Finite Element Analysis

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Lecture No. # 02

Before, we start with today's class, let me summarize, what we have done in the last class. In the last class, we have noted that, all the physical phenomena encountered in engineering mechanics are model by differential equations and the problem addressed is too complicated to be solved by classical analytical methods.

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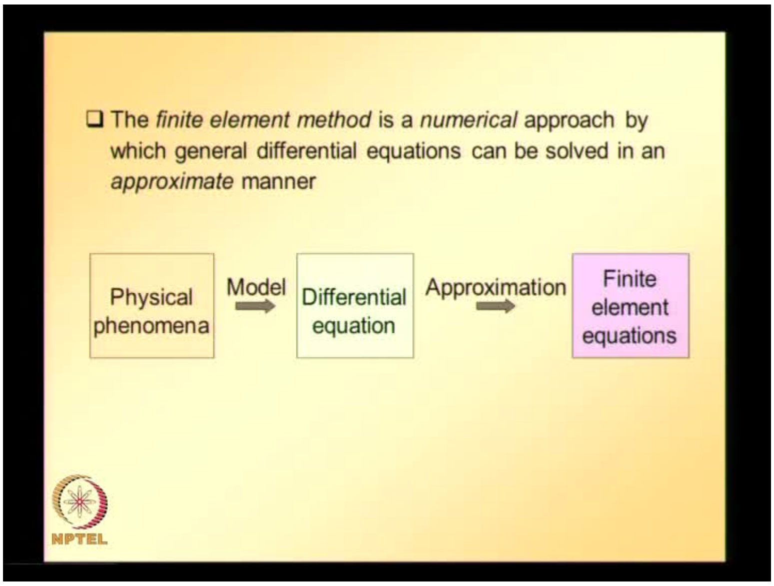

So, basically, finite element method is a numerical approach by which the general differential equation can be solved in an approximate manner. So, finite element method basically, what we will be doing is we will be solving a differential equation, which is valid over a domain subjected to some boundary conditions and please note that the solution is approximate. So, basically these are the steps that are involved in solving a physical problem, first the physical phenomena needs to be expressed in a mathematical form is a differential equation, which needs to be satisfied over certain domain subjected to some boundary conditions and this approximate solution of this differential equation, we can obtain through finite element method. So, by using approximation on this, so, there is using some of the methods that we will see some of the methods which we have already seen and some of the methods that we are going to see in today's class we are going to use some of those methods and get the finite element solutions from the given differential equation.

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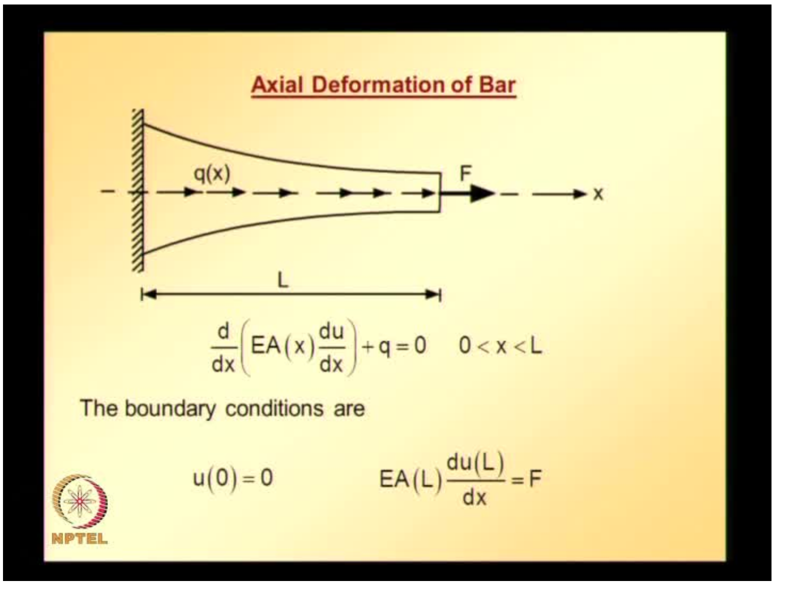

So, in the last class, we have seen how to get the governing differential equation and boundary conditions for axial deformation of bar and this is a bar which we have taken subjected to somebody force or some distributed force q and a point load at the tip and last time we went through the details of how we got this differential equation which needs to be valid over the domain that is, x going to zero to l subjected to the boundary conditions that is displacement evaluated at x is equal to zero is zero displace and force at x is equal to l is point four c f. So, we need to solve this differential equation subject to to these boundary conditions to solve basically, this problem of axial deformation of bar.

And also we noted that one of the boundary condition is essential, another boundary condition is natural. Here, two boundary conditions are required, because this is a second order boundary value problem and also last class we noted a thumb rule to classify boundary conditions it essential and natural, based on the order of the boundary value problem.

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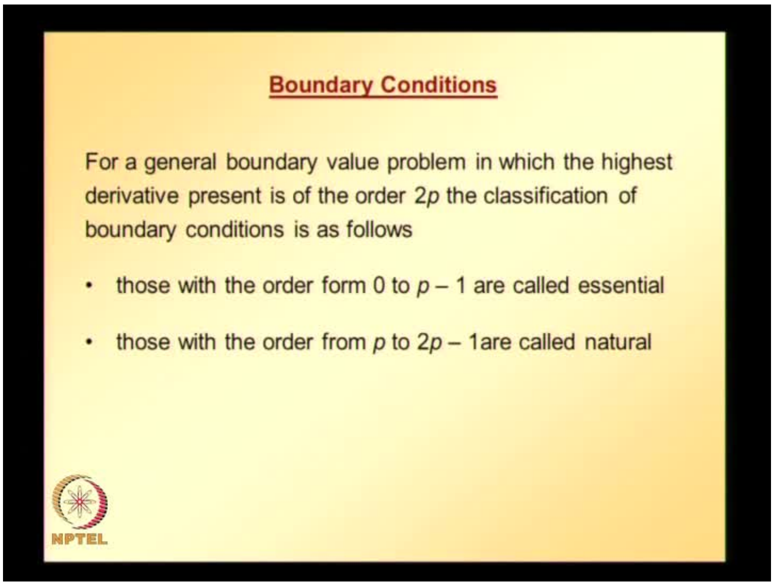

So, for a general boundary value problem in which highest derivative present is a of order two p the classification of boundary conditions is as follows, those boundary conditions of order zero to p minus one are essential; and those boundary conditions with order p to two p minus one are natural boundary conditions. So, this is how we are going to classify the boundary conditions for a general boundary value problem, if no information is given about the physical information is given about the physical phenomena, which that particular differential equation is representing.

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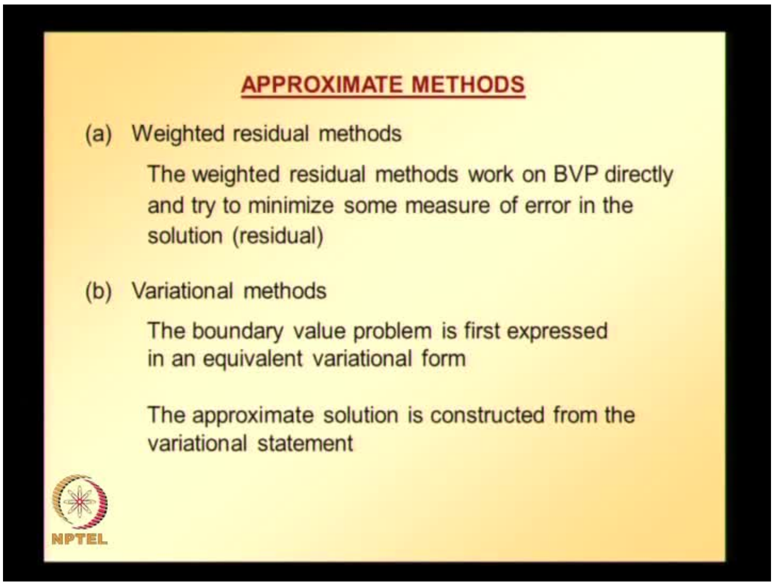

And as a part of approximate methods, we looked at what are these, how classical solution techniques are classified? Basically, these solution techniques can be classified into two categories weighted residual methods and in weighted residual methods, the weighted residual they are the boundary value problem is actually we define certain measure of error in some sense and try to minimize that error. So, the weighted residual methods work on the boundary value problem directly and try to minimize some measure of error in the solution or residual. And we looked at length some of the weighted residual methods and also we noted that in variational methods. The boundary value problem is first expressed in an equivalent variational form the approximate solution is constructed from the variational statement. And both classes of the both classes of these methods require a general form of solution to be assumed first, and this solution that we start out with can be of any form or it can consists of function of any type that you can hope.

But, usually polynomial based trial solutions are more preferable, because basically they are easy to integrate and differentiate and also if the trial solution or assumed solution, if we want to increase the order of a solution, we can easily add more terms to that. So, usually polynomial based trial solutions are assumed. And one of the requirement is this assumed solution are should have some unknown coefficients, that needs to be determined based on the techniques that are established in each of these methods and also we looked at length weighted residual methods.

