Finite Element Analysis

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Lecture No. # 21

In today's class let us see how to solve two-dimensional boundary value problems using classical methods or how to get approximate solution of two-dimensional boundary value problems using classical methods.

The two methods that we are going to look in are modified Galerkin Method and Variational method, and both these methods we have already looked at when we are solving or when we are trying to find approximate solution of one-dimensional boundary value problems. And both these class of methods requires us to start with a general form of solution or we need to assume a general form of solution to start with and this general form of solution should contain some unknown coefficients or parameters, which are found by techniques established by the corresponding methods.

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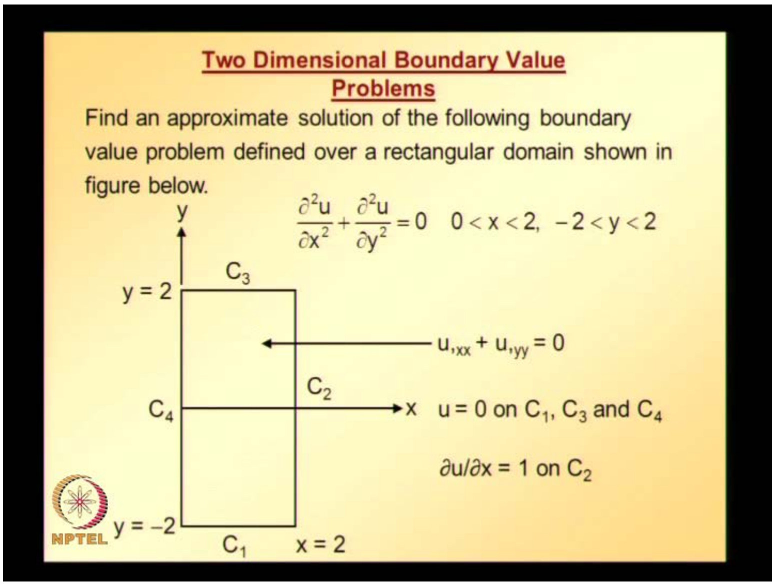

So, as a part of today's lecture, let us look at this problem two-dimensional boundary value problem find an approximate solution of the following boundary value problem defined over a rectangular domain shown in the following figure or shown in the figure below. Basically we need to solve this second order differential equation over a domain where x goes from 0 to 2, and y goes from minus 2 to 2 which is a rectangular domain and the domain is shown along with the boundary conditions u is specified, u is specified as equal to 0 on the boundary C 1, C 3 and C 4 and derivative of u with respect x is specified as 1 on the side or on the boundary C 2.

Some of these boundary conditions are essential boundary conditions, some of these boundary conditions are natural boundary conditions, and actually this whatever exercise that we are doing today that is solve finding trying to find approximate solution of two dimensional boundary value problem using classical methods. Actually it forms a basis for the topic that we are going to that we are going to see in the next class that is two dimensional boundary value problems using finite element methods are finding solution of two-dimensional boundary value problems using finite element methods.

Basically this exercise actually gives us more insight into the solution process, and also clearly demonstrates the little difference between the classical techniques, and finite element method So, that is the motive behind today's lecture, and before we proceed to solve this problem this two-dimensional boundary value problem which is shown here using the two techniques that is modified Galerkin Method, and Variational methods. Let us briefly review what we have done in the last classes. How to classify these boundary conditions, and what are the salient features of the two methods that is modified Galerkin Method and Variational method, and also we need to note some of the important points associated with those methods before we start solving this problem using any of these methods.

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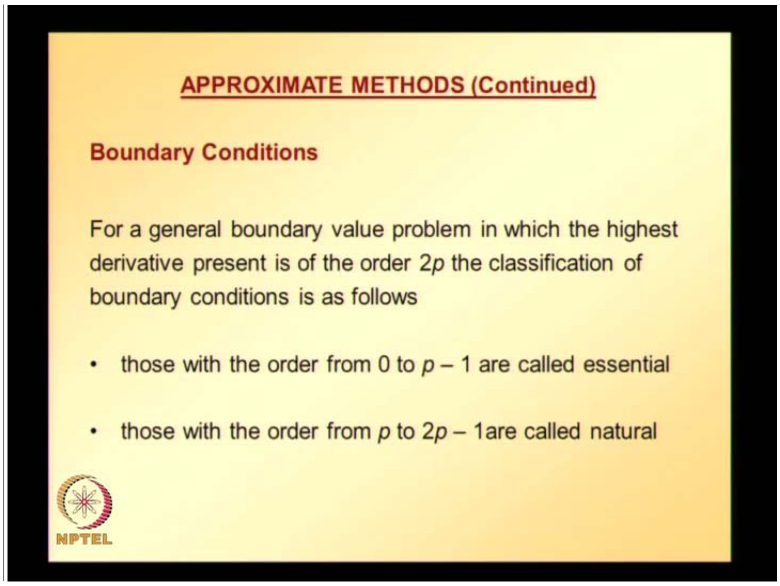

If you recall earlier, we noted that for a general boundary value problem in which the highest derivative present is of order 2 p the classification of boundary conditions is as follows, those boundary conditions with order from 0 to p minus 1 are essential, and those boundary conditions with order p to 2 p minus 1 are natural boundary conditions. This is how we classify the boundary conditions. So, if you look back the at the problem that we have taken to solve in today's class it is a second order differential equation, so the order is 2 p is equal to 2 and those boundary conditions of order 0 to p minus 1.

So those boundary conditions of order 0 to p is 1, so 1 minus 1 are essential so the boundary conditions that u is equal to 0 along C 1, C 3 and C 4 that is essential boundary condition, and those boundary conditions of order p 2 p minus 1 are natural boundary conditions. p is equal to 1 in this case so it is going to be 1 minus 1. So, that is those equations of order one are natural boundary conditions, so if you see the boundary conditions that are given for the problem that we are looking at derivative of u with respect x along C 2 is given as 1, so that is going to be the natural boundary condition. This is how we classify boundary conditions.

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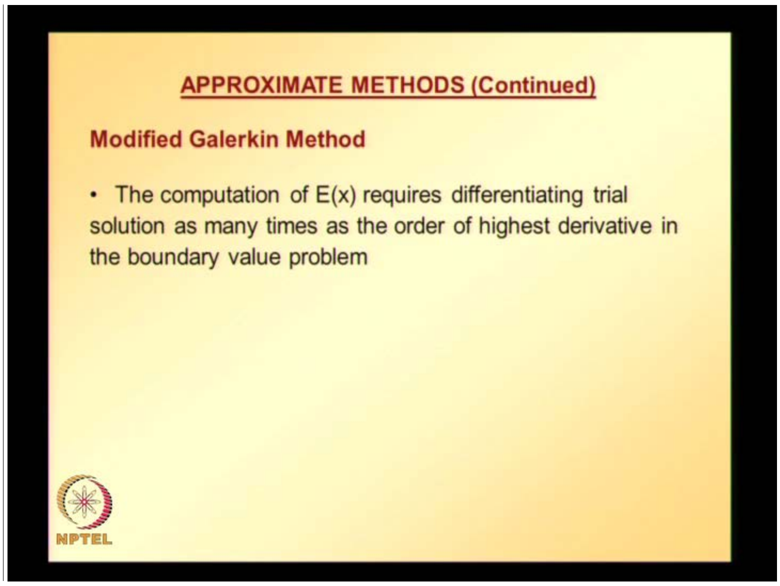

Now, let me briefly review modified Galerkin Method some of the points associated with this, so if you call in modified Galerkin Method or in Galerkin Method. First basically we need to understand why we are? Why we have this modified Galerkin Method? The computation of residual which is required for finding the total weighted residual the computation of residual e requires differentiating trial solution as many times as the order of highest derivative in the boundary value problem.

Therefore, if the basic Galerkin criteria is used directly it would mean that you must use at least a quadratic polynomial as trail solution for second order boundary value problem. If you recall the basic Galerkin criteria is multiply the residual with weight function which is for basic Galerkin criteria weight function or for Galerkin criteria weight function is partial derivative trial function with respect to the unknown parameters or coefficients. We need to multiply the residual with the weight function integrate over the problem domain, so that gives us total weighted residual and that residual, we are going to find these unknown parameters or coefficients such a way that total weighted residual is minimum.

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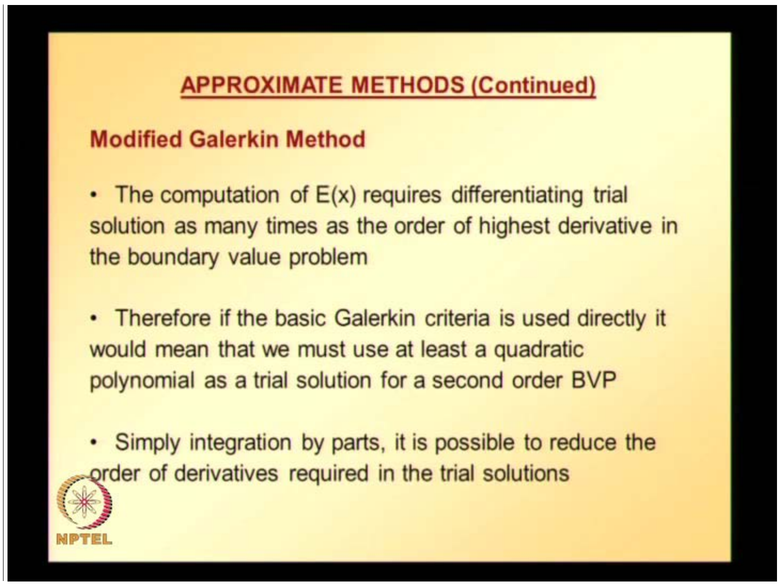

That is the process that we follow in this Galerkin criteria, so if basic Galerkin criteria is used directly, it would mean that we must use at least quadratic polynomial as trail solution for second order boundary value problem, because this residual is going to be second order for second order boundary value problem.

