Finite Element Analysis
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Module No. # 01
Lecture No. # 13

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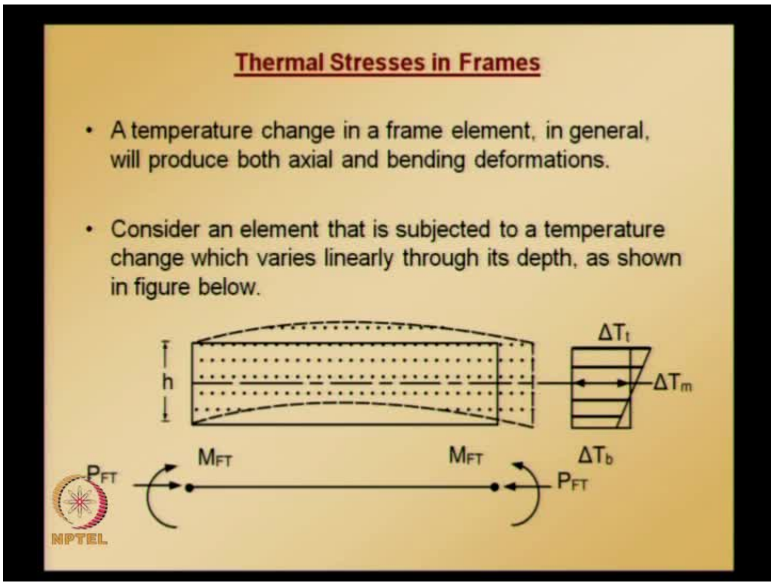

In the last class, we have seen how to solve 2D plane frames. Basically, a plane frame or even space frame is a combination of several effects, like, if it is a 2D plane frame, it is a combination of axial force effects and bending effects; and will see later if it is a 3D space frame, even torsional effects will come into picture. So, when we are taking or solving a plane frame problem assuming small deformation conditions and assuming all these effects to be uncoupled. What we did in the last class is we derived element equations and also we looked at a problem - how to solve a 2D plane frame problem subjected to some distributed load in the last class.

In today's class, will see thermal stresses in frames - how to analyze the stresses and displacements due to temperature changes; and, we have seen a similar kind of thing when we are actually solving beam problems - beam bending problems - subjected to thermal stresses due to temperature change. We have seen that a beam element because of temperature change will be subjected to curvature, and because of that it will be subjected to bending moment, which is uniform, that is, which is constant along the a length of the beam; and, these thermal stresses will not produce any shear forces.

Here, in frames in addition to this bending moment - again no shear forces will be there even in frames. If it is a plane frame we are going to get - because of these thermal temperature changes we are going to even have the axial force that is what we are not going to see in today’s class.

So, temperature change in a frame element in general will produce both axial and bending deformations. Now, let us consider an element that is subjected to temperature change that varies linearly through its depth as shown in figure below.

Similar kind of thing we have done even when we are looking at a thermal stresses in beams. So, this is a frame element of depth h subjected to some temperature change; the top surface temperature change is a delta capital T subscript small t and change in the surface temperature - bottom surface - is delta capital T subscript p and delta T m is the mean temperature, that is, average of top surface temperature and bottom surface temperature.

Because of these temperature changes - top surface and bottom surfaces - we expect the element to experience some curvature; because of curvature, moment will develop. And, in addition to that, if these ends are constrained - the ends of the frame element are constrained - for any moment or they are totally fixed, in that case, fixed end moments will be developed in the direction which is shown in the figure; and, in addition to that, in frame element axial forces will be developed at the ends, because if both ends are constrained reactions will be there.

The direction in which these axial forces act also is shown in the figure. Please note that the deformation shown in the figure is for the condition that the top surface temperature is higher than bottom surface temperature.

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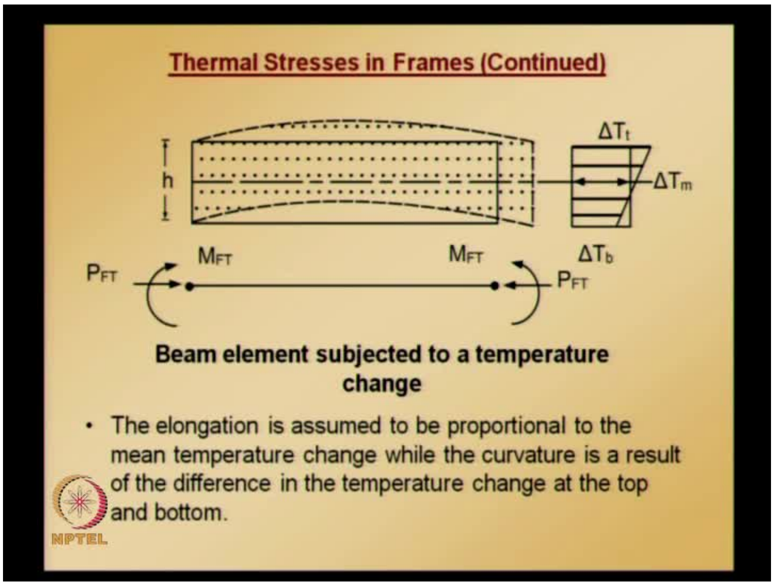

The elongation is assumed to be proportional to mean temperature change while curvature is result of difference in temperature change at the top and at the bottom. So, elongation - how to find elongation of this element when it is subjected to temperature change? You already know that from your mechanics of material background.