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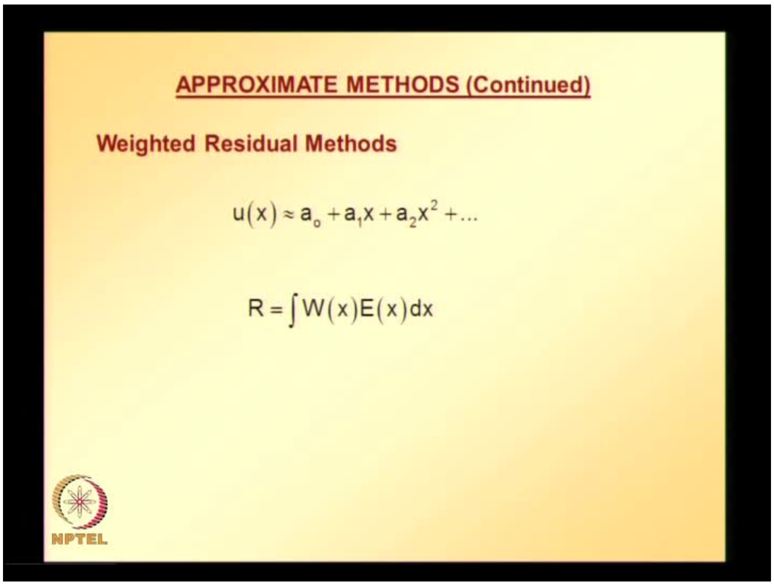

Basically, weighted residual concept can be explained by taking a trial solution like this when this trial solution is substituted into the given boundary value problem. Since this is a trial solution which is approximate the boundary value problem is not going to satisfy or the governing equation corresponding to the boundary value problem is not going to satisfy this solution exactly. So, there will be an associated residual or error there is a difference between left hand side and right hand side of the governing differential equation and that residual basically in weighted residual methods.

We are going to multiply with the weight function and integrate over the problem domain and that is what is defined as total residual. Total residual is this one weight function multiplied by or residual multiplied by weight function integrated over the problem domain and these coefficients of the trial solution a naught a 1 a 2 such kind of coefficients can be or unknown parameters, can be determined by minimizing this total weighted residual weighted residual, because we are multiplying residual with a weight function and we are integrating over the entire solution domain. So, that is why its total weighted residual.

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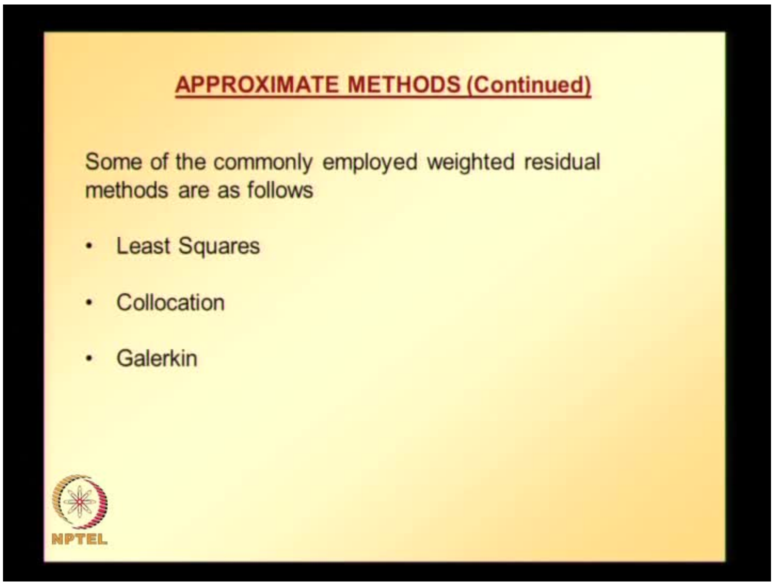

And depending on the weight function that we select, we can categorize this weighted residual methods into three types some of the commonly employed weighted residual methods are: least square weighted residual methods, collocation weighted residual methods and Galerkin weighted residual methods.

We looked at length each of these methods, what are the details involved in getting the approximate solution starting with a trial solution, quadratic trial solution, in the last class.

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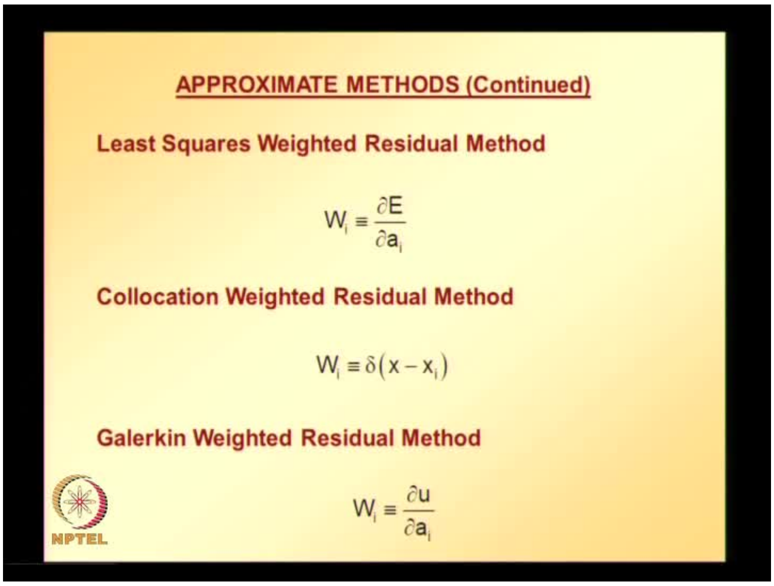

So, just to summarize once again in least square weighted residual each of these weighted residual methods differ in the way weight function is defined. So, least square weighted residual method. Weight function is defined as partial derivative of residual with respect to the unknown parameters and in collocation weighted residual methods, weight function is same as dirac delta function and in Galerkin weighted residual method, weight function is partial derivative of trial solution with respect to the unknown parameters are unknown coefficients. So, this is a weight functions are deter weight functions are defined. So, once we decide what is the trial solution or the start with a assumed polynomial based trial solution, we can actually substitute the assume trial solution into this and try to minimize the total residual and calculate the unknown parameters are coefficients.

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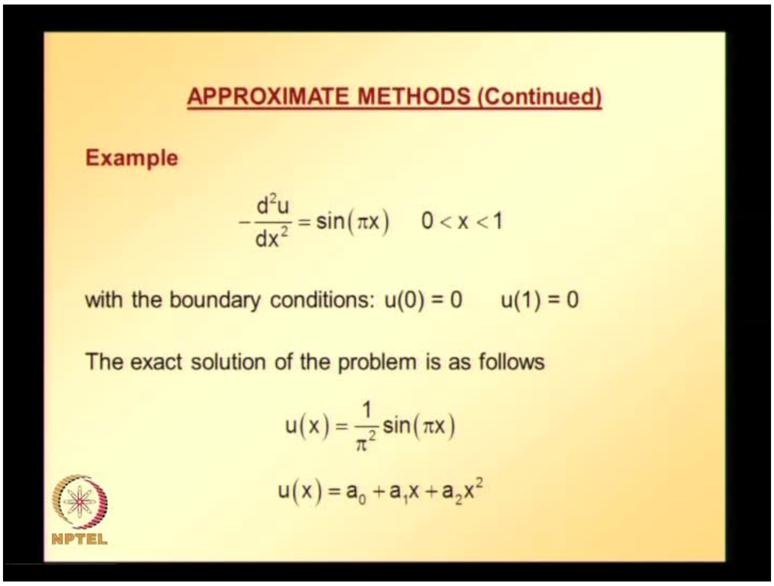

So, to illustrate all the details of this methods and also for comparison purposes, we have selected a second order boundary value problems solution of which we know are the exact solution of which we know… So, this is a second order differential equation that we have taken or second order boundary value problem, which needs to be solved over the domain x going from zero to one subjected to the boundary conditions and these problem can be solved exactly the exact solution is this one.

We noted all these in the last class and now our objective is to find approximate solution to this. So, what we did in the last class is we started out with quadratic trial solution a naught plus a 1 x plus a 2 x square and before we substitute this trial solution into the residual or into the differential equation and get the residual, we need to make sure that this trial solution is admissible trial solution. So, we need to make sure that it satisfies the essential boundary conditions. So, basically that is what we did in the last class, we substituted at the essential boundary conditions that the two essential boundary conditions that are given for this particular problem u evaluated at x is equal to zero is zero u evaluated at x is equal to one is zero and substituting this, we can express some of the unknown parameters in terms of others and some of the unknown parameters can be the values can be evaluated.

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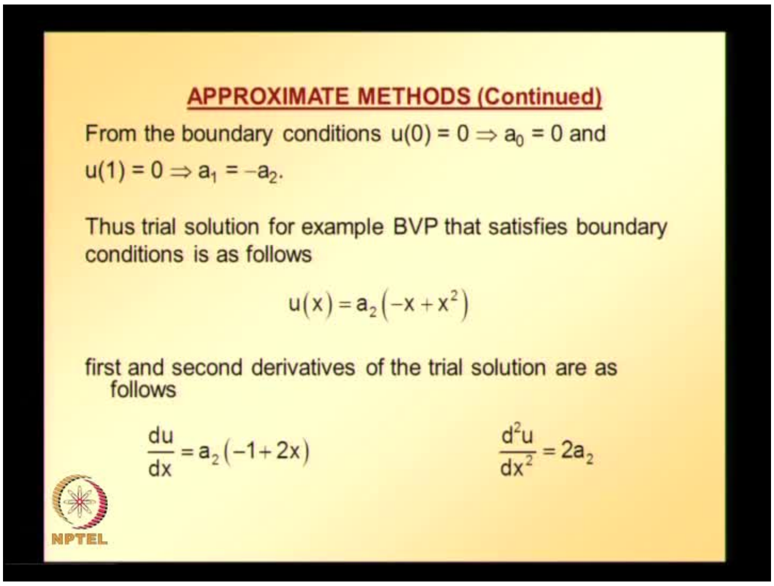

So, substituting the boundary conditions it turns out that a naught is equal to one a one is equal to minus a two and substituting this into the trial solution, we get admissible trial solution as u is equal to a two times minus x plus x square and to before we proceed with weighted residual method, we require second derivative of this trial solution.