An alternative for this is or a remedy for this not requiring higher order polynomial is integration by parts, so simple integration by parts by applying simple integration by parts, it is possible to reduce the order of derivatives required in the trial solution. We can get or we can solve a problem second order boundary value problem even with a linear polynomial, because once we apply integration by parts we can reduce a second order boundary value problem or the highest derivative that is present, which is going to be two for second order boundary value problem to an order less to an order one less.

So, we are going to get a differential equation or the residual is going to be a first order it is going to contain first order derivative, so we can start with a linear polynomial as trail solution. These are the points that we already noted when we are looking at modified Galerkin Method. That is the reason why we are going to use integration by parts in modified Galerkin Method compared to the Galerkin Method or basic Galerkin criteria. And as part of this modified Galerkin Method when we are looking at one dimensional bound value problems we looked at some examples, so let me quickly go through those some of the important points associated with those examples.

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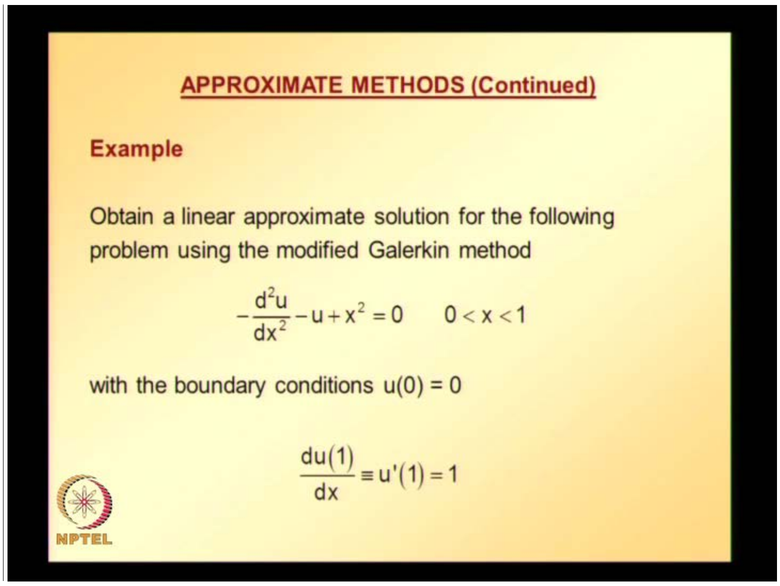

This is one of the examples that we looked at obtained a linear approximate solution for the following boundary value problems using modified Galerkin Method. So, this is the differential equation we need to solve this differential equation or we need find approximate solution of u for this, which satisfies this equation over the domain x going from 0 to 1 subjected to these boundary conditions. And if you apply the rule that we already noted it turns out the first boundary condition, that is u evaluated at x is equal to 0 is 0 and derivative of u evaluated at x equal to 1 is 1.

The first boundary condition turns out to be essential, second boundary condition turns out to be natural boundary condition. Basically in modified Galerkin Method what we are going to do is we are going to take the given differential equation. And we are going to define what is residual, and that residual, we are going to multiply with the weight function, which is going to be partial derivative of trial solution with respect to unknown parameters, and so then we are going to integrate that. That is residual multiplied by the weight function integrate over the problem domain in this case it is going to be from 0 to 1 and equate it to 0, before we substitute the trial solution what we are going to do is we are going to apply integration by parts, and reduce the highest order derivative present in the equation to as lowest order as possible.

So, as a result of integration by parts modified Galerkin criteria always involves boundary terms when we suppose, if you take this equation when we apply integration by parts second order different the term which is having second order derivative at reduces to first order derivative, and if you see the natural boundary condition we have first order derivative there, that is what is going to happen as a result of integration by parts modified Galerkin Method always involves boundary terms. If a non zero natural boundary condition is specified then that is used to simplify the terms further, if essential boundary condition is specified then boundary term does not contribute anything to the functional, because waiting function at that point is going to be 0.

So, this is consequence of using admissible functions for trial solution thus when using modified Galerkin Method only boundary terms that needs to be considered the residual or those that correspond to the natural boundary conditions. This observation is particularly useful in simplifying the evaluation of boundary terms in two and three dimensional boundary value problems. All these points we are going to see in the problem second order boundary value problem that we are going to solve using this modified Galerkin Method before we do that the important point that I want to emphasis, here is where ever essential boundary condition is specified at that location, if we start with admissible trial solution the weight function is going to be zero. So, that is one of the important thing that we need to keep in mind when we are simplifying the total residual total weighted residual, and there after integration by parts and we are simplifying we need to use this condition.

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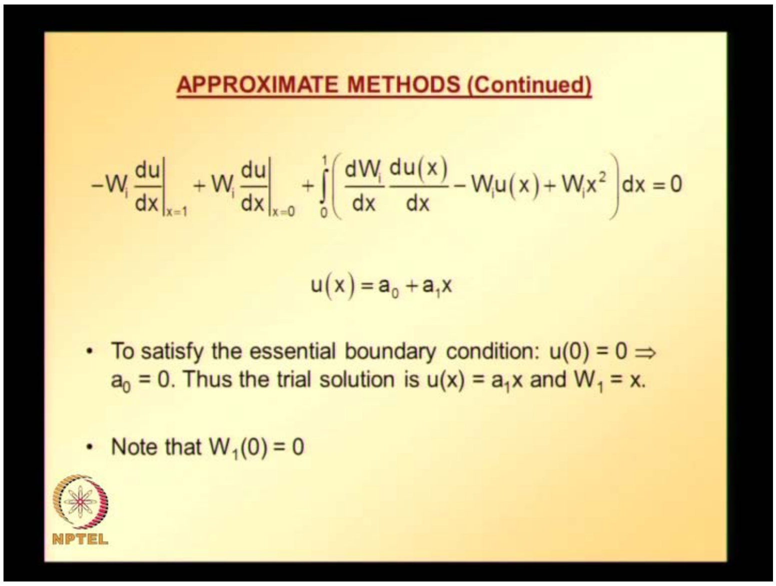

Now, let us look at what we what how the equation is going to be after doing integration by parts over the total weighted residual. This is what we are going to get when we apply modified Galerkin Method. And if you see the first term, first term needs to be evaluated at x is equal to 1 where natural boundary condition is specified, and second term needs to be evaluated at x is equal to 0, where essential boundary condition is specified and the third term is the integral term. If you see the first term - first term can be further simplified, because it is given that derivative of u with respect to x is equal to 1, evaluated along or at x is equal to 1. That value is given derivative of u with respect to x evaluated at x is equal to 1 is 1.

That information is given, so first term can be simplified whereas the second term it needs to be evaluated at x is equal to 0, and we noted that weight function if we start with an admissible trial solution weight function at the location where essential boundary condition is specified is 0. So, with that reasoning second boundary condition is going to drop off. Let us verify first how weight function evaluated at x is equal to 0 is 0.

So, we start let us say we start with this linear trial solution, and to satisfy the essential boundary condition u evaluated at x is equal to 0 is 0, when we impose that condition it turns out that a 0 is equal to 0. The admissible trial solution turns out to be u is equal to a 1 x, so the weight function is going to be the partial derivative of admissible trial solution with respect to the unknown parameters or coefficient. Then it turns out to be W 1 is equal to 1 there is equal to be only 1 weight function. It is W 1 which is going to be x. And when we evaluate W 1 at x is equal to 0 we can easily verify that it is going to be 0. This is how the boundary term where essential boundary condition is specified is going to drop, because weight function is going to be 0 where ever essential boundary condition is specified, if we start with admissible trial solution.

