is global node 4. With this understanding also comparing the given differential equation with general boundary value problem equation, we get this k x, k y, P Q, which is similar to element 1 and natural boundary condition. If you see the natural boundary condition that is specified along side 4 1, that is local side 3 1. We get this information alpha is equal to 0, beta is equal to 0, for this particular element k alpha is going to be 0 and, r beta is also going to be 0.

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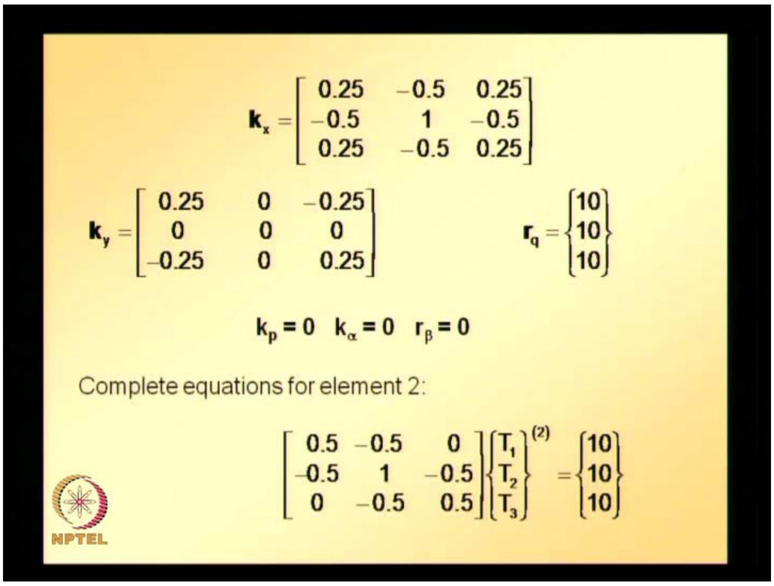

Once we have all that information following the procedure similar to that for element one we get k x k y and r q. Rest of the quantities k p, k alpha, r beta are 0. Using all this information, we get the complete equations for element 2 like this.

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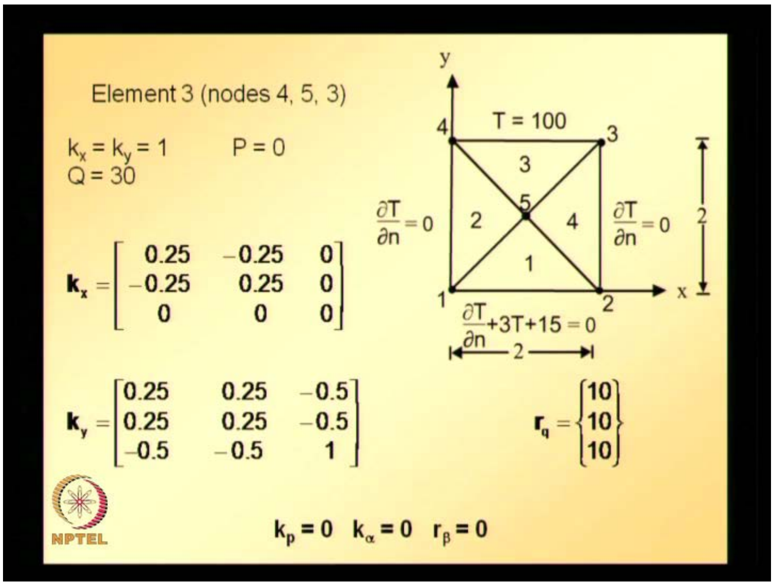

Similar operation can be repeated for element 3. Noting down local node 1 is same as global node 4, local node 2 is global node 5, local node 3 is global node 3 and noting down k x, k y, P Q. We get k x, k y, r q along any of the sides of this element, natural boundary condition is not specified; k alpha r betas are going to be 0. P is 0 that is why k p is equal to 0.