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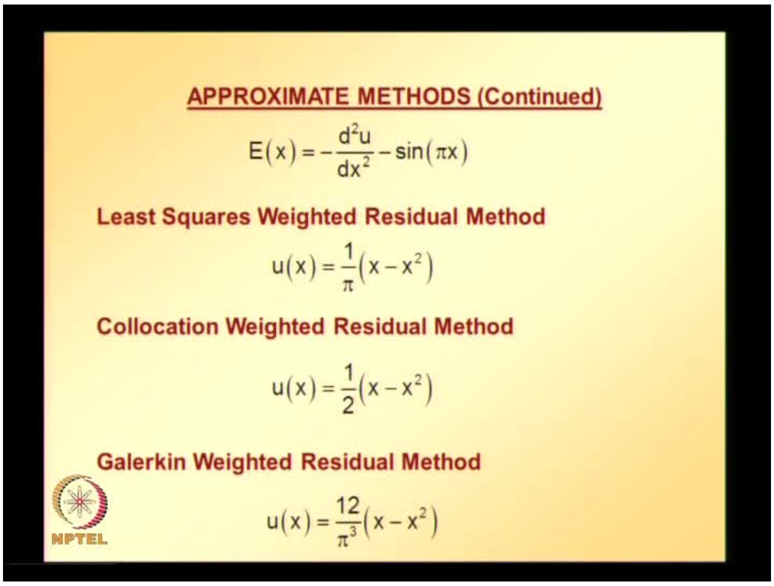

After this we define what is residual and once we have this residual and we can multiply the weight function appropriate for the corresponding method, if least square method is partial derivative of residual with respect to the unknown parameter if it is collocation method it is dirac delta function. This Galerkin method it is partial derivative of trial solution with respect to the unknown parameter using that weight function and integrating, multiplying that weight function with this residual and integrating over the problem domain and trying to minimize. If you are try to minimize the total residual total weighted residual, we can determine, what is the two unknown parameters in the admissible trial solution and once we find a two substitute back into the admissible trial solution, where colligate at this one for least square weighted residual methods, method this is a solution that we obtained starting with quadratic trial solution.

For collocation weighted residual method, this is a solution that we obtained and Galerkin weighted residual method is solution. This is a solution that we obtained note that for all these methods, we started with a quadratic trial solution and we can see how this solution matches with exact solution.

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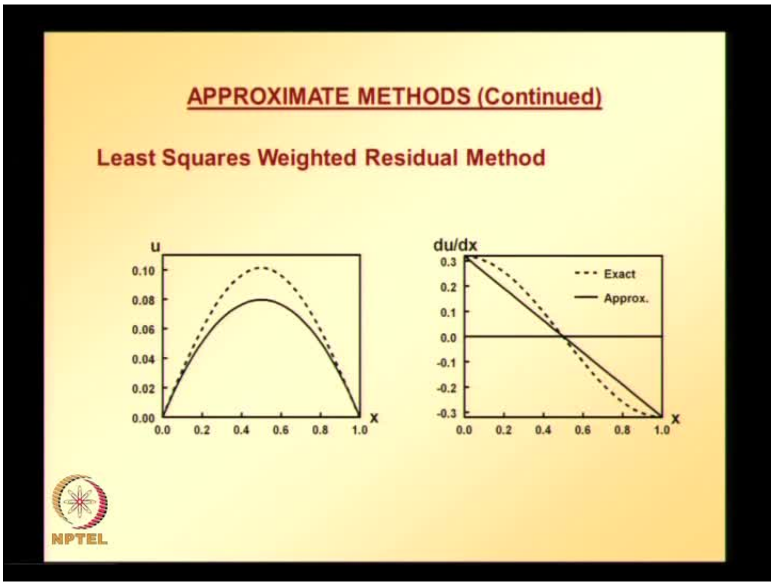

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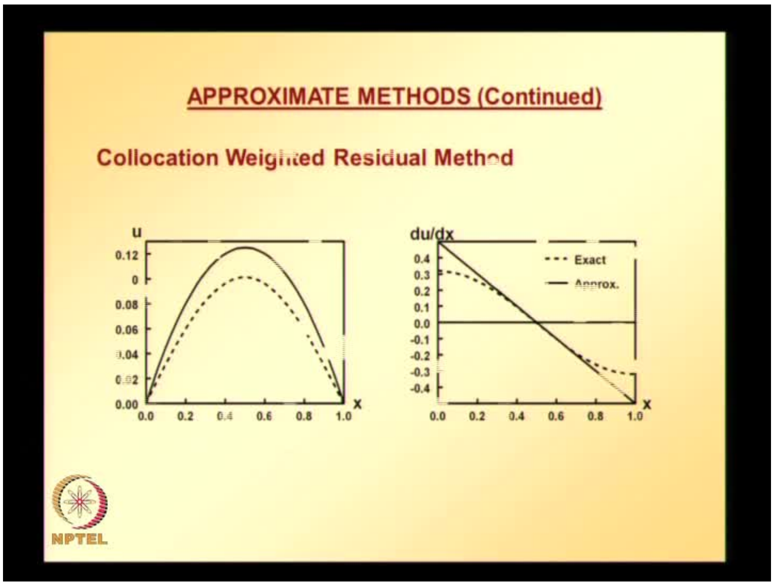

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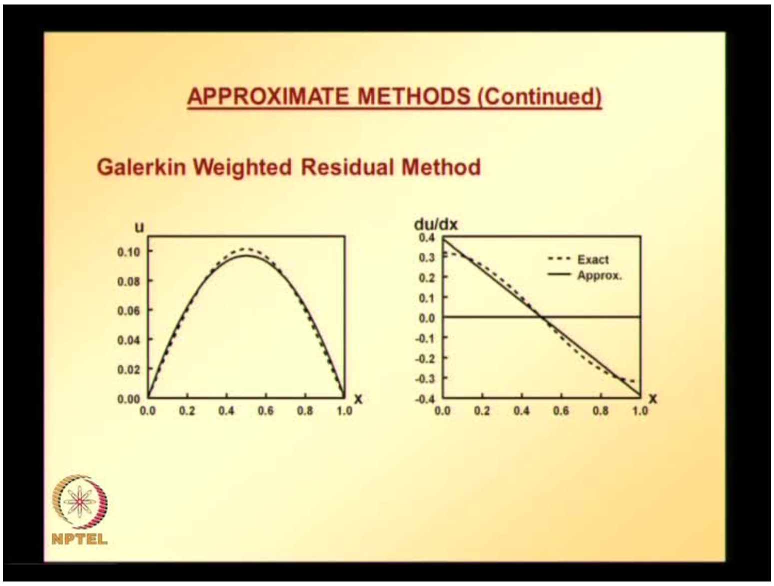

Last class, we have also seen a comparative plot showing comparison between exact and approximate both the solution and derivative solution. This is for least square weighted residual method, collocation weighted residual method, and Galerkin weighted residual methods. So, basically, when we compare this least square weighted residual method collocation weighted residual method and Galerkin weighted residual method the solution obtained by each of these methods, if we compare with exact what we noticed is Galerkin weighted residual method is superior to rest of the two methods.

And also even the small error that is showing up in the approximation of the weighted of the solution, that is, obtained using weighted Galerkin method. Weighted Galerkin residual method can be improved by starting with higher order trial solution starting with cubic or quartic or quintic trial solution and determining the some of the coefficients using the boundary condition, rest of the unknown parameters using the minimization of the total weighted residual.

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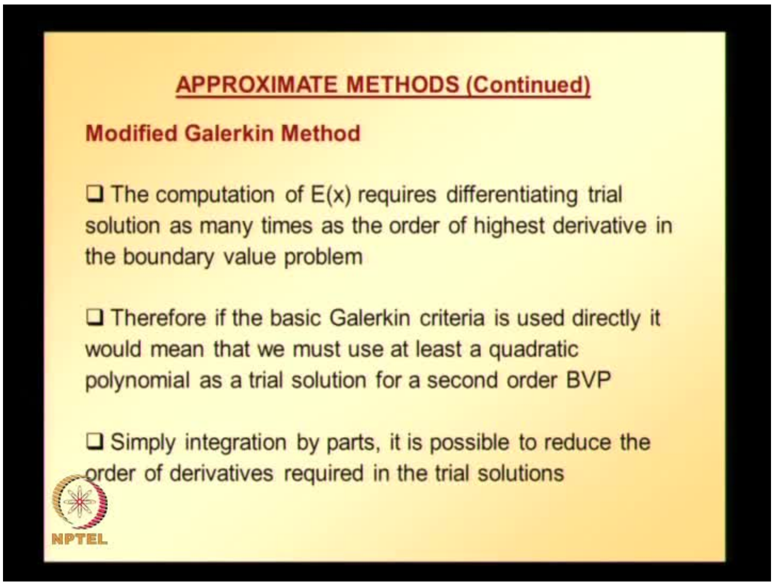

So, also we noted in the last class, that if you see the basic Galerkin criteria the computation of residual requires differentiating trial solution as many times as the order of highest derivative in the boundary value problem. Suppose, if you are start solving second order boundary value problem, we need to differentiate the trial solution for calculation of residual twice and if you are have a fourth order boundary value problem, then we need to differentiate. This trial solution four times to evaluate this residual. So, in a way, we are actually imposing some restriction on the initial trial solution that we can start out for solving that particular problem.