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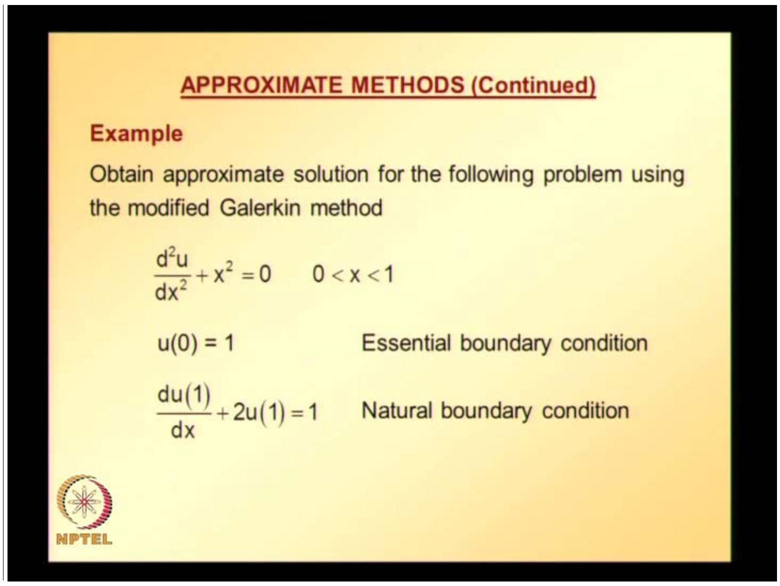

Same thing is with the other example that we looked at last time or earlier class, so this is the other example that we looked at when we are solving one dimensional boundary value problems using modified Galerkin Method. The problems statement is like this obtained approximate solution for the following boundary value problem using modified Galerkin Method. So, second order differential equation we need to solve that over the domain x going from 0 to 1, and two boundary conditions are given one boundary condition turns out to be essential boundary condition. Second boundary condition turns out to be in the natural boundary condition to solve this problem using modified Galerkin Method what we did is we defined residual multiply residual with a weight function, which is partial derivative of admissible trial solution or partial derivative of admissible trial solution with respect to unknown parameter integrate over the problem domain equate it to 0.

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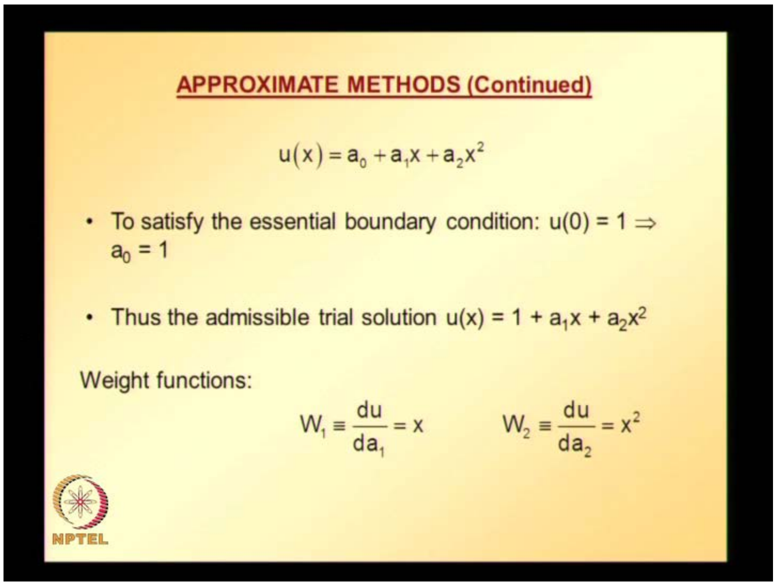

And then apply integration by parts to reduce the highest order derivative present to as low as possible. We started out earlier when we are solving this problem, we started out this with this poly quadratic trial solution and essential boundary condition is imposed on this essential boundary condition says that u evaluated at x is equal to 0 is 1. That a result in a 0 is equal to 1, so admissible trial solution is going to be 1 plus a 1 x plus a 2 x square. So, here there are two unknown parameters that needs to be determine, so we are going to get two weight functions and the weight functions are given by partial derivative u with respect to a 1 partial derivative of u with respect to a 2.

So, one of the weight functions turns out to be x another weight function turns out to be x square. These are the weight functions and again these weight functions we can easily verified that this weight function turns out to be 0 at the location where essential boundary condition is specified for this particular problem. Essential boundary condition for this problem is specified at x is equal to zero, so it can be easily verified that these two weight functions are zero when evaluated at x is equal to 0. These are some of the points that we need to keep in mind even when we are solving second two dimensional problems.

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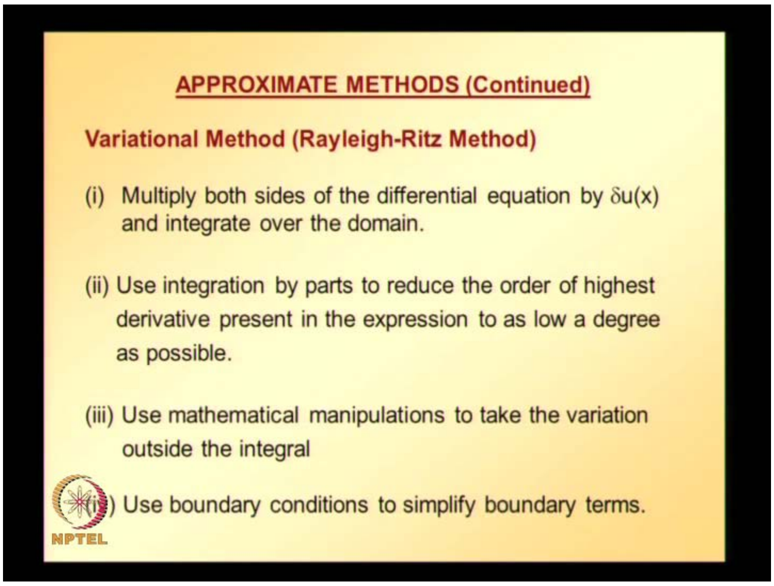

Let me briefly review the important points associated with Variational method. Variational method or Rayleigh-Ritz Method, in this method the steps involved are like these multiply both sides of differential equation whether it is one dimensional or two dimensional with variation of u, if it is one-dimensional problem we multiply it with variation of u is going to be function of x alone, where as if it is two-dimensional problem it is going to be function of x and y. Multiply both sides of differential equation by variation of quantity that we are looking of that we are looking for and integrate over the domain use integration by parts to reduce the order of highest derivative present in the expression to as low degree as possible. That is the second step and then uses mathematical manipulations to take Variational variation, Variational operator outside the integral and use boundary conditions to simplify boundary terms.

In the third step we are going to use some of the Variational identities, because the final objective in Variational method is to bring the given boundary value problem into the form variation of some quantity inside the bracket is equal to 0. When we have in that form variation of something inside bracket is going to be zero is possible only if whatever quantity is there inside the bracket, if the partial derivative of that quantity with respect to the unknown parameters is equal to 0. By applying that condition we are going to get as many number of equations as unknowns present, so that we can solve those equations and determine the unknown, so that is how this Variational method procedure goes. These are the steps involved in Variational method.

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Now, let us go back to the two dimensional boundary value problem that we intend to solve. This is the problem statement we start with modified Galerkin Method. In modified Galerkin Method we multiply the residual with a weight function integrate over the problem domain here the problem domain is a rectangle. Integral is going to be in area integral so before that let us make a note of all the boundary conditions along with the limits on x and y. The boundary conditions are u evaluated u evaluated along x is equal to zero where y goes from minus 2 to 2 u is equal to 0 and u is equal to 0 along x going from 0 to 2 and y is equal to minus 2. Also u is equal to zero along x going from 0 to 2 y is equal to 2 and also first derivative of u with respect to x is equal to 0 or partial derivative of x with respect to partial derivative of u with respect to x is equal to 1 along x is equal to 2 where y goes from minus 2 to 2. We need to solve the given second order differential boundary value problem subjected to these boundary conditions, so if you go back and see the point that we noted that weight function is going to be 0.

If we start with the admissible trial solution admissible - trial solution weight function is going to be zero along the boundaries on which essential boundary condition is specified. So, essential boundary condition is specified along C 1, C 3 and C 4, so if we start with admissible trial solution weight function is going to be 0 along C 1, C 3 and C 4.

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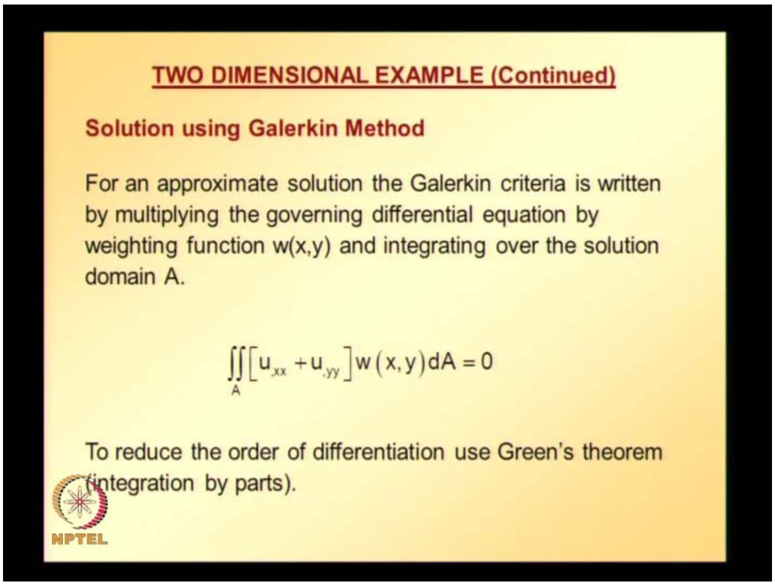

So, solution using Galerkin Method here Galerkin Method means modified Galerkin Method for an approximate solution the Galerkin criteria is written by multiplying the governing differential equation by weight function and integrating over the solution domain, this is the first step. And the second step is to reduce the order of differentiation using Green's theorem or integration by parts here we need to use integration by parts in two dimensions.