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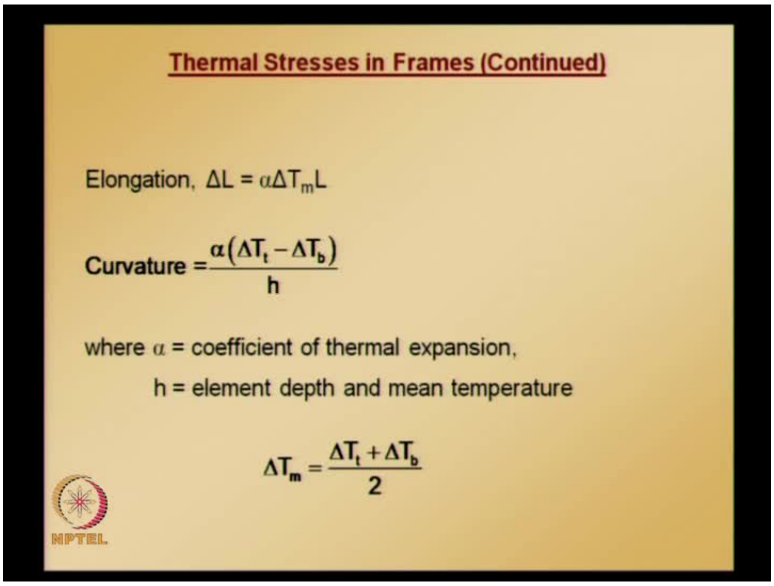

Elongation as given by alpha delta T m L - where L is length of frame element; curvature is given by this; this formula is a familiar to you - it is the same as what we use for beam element; and, here in these equations for elongation and curvature alpha s square into thermal expansion, h is element depth and delta T m is the mean temperature, which is an average of top surface temperature and bottom surface temperatures, and these are the elongations and curvature.

We know that once we know elongation we can find what is strain and once we know strain we can find what is the axial force.

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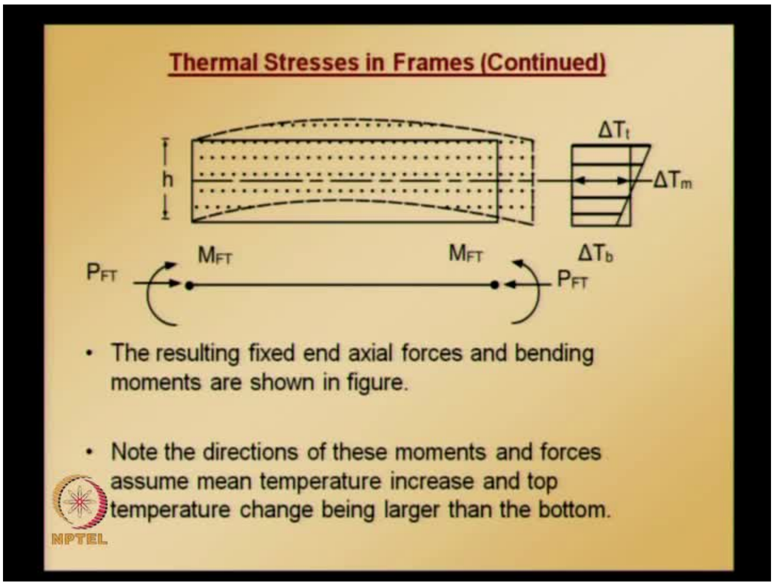

Similarly, we know what the relationship is between bending moment and curvature; since we know curvature we can find what is bending moment. So, the resultant - the resulting - fixed end axial forces and bending moments - this figure you have already seen it has been shown here - the resulting fixed and axial forces and bending moments.

And, the direction of these moments and forces assume mean temperature increase, and top temperature change being larger than bottom surface temperature.

So, suppose if top surface temperature is less than bottom surface temperature, then the direction in which fixed end moment is going to act is in the opposite direction to the arrows indicated there; similarly, if the average temperature is, in total, the average of top surface and bottom surface put together, if it increases then the axial forces will be acting in the direction shown in the figure; otherwise, the direction will be in the opposite; so, this is what is pointed out there in the slide.

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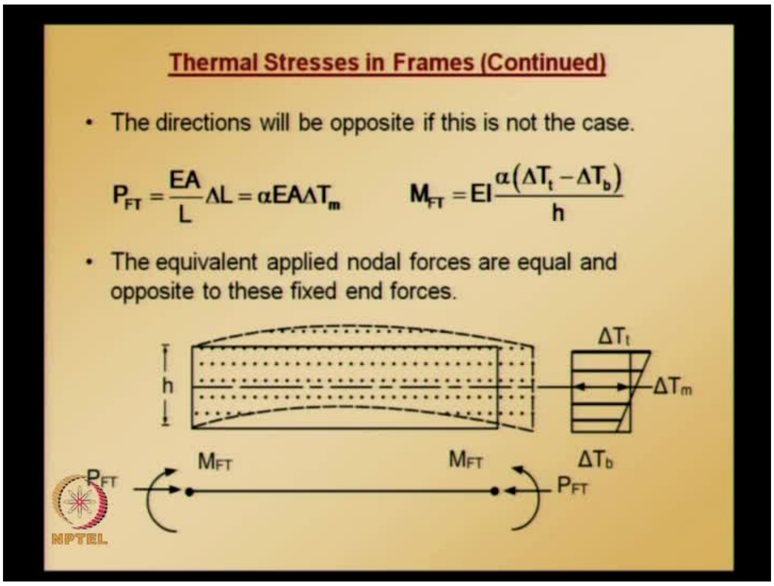

The direction will be opposite if this is not the case; now, let us see what are these values and how we get - in the figure we have indicated or we have seen how the force - these axial forces - and bending moments are acting - fixed and bending moments. But, how to compute those from the elongation and the curvature that we have? This is - you know that strain times Young's modulus gives you stress times area of cross section, gives you axial force; so, that is how P FT can be calculated which is given there.