So, some how can we reduce this demand on the initial trial solution or can we somehow reduce the order of differentiation of residual. Therefore, if the basic Galerkin criteria is used directly it would mean that you must use at least quadratic polynomial as trial solution for second order boundary value problem and the solution for this problem is simple integration by parts, by simple integration by parts it is possible to reduce the order of derivatives required in the trial solution. So, that is the idea, that is used to modify this basic Galerkin method and we use integration, when we use integration by parts, we get what is modified Galerkin method in the last class we looked at length what are the details and we also noted all these points. So, now coming to today's lecture variational methods basically in variational methods the boundary value problem is first expressed in an equivalent variational functional an approximate solution is constructed from the variational statement.

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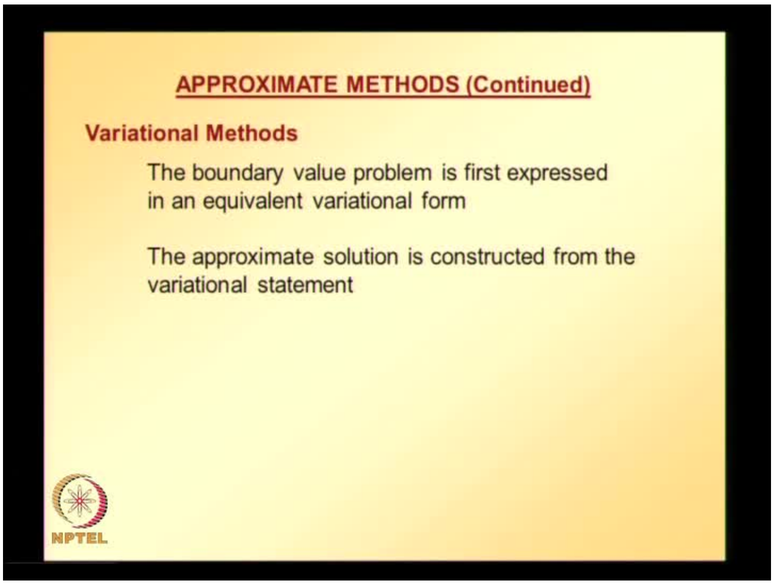

please note that for certain, classes for certain class, of boundary value problem it is possible to derive equivalent variational form, but for other sort of problems other kind of other class of boundary value problems it is may not be possible to derive equivalent variational form basically equivalent variational form is written in terms of integral over the solution domain and it is convenient for developing approximate solutions. And this integral, in the integral over the solution domain the trial solution is substituted and it is then minimized to get the system of equations relating the unknown parameters.

So, this is basically the broad outline of this procedure and the thing is not all problems, we can solve using this or for not all problems, do not necessary have equivalent variational form. So, that is the limitation of this method.

And also for some physical problems it is may be possible to develop variational form from physical properties. So, called energy methods of structural mechanics are based on developing functional that is potential energy functional.

And here, without starting with governing differential equation, we directly use some methods to get the equivalent variational functional or potential energy functional first and the governing differential equations can then be obtained by imposing the necessary condition, for are necessary conditions for functional to be minimum.

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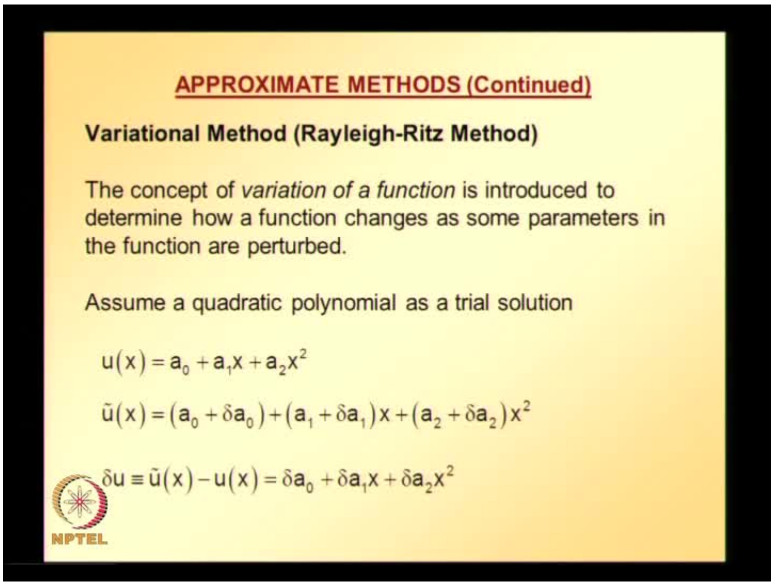

So, before we proceed with the details are to see the steps that are involved into, in this variational method we need to know what is concept of variation of a function and here the concept of variation of a function is introduced to determine, how a function changes as some parameters in the function are perturbed and variational method or Rayleigh-Ritz method is one of the variational method. So, that is why it is written in brackets rayleigh-ritz method. So, assume a quadratic polynomial as trial solution like this and let us say this parameters are perturbed by certain amount and the perturbation is a nonzero value.

So, a naught is perturbed by delta a naught a 1 is perturbed by delta a 1 a 2 is perturbed by delta a 2. So, the perturbed function is shown here it is denoted with u tilde the difference between u tilde and u is what is variation of u and if we simplify substituting u tilde and u the last equation it simplifies to what we shown on the right hand side of that equation that is variation of u is equal to delta a naught plus delta a 1 x plus delta a 2 x square.

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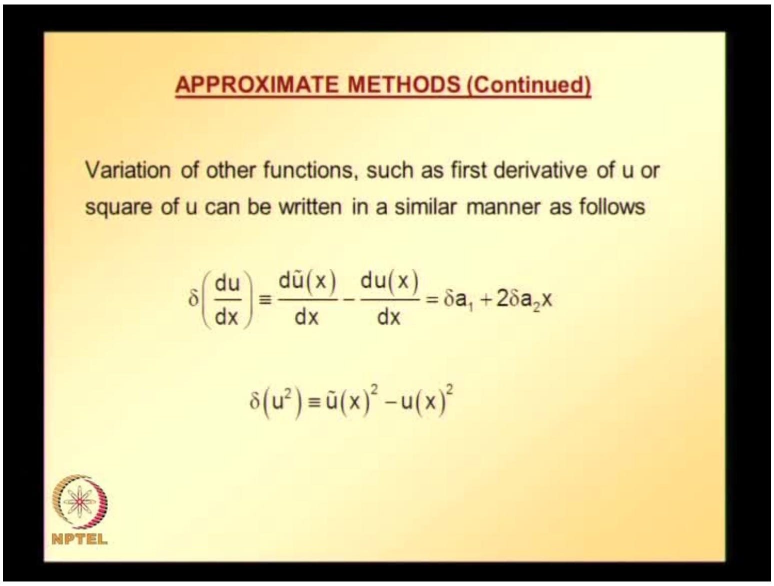

And variational of other functions, such as first derivative of u square of u can be written in a similar manner this is by definition what is a variation of derivative of u variation of derivative of u is - derivative of u tilde minus derivative of u. And when we know what is u tilde when we take derivative of u tilde with respect x and when we take derivative of u with respect x and substitute those two quantities into this equation. We get variation of derivative of u with respect x as delta a 1 plus 2 delta a 2 x similarly, variation of u square is u tilde square minus u square and substituting we can get what is variation of u square.

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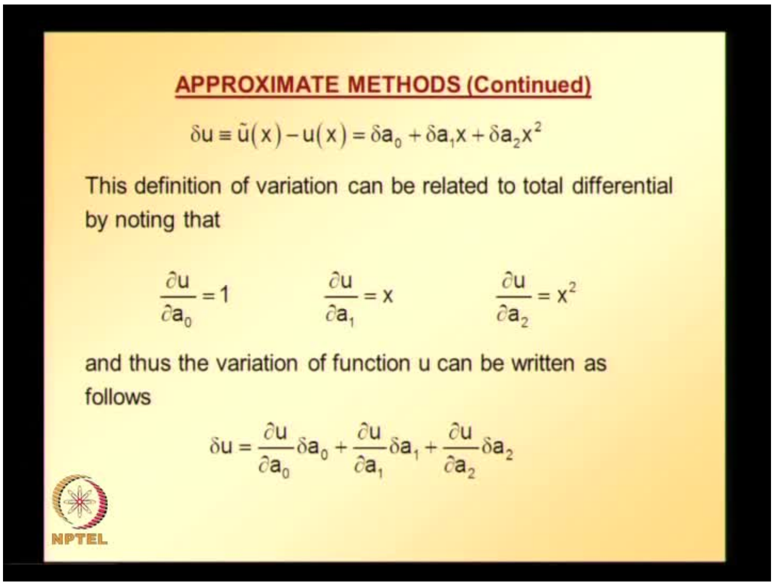

And we noted that variation of u is delta a naught plus delta a 1 x plus delta a 2 x square, please note that the coefficient of delta a naught is 1 and coefficient delta a 1 is x, coefficient of delta a 2 is x square. So, with this understanding, the definition of variation can be related to the total differential. And please note that partial derivative of the trial solution that we started out with, that is, u is equal to a naught plus a 1 x plus a 2 x square partial derivative of u with respect to a naught is going to be one partial derivative of u with respect to a 1 is going to be x partial derivative of u with respect to a 2 is going to be x square. So, if you see, if you substitute this information into the first equation that is variation of u is equal to delta a naught plus delta a 1 x plus delta a 2 x square coefficient of delta a naught is 1, I will replace one with partial derivative of u with respect to a naught coefficient of delta a 1 is x replace partial derivative of u with respect to a 1 replace x with partial derivative of u with respect to a 1 and coefficient of delta a 2 is x square replace x square with partial derivative of u with respect to a 2 doing that at the variation of function u can be written as like this.