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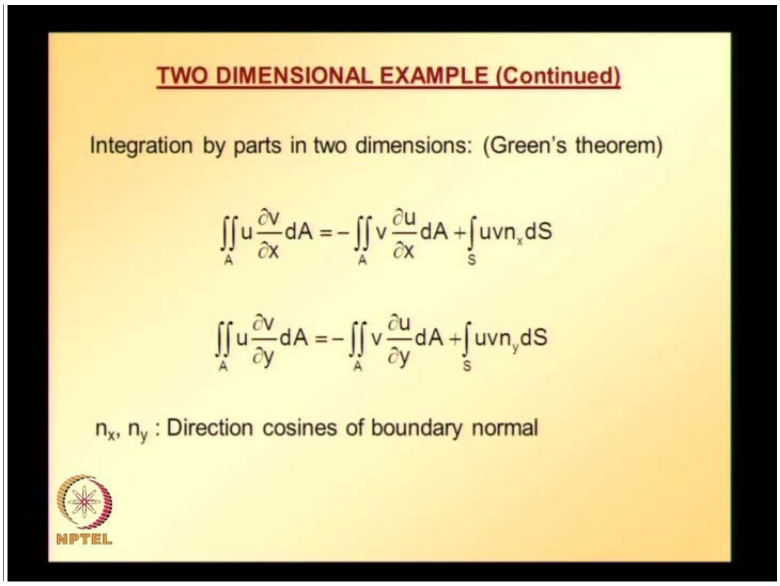

Let us briefly review the integration by parts in two dimensions, this is the formula where n x n y are direction cosines of boundary normal. So, using these two formulas we can simplify the two terms appearing on the right hand side of the previous equation.

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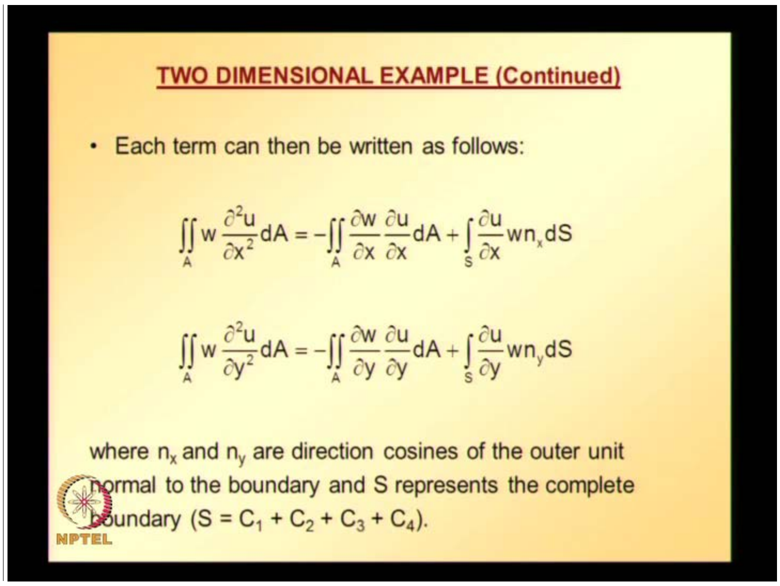

Each term then can be written as follows. The first term using the previous integration by parts formula's in the second term. So, substituting these two terms, here n x n y are the direction cosines of outer unit normal to the boundary where s is total boundary it comprises C 1, C 2 plus C 3 plus C 4, and if you see second term in each of these equations it consists of w. Basically this integral over s reduces to integral along c two alone because along C 1, C 3 and C 4, W is equal to zero, and weight function is equal to zero.

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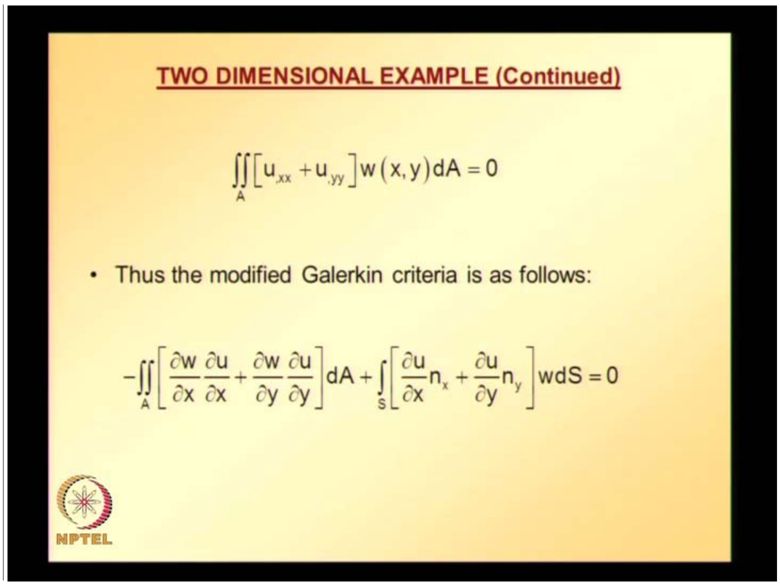

Substituting these two equations in to the previous equation that we initially started out with, so this is the equation that we started out with in this equation substitute the previous two equations then modified Galerkin criteria becomes this one.

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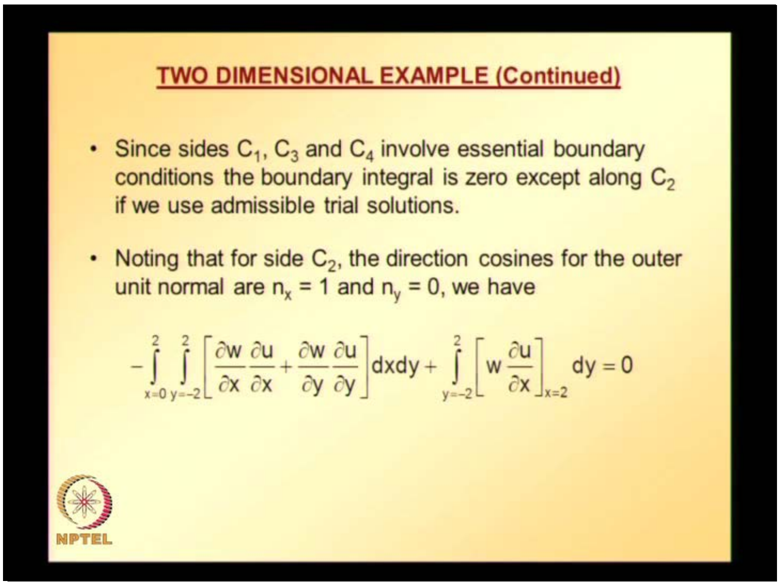

Since, sides C 1, C 3, C 4 involve essential boundary conditions. The boundary integral is 0 except along C 2, if we use admissible trial solutions so noting that for side C 2 direction cosines for outer unit normal C 2. What is C 2? C 2 is a line along x is equal to 2, so direction cosines for outer unit normal are n 1 or n x is equal to 1 and n y is equal to 0. So we need to know these direction cosines to simplify the boundary term. Substituting this information we are going to get this one, and if you see the second term it involves boundary integral along x is equal to two y going from minus 2 to 2 along C 2 that is along x is equal to 2, it is given partial derivative of u with respect x is equal to 1.

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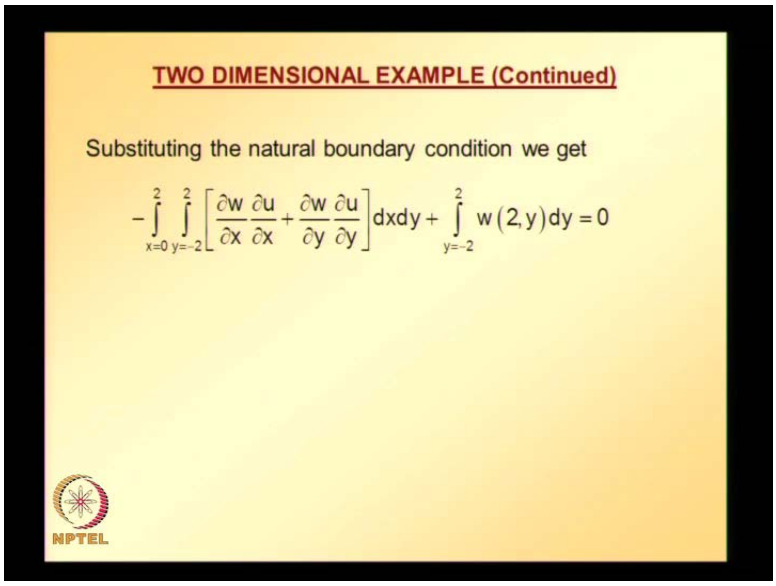

This boundary condition is given, so substituting the natural boundary condition we get this one. Before we proceed further we need to substitute in this equation admissible trial solution, so an admissible trial solution must satisfy essential boundary conditions.

What are the essential boundary conditions essential boundary conditions are u is equal to zero along C 1 C 3 and C 4. In this case it means that we need to pick a function such that it is zero along side C 1, C 3 and C 4. So, before we come up with this function let us go back and see the domain.