We know that bending moment is related to curvature, through this relation using modulus of flexural rigidity EI, and once we have this equivalent applied nodal forces are equal and opposite in direction to these fixed end forces; we can easily assemble what is the equivalent nodal force vector.

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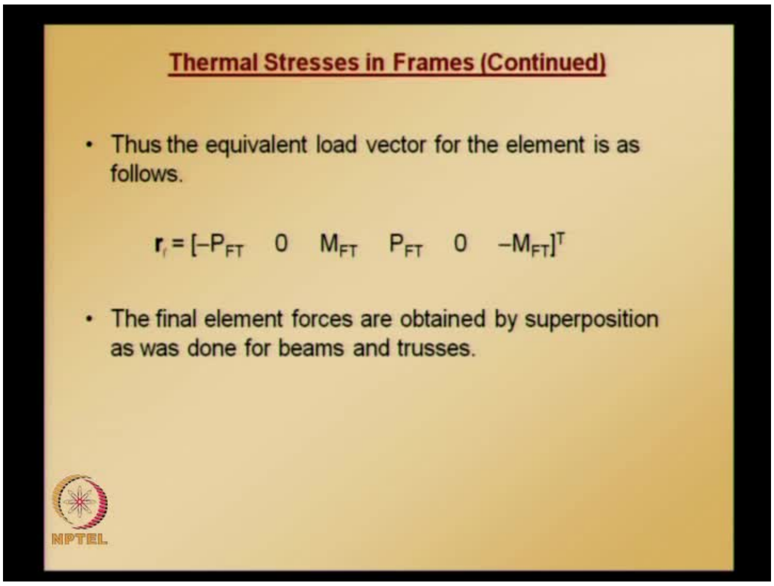

This equivalent nodal force vector for element is given by this; so, for a problem which is subjected to temperature changes in frames, what you need to do is we need to - rest of the procedure for assembling the element equations corresponding to the stiffness part is similar to what we have seen in the last class; that is, using the combination of axial effects and bending effects we can get the stiffness part; only difference is that if there is a temperature change then this is how equivalent load vector can be assembled.

Final element forces can be obtained once we solve for the nodal values using regular methods like - because of these temperature changes the forces that are developed are uniform throughout the length of the frame element; we need to apply so preposition method.

All these details will be clear if we solve a problem; so, final element forces are obtained by superposition as was done for beams and trusses.

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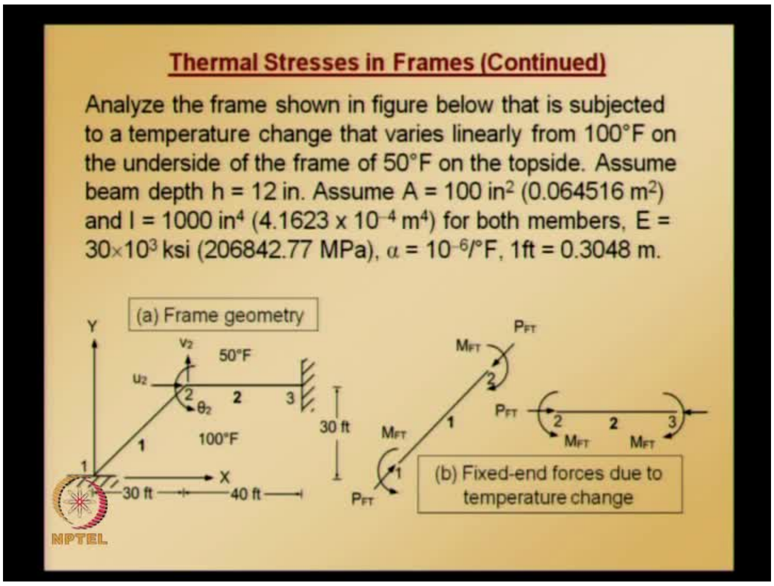

Now, let us take an example; basically, this example is what you have already seen when we were solving 2D plane frames; only thing is earlier it was subjected to uniformly distributed load, here the same structure is subjected to temperature changes.

Analyze the frame shown in figure below that is subjected to a temperature change that varies linearly from 100 Fahrenheit on the underside of the frame to 50 Fahrenheit on the top side.

Assume a beam depth and also cross sectional area and moment of inertia and Young's modulus value, coefficient of thermal expansion - all these values are given both end FPS units and SI units; as you can see, these values are round numbers in FPS units whereas in SI units there all having some decimal places.

It is the convenient to work in FPS units, so let us work this problem out in FPS units. The frame is shown in the first figure and the node numbers and element numbers are shown, as you can see from the figure that node 1 and node 3 are constrained.

All the degrees of freedom at node 1 and node 3 are 0; the second figure shows the fixed end forces due to temperature change. This is different from what we have seen when we are looking at the concept just few minutes back; there, the top surface temperature is assumed to be higher than bottom surface temperature, here the top surface temperature is less than bottom surface temperature; so, all the fixed end forces, that is - and overall temperature change rises; so, the axial forces will be in the same direction as we have seen when we were looking at this concept.

But, the bending forces or the fixed end bending moments will be opposite direction; because, now the bottom side temperature is higher than top side temperature; so, all the fixed end forces due to this temperature change - the direction in which they will be acting are shown in figure b.