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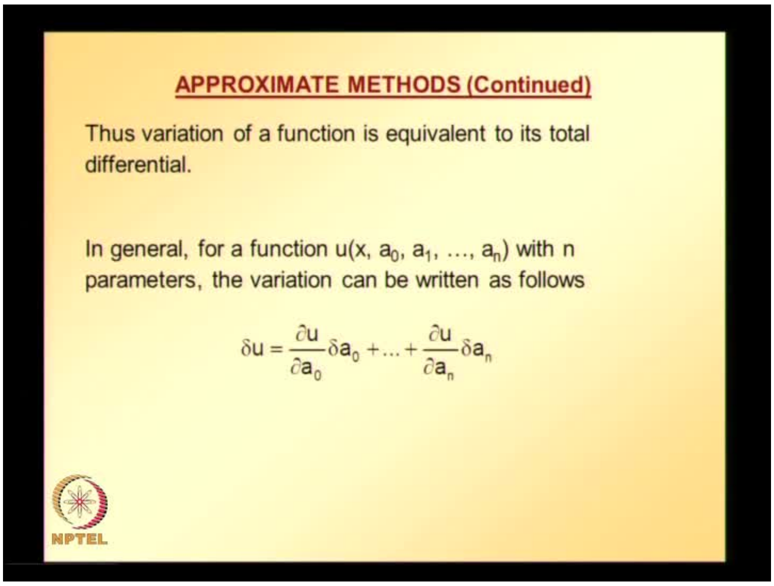

So, this is how variation can be related to the total differential thus variation of a function is equal to its total differential. And we can extend this definition to a function containing n number of parameters and variation of that function can be written in this manner. Variation of u is equal to partial derivative of u with respect to a naught times delta a naught and so on plus partial derivative of u with respect to a m a n times delta a n. one important point, I want to infer from this equation, suppose if delta sorry, suppose if variation of u is equal to zero, variation of u is equal to zero is possible, only if partial derivative of u with respect a naught, partial derivative of u with respect to a 1, and so on and partial derivative of u with respect a n are equal to are independently equal to zero.

So, this is one of the important points, because this is because delta a naught delta a 1 and delta a n all these perturbed parameters are nonzero. So, if variation of u is equal to zero it is possible that variation of u is equal to zero only if partial derivative of u with respect to the unknown parameters are these parameters a naught to a n are zero independently. So, this is one of the important point that you should keep in mind which will be using later during this variational approach.

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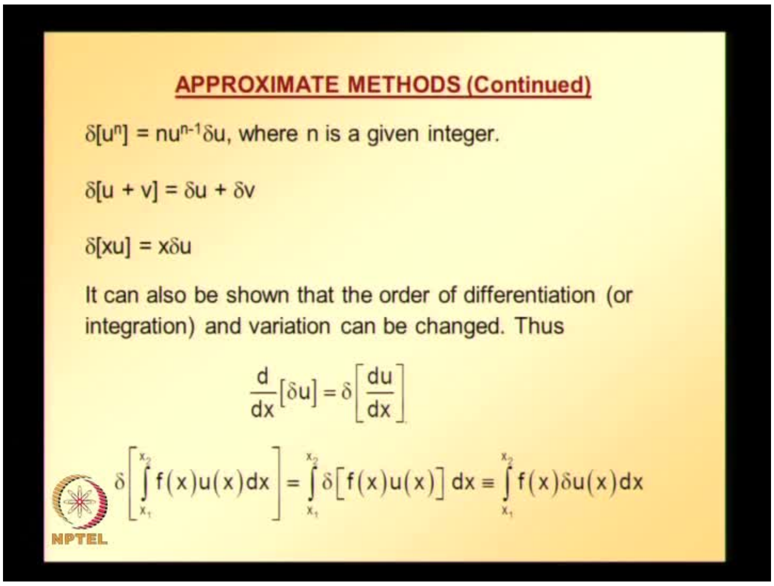

And some more identities variation of u power n, n times u power n minus one variation of u where n is integer, variation of u plus v is variation of u plus variation of v and variation of x times u x is not a function of any of the parameters. So, variation operator is going to operate only on u, so, x times u and it can also be shown that the order of differentiation and integration or the differentiation or integration and variation can be interchanged. So, the order of differentiation and variation or order of difference integration and variation can be changed that is, derivative of variation of u with respect x is same as variation of derivative of u with respect x and similarly, variation of integral x 1 to x 2 f x u x d x is you can take this variation operator inside integration and since f is not a function of any of the parameters variation operator is going to operate only on u.

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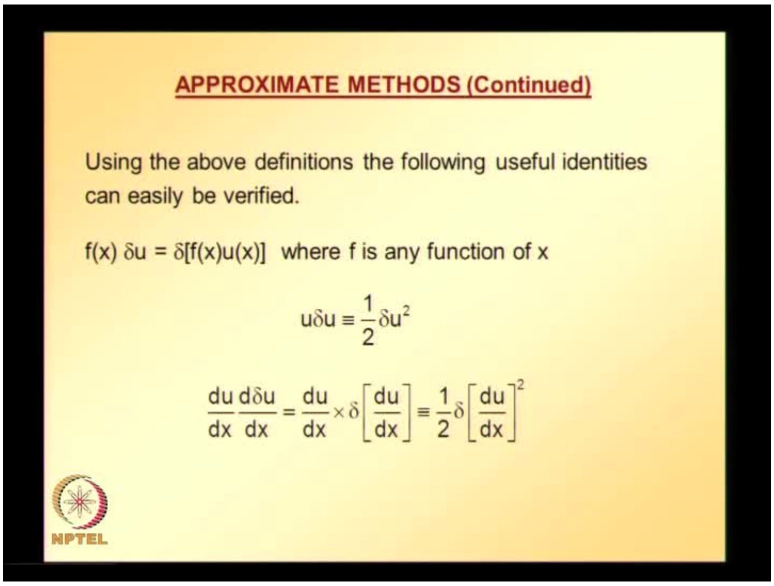

And using this definition, the following identities can easily be verified. Suppose, f times variation of u is there any where in the derivation we can replace that with variation of f times u, because of the reasoning that we have seen earlier, that is, f is independent of any of the parameters a naught a 1, so variation operator is going to operate only on u. Where f is any function of x similarly, you times variation of u can be replaced with half variation of u square and this definition can further be extended to the derivatives, also derivative of u with respected x times derivative of variation of u with respect x, can be replaced with half variation of derivative of u with respect x square, whole square, sorry. So, once we are comfortable with this concept of variation of function; we can apply these some of these and get the equivalent variational functional.

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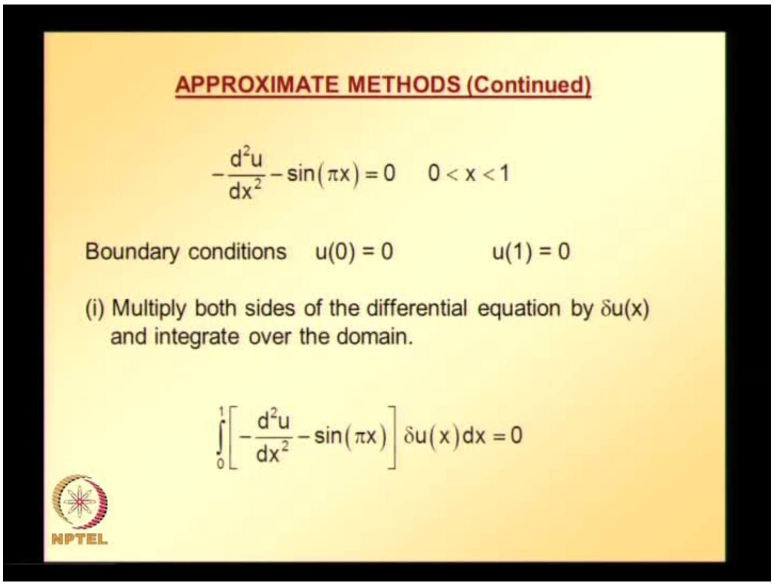

So, now let us, go back to the boundary value problem that we are looking at this is the boundary value problem that we are looking at and subjected to these boundary conditions. So, here I am going to present the derivation of equivalent variational form for this particular boundary value problem, so whatever the steps, we are going to look at these steps are in general, they are going to same even for other class of problems, like any of these steps are applicable for any second order or forth order or any boundary value problem, for those problems. The equivalent variational functional can be obtained by following the same steps. So, the first step is - multiply both sides of the differential equation by del variation of u and integrate over the domain. So, why we are multiplying here? The differential equation with variation of u, because we are interested in finding the approximate solution of u.

So, whichever quantity, the approximate solution of which you want to find. So, multiply the given differential equation with variation of that particular quantity and integrate over the domain. So, you do this on both sides. So, this is a first step and similar to modified Galerkin method that is, the difference between Galerkin method and modified Galerkin method is incorporation of this integration by parts to reduce the highest derivative present in the equation same idea is used here.