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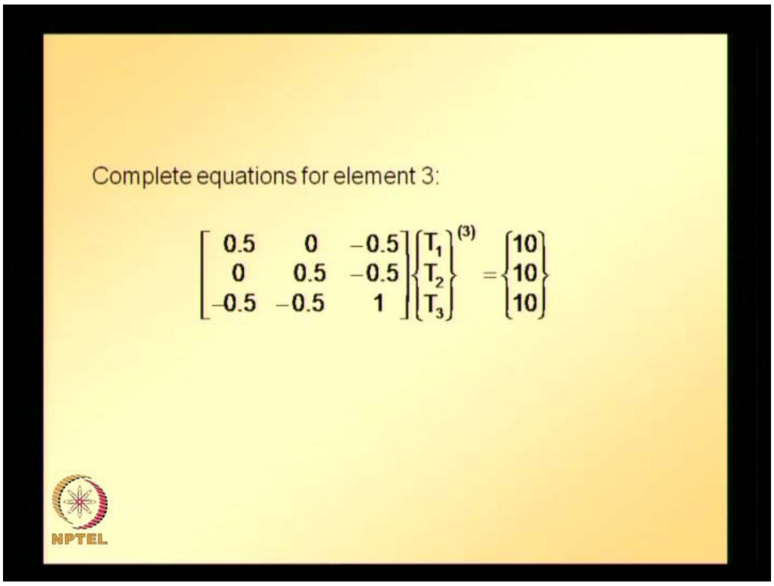

Complete equations for element 3 are given by this. Please note that for element 3 even though along edge 3, 4. Essential boundary conditions are specified at this stage, we have not used that information but, later when we assemble the global equations based on the element equations for all the four elements. Once we get the global equation at that stage, before we solve the global equation system we impose this information. That is alongside 3 4, T value is 100 at that stage we are going to impose.

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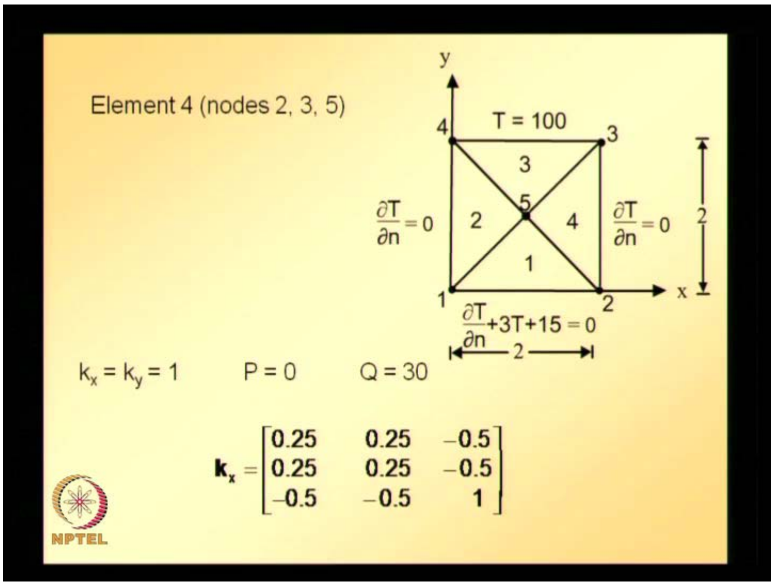

Let us go to the element 4 identifying local node 1 as global node 2, local node 2 as global node 3, local node 3 as global node 5 and rest of the information similar to the other three elements.

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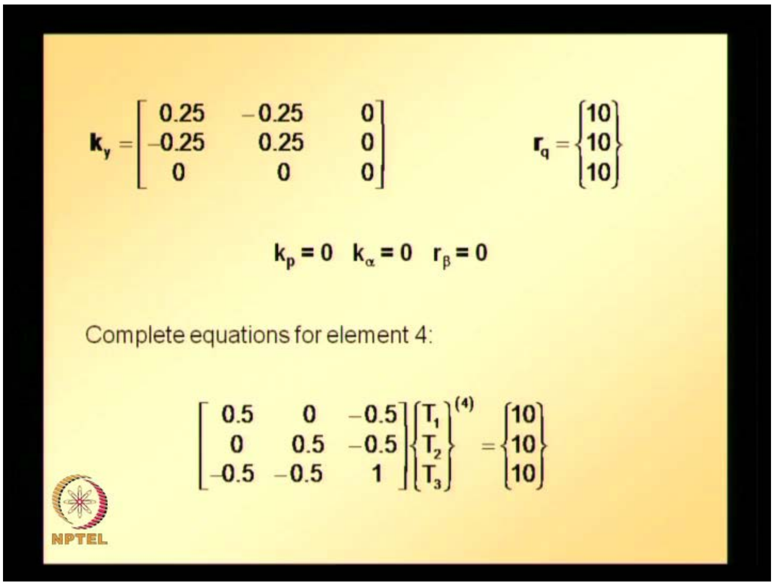

We can get k x, k y, r q and rest of the matrices and vectors are going to be 0, because P is 0, alpha is 0, beta is 0 complete element equations for element 4 are these now we have element equations for element 1, 2, 3, 4.