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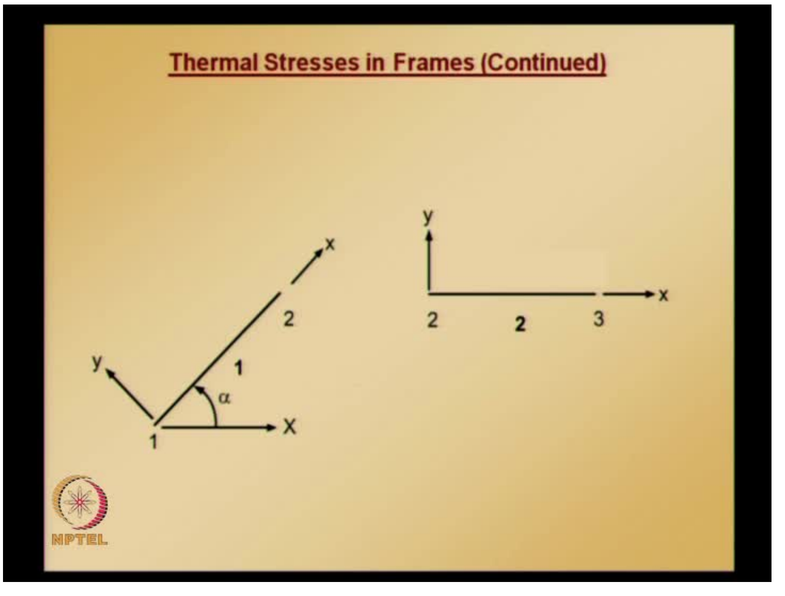

Now, by choosing arbitrarily, element 1 to go from node 1 to node 2 and choosing arbitrarily element 2 to go from node 2 to node 3; here, the local x axis and y axis are shown and also alpha is the angle between global x axis and local x axis measured in the counter clockwise direction that is also shown; alpha value can be calculated from the dimensions of the plane frame given; and you can see that element 2 is oriented along in the same direction as a global x axis.

The angle between local and global x axis is 0; so, in this - for second element - the transformation matrix is going to be an identity matrix and whatever element quantities, that is, the element stiffness matrix in the local coordinate system will be same as in the global coordinate system; and also, the local the force vector calculated in local coordinate system for element 2 will be same as in the global coordinate system.

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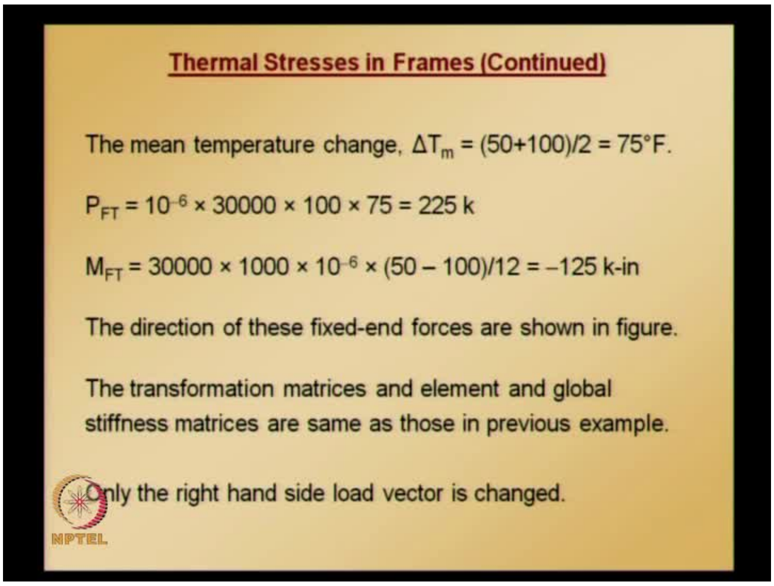

Now, let us note down something; what is the mean temperature? This information is given - top side and bottom side temperature different change; so, using those we can calculate what the mean temperature change is; and, once we have this we can calculate what is P FT; please note, that this P FT it is not dependent on length of the element; P FT value is going to be the same for the both element 1 and element 2.

Here, the calculation for fixed end moment is also given, and if you carefully observe even fixed end moment equation there is no length quantity when you are calculating; basically - so fixed end axial forces and fixed end axial moment are independent of length of a element.

So, these values remain same for both element 1 and element 2; using these values we can get the equivalent load vector. Direction of these fixed end forces are already shown in the figure - the transformation matrices and element global stiffness matrices are same as those in element stiffness matrix and local coordinate system and global coordinate system are same as those what you have already got when we were solving this problem subjected to uniformly distributed load in the last class.

But, for completeness let us repeat those things again once again here. Now, let us start doing that and also note that only the right hand side load vector is going to be different, because now it is subjected to temperature changes.

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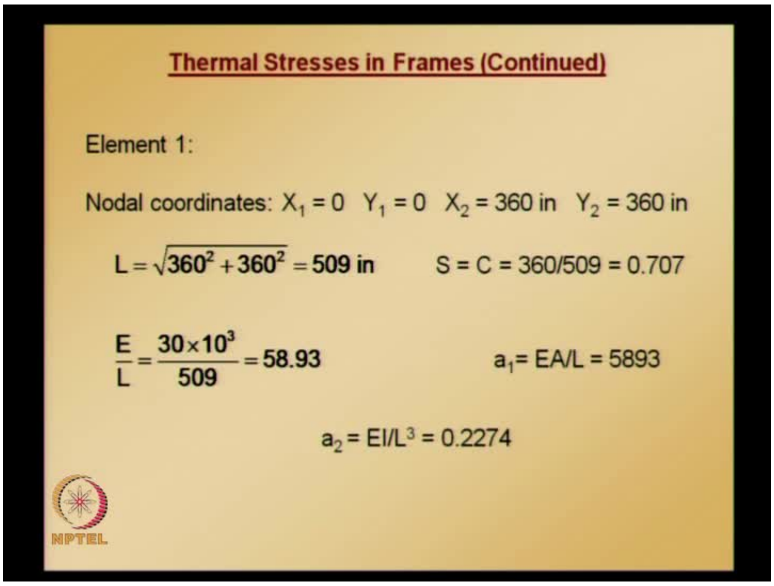

So, element 1 how can we calculate element equations the local coordinate system? We need to keep a note of what the nodal coordinates are. Once we know the nodal coordinates, we can calculate length of this element and also direction cosines - once we have these we can calculate what is the transformation matrix.