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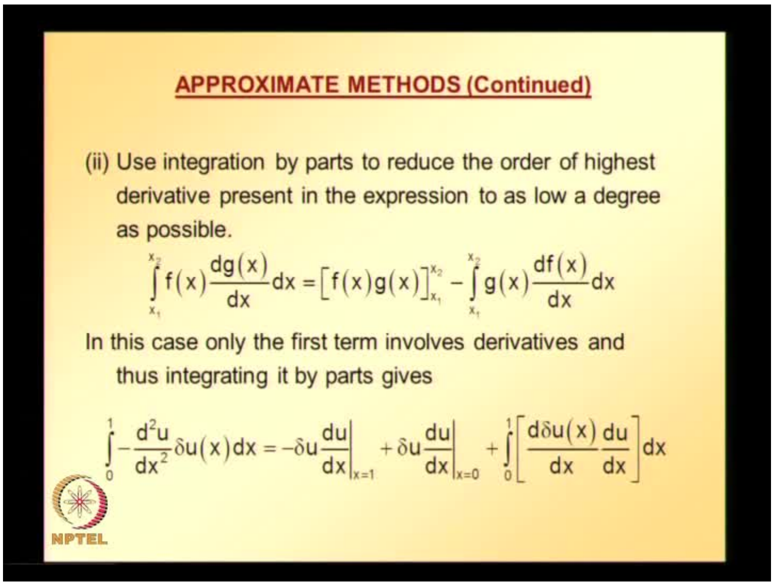

So, the second step is use integration by parts to reduce the order of highest derivative present in the expression to as low a degree as possible. So, this is second step; so, if you see the previous equation we have highest derivative two. So, we can reduce it to one order less by using integration by parts suppose a forth order derivative is appearing. So, we need to use integration by parts twice to reduce it to first order derivative, because once we have forth order first time we use it becomes third and second time we use, second time we use integration by parts it become second order differential equation again once again, sorry, once again third time integration by parts we are going to get first order differential equation. So, in that manner use integration by parts to reduce the order of highest derivative present in the expression to as low degree as possible and what is integration by parts that is given here integral x 1 to x 2 f x derivative of g with respect to x d x can be written as f x g x evaluated within the limits x 1 x 2 minus integral x 1 to x 2 g x derivative of f x with respect to x times d x.

So, this is the formula integration by parts. These we can apply on the first term, since only the first term involves second derivatives. In this case, only first term involves derivatives and thus integration did by integrating it by parts gives us this.

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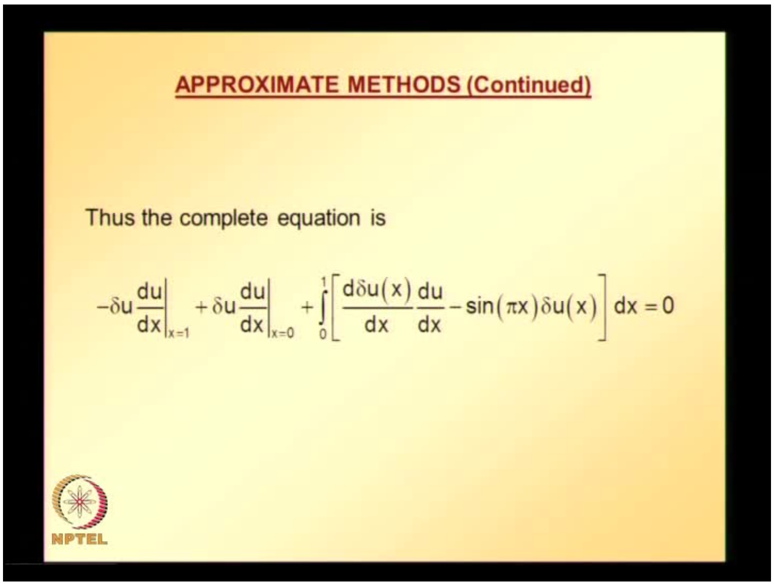

So, substituting this back into the equation that we got into in the first step, we get this equation. So, we can use some of the identities that we looked at before we started out with getting the equivalent variational functional for this particular problem, we looked at some identities, we can use those identities and simplify the first the two terms that are appearing inside the integral.

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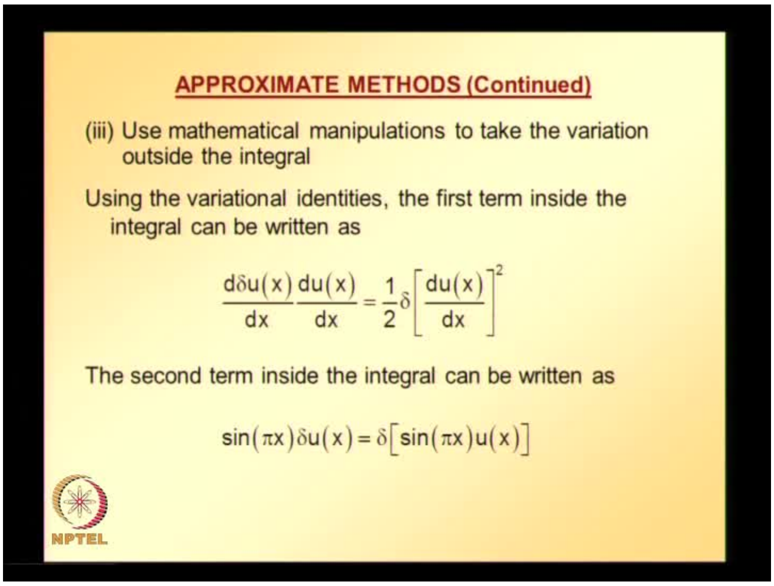

So, now let us use, mathematical manipulations to take the variation outside the integral. So, this is third step before that using the variational identities the first term inside the integral can be written, if you recall we already note done of the identities as this and also second term inside the integral sin pi x is like f x. so, it is not function of any of the parameters .

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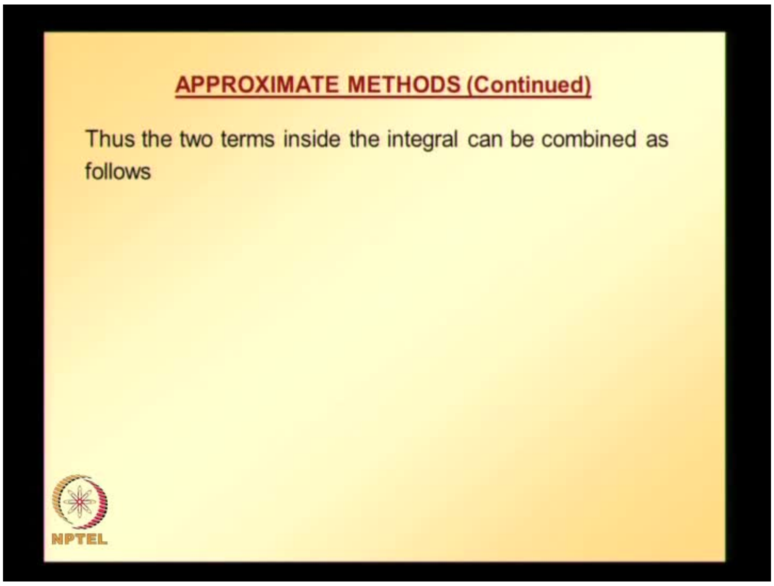

So, we can write it as variation of sin pi x times u x. So, substituting back these two terms and also these two terms in turn they are in the form variation of u plus v, which is variation of u plus variation of v. So, we can write these two terms together in this manner. So, substituting back this information into the previous equation substituting this expression interchanging order of integration and variation completes equation complete equation can be written in this manner. Basically, in this equal getting this equivalent to variational functional or equivalent variational form for a given boundary value problem the basic idea is to bring the given boundary value problem into the form variation of some quantity inside the bracket is equal to zero.

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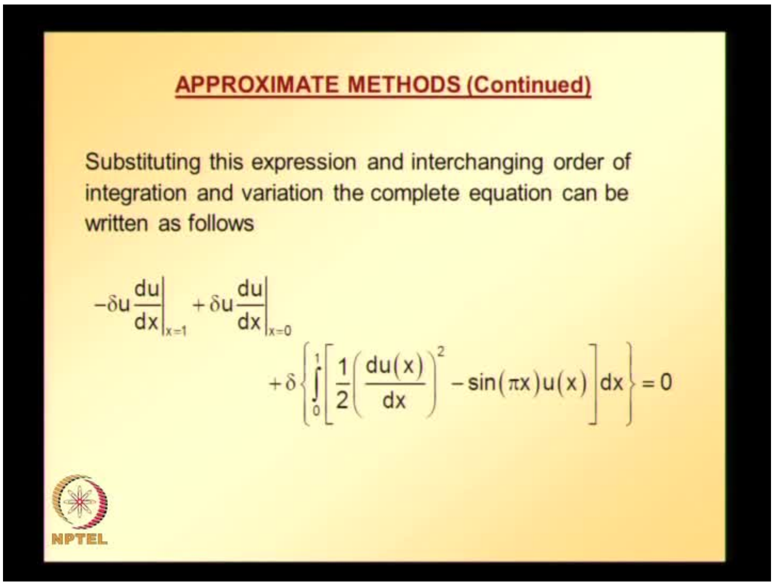

So, if you see this equation if we somehow get rid of first two terms, we have what is desired that is variation of some quantity inside is equal to zero. So, the next step is use boundary conditions to simplify boundary terms. So, if you see the first two terms in this equation they are related to boundary conditions x is equal to zero x is equal to 1. So, the next step is use boundary conditions to simplify boundary terms. Please note that whatever trial solution is start out with before we substitute that trial solution into either equivalent variational functional or at the total weighted residual, we need to make sure that it is admissible trial solution also note that admissible trial solution satisfies essential boundary conditions.

And satisfying essential boundary conditions means, for any value of parameters a 1 a 2 a n the admissible trial solution satisfies essential boundary conditions that means, variation of u evaluated at the essential the points at which are the location at which essential boundary conditions are specified at those locations variation of trial admissible trial solution is going to be zero. So, what it means, if you use admissible trial solution variation of u evaluated at x is equal to zero is zero variation of u evaluated at x is equal to 1 is zero since here for this particular boundary value problem. Two essential boundary conditions are specified and those two boundary conditions are specified at x is equal to zero and at x is equal to 1 variation of admissible trial solution at x is equal zero at x is equal to 1 at both these points variation of u is going to be zero.