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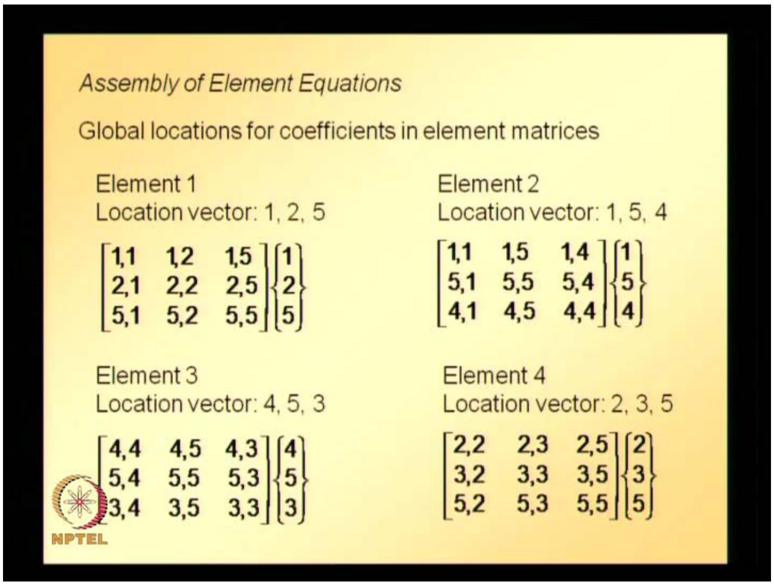

Now, we need to assemble the global equations for that we require global location coefficients and the element matrices. Once again we look at the element connectivity or the nodal connectivity for each of the element for element 1, the nodal connectivity is like 1, 2, 5. For element 2, 1, 5, 4 is the nodal connectivity. With this information we can easily identified at what locations the contribution of each of the elements 1, 2 goes into the global equation. Element 1 contribution goes into the locations that are given under element 1, element 2 contribution goes into locations that are given under element 2 into the global equations from the local equations. Similarly, for element 3 and element 4, element 3, the node nodal connectivity is 4 5 3. Element 4 nodal connectivity is 2, 3, 5. We get these global locations for coefficients in element matrices.

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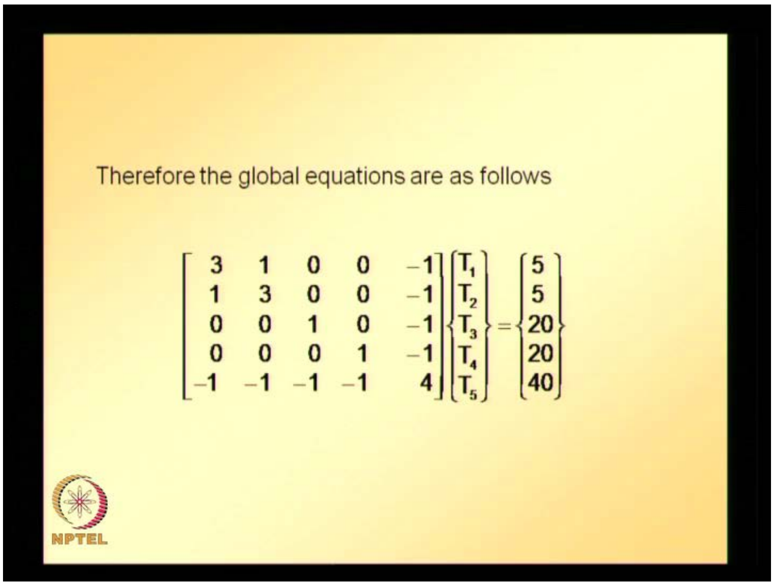

Using this information, we can easily assemble the global equations by plugging in the corresponding locations. The local element equations contributions into the global equations. Plugging in based on the information that we just have, we get this global equations. Before we proceed further we need to impose that T value along a side 3 4 is 100, that is T 3 is going to be 100, T four is going to be 100. We need impose that condition.