Also we require some coefficients for stiffness matrix calculation, and please note that there is no load - distributed load - that is, uniformly distributed load applied on this member unlike in the previous example - only thing is now it is subjected to temperature change; so, these are the coefficients that are required for calculating the stiffness element stiffness matrix in the local coordinate system.

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Thermal Stresses in Frames (Continued)

$$\mathbf{k}_\ell = \begin{bmatrix} 5893.0 & 0. & 0. & -5893.0 & 0. & 0. \\ 0. & 2.728 & 694.4 & 0. & -2.728 & 694.4 \\ 0. & 694.4 & 2.357\times10^5 & 0. & -694.4 & 1.179\times10^5 \\ -5893.0 & 0. & 0. & 5893.0 & 0. & 0. \\ 0. & -2.728 & -694.4 & 0. & 2.728 & -694.4 \\ 0. & 694.4 & 1.179\times10^5 & 0. & -694.4 & 2.357\times10^5 \end{bmatrix}$$

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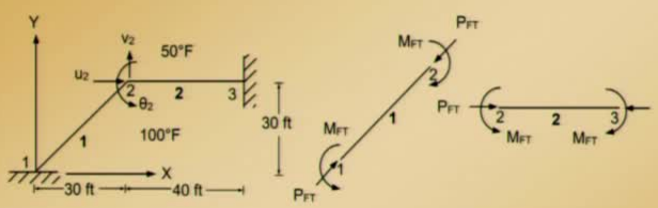
Thermal Stresses in Frames (Continued)



Equivalent load (directions opposite to those shown in figure).

r_ℓ = [–225   0   –125   255   0   125]^T

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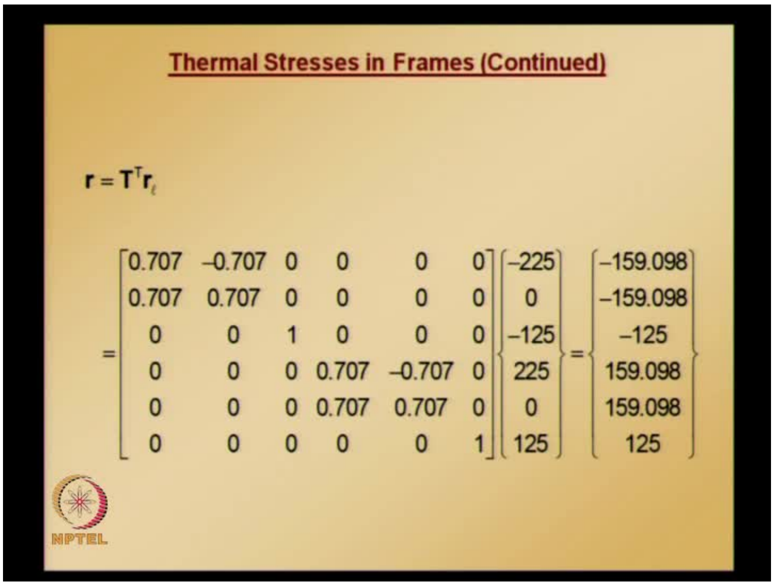

Using these values we can calculate what the transformation matrix is, and we can also calculate what the stiffness matrix in the local coordinate system is; equivalent load vector using the values of P FT and M FT that we calculated we can get this equivalent load vector. This is in the local coordinate system and the equivalent load vector in the global coordinate system can be obtained using transformation matrix.

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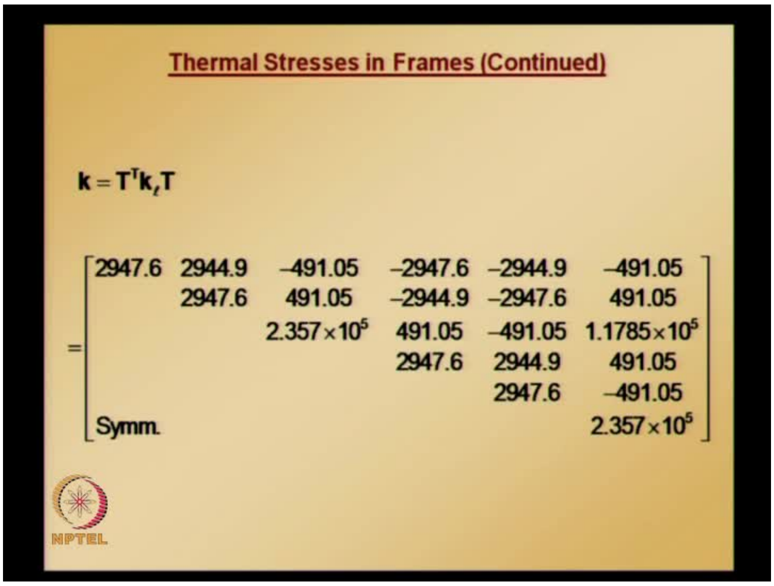

Similarly, element stiffness matrix in the local coordinate system - we can convert into the global coordinate system using the transformation matrix using this equation; only the upper triangular portion of this stiffness matrix is shown and lower triangular portion is similar to the upper triangular portion.