So, with that reasoning we can get rid of the first two terms. So, first two terms becomes zero because basically first term is variation of u evaluated at x is equal to one times derivative of u evaluated at x is equal to 1 and second term is variation of u evaluated at x is equal to zero times derivative of u evaluated at x is equal to zero.

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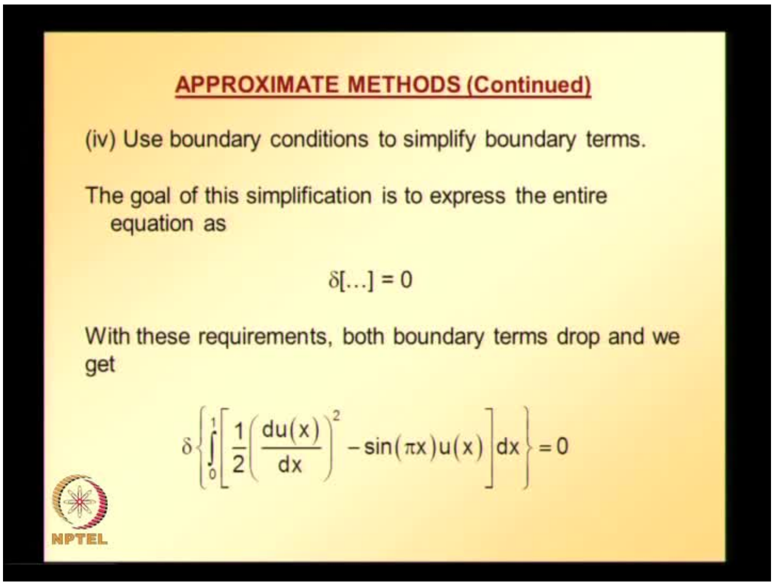

So, we can get rid of these two terms based on that condition, I hope this concept is clear. So, use the boundary conditions to simplify boundary terms, so the goal of the simplification is to express the entire expression as variation of some quantity inside the brackets is equal to zero and with these requirements both boundary terms drop and we get this one. And now, we can substitute admissible trial solution into this, and now and then we can use the condition that earlier, we have seen when we are looking at the relationship between variational and a total differential, we noted that variation of u is equal to zero means partial derivative of u with respect to each of the unknown parameters is equal to zero.

So, once we substitute admissible trial solution into this equation, last equation everything is going to be a function of unknown parameters and you have a function variation of that function is equal to zero that means partial derivative of that function with respect to unknown parameters is equal to zero. So, we are going to get or we are going to determine the unknown parameters by solving those equations that is, partial derivative of that function, which is in the bracket after substituting the admissible trial solution, by taking partial derivative of that with respect to unknown parameters and we are going to get so many equations as the number of unknown parameters that needs to be determine. So, that is how we can evaluate the unknown parameters. So, one more important thing is it is not possible to bring into this form for all kinds of boundary value problems, so that is the limitation of this variational method. And sometimes, this equivalent variational functional can directly be obtained especially for structural mechanics problems, we can use some of these. So, called energy methods without going through all these steps that is step one, two step, four can be directly avoided by writing the equivalent variational functional using potential energy principles or how to if you know, how to get potential energy functional based on energy methods.

So, we can directly use that instead of going through all these process, if somebody gives you equivalent variational functional directly and if you want to verify whether that equivalent functional, equivalent variational functional is corresponding to the boundary value problem that you are looking, you can back check or you can verify by substituting the equivalent variational functional into this equation and then working in the backward direction and we can see whether after substituting the boundary conditions, we can substituting the condition that variation of the trial solution at the locations at which essential boundary conditions are specified is equal to zero substituting and working backwards all that we can, if we can verify that we can retrieve the governing differential equation of the given boundary value problem and the boundary conditions.

Then, if we arrive at those, then that is going to be correct equivalent variational functional for that problem so that is how verification of variational equivalent variational functional can be perform.

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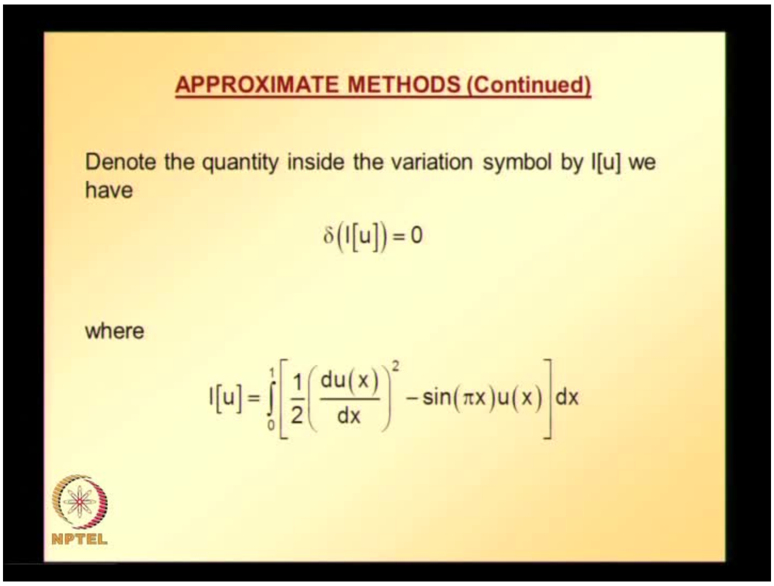

So, now coming back to a problem that we are looking at so denote the quantity inside the symbol by i u, because it is an integer also we can replace whatever that integral is here with some a single quantity i as a function of u i is nothing but, it is something like its denoting that integral and it is i is a function of trial solution u where i u is this one.

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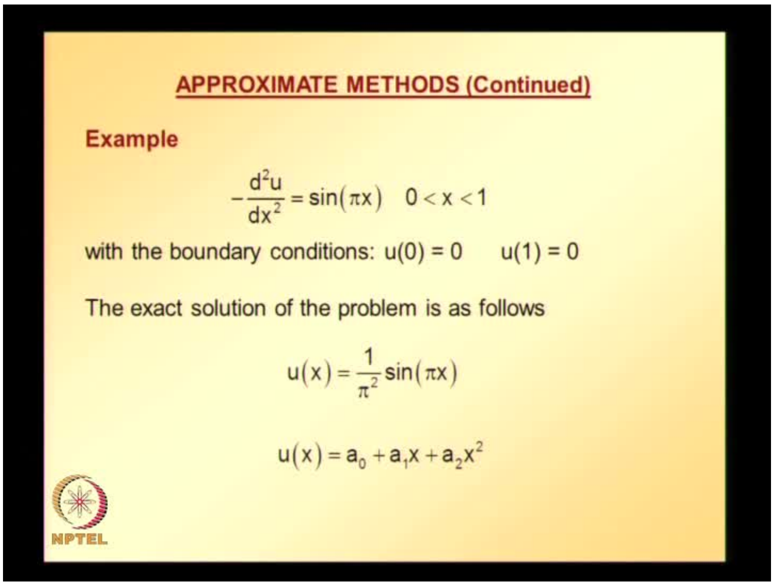

So, now let us go back to the problem, so, these are the four steps which are common for any boundary value problem once we follow these four steps we can get the equivalent variational functional, once we get equivalent variational functional we can substitute admissible trial solution into it and minimize variation of that particular function or we need to apply the condition variation of that potential energy functional is equal to zero that means, we need to take partial derivative of that functional with respect to the unknown parameter and so all for the unknown parameters.

So, this is the bound value problem that we are looking at subjected to these boundary conditions and exact solution for this problem is like this and also we already noted for the other weighted residual methods that we already looked at.

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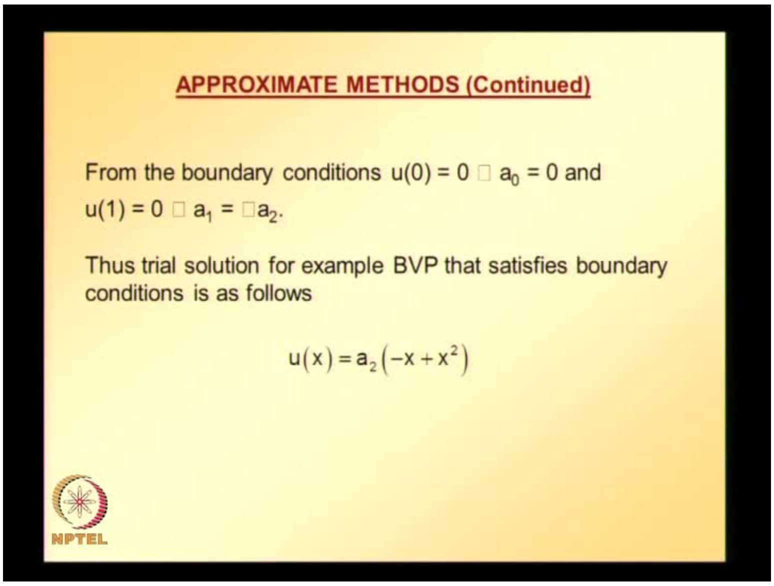

Starting with quadratic trial solution like this, after substituting the boundary conditions u evaluated at x is equal to zero is zero that leads to a naught is equal to zero u 1 evaluated at x, x is equal to u, u 1 sorry u evaluated at x is equal to 1 is zero that leads to a 1 is equal to minus a 2, we already noted this. Please, excuse me, here there is some problem with the font. And the, trial solution for the boundary value problem also we noted that this is any way this is this equation is correct trial solution u is equal to a two minus x plus x square.

