Finite Element Analysis

Prof. Dr. B. N. Rao

Department of Civil engineering

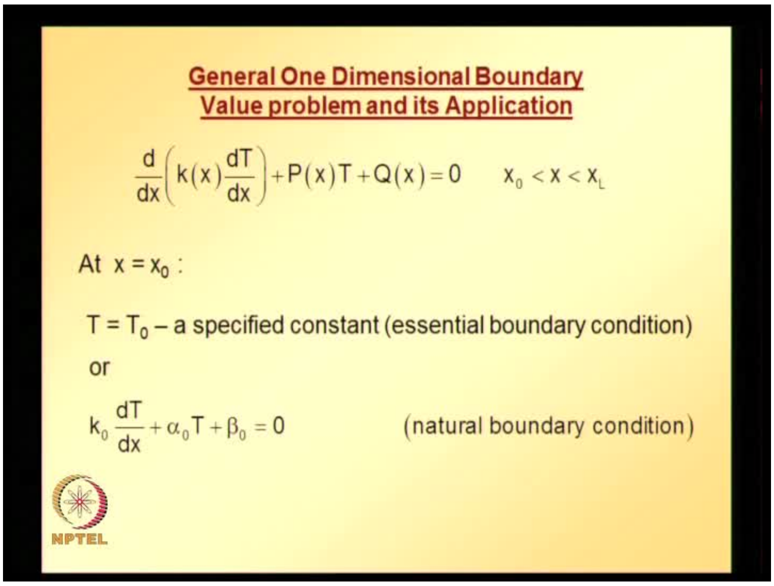
Indian Institute of Technology, Madras

Module No. # 01

Lecture No. # 17

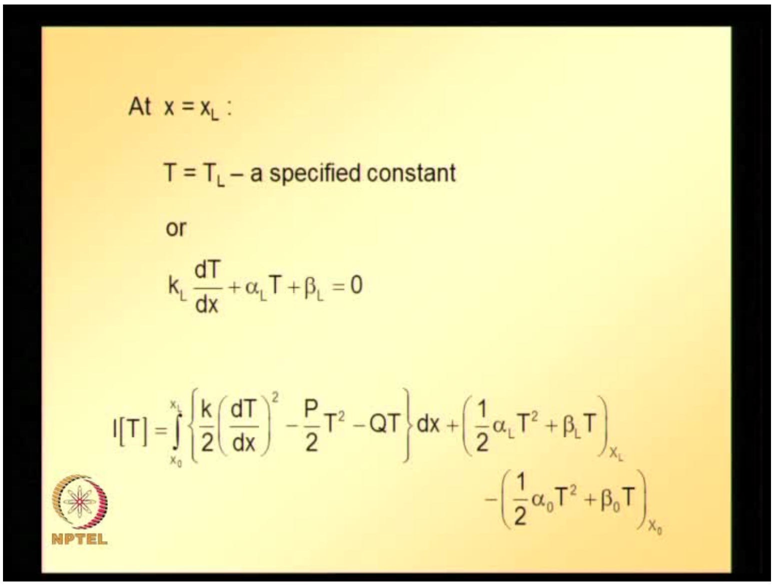
In the last class, we were looking at this general one-dimensional boundary value problem and also we looked at some applications like steady state heat flow problem, and let me summarize what we have done in the last class.

(Refer Slide Time: 00:34)



This is an equation representing general one-dimensional boundary value problem and the domain is also indicated there, x is going from x 0 to x L. And these are the boundary conditions: at x is equal to x 0, where T is the field variable here; if it is a steady state heat flow problem, then T is temperature; in the last class, we have looked at steady state heat conduction problem.

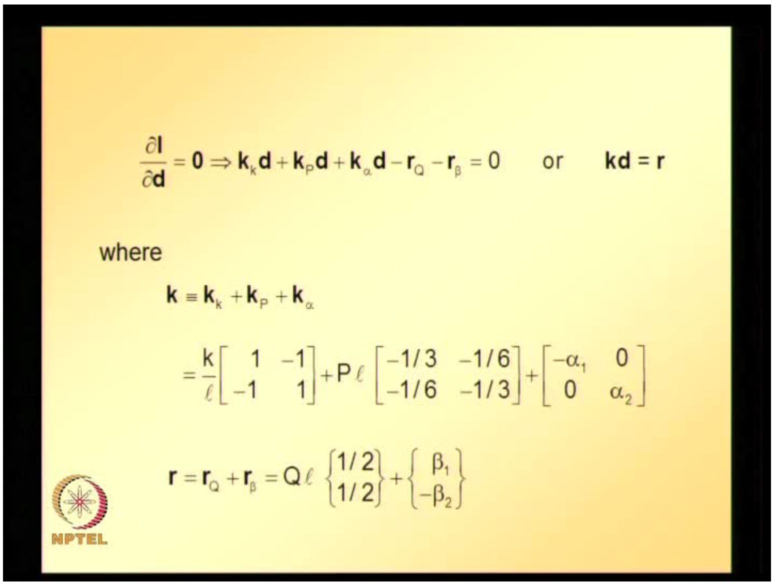
(Refer Slide Time: 01:45)



Here, either the boundary conditions can be essential boundary conditions or natural boundary conditions. These are the boundary condition at x is equal to x 0, where k naught is k value evaluated at x is equal to x 0. The boundary condition at the other end at x is equal to x L is given by this one, where k L is k evaluated at x is equal to L, either the boundary conditions can be either essential boundary conditions or natural boundary conditions. So, what we did is, we applied Rayleigh-Ritz method and we found equivalent functional before we proceeded and substituted finite element approximations in terms of two node linear element, finite element approximation of T and derivative of T.

So, equivalent functional that we obtained in the last class using Rayleigh-Ritz method is given here and then, what we did is, we substituted finite element approximations of T and also derivative of T. We have taken a two node linear element and when we made substitution of finite element approximations, we get I equivalent functional in terms of the nodal values.

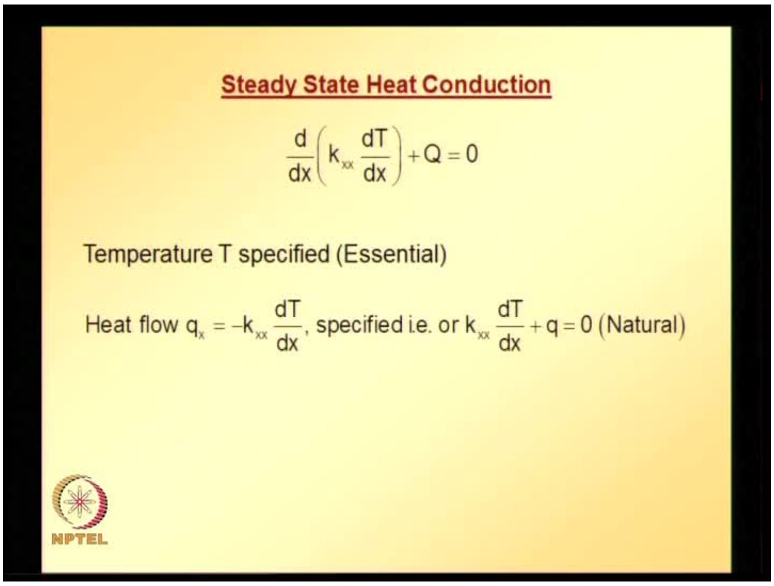
(Refer Slide Time: 03:20)



So, after that what we did is, we applied stationarity condition, that is, partial derivative of I with respect to d should be equal to 0, where d is a vector consisting of nodal values. We obtained this equation where the contributions to the stiffness like matrix is coming from various components k k, k p, k alpha and force like vector contribution is coming from r Q and r beta which are given or which are shown on the slide there.

So, this is the element equations for a two node element for general one-dimensional boundary value problem that we derived in the last class. We obtained this element equations using Rayleigh-Ritz method, but we can also apply Galerkin method to get similar kind of equations.

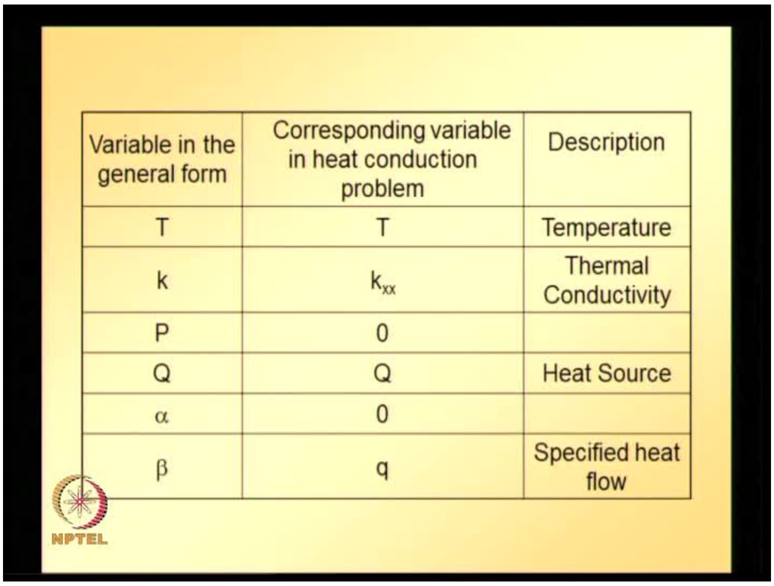
(Refer Slide Time: 04:34)



So, now, later what we did in the last class is we applied whatever we developed element equations for general one-dimensional boundary value problem using two node linear element, we applied to steady state heat conduction problem.

The governing equation for steady state heat conduction problem is given by this differential equation where k x x is thermal conductivity, T is the temperature, Q is the heat generator per unit volume. These are the boundary conditions either T can be specified or heat flow can be specified, that is, either essential boundary conditions or natural boundary conditions can be specified.