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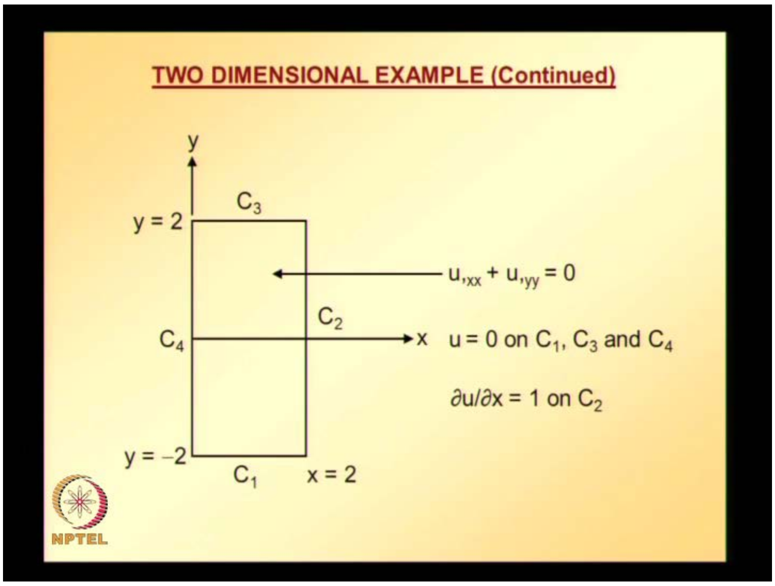

This is the domain, so the admissible trail solution that we are going to assume it needs to be zero along C 1, C 3 and C 4, and if you see equation of C 1 equation C 1 is going to give y is equal to minus 2, and equation of C 3 is going to be y is equal to 2, and equation of C 4 is x is equal to 0. The admissible trial solution should consist of these terms that are y plus 2 y minus 2 and x

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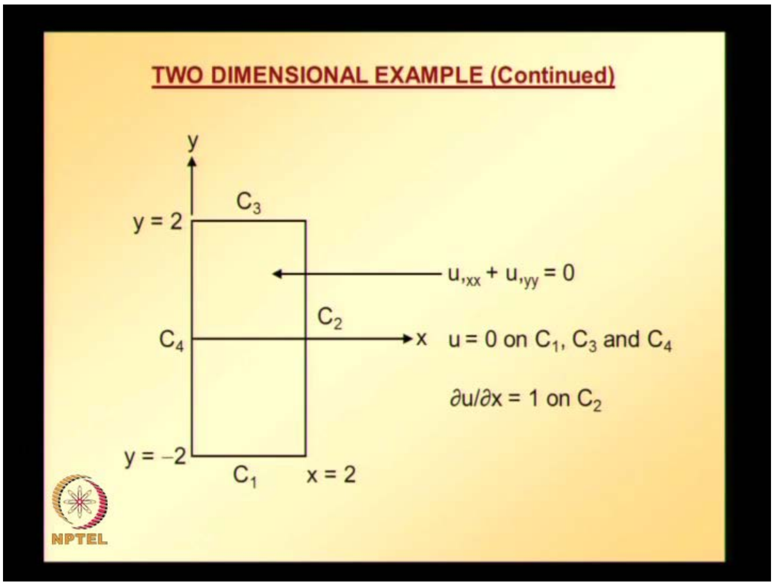

With these information one possible function that meets this criteria is as follows, if you see this function it consists of term x is equal to 0 that is x, and also it consist of y minus

2 and y plus 2 product of y minus 2, y plus 2 is y square minus 4, and also this trial solution can easily be verified. That it satisfies the essential boundary conditions and taking derivatives of these with respect to x, we get this equation with respect to y we get this equation, because this two derivatives are required for us to simplify the equation that we are looking at.

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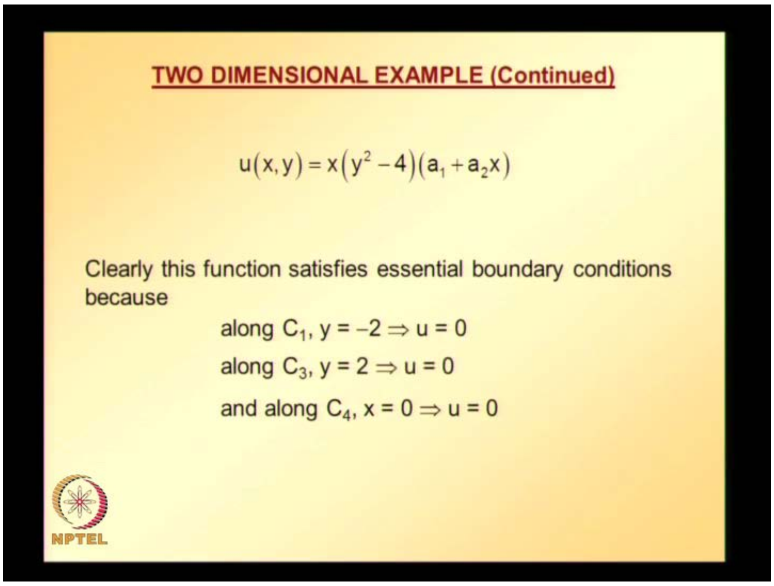

Let us verify, whether this satisfies those boundary conditions clearly this functions satisfies essential boundary conditions, because along y is equal to minus 2 it is 0 along y is equal to 2 it is 0 and along x is equal to 0 this function is 0.

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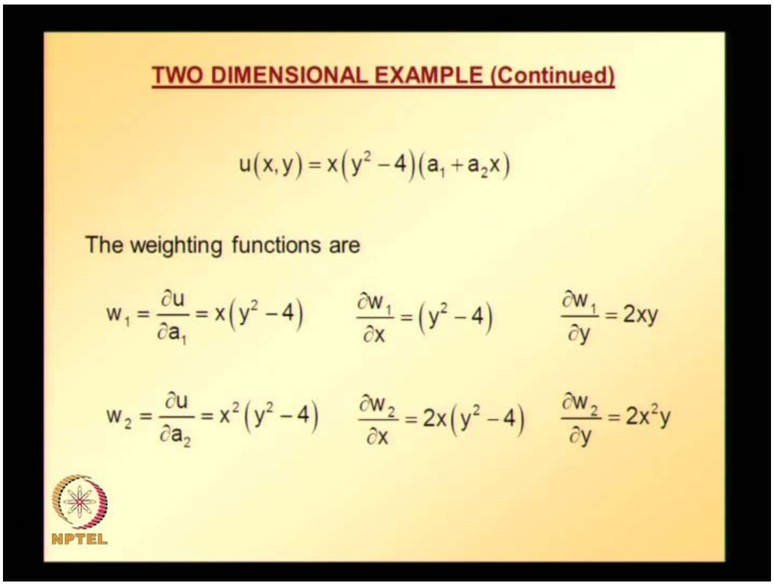

Now let us look at weight functions if you see or if you recall weight function is defined as or Galerkin criteria weight function is defined as derivative of partial derivative of trial solution with respect to the unknown parameters. We have two unknown parameters a 1 and 2, so first weight function is given by partial derivative of u with respect to a one second weight function is given by partial derivative of u with respect to a two. So, weight functions are like this w 1 and derivative of w 1 with respect to x w derivative of w 1 with respect to y similarly, w 2 derivative of w 2 with respect to x derivative of w 2 with respect to y. And if you see these equations w 1 and w 2 it can be easily verified that this weight functions w 1 and w 2 are 0 along y is equal to minus two y is equal to 2 and x is equal to 0.

Those equations corresponds to C 1, C 3 and C 4 along which essential boundary conditions are specified so whatever point that we noted when we are looking at or when we are briefly reviewing in today’s in the beginning of today’s class that for modified Galerkin criteria the weight function turns out to be 0, along the locations or along the boundaries on which essential boundary conditions are specified. That is what is verified here note that these weighting functions are 0 along C 1, C 3 and C 4, thus numerically verifying that we need to evaluate boundary integral only along C 2 on which natural boundary condition is specified.

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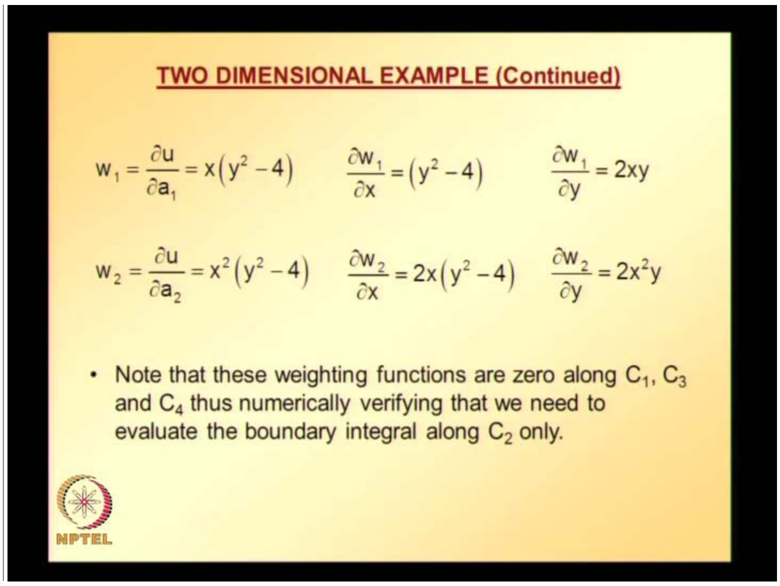

The system of equations is obtained by using these weighting functions in the Galerkin weighted residual expression, so substituting w 1 into the previous equation of Galerkin weighted residual we get this which can be simplified. And we get this equation which consists of a 1 and a 2, and now we get another equation when you substitute second weight function in the Galerkin weighted residual expression.