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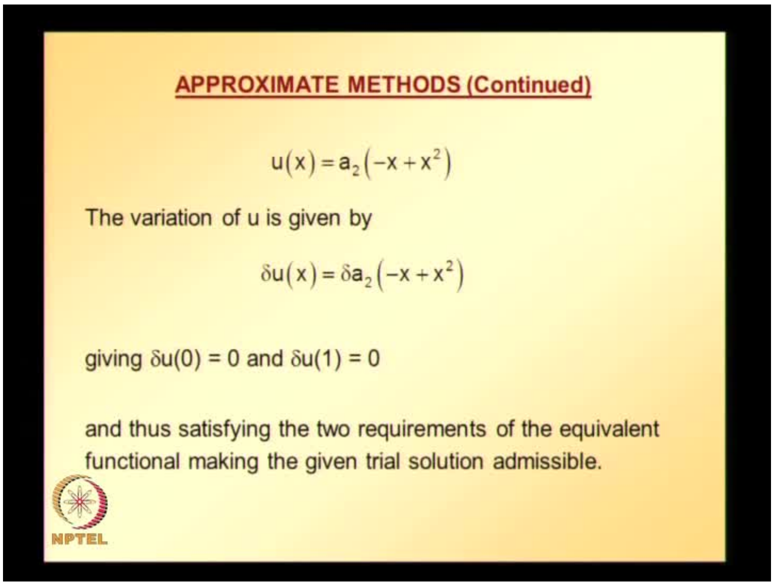

So, this trial solution substituting this into the equivalent variational functional that we already derived, we can determine a2 before that we need to are I mentioned that admissible trial solution satisfies a condition that variation of u is equal to zero at x is equal to zero and at x is equal to one . So, let me show you whether that is true or not. So, here I took admissible trial solution and variation of that is delta a 2 times minus x plus x square substitute x is equal to zero and x is equal to one in both cases variation u turns out be zero.

So, the reasoning with which we cancelled out the first two terms to arrive at the equivalent variational functional, that is, actually indeed it is correct or it is true now. So, thus satisfying, the two requirements of equivalent functional making the trial solution admissible.

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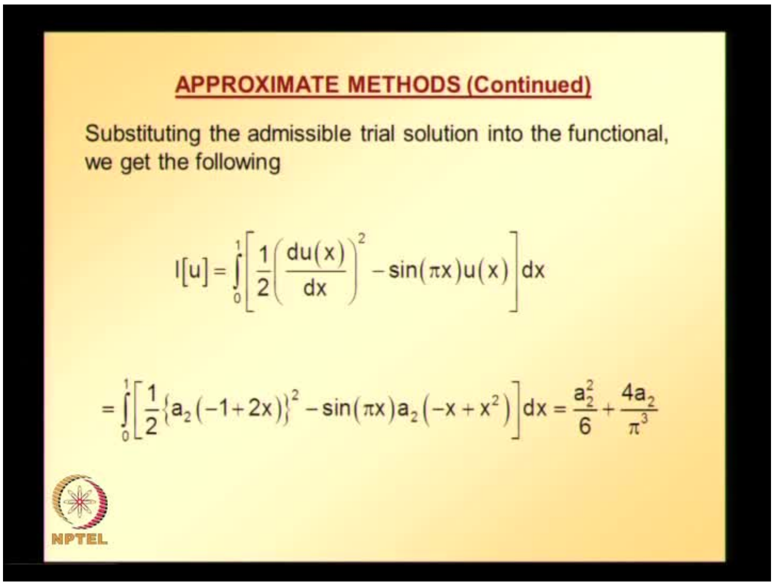

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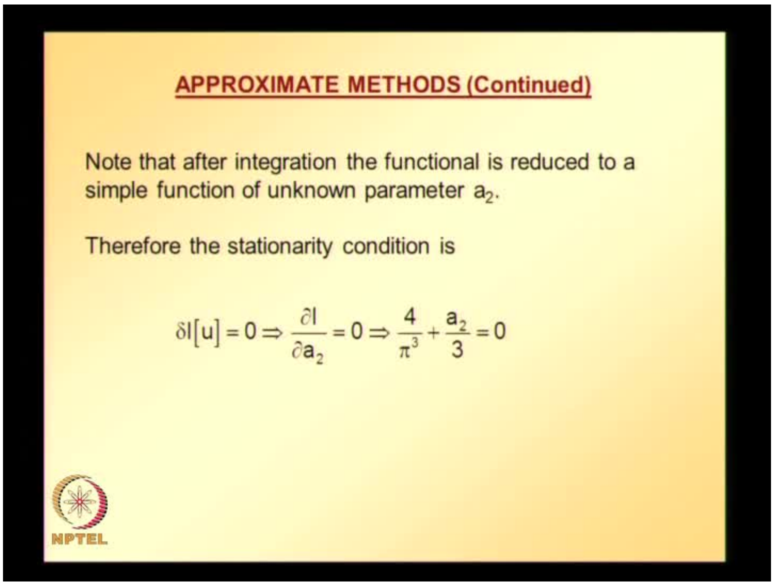

Substituting admissible trial solution into the functional, into this functional, we get this equation and there is only one equation, sorry we get this equation and note that after integration the functional is reduced to a simple function of unknown parameter a 2 and we want variation of that function is equal to zero that means, partial derivative of that function with respect to the unknown parameter is equal to zero.

So, we get one equation and one unknown, we can solve for a 2 suppose, if we have more number of unknowns we are going to get as many number of unknowns we are going to get as many equations, as the number of unknowns that needs to be determine solving these equations we can determine these unknown parameters.

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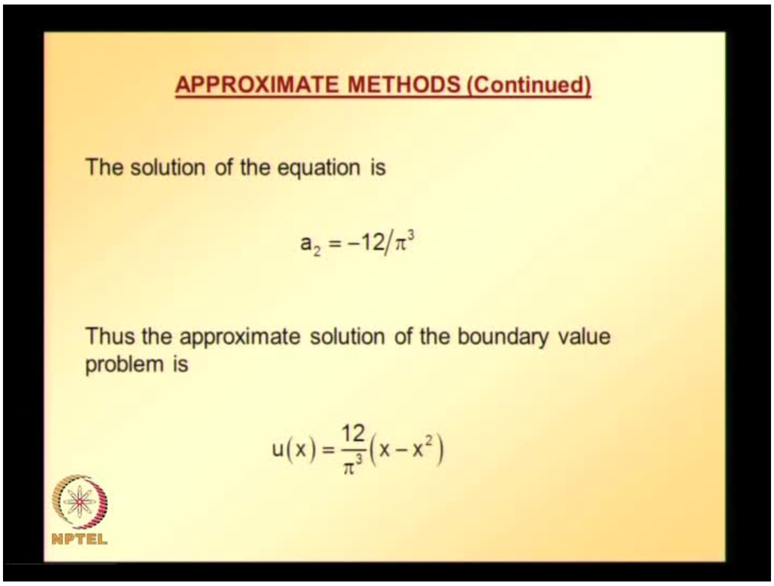

Solving a two solution turns out to be this one and if you recall and if you go back and see the solution that we obtained using Galerkin method, this is exactly what we obtained using Galerkin method and also comparison of this solution with exact solution is shown here.

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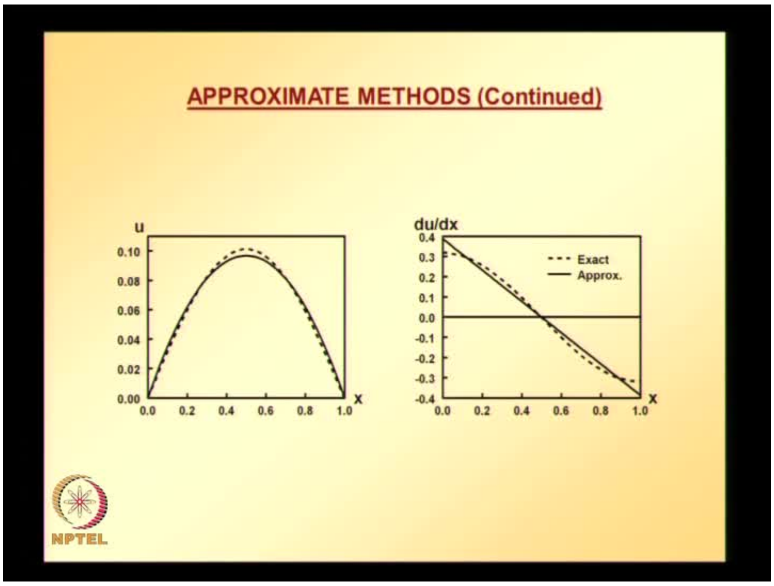

Solution both solution and derivative of solution comparison with exact is shown in this part. So, what we can conclude from today's lecture is, for a particular boundary value problem we are able to find equivalent variational functional and if we start with the same trial solution, as in this case, it is quadratic trial solution and following same steps forgetting admissible trial solution substituting, essential boundary conditions and reducing the some of the unknown parameters or expressing some parameters in terms of other parameters.

After doing all lets substituting admissible trial solution into the equivalent variational functional and minimizing functional with respect to each of the unknown parameters, we can solve for the unknown parameters and substituting back these unknown parameters into the admissible trial solution, we are going to get approximate solution and whatever solution we are going to get using variational method is going to be exactly same as what we get using Galerkin method. Even though, it two approaches are different, but the problem with variational method is not all boundary value problems will be able to bring into the form of variation of some quantity inside bracket is equal to zero.But if a particular problem can be brought into that form the solution that we get using variational method is going to be same as Galerkin method.