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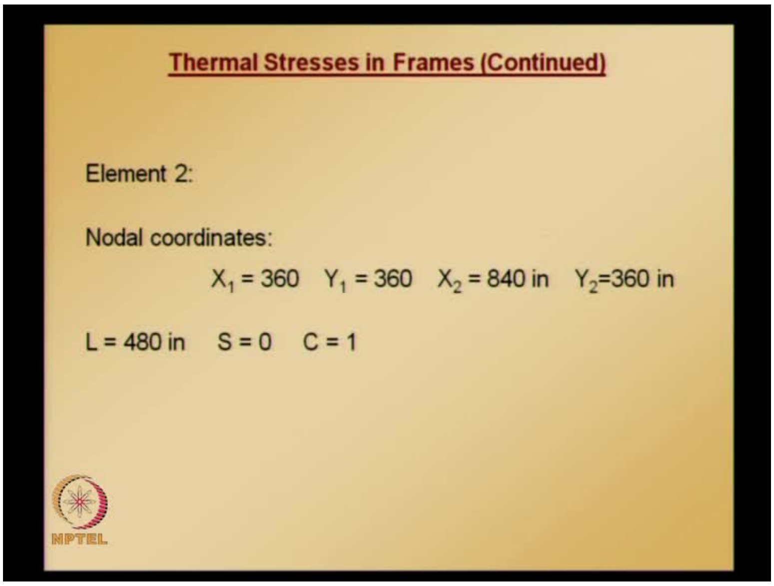

Similar calculations we need to repeat for element 2 - nodal coordinates for element 2; length, direction cosines and these are the coefficients that are required for assembling stiffness matrix.

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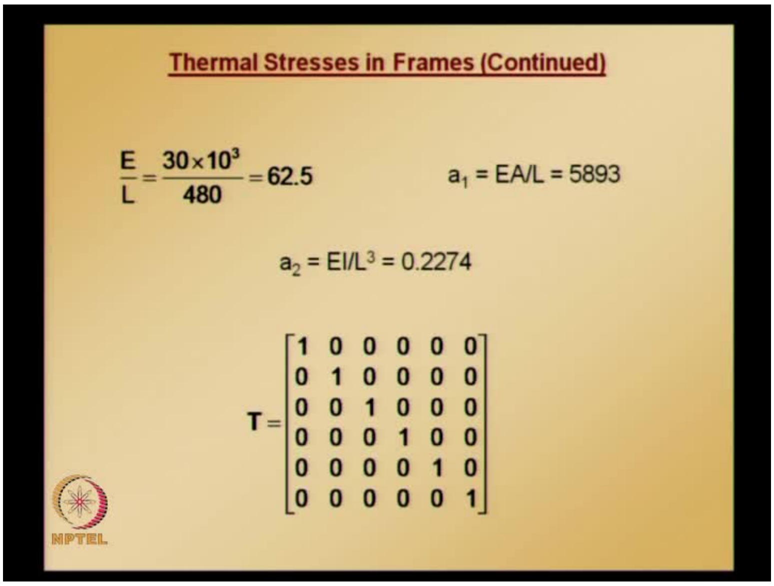

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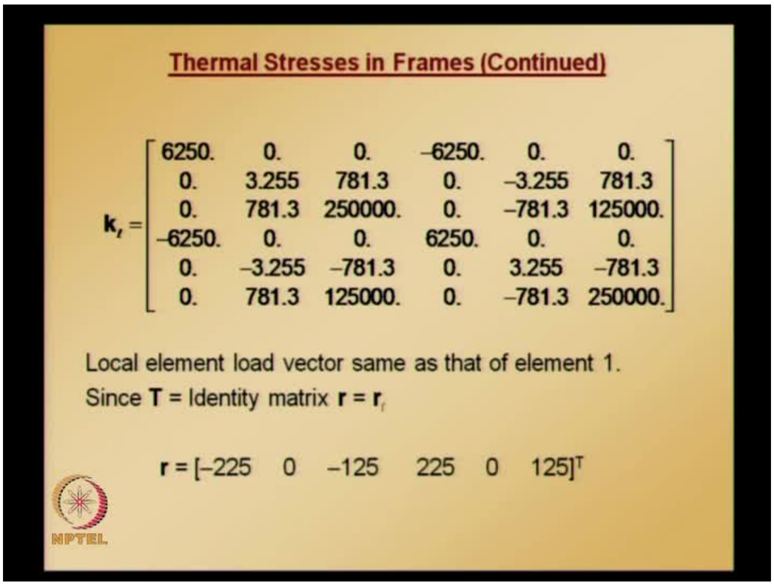

Using the direction cosines, this is what is the transformation matrix that we get. Please note that the load vector is same for both element 1 and element 2; using the coefficients for stiffness matrix - this is the element stiffness matrix in the local coordinate system.

This is the load vector and since the transformation matrix is identity matrix, local stiffness matrix - stiffness matrix in the local coordinate system - will be the same as stiffness matrix in the global coordinate system.

Similarly, load vector - element load vector - in the local coordinate system will be same as in the global coordinate system; now, we have got all the information. And also note that this node numbering for this problem - only node 2 is not constraint, node 1 node 3 are constraint and element 1 is assume to go from node 1 to node 2 and element to go from node 2 to node 3.