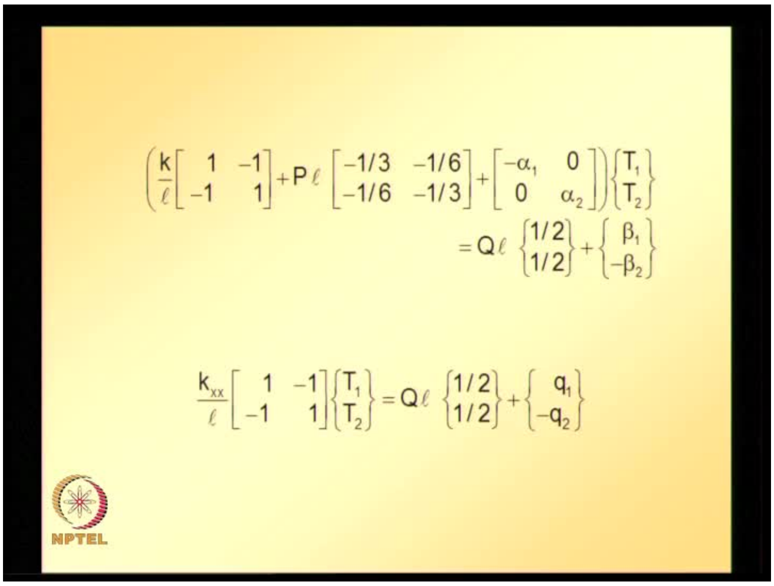
(Refer Slide Time: 05:33)



Variable in the general form	Corresponding variable in heat conduction problem	Description
T	T	Temperature
k	k_{xx}	Thermal Conductivity
P	0	
Q	Q	Heat Source
α	0	
β	q	Specified heat flow

So, these are the boundary conditions and the differential equation shown there; so what we did in the last class is we made a comparison between the corresponding quantities in the general one-dimensional boundary value problem and the corresponding variables in the steady state heat conduction problem. Actually, we came up with this comparative table; so, what we can do is once we have this comparative table, we can take the element equations that we have for one-dimensional boundary value problem and make substitution with the corresponding variable for a specific problem, here it is heat conduction problem.

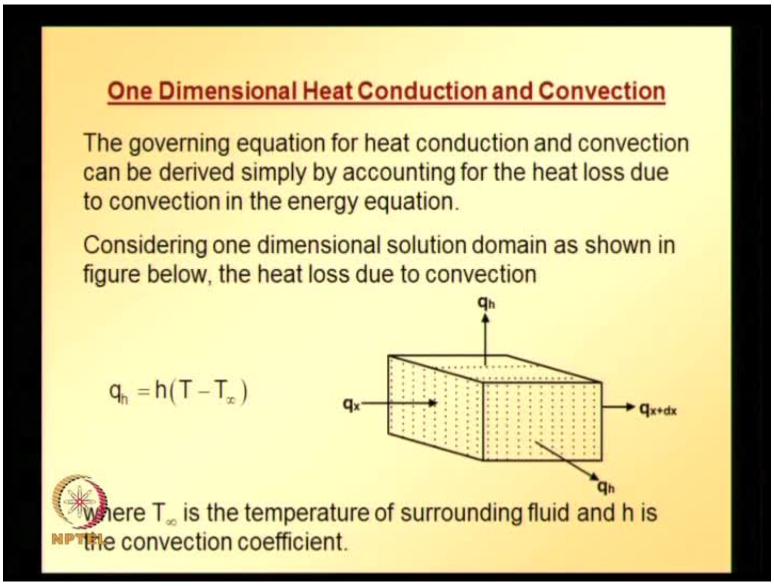
(Refer Slide Time: 06:22)



So, once we do that, we get the element equations for heat conduction problem, so we do not need to repeat the entire procedure each time. So, this is an element equation that we have for general one-dimensional boundary value problem. Now, we need to make substitution of the corresponding variables corresponding to the steady state heat conduction problem that we are looking at and when we do that substitution, we get these element equations.

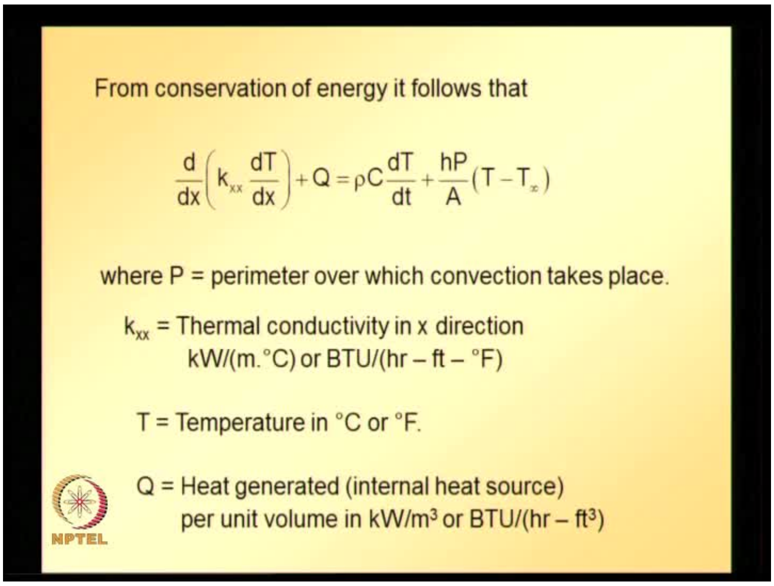
So, this is what we obtained in the last class and using this we also solved one example to see the application of this whatever element equations that we developed. So, now, in this class, we are going to continue further look at some more applications. So, in today's class, we will look at one-dimensional heat conduction convection problem.

(Refer Slide Time: 07:14)



The governing equation for heat conduction and convection can be derived simply by accounting for the heat loss due to convection in the energy equation. Considering one-dimensional solution domain as shown in figure, the heat loss due to convection, q h is equal to h times T minus T infinity, where T infinity is temperature of surrounding fluid, h is the convection coefficient.

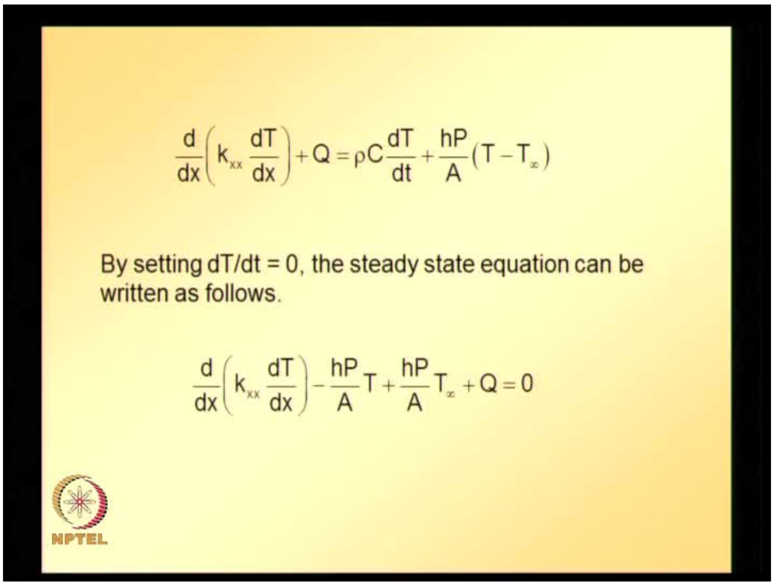
(Refer Slide Time: 08:05)



So using this, from conservation of energy we can write this differential equation for this heat conduction and convection case, where the coefficients k xx, T or the way the parameters k xx TQ, all have same meaning as what we have seen already for steady state heat conduction problem.

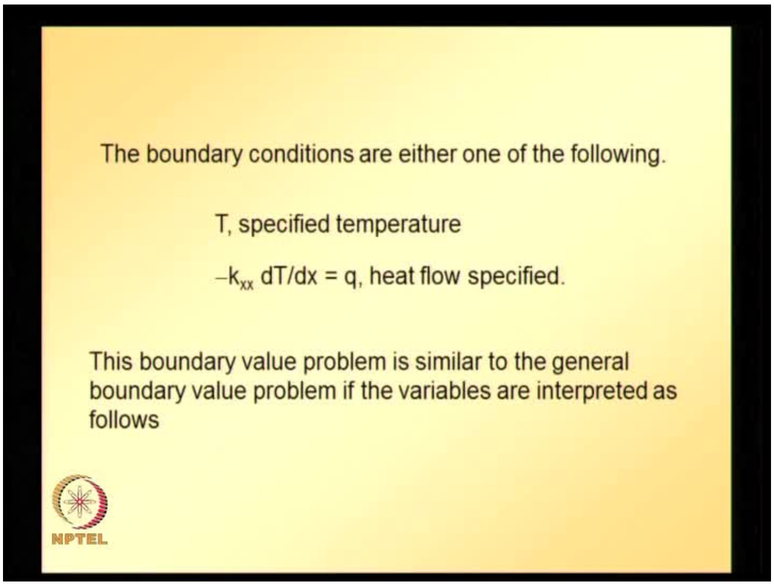
In this equation P is parameter over which convection takes place and other quantities have similar kind of meaning as what we have used for steady state heat conduction problem, k xx is thermal conductivity in x direction because this is 1D problem, T is temperature and Q is heat generated per unit volume.

(Refer Slide Time: 09:25)



So, now, in this differential equation if we set derivative of T with respect to time equal to 0, we get in this equation by setting dT d capital T over d small T equal to 0, the steady state equation can be obtained which is shown there.

(Refer Slide Time: 09:55)



So, now, let us look at what are the boundary conditions, boundary conditions can be one of the following either temperature specified or heat flow specified. First one is essential boundary conditions; second one is natural boundary condition.

So, now, for this particular case of heat conduction and convection, we can obtain element equations by making a comparative table between the corresponding quantities in the general one-dimensional boundary value problem and the variables in this particular case. So, this boundary value problem is similar to general boundary value problem, if variable are interpreted as shown in the table here.

(Refer Slide Time: 10:46)

Variable in the general form	Corresponding variable in heat flow equation	Description
k	k_{xx}	Thermal Conductivity
P	$-hP/A$	Convection term
Q	$Q + (hP/A)T$	Heat generated
α	0	
β	q	Specified heat flow at ends

So, once we identify the corresponding quantities, we can make substitution or plug-in into the element equations that we have for general one-dimensional boundary value problem.