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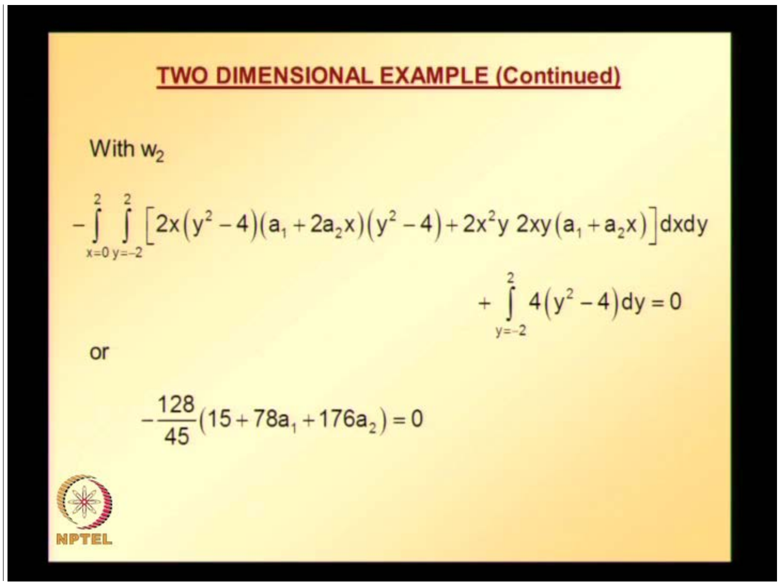

And substituting w 2, we get this equation which can be simplified, and we get this equation. So, we got two equations and two unknowns the two equations that we obtained or in terms of a 1 and a 2. These two equations can be solved for a 1 and a 2, when we substitute this a 1 and a 2 back in to the admissible trial solution that is x times y square minus 4 times a 1 plus a 2 x into that equation if you substitute we get the approximate solution.

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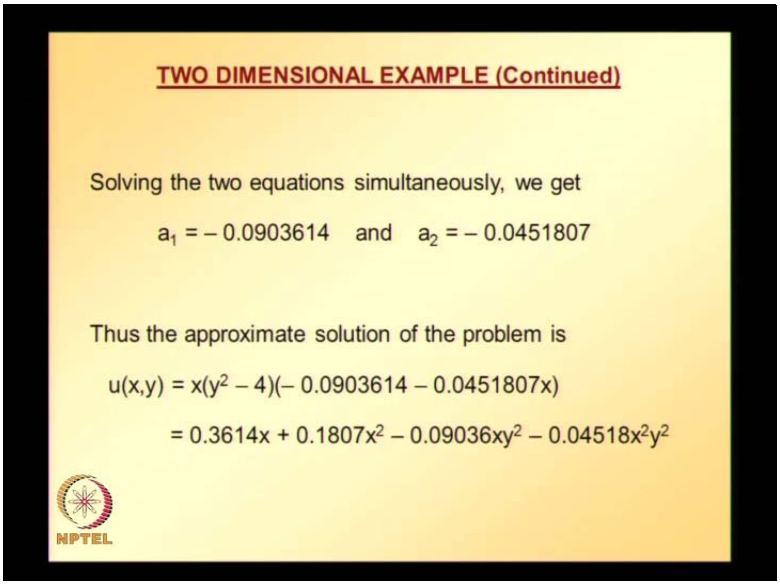

Solving those two equations simultaneously we get a 1 is equal to minus 0.093614, and a two as minus 0.0451807 and substituting this into the admissible trial solution we get this approximate solution. So, this is how the procedure goes for solving two-dimensional boundary value problems using modified Galerkin Method in conjunction with classical approximate solution techniques and this can be further simplified. And finally, u turns out to be 0.3614 x plus 0.1807 x square minus 0.09036 x y square minus 0.4518 x square y square. As we observed for one-dimensional boundary value problems if we start with the same trial solution, and if we solve this problem using Variational method we can exactly same solution.

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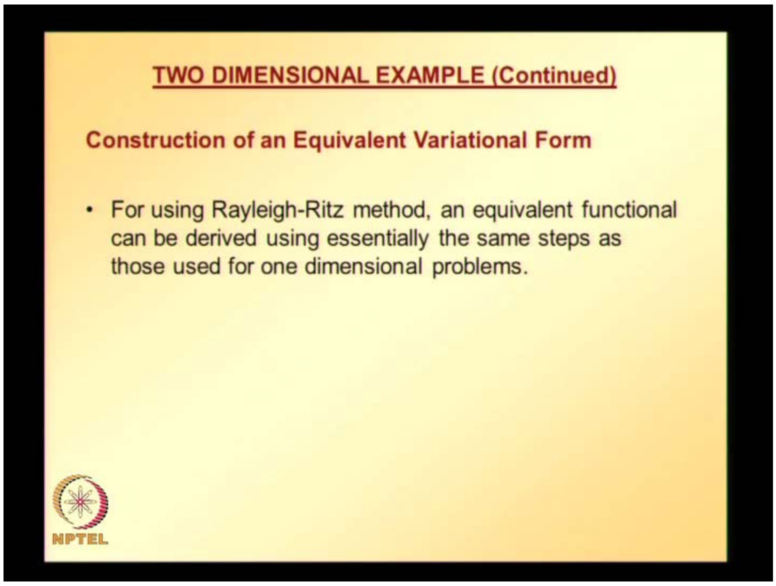

Let us see whether we are going to get that are not. So, before you go for construction of equivalent Variational form which involves or for using Rayleigh-Ritz method an equivalent functional can be derived essentially using essentially same steps as those used for one-dimensional problem. So, before we proceed further let us look back what are the various steps that we followed for getting equivalent functional for one dimensional boundary value problems.

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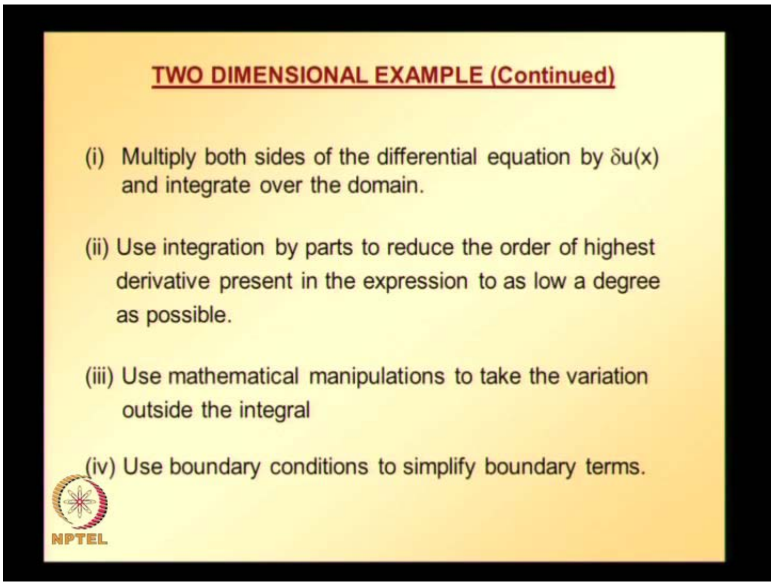

These are the steps, first step is multiply both sides of differential equation by variation of u , its shown as a function of x what for two-dimensional boundary value problem case it is going to be function of x and y and integrate over the domain. This is the first step and the second step is similar to modified Galerkin Method. That is use integration by parts to reduce the order of highest derivative present in the expression to as low a degree as possible and use mathematical manipulations to take variation outside the integral, and use boundary conditions to simplify boundary terms. These are the steps involved in this Variational method or Rayleigh-Ritz method.

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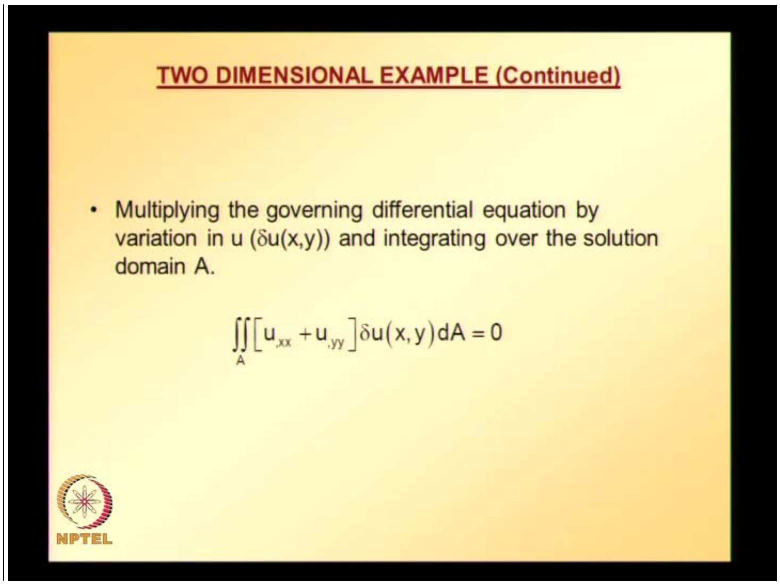

Let us start multiply the governing differential equation by variation of u. Since, u is function of x and y it is going to be variation of u in bracket's x and y, and integrate over the solution domain. Next step we need to use integration by parts that is nothing but applying green's theorem that we already did when we are solving this problem using modified Galerkin Method.