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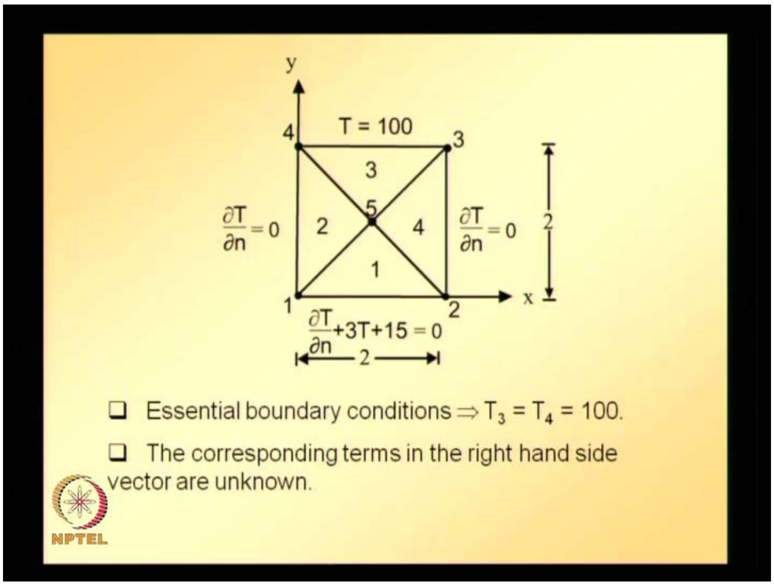

That condition we are getting from the problem that is given to us, essential boundary condition is T 3 is equal to 100, T 4 is equal to 100. Corresponding terms in the right hand side vector are unknown. Whenever essential boundary condition is specified in the global equation system, the corresponding terms in the right hand side vector becomes unknown.

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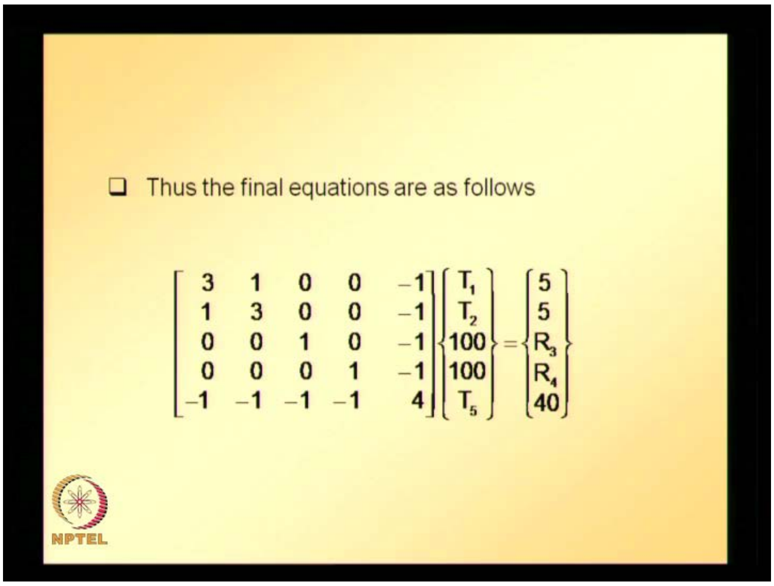

That we already have seen, we have seen that kind of thing earlier when we are solving the previous problems. A final equation after imposing T 3 is equal to 100, T 4 is equal to 100. To solve for the three unknowns T 1, T 2 and T 5, we need to rearrange this equation system the three unknown. T 1 T 2 and T 5 can be obtained from equations 1, 2, 5.

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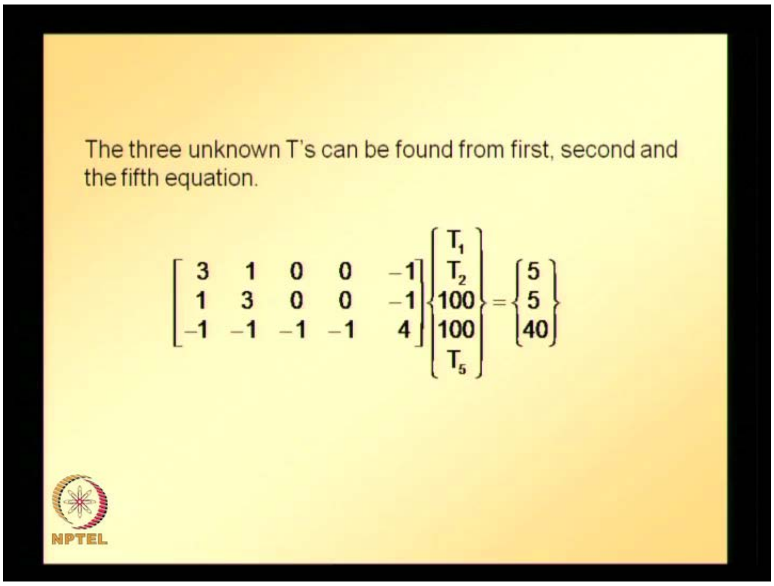