(Refer Slide Time: 11:11)

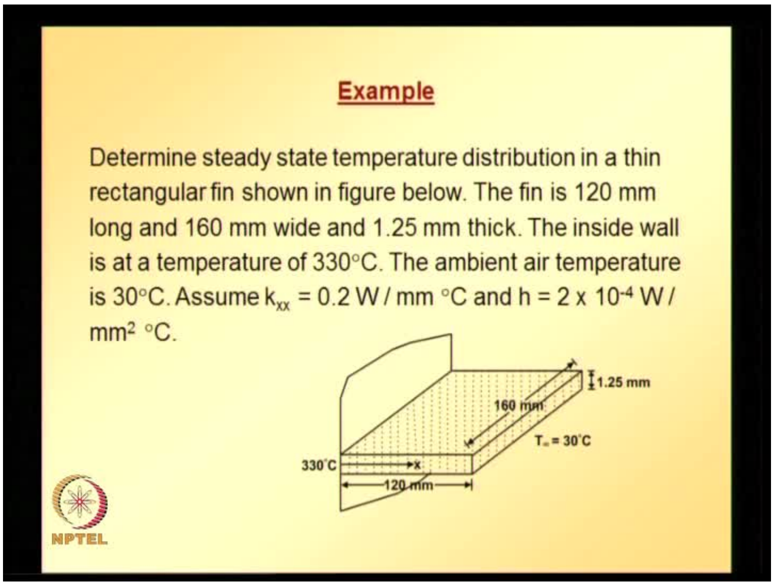
$$\left(\frac{k}{\ell}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+P\ell\begin{bmatrix}-1/3 & -1/6\\ -1/6 & -1/3\end{bmatrix}+\begin{bmatrix}-\alpha_1 & 0\\ 0 & \alpha_2\end{bmatrix}\right)\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}=Q\ell\begin{Bmatrix}1/2\\ 1/2\end{Bmatrix}+\begin{Bmatrix}\beta_1\\ -\beta_2\end{Bmatrix}$$

The following finite element equations can be written directly from the general equations.

$$\left(\frac{k_{xx}}{\ell}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\frac{hP\ell}{A}\begin{bmatrix}1/3 & 1/6\\ 1/6 & 1/3\end{bmatrix}\right)\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}=\left(Q+\frac{hPT_\infty}{A}\right)\begin{Bmatrix}\ell/2\\ \ell/2\end{Bmatrix}+\begin{Bmatrix}q_1\\ -q_2\end{Bmatrix}$$

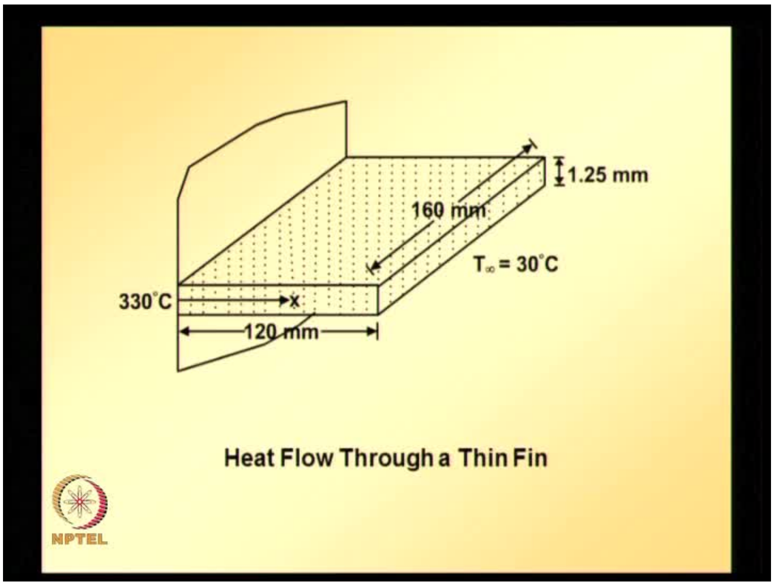
This is the element equation that we have for general one-dimensional boundary value problem. Now, we need to make substitution of the corresponding variables for this particular heat conduction convection problem. When we make that substitution the following finite element equations can be written directly from general equations after making substitution. So, this is the element equation for heat conduction convection problem, if we adapt two node finite elements for discretization. So, now, let us look at the application of this equation by looking at a problem, let us take an example.

(Refer Slide Time: 12:10)



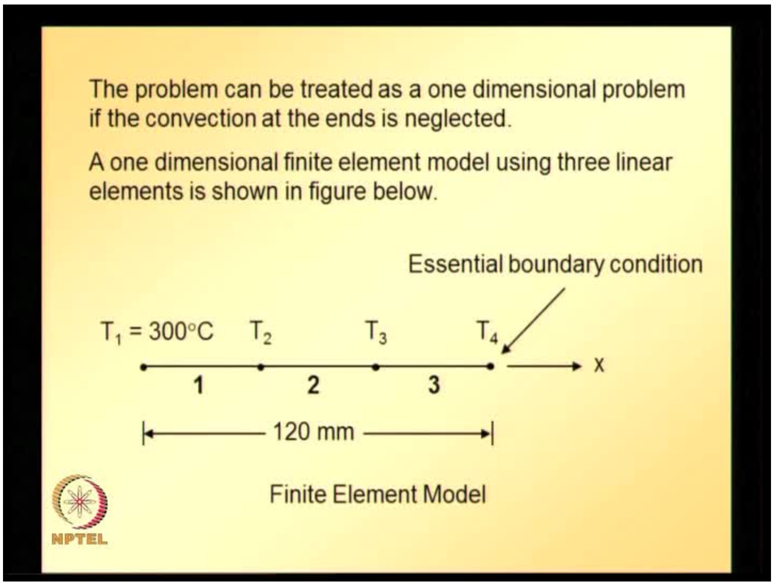
Determine steady state temperature distribution in a thin rectangular fin shown in figure below, which is shown there. The fin is 120 mm long and 160 mm wide and 1.25 mm thick. The inside wall is at a temperature of 330 degree centigrade. The ambient air temperature is 30 degree centigrade and other quantities like thermal conductivity, convection coefficient are given.

(Refer Slide Time: 13:08)



So, now, we need to discretize this problem before we apply the element equations that we derived for two node finite element, so this is a figure which is showing the rectangular fin with all detail.

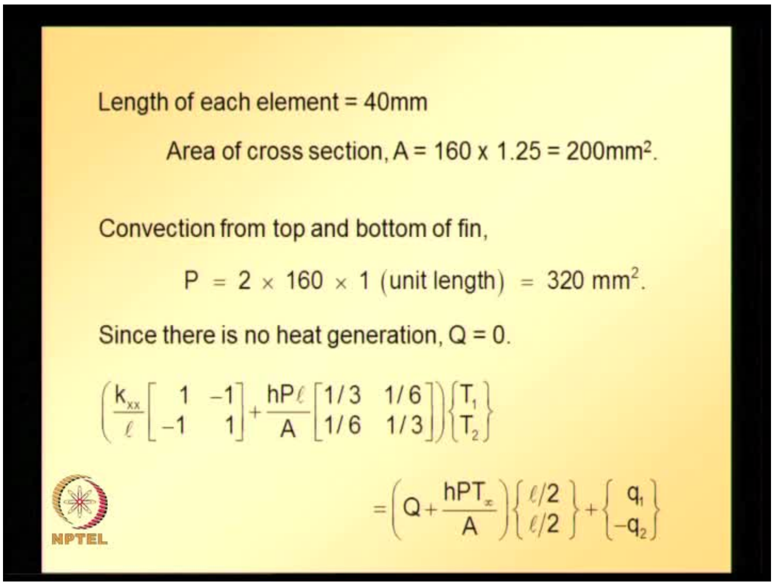
(Refer Slide Time: 13:26)



And now, before we discretize, the problem can be treated as one-dimensional problem if convection at the ends is neglected, so this is one of the assumptions. So, once we treated as one-dimensional problem, a one-dimensional finite element model using three linear elements is shown. Here, T 1 is shown as 300 degree centigrade, it is a mistake it should be 330 degree centigrade, and the other end essential boundary condition is specified T infinity is equal to 30 degree centigrade. So, at T 1 we have 330 degree centigrade, at T 4 we have 30 degree centigrade and total length of rectangular fin is 120 millimetres. It is discretized using three elements and each element has two nodes and length of each element is 120 divided by 340 millimetres.

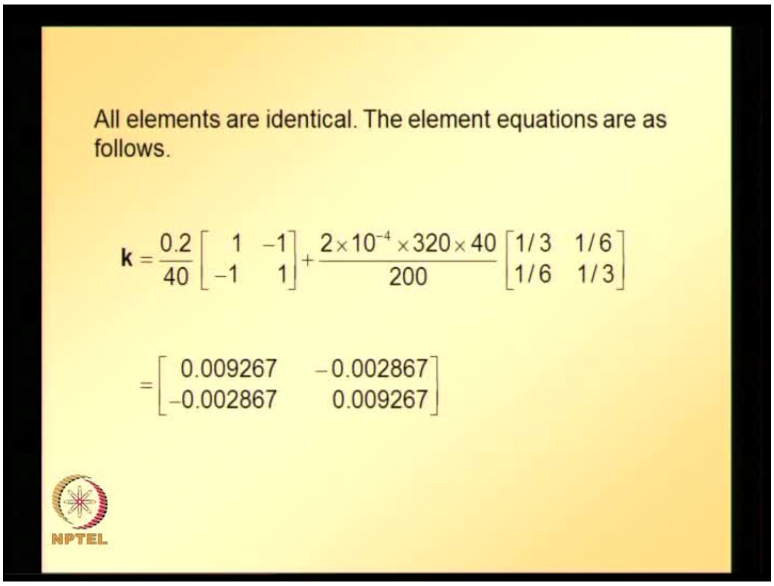
Since the material properties like thermal conductivity, convection coefficient and also since the length of each element is same, element equations are going to be identical for all the three elements.

(Refer Slide Time: 15:13)

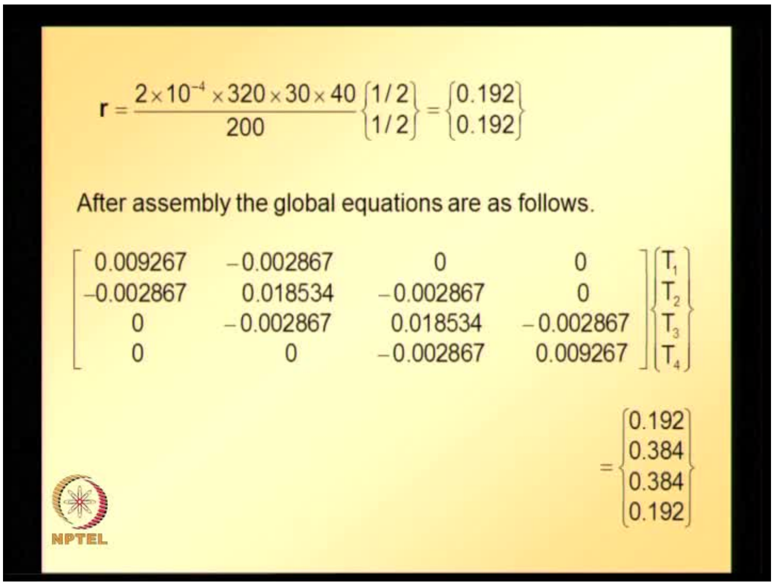


So, now, let us look at each of the element, length of each element 40 millimetres, area of cross section 160 multiplied by 1.25 millimetres square. Since convection at the ends is neglected, convection at the top and bottom of fin is going to depend on perimeter P and here perimeter value per unit length is given or calculated. Here, there is no heat generation, so capital Q is equal to 0. So, making substitution of all these things into the element equations for heat conduction convection problem that we already have, we get element equations for this particular problem.