If you see the - imagine the final global equations - the global equation system - it is going to be 9 by 9 equation system; so, instead of assembling all the entire global equation system we can a smartly assemble or directly get the reduced equation system; and, the contribution from reduced stiffness matrix comes from the bottom or the lower quadrant of the element 1 stiffness matrix. To these reduced equations we get - or reduced stiffness matrix - we get contribution from upper quadrant of the element 2.

Because of the boundary condition all the degrees of freedom at node 1 and node 3 are 0 there is no need to assemble corresponding rows and columns; therefore, since the specified values are 0 the corresponding columns will not contribute anything either; thus, only in the following global equations only the terms associated with node 2 are written.

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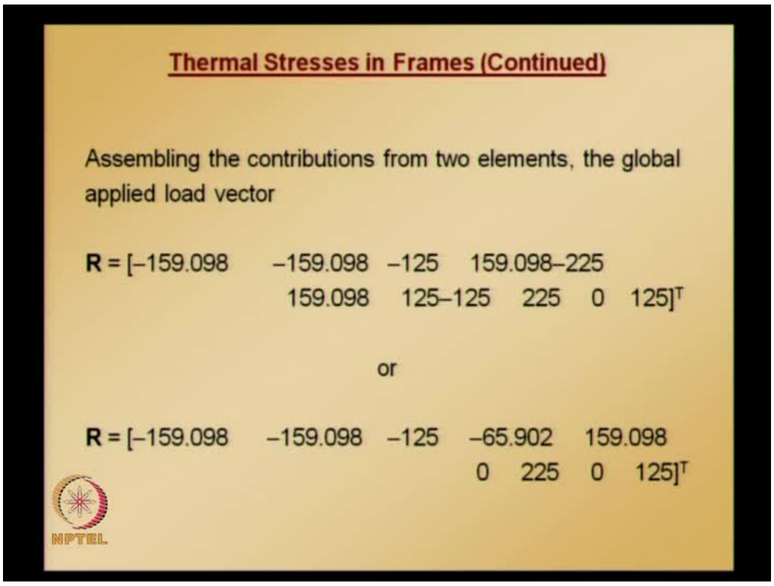

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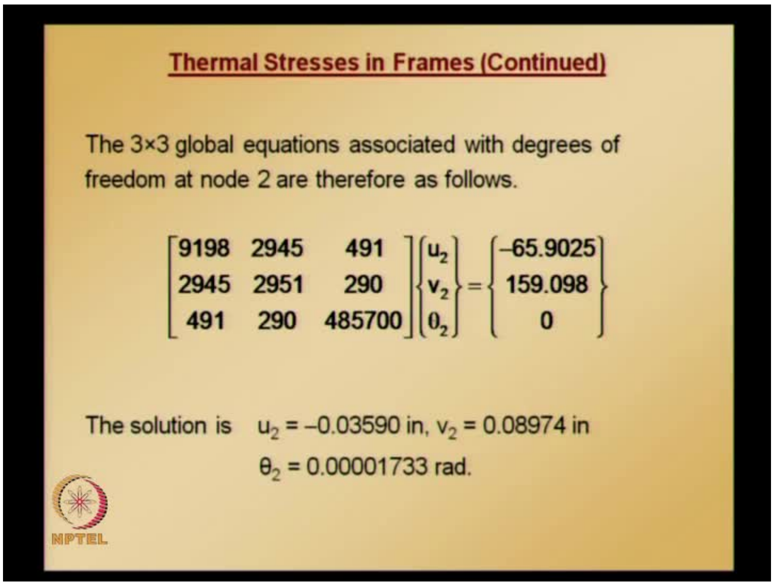

This is a assembles for a load vector assembling contribution from two elements; so, global applied load vector is this and this can be simplified and we get this contribution from both elements to the global load vector; the 3 by 3 global equations associated with degrees of freedom at node 2 are therefore as follows: so, in the global load vector that we got, whatever is there - the value at locations 4 5 6 - that is what is taken as the load vector for the reduced equations; and, the stiffness part is obtained by adding the lower quadrant of element 1 stiffness matrix and the upper quadrant of element 1 stiffness matrix; because, that is what is the contribution that goes into the locations of 4 5 6 rows and columns in the global equation system; so, using these things we get this reduced equation system which we can solve for u 2 v 2 and theta 2 which are the unknowns at node 2.

So, these are the values; now, we have got all the nodal values; so, the element forces can easily be obtained by, first determining the local displacements and then using the shape functions and their derivatives.

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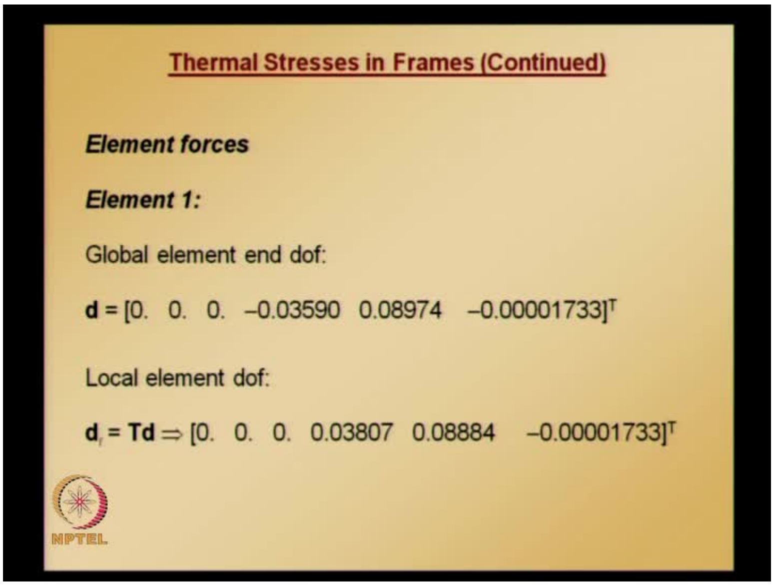

So, for element 1 the forces - element forces - this is the global element degrees of freedom; but, we want to calculate the element forces using shape functions and their derivatives; we need local degrees of freedom, so we can use transformation matrix and get the local corresponding local degrees of freedom.