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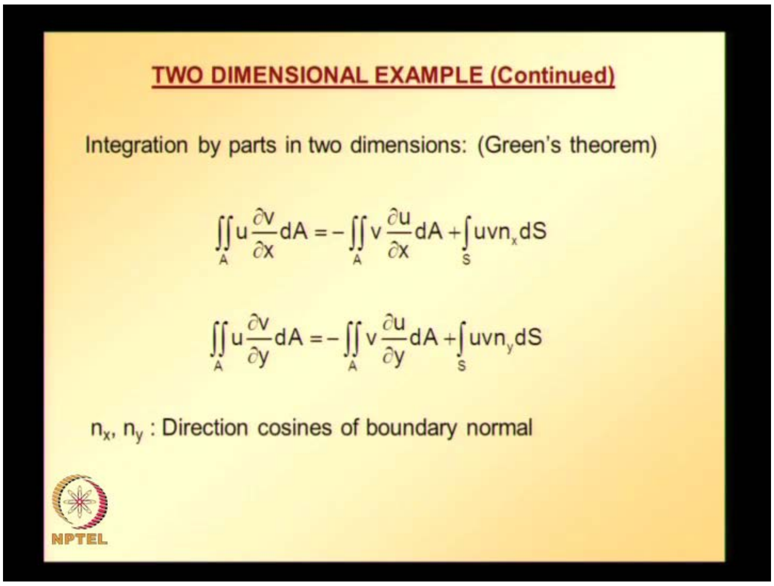

Let me briefly show those formulas integration by parts in two dimensions, where n x n y are the direction cosines, so boundary normal. So, these are the two equations, so the two terms that are appearing in the previous equation can be simplified using these formulas.

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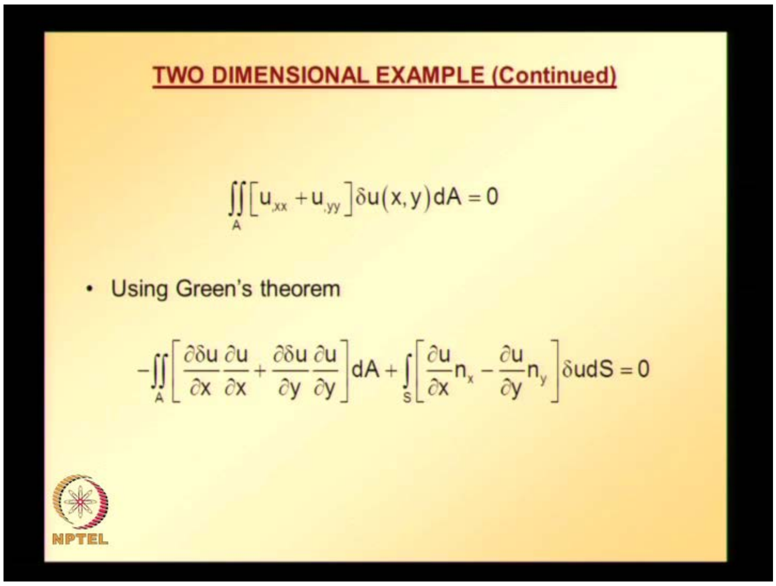

Substituting the first two terms after applying integration by parts, we get this equation; again if you see this second integral of this equation, it is a boundary integral which needs to be evaluated along s compresses of C 1, C 2, C 3, and C 4 and please note that for Variational method wherever essential boundary conditions are specified along that boundary or at that location variation of u is going to be 0.

If you see this second integral which needs to be evaluated along s, it turns out that that integral is going to be zero along C 1, C 3 and C 4. We need to evaluate that integral only along C 2.

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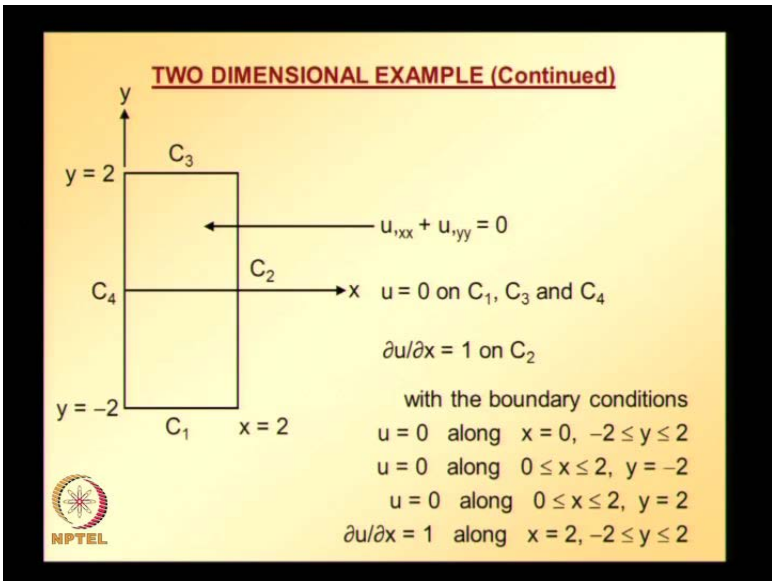

This is the given boundary value problem, and these are the boundary conditions we already noted.

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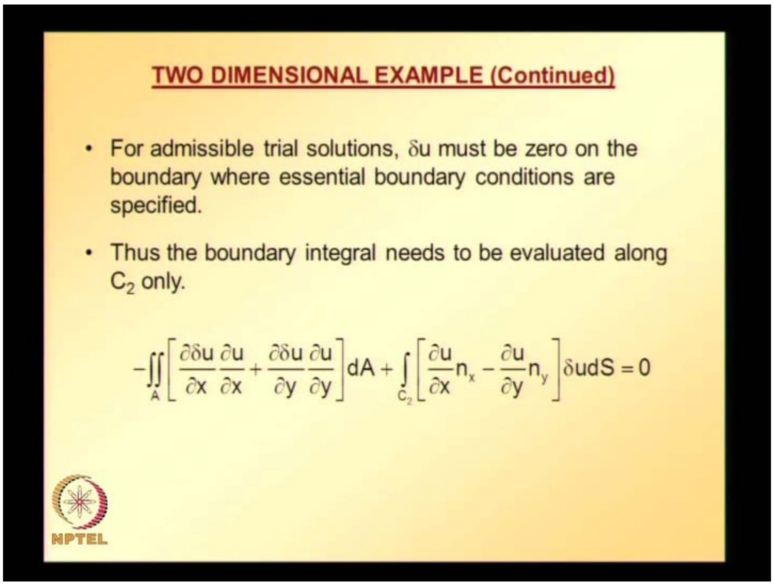

For admissible trial solutions variation of u must be 0 on the boundary, where essential boundaries conditions are specified. Thus the boundary integral that we looked in the previous equation needs to be evaluated along C 2 only. So, the boundary integral reduces to this equation.

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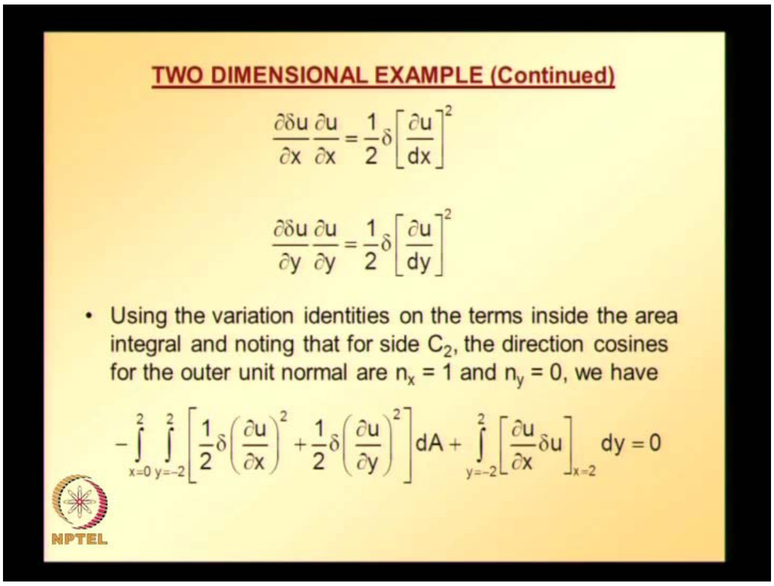

To further simplify the previous equation, we need to use some Variational identities as a part of one-dimensional boundary value problems when we are looking at this Variational method. We noted these Variational identities this is one of the identities that we noted. Similarly, we can write the second identity given in the second equation so using the variation identities on the terms in the previous equation of equivalent Variational form the terms inside the area integral. We will be using this Variational identity on the terms inside the area integral and nothing that for side C 2. The direction cosines for outer unit normal are n x is equal to 1 n y is equal to 0, we get this equation.