(Refer Slide Time: 16:30)



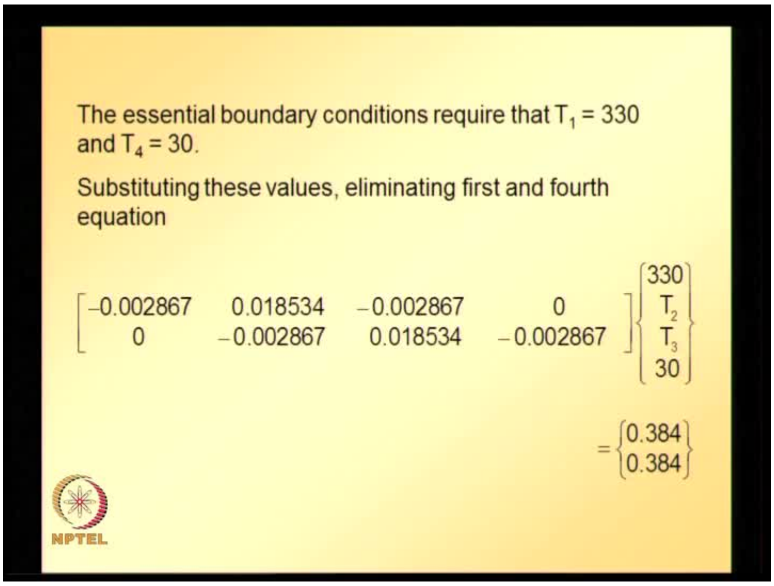
(Refer Slide Time: 17:01)



And as I mentioned all elements are identical, so element equations are as follows, where k is stiffness like matrix making substitution of all the quantities, and simplifying it further we get this k and r. Again making substitution of all quantities like convection coefficient perimeter, ambient temperature, area of cross section, length of each element, we get this r. Now, this k and r are going to be same for all elements because all elements are identical, so now assembling global equations we get this. There are three elements and each element has two nodes, element 1 contribution goes into one and two rows and columns, element 2 contribution goes into two and three rows and columns, element 3 contribution goes into three and four rows and columns, so with that understanding after assembling the global equations looks like this.

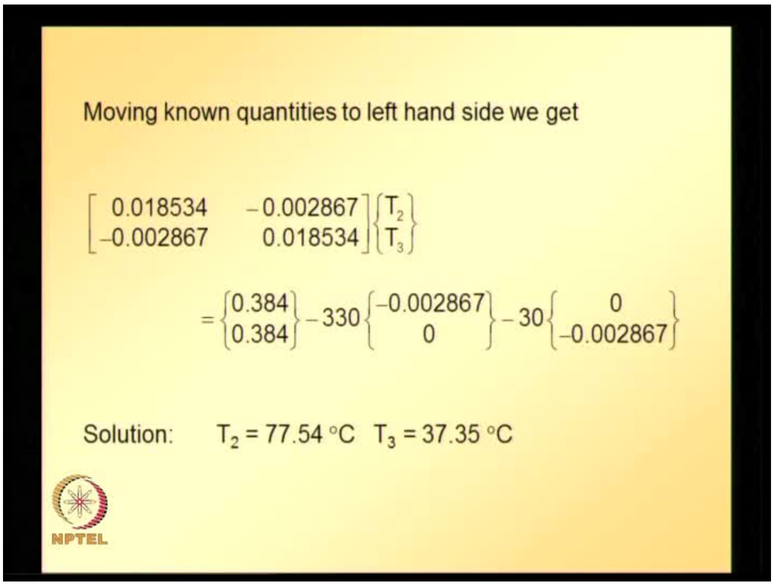
And now, making or imposing the essential boundary condition that is T 1 is equal to 330 degrees centigrade and T 4 is equal to 30 degree centigrade which is given.

(Refer Slide Time: 18:25)



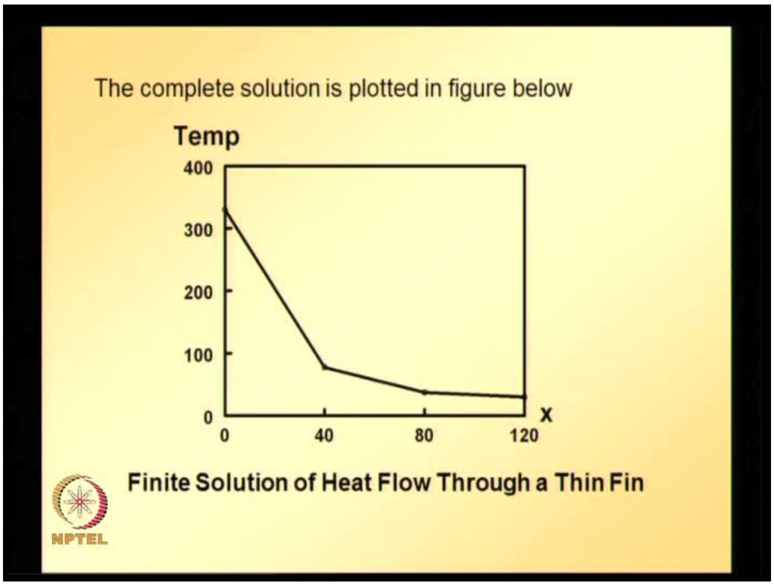
Essential boundary conditions require that T 1 is equal to 330 degree centigrade and T 4 is equal to 30 degree centigrade.

(Refer Slide Time: 19:04)



So, substituting these values and since the temperature value is specified at T 1 at node 1 and node 4, that is, T 1 and T 4 are known, we can eliminate first and 4 th equations and we can get this reduced equation system which we can manipulate by moving the known quantities to the left hand side. We obtain this equation and by solving this equation we can obtain what is T 2 and T3. So, solution of T 2 temperature at node 2 and temperature at node 3 are given in the slide there.

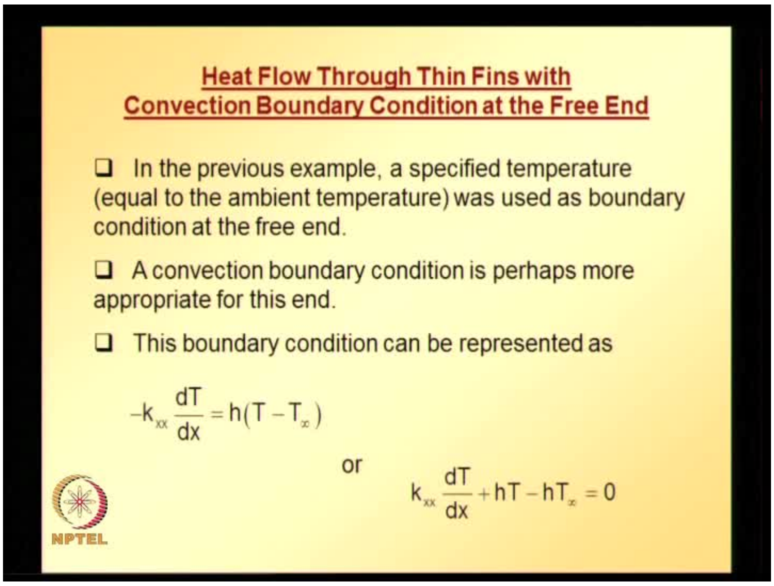
(Refer Slide Time: 19:56)



So, now, we solve this problem and we just calculate it T 2 and T 3, and T 1 T 4 are imposed, so complete solution we can plot. Complete solution is plotted in figure below that is T 1, T 2, T 3 and T 4 values, temperature on y axis and distance on x axis.

So, now, this is one application of general one-dimensional boundary value problem that we are looking at. Here, if you see this problem at node 4 temperature is specified, instead of that we can also have convection boundary condition prescribed. So, now, that is, in the previous problem at T 4, we have essential boundary condition specified. Now, what we will do is, we look at the case when natural boundary condition is specified at node 4.

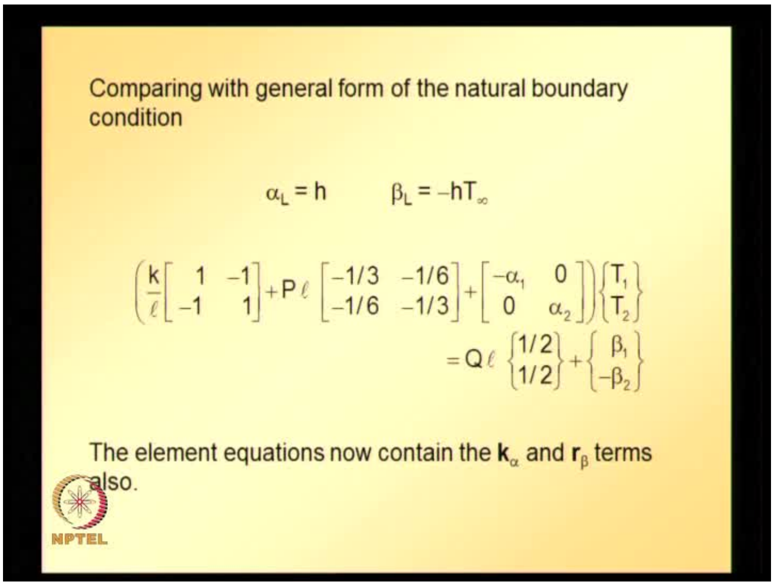
(Refer Slide Time: 21:08)



So, heat flow through thin fins with convection boundary condition at free end. So, in the previous example, a specified temperature equal to the ambient temperature was used as boundary condition at the free end. A convection boundary condition is perhaps more appropriate for this end. So, in this we are going to assume or this boundary condition can be represented as - this is the convection boundary condition. So, instead of T 4 being specified as essential boundary condition, if T 4 is specified as natural boundary condition then, the element equations are going to change a little bit.

Since, node 4 corresponds to element 3 and element..., 1 and 2 equations remain same as what we have seen in the last example and element 3 which consists of node 4. We need to have these boundary conditions, so element equations for that particular element; element 3 are going to be different little bit, because we need to include these boundary conditions.