The previous global equation to solve for T 1, T 2 and T 5 needs to be re arranged like this and which can be simplified and solved for T 1, T 2 and T 5.

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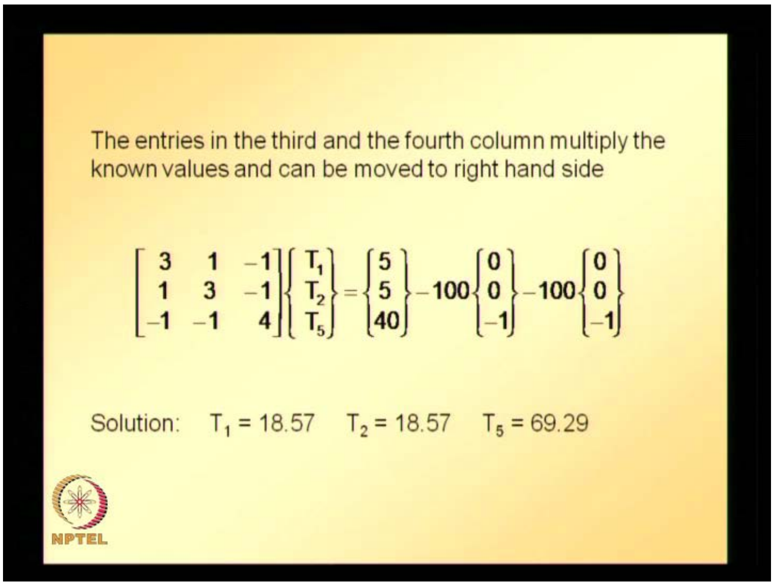

Entries in the third and fourth column multiply the known values and can be moved to the right hand side. Entries in the third and fourth column multiply by the known values that is 100. 100 can be moved to the right hand side, the previous equation can be re arranged like this to solve for T 1, T 2 and T 5. Once this equation system is solved, we get solution T 1, T 2, T 5 like this.

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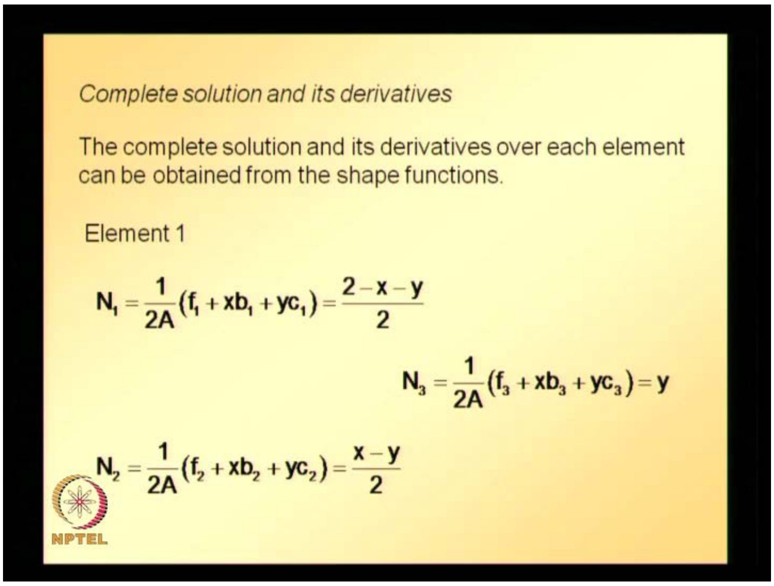

Once we obtain solution for T 1, T 2, T 5 we can go to each element and do post processing complete solution. Its derivative in each of the element can be obtained over each element, can be obtained from the shape functions. Shape functions corresponding to element 1, N 1, N 2, N 3. T at any point in the element is given by N 1 times T 1 plus N 2 times T 2 plus N 3 times T 3. For element 1, we know what the nodes for element 1 are, the nodes are 1, 2, 5.