So, basically what we did is after applying integration by parts whatever equation we have in that we applied or in that we substituted Variational identities, and also noted that the boundary integral needs to be evaluated only along C 2 thus variation of u is going to be 0 along C 1, C 3 and C 4. And then along C 2 we also noted what are the direction cosines, and we substituted all that information; finally, we got this equation and here if you again boundary integral term it involves derivative of u with respect x along x is equal to 2. That information is already given as a part of natural boundary condition, so substituting that information that is derivative of u along derivative of u with respect x along x is equal to 2 that is C 2 is equal to 1.

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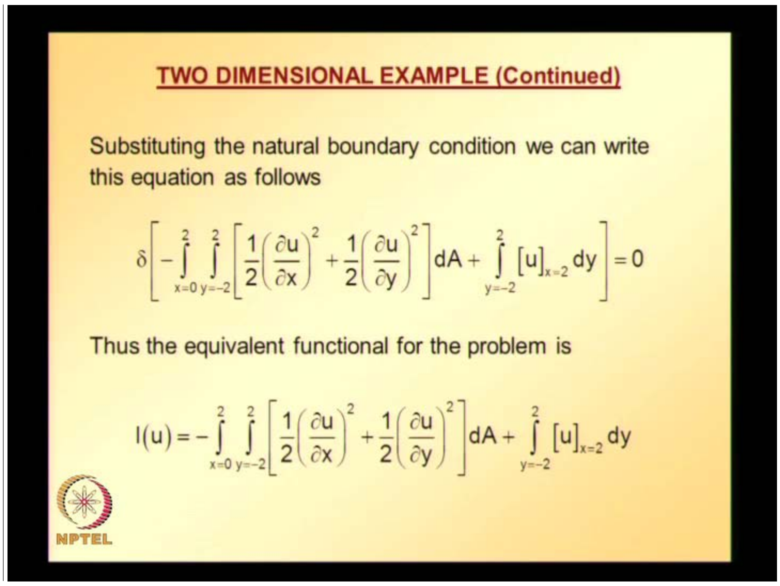

Substituting the natural boundary condition, we can condition we get we can write this previous equation like this we can take the Variational operator outside, and we get this equation. The equivalent functional for this problem is this one, whatever is inside the bracket square bracket. In this equation, in this equivalent functional we need to substitute admissible trial solution, and admissible trial solution should contain some unknown coefficients or unknown parameter that needs to be determined. So, we already came up with admissible trial solution is a trial solution which is going to be zero along C 1,C 3 and C 4. We already noted it is going to be u is equal to x times y square minus 4 times a 1 plus a 2 x.

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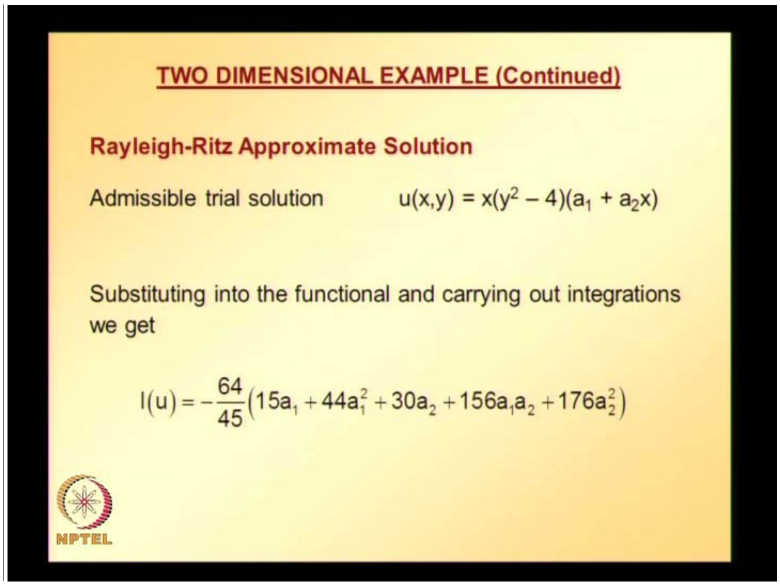

Rayleigh-Ritz approximate solution the admissible trial solution is u is equal to x times y square minus 4 a 1 plus a 2 x, and substituting this admissible trial solution into the equation of i that is previous equation which is equivalent functional and carrying out required integrations. We get this equation which is in terms of a 1 and a 2 and variation of this i needs to be zero variation of i is going to be zero only if partial derivative of i with respect to a 1 and a 2 is equal to 0 independently. Applying that condition of stationary we get two equations in terms of a 1 and a 2, so we can solve these two equations for a 1 and a 2. And finally, once we get a 1 and a 2 we can substitute back a 1 and a 2 into the admissible trial solution to get the approximate solution using Rayleigh-Ritz method.

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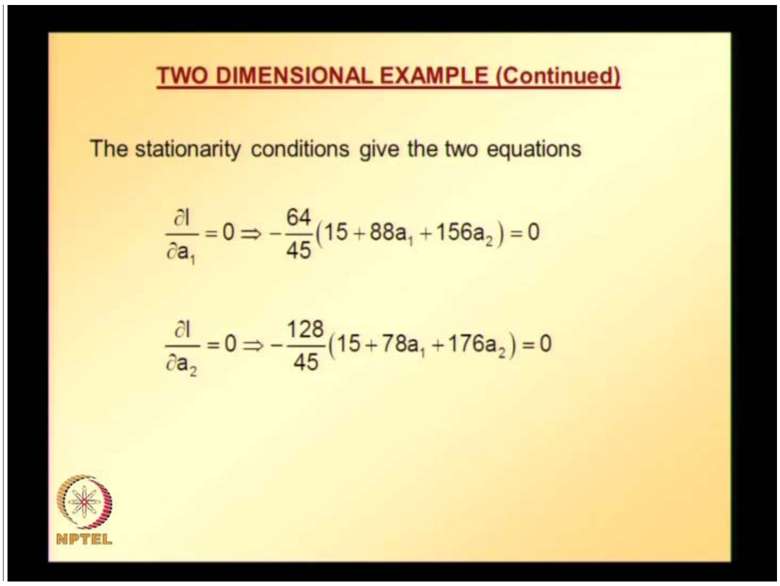

So, applying stationary conditions partial derivative of 5 with respect a 1 is equal to 0 that gives us this equation, and partial derivative of 5 with respect a 2 gives us this equation and if you see these two equations is exactly same as those we obtained using modified Galerkin Method. So, as expected these are the same equations as those obtained from modified Galerkin Method and living these two equations we are going to get a 1 and a 2 which are going to be same as that we obtained using modified Galerkin Method.

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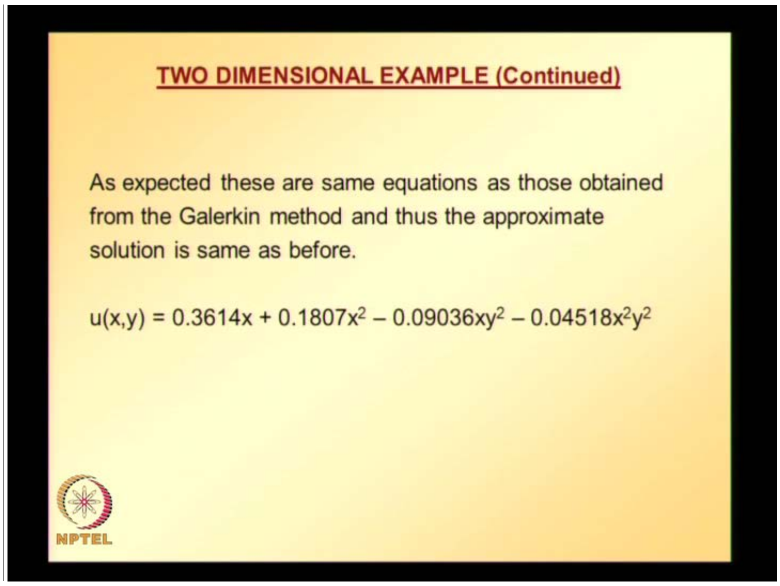

As expected these are same equations as those obtained from Galerkin Method and thus the approximate solution is same as before. This is the approximate solution that we are going to get using Rayleigh-Ritz method, so this is what we noted even when we are solving one dimensional boundary value problem when we start out with same admissible trial solution whether we use modified Galerkin Method or whether we use Galerkin Method or Rayleigh-Ritz method. We are going to get exactly same solution even though the procedure is the steps involved in both procedures are different.

Basically this exercise of solving two-dimensional boundary value problems using these two methods helps us to get more insight into the solution process, and also when we looked at solving these problems or solving two-dimensional boundary value problems using finite element method, we get or we can get a clear idea, what is the difference between classical approximate methods and finite element method that we are going to see in tomorrows class. Steps involved are same exactly or going to be exactly same except that when it comes to substituting admissible trial solution in case of finite element approximation. We are going to substitute the trial solution in terms of finite element shape functions and nodal parameters so that is going to be the only difference, and then after applying similar conditions we are going to determine what are these nodal parameters or nodal unknowns.

Once we get nodal unknowns, we can use again finite element shape functions, and interpolate solution at any point where we are interested, and also we can do all kinds of post process like finding stresses and strains and such kind of things.