(Refer Slide Time: 22:50)



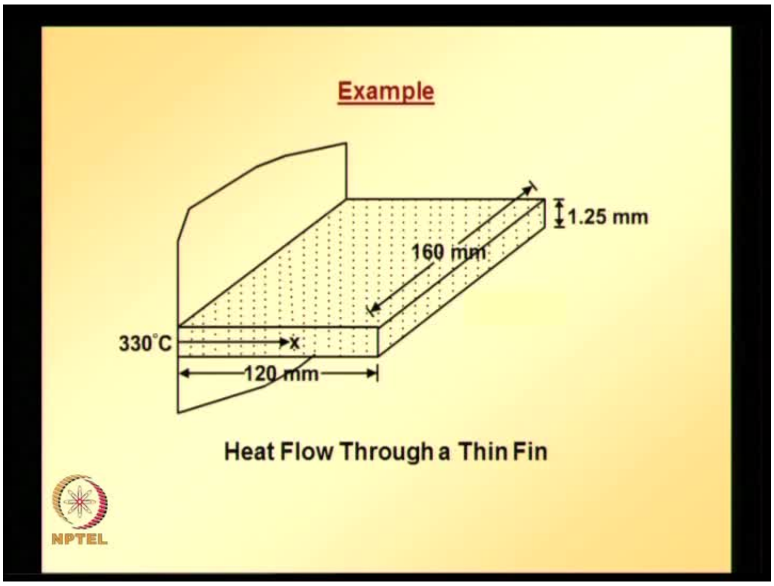
Comparing with general form of a natural boundary condition, we have already the element equations for general one-dimensional boundary value problem. So, now, with this natural boundary condition that is, convection boundary condition at node 4 or at one of the ends what we can do is, we can make a comparison with general one-dimensional boundary value problem and identify the corresponding variables and then, we can develop the element equations for this particular problem.

So, the equation that we obtained for general one-dimensional boundary value problem with k PQ alphas and betas are shown. Now, making a comparison alpha L is nothing but alpha 2 and beta L is nothing but beta 2, so making and alpha 1 and beta 1 are going to be 0. So, making the substitutions into the general one-dimensional boundary value problem element equations, we notice that now element equations consists of k alpha and r beta terms because in the previous example where we do not have this convection boundary condition alpha 1, alpha 2 both are 0 and beta 1, beta 2 both are 0.

So, those two that matrix consisting of alphas and vector consisting of betas contribution of those we have not seen, but for this particular problem where alpha L, which is alpha

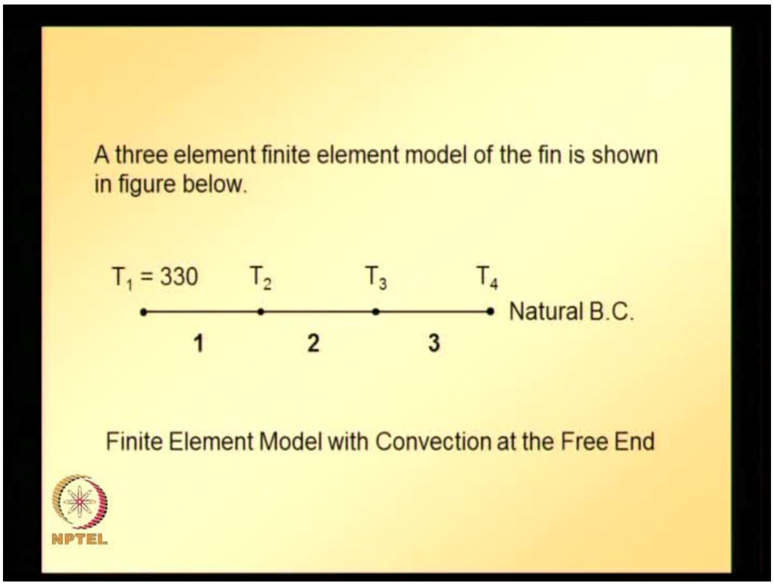
2 and beta L, which is beta 2 are given, so these are non-zero, so element equations now consists contain k alpha and r beta terms.

(Refer Slide Time: 25:17)



Now, with that understanding we will go back to the previous example and solve it except that. Now, we have the convection boundary condition which is natural boundary condition specified at one of the ends.

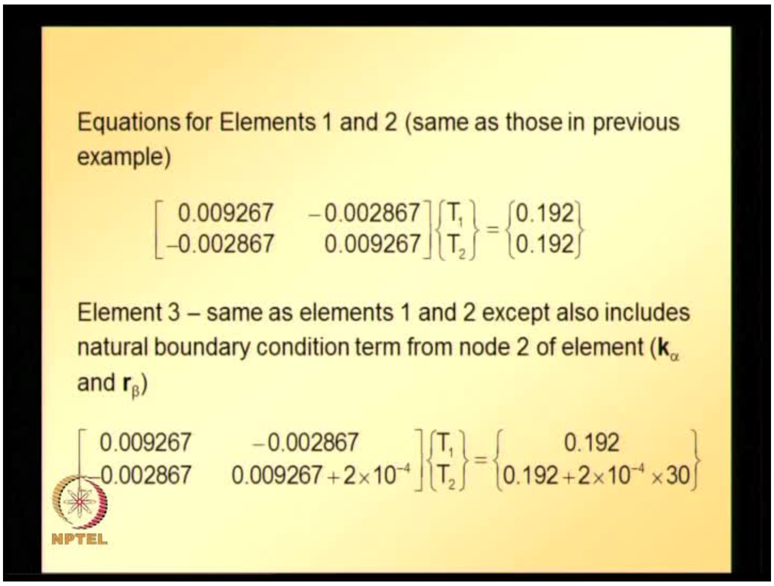
(Refer Slide Time: 25:42)



So, now, let us take the same discretization, three element finite element model for the fin is shown in figure and T 1 is equal to, T 1 is given already, 330 degree centigrade and T 4 now we need to apply this convection boundary condition which is natural boundary condition.

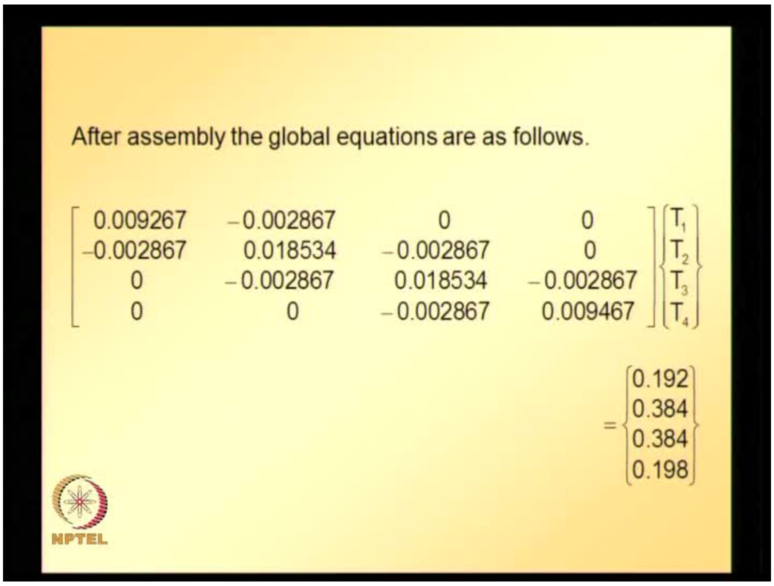
And Element 1 and element 2, the element equations are going to be identical to what we already have for the previous example and element 3, element equations are going to get modified little bit because of contribution from k alpha and r beta.

(Refer Slide Time: 26:26)



So, element equations for element 1 and 2 just repeated here which are obtained in the previous example. Element 3 same as element elements 1 and 2 except also includes natural boundary condition term for node 2 of element k alpha and r beta, so making this small change we get the element equations corresponding to element 3.

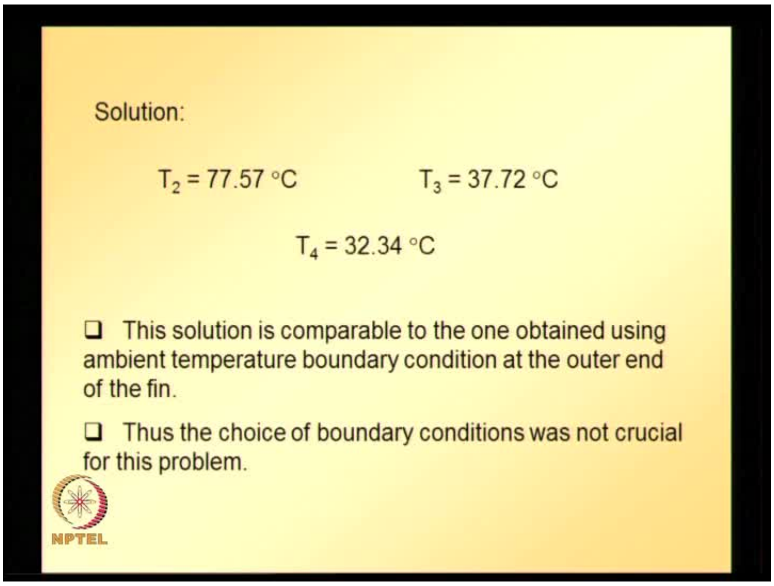
(Refer Slide Time: 27:02)



Now, with the same reasoning as what we have for the previous example, element 1 contribution goes into 1 and 2 rows and columns, element 2 contribution goes into 2 and 3 rows and columns, element 3 contribution goes into 3 and 4 rows and columns, and putting at the appropriate location the corresponding contribution we get this global equations.

And now, only one essential boundary condition is given which is T 1 is equal to 330 degree centigrade, essential boundary condition requires or require that T 1 is equal to 330 degree centigrade.