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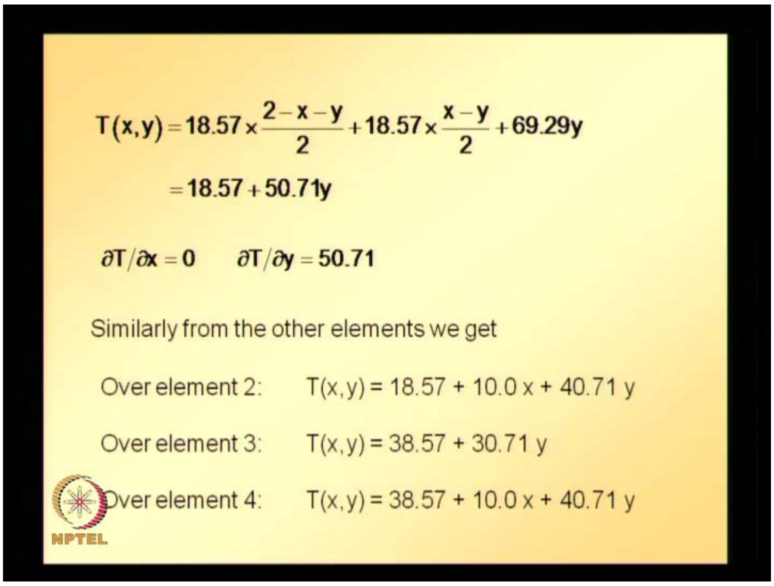

Using the nodal values T 1, T 2 and T 5 we can get T value in over element 1. Using this which can be further simplified and once we have this, we can easily calculate derivative of T with respect to x, derivative of T with respect to y. This gives us information about derivative of T over element 1. Similar operation can be repeated for other elements. Similarly, for other elements we get T is equal to this for over element 2, over element 3, T is this, and over element four T is this.

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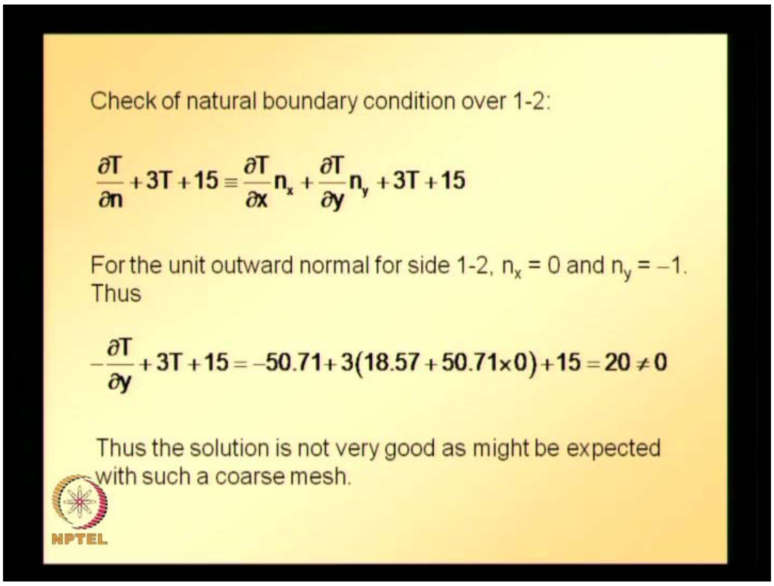

Also we can check whether the solution for T 1, T 2 and T 5, that we obtained. How accurate is that by sub plugging in into the natural boundary condition alongside 1 2. Check for natural boundary condition along side 1 2. This is the natural boundary condition that is given alongside 1 2 and partial derivative of T with respect to n, is defined as partial derivative of T with respect to x times direction cosine component n x plus, partial derivative of T with respect to y times direction cosine component n y. We can write the given natural boundary condition like this. For side 1 2 n x is equal to 0, n y is equal to minus 1. Substituting that we get this, now plugging in derivative of T value we can see that the equation is not satisfied exactly.

We can conclude that solution is not very good and this is expected. Since, we used only four linear elements. Please note that 3 node triangular element is a linear element for two-dimensional problems we used four linear elements, which may not be good enough to get the accurate solution for this problem. To get more accurate solution we need to use much finer refinement for this problem. So, with this we will continue in the next class.