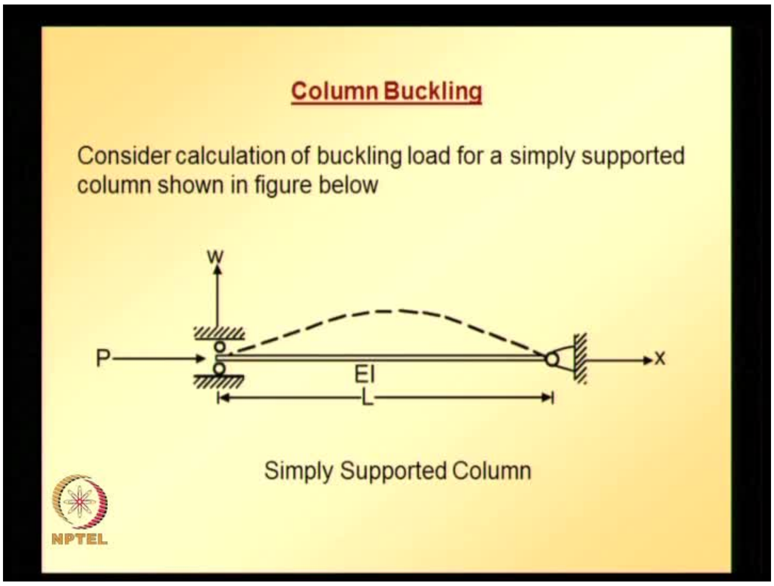
(Refer Slide Time: 28:11)



So, removing the first equation we get, and rearranging it and solving we can obtain what is T 2, T 3 and T 4, so solution of T 2, T 3, T 4 is given. If you see or if you compare this solution with what we already have for the case when T 4 at node 4 we have essential boundary condition specified ambient temperature in the previous example, if you compare solution of both these cases, this solution is comparable to one obtained using ambient temperature boundary condition at the outer end of the fin both are almost equal, thus this choice of boundary condition was not crucial for this particular problem.

So, may be in this particular case the choice of boundary condition, that is, whether you apply convection boundary condition or ambient temperature boundary condition almost same equation can be obtain.

(Refer Slide Time: 29:20)



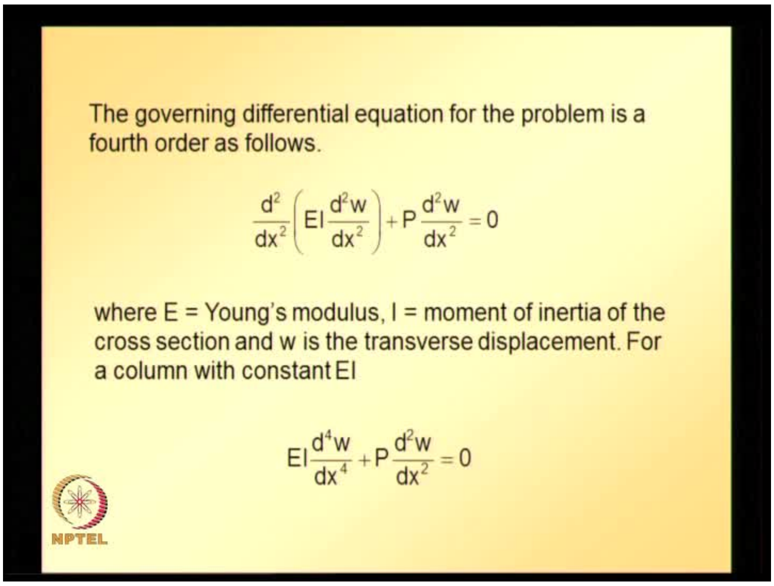
So, now, let us look at another example or another application of what we have already seen, that is, general one-dimensional boundary value problem and application to structural mechanics problem.

So, in this case, what we will be doing is, will be considering a column buckling problem. Consider calculation of buckling load for a simply supported column shown in figure below. So, a member which is subjected to axial compressive load like this can be classified as a column.

So, length of column is L, P i is constant and x axis is shown along centroidal axis of the member and w is shown perpendicular to x axis and P is the load that is applied at one end, so this is simply supported column. And if you observe the boundary conditions, w at both ends is equal to 0 and also curvature at both ends is going to be 0. Now, if you look at the governing differential equation for this particular problem, it is going to be fourth order equation. So, what we will be doing is, we have the element equations corresponding to general one-dimensional boundary value problem, we will see if the differential equation for this particular problem.

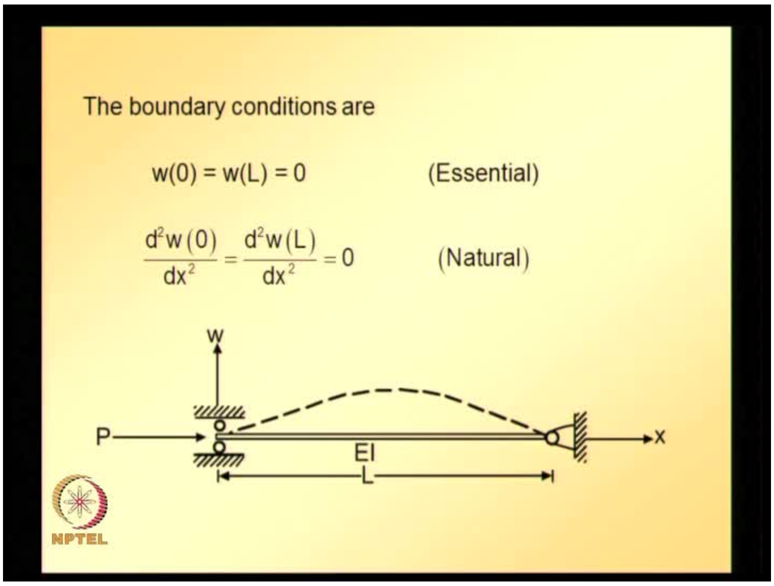
We can put into the form similar to general one-dimensional boundary value problem and identify the corresponding variables and then, we can develop the element equations for this particular column buckling problem. Once we decide the discretization, how many numbers of elements we want to use, we can get the element equation for each of the elements and then, we can assemble the global equations and apply the boundary conditions, we can solve for the problem.

(Refer Slide Time: 32:10)



So, now, let us look at the governing differential equation for this problem which is a fourth order differential equation, where E is young's modulus, I is moment of inertia of cross section, w is transverse displacement and if the column has constant E I then, we can take EI out of the differentiation operator and this equation becomes this.

(Refer Slide Time: 33:09)



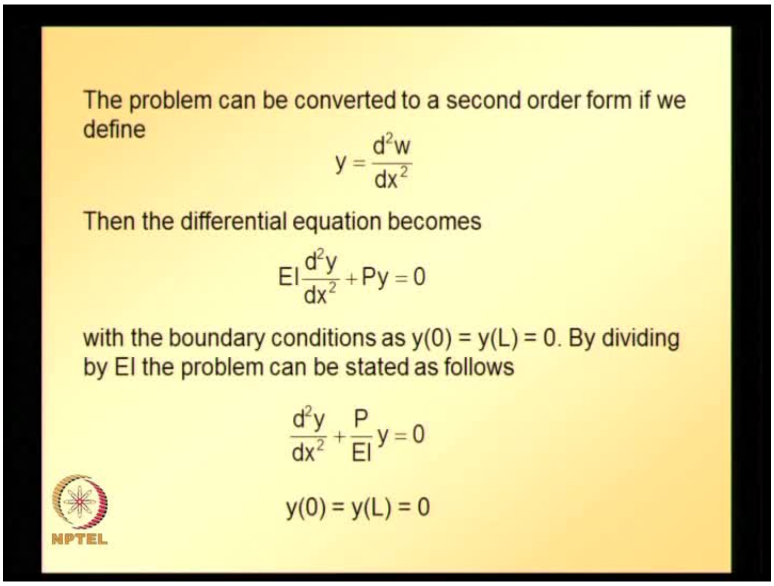
And now, we got the differential equation, this differential equation has to be satisfied from x going from 0 to L. The boundary conditions, if you look at the problem or the schematic once again, we can write the boundary conditions w at x is equal to 0, w at x is equal to L is equal to 0 and second derivative of w at x is equal to 0, second derivative of w at x is equal to L also should be equal to 0.

The first boundary condition is essential boundary condition, second boundary condition is natural boundary condition, you can easily recall the thumb rule that we discussed in the first class. If a differential equation is of order 2P those boundary conditions of order 0 to P minus 1 are essential and those boundary conditions from P2, 2P minus 1 are natural boundary conditions.

So, here, differential equation is of order 4, so 2P is equal to 4P, P is equal to 2, so those boundary conditions of order 0 to P minus 1, which is 1 are essentially those boundary conditions of order P which is 2 to 2P minus 1, which is 3 are natural boundary conditions. So, they can also verify that thumb rule once again for this particular problem.

So, now, we got the boundary conditions and we have the differential equation for this problem, but only thing is this is a 4 th order differential equation, whereas the general one-dimensional boundary value problem that we looked at is a second order differential equation, so somehow we need to make some substitution here to bring this fourth order to second order.

(Refer Slide Time: 35:07)

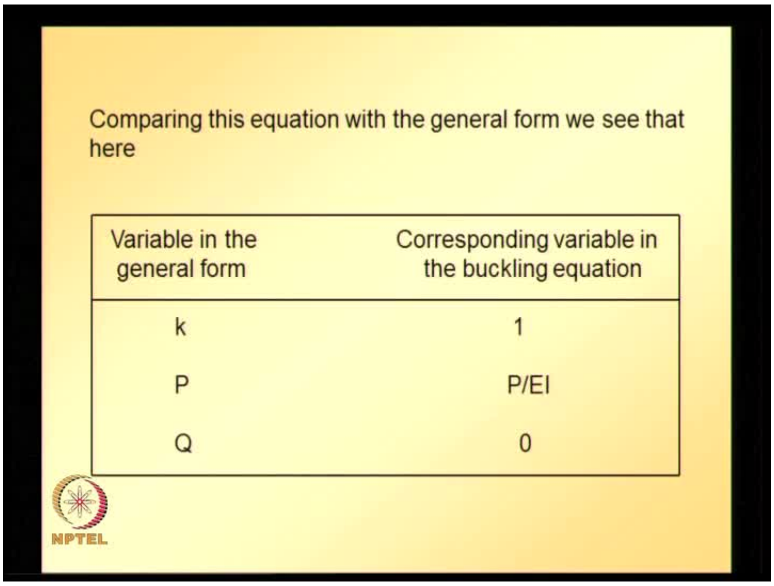


The problem can be converted into second order form if we define y is equal to second derivative w with respect to x square, so then the given differential equation or the differential equation for this particular problem, becomes after making the substitution becomes this one. And this is we can make a comparison of this with general one-dimensional boundary value problem, because both are second order differential equations.

But now, when we make the substitution boundary conditions also changes, so the corresponding boundary conditions in terms of y are given here. We already know that second derivative w with respect to x square at x is equal to 0, x is equal to L are 0. So, making the substitution we get y at x is equal to 0, y at x is equal to L is equal to 0. Now, the given differential equation we can rearrange it by dividing the given differential equation or the differential equation with EI, we can write the problem statement in this manner.

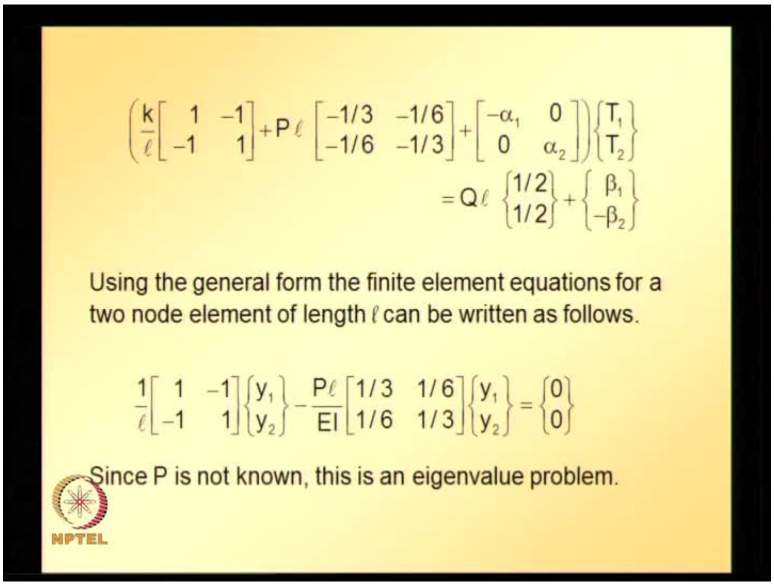
Second derivative of y with respect to x square plus P i, P over EI times y is equal to 0, that is, the differential equation that needs to be satisfied between x is equal to 0 and L including x is equal to 0 and x is equal to L and the boundary conditions are y evaluated at x is equal to 0, y evaluated at x is equal to L is equal to 0.

(Refer Slide Time: 37:08)



So, now, what we need to do is, we need to make a comparison between this and general one-dimensional boundary value problem. Comparing this equation with general form we see that here, we can identify the corresponding variables, variable in general form and the corresponding variable in the buckling equation. So, once we have this comparison, we can make substitution of the corresponding variable values or corresponding variable quantities into the element equations that we have for general one-dimensional boundary value problem.

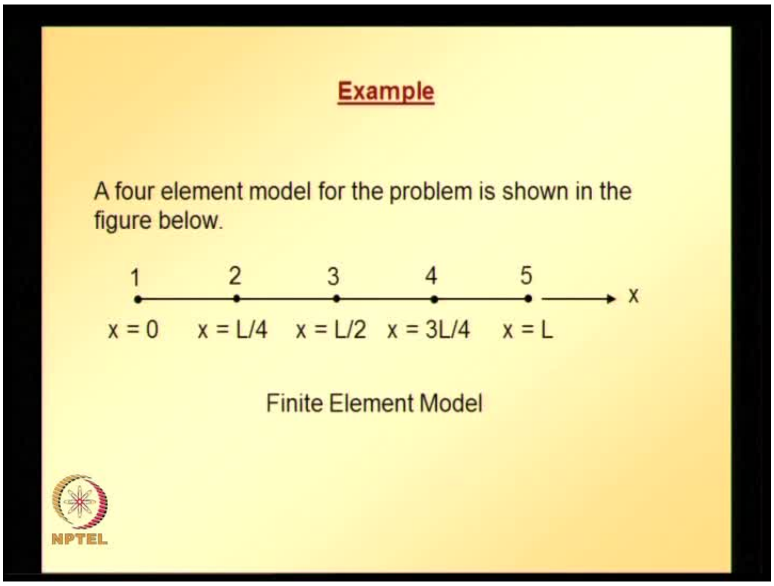
(Refer Slide Time: 37:50)



So, making this is the general one-dimensional boundary value problem element equation. Now, making substitution of k PQ into this equation which is corresponding to the buckling problem, we get the finite element equations for a two node element of length L for buckling problem can be written like this.

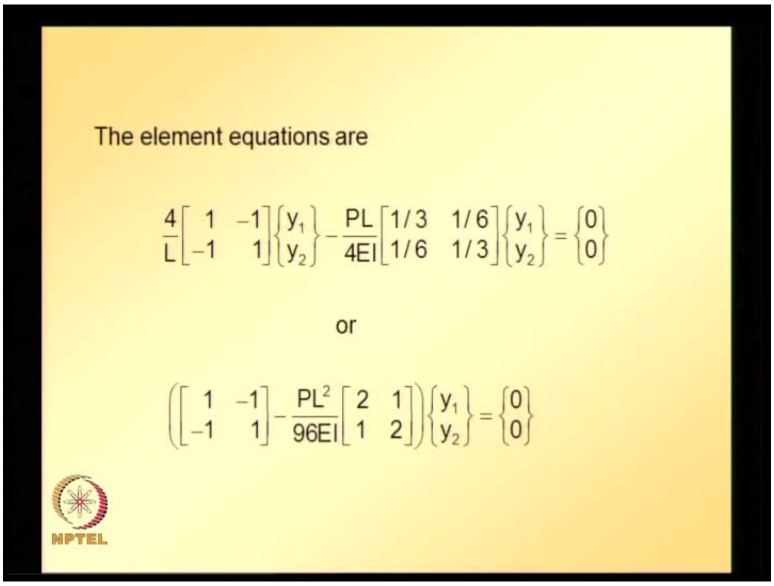
And here, we do not know what P is, we need to determine P; since P is not known, this is an Eigen value problem. So, this equation gives us an idea about element equations for a two node finite element for buckling problem. So, now, the given simply supported column of length L we can discretize using some number of finite elements and then, we can use these element equations and get the global equations and solve for the problem.

(Refer Slide Time: 38:59)



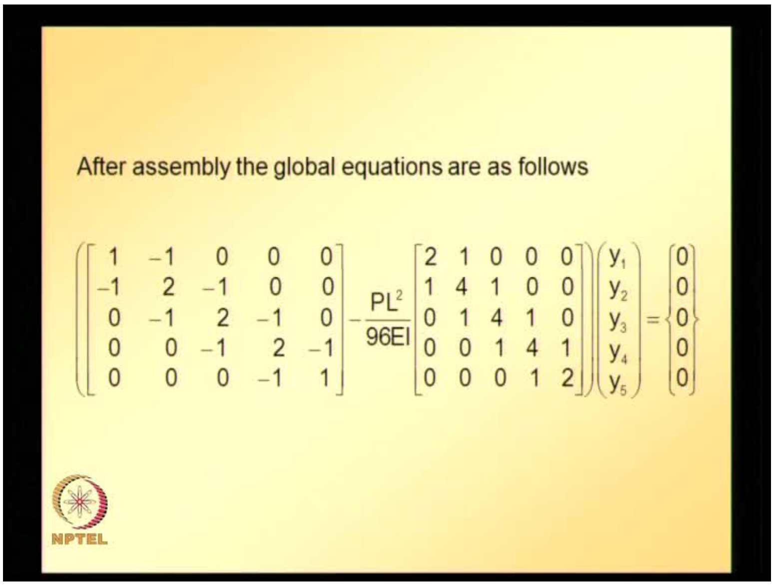
So, now, let us take an example on column buckling, a four element model for the problem is shown in figure. So, simply supported column of length L is discretize using four elements, four linear elements, are 5 nodes in total for this problem and each of the elements are of same length and length of each element is L over 4.

(Refer Slide Time: 40:10)



And since length of each element is same and EI is constant, the element equations for all the elements are going to be identical and we already have the element equations for column buckling problem for two node element - linear element. So, now we can make substitution of the values into that equation, the element equations, for one element by making substitution into the element equations that we obtained for column buckling problem from general one-dimensional boundary value problem.

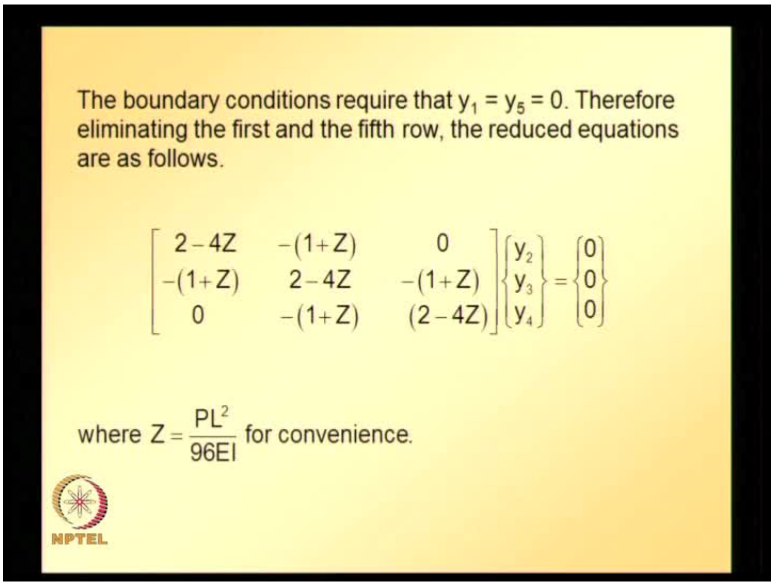
(Refer Slide Time: 40:55)



We get these element equations for 1 element and all elements are identical and we can rearrange this element equation in this manner for convenience. So, this is for a 1 element and global equations after assembly the global equations looks like this.

For this particular problem we have taken four element model, element 1 contribution goes into 1 and 2 rows and columns element 2 contribution goes into 2 and 3, element 3 contribution goes into 3 and 4, element 4 contribution goes into 4 and 5 rows and columns with that understanding, if we assemble after assembly the global equations looks like this.

(Refer Slide Time: 41:46)

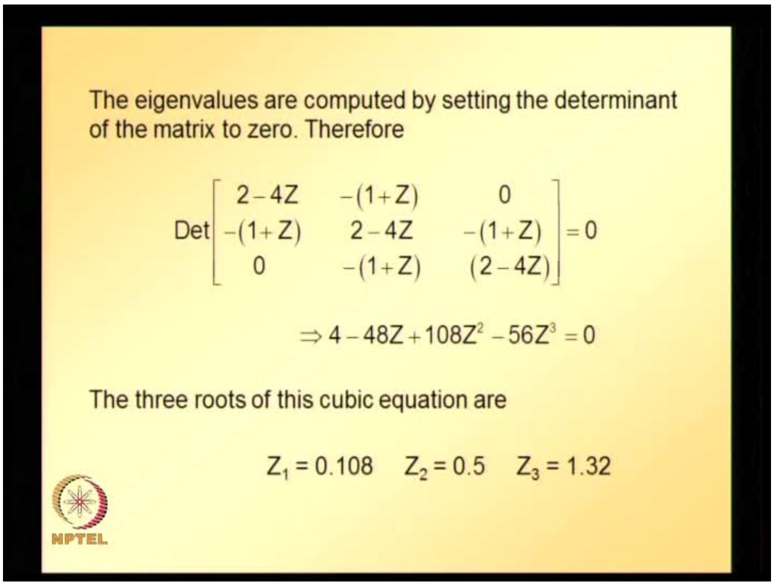


And now applying the boundary conditions, boundary conditions require that y 1 is equal to 0 and y 5 is equal to 0. Therefore, eliminating first and 5 th rows, the reduced equations can be obtained from the global equations and here a substitution made Z is equal to PL square over 96 EI and this can see here is an Eigen value problem.

So, the Eigen values can be computed by setting determinant of the matrix is equal to 0, determinant of matrix consisting of Z is equal to 0, because to have a non-trivial solution determinant of matrix should be equal to 0, because trivial solution is y 2, y 3, y 4 is equal to 0.

That is not what we are looking for; we are looking for non-trivial solution, so determinant of matrix should be equated to 0. Since this is a 3 by 3 matrix, we are going to get 3 roots or 3 Eigen values.

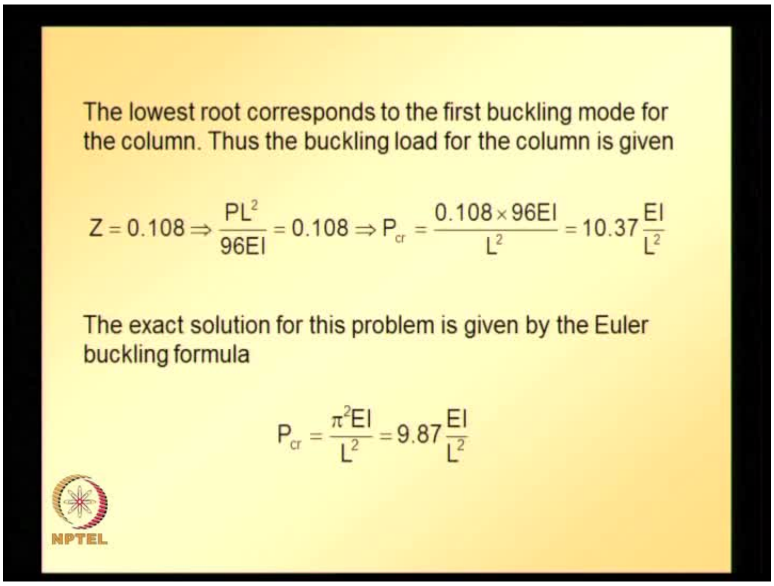
(Refer Slide Time: 43:07)



Eigen values are computed by setting determinant of matrix equal to 0, therefore we get this, when we do that we are going to get a cubic equation, so they are going to get three roots three roots for this cubic equation and the lowest root corresponds to the first buckling mode of the column.

So, now, Z is equal to PL square over 96 EI, so what we will be doing is, we calculate buckling load of the column. We equate the lowest Eigen value to PL square over 96 EI by calculating what is P that is going to be the load at which column is going to buckle first.

(Refer Slide Time: 44:21)

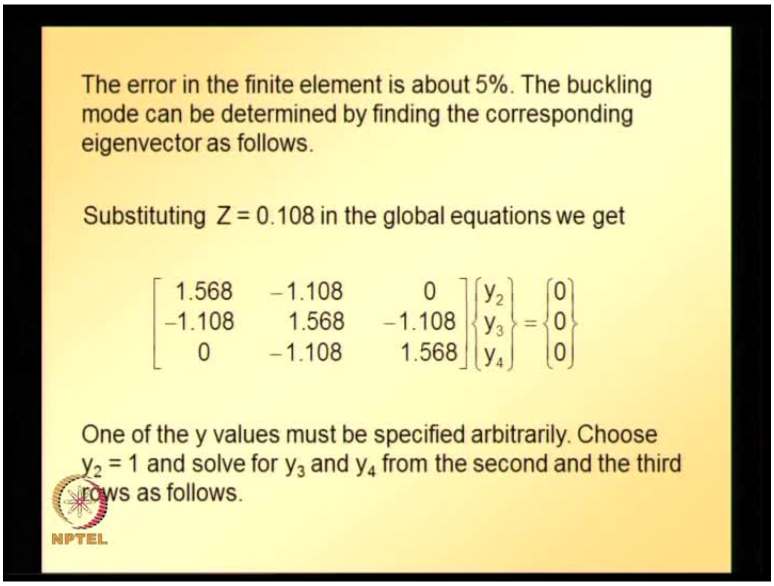


The lowest root corresponds to the first buckling mode of the column. Thus buckling load for the column is given by equating the lowest Eigen value to PL square over 96 EI and back calculating what P is, which is going to be the critical load at which column is going to buckle first.

And if you compare this with the exact solution for this particular problem, which is same as for which is same as Euler buckling load for pin pin conditions of a column. So, the exact solution for this problem is given by P critical is equal to pi square EI over L square and if you calculate what is this pi square? It is going to be 9.87 P i over L square.

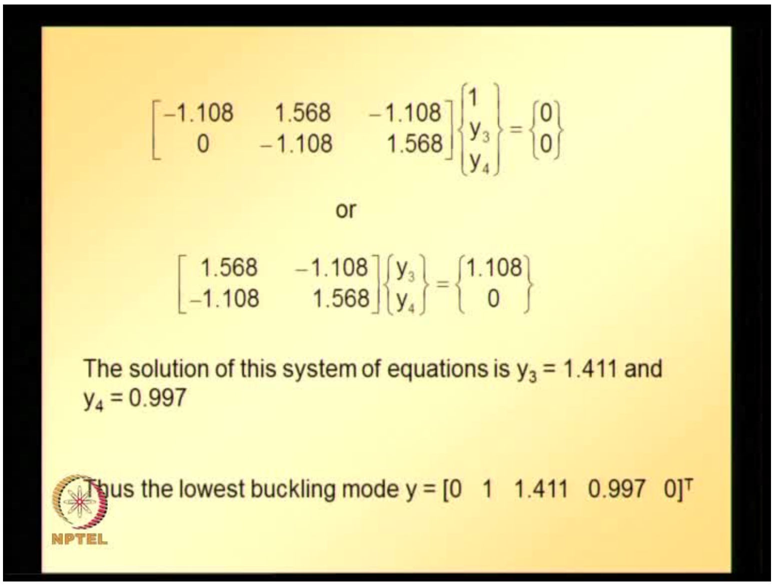
So, this is the critical theoretical value, critical load or theoretical value of critical load for this particular column with the end conditions as pin pin is given by this one and whereas, when we apply finite element method with 4 elements 4 linear elements. We obtained P critical as10.37 EI over L squared and if you find error, error infinite element solution that we obtained is about 5 percent.

(Refer Slide Time: 46:16)



Error infinite element solution is about 5 percent the buckling mode can be obtain can be determined by finding the corresponding Eigen vectors. So, we got the Eigen values by making substitution of each Eigen value back into the equation. We can determine what the corresponding Eigen vectors are, substituting Z is equal to 0.108 in the global equations we get. And again, when we are solving for Eigenvectors one of the y values must be specified arbitrarily. So, we choose y 2 is equal to 1 and solve for y 3 and y 4 from the second and third rows.

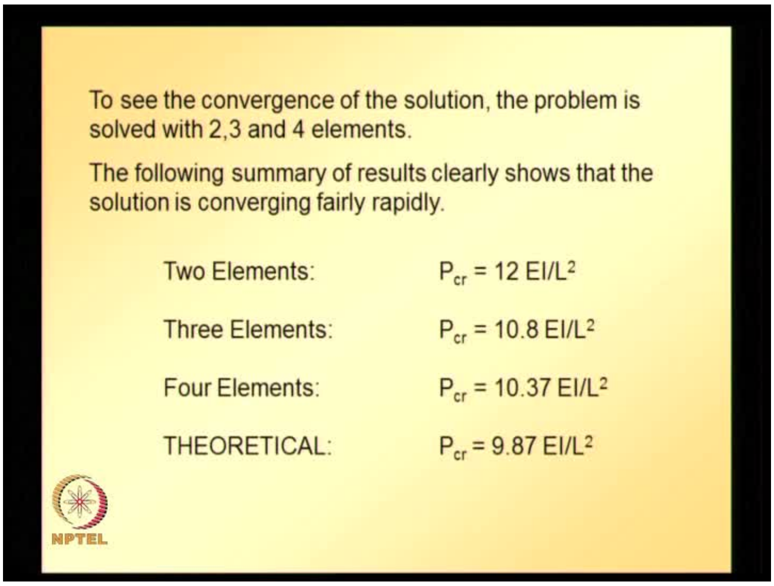
(Refer Slide Time: 47:14)



And when we do that we get this equation by setting y 2 is equal to 0, and eliminating or removing first equation we can use the other two equations to solve for y 3 and y 4. Solution of this system of equations is y 3 is equal to this and y 4 is equal to this one.

So, the lowest buckling mode by putting all the values y 1, y 2, y 3, y 4 and y 5 values in a vector we get the first or the lowest buckling mode vector like this. And similar calculations can be repeated for other Eigen values, but since we are interest in critical load we are looking at only the lowest Eigen value and corresponding buckling mode.

(Refer Slide Time: 48:45)



So, now, if we repeat this problem using 2 elements, 3 elements and 4 elements and this is how convergence looks like. To see the convergence of the solution the problem is solved with 2, 3 and 4 elements, the following summary of results clearly shows that solution is converging fairly, rapidly.

And if you instead of 4 elements, if 2 elements are taken P critical are 12 EI over L square that is what we obtained using two linear finite elements for this problem. If you adapt three linear elements, we get P critical as 10.8 EI over L square 4 elements as the solution finite element solution gives as P critical as 10.37 EI over L square, whereas a three theoretical value is P critical is equal to pi square P i over L square which is going to be 9.87 EI over L square.

So, we can see the solution is converging fairly well and here, if you want more closer to 9.87 EI over L square, we can try this problem using five linear elements or we can adopt higher order elements; instead of linear element we can adopt quadratic elements, that is, instead of two node elements we can go for three node elements. So, in the next class, we will be looking at higher order elements.