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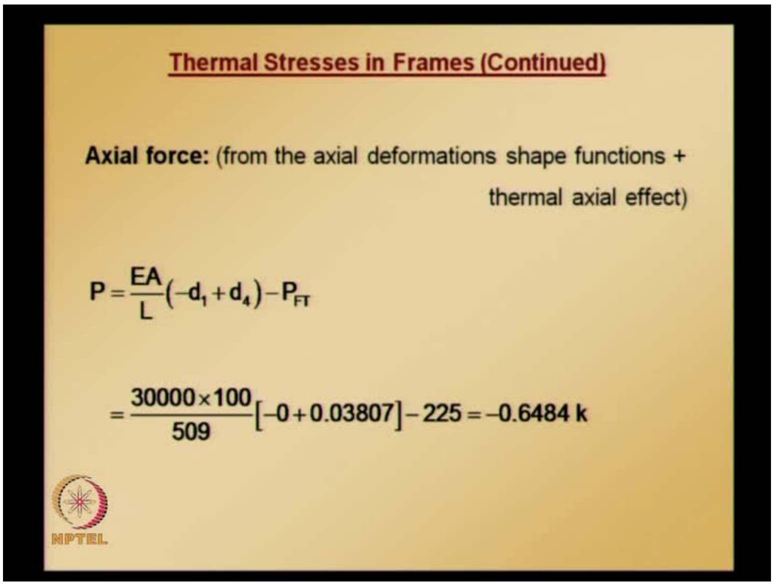

Once we get this d vector we can calculate axial effects, we can calculate using the components in the d vector, and the finite element shape functions; but, since the axial force is distributed or it is constant over the length of the element we need to apply fixed end correction - that is, thermal axial effect needs to be also taken into account when we are calculating axial force; so, using this formula we get axial force in element 1.

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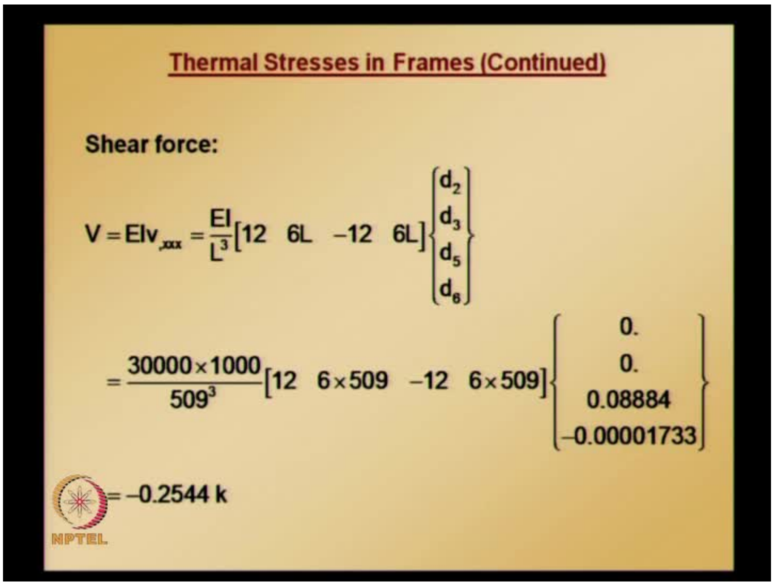

Similarly, we can calculate rest of the other two - that is, shear force and bending moment. Shear force, as I mentioned, because of thermal effects only bending moment and axial forces will be developed which are going to be uniform or constant over the span of the element; no shear forces will be induced so no shear correction will be there - there is no fixed and shear correction for when you are calculating shear force; so, substituting all the values we get shear force with this value.

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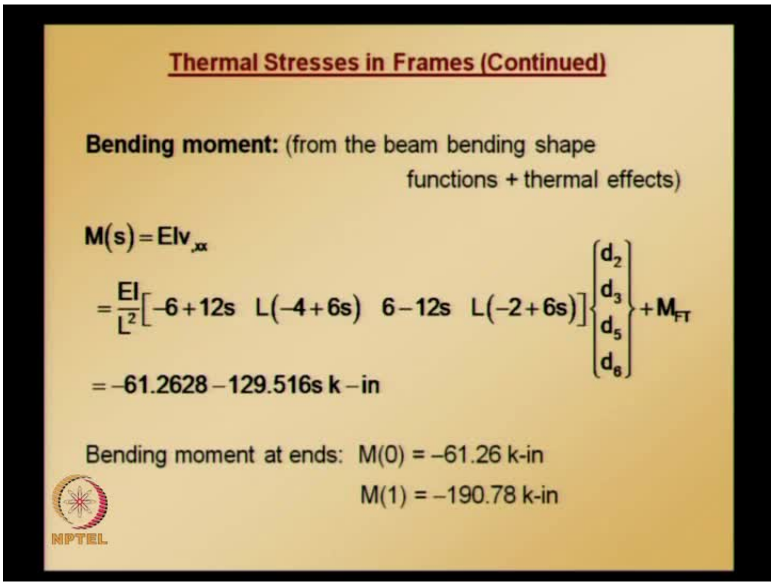

Using these sign conventions for internal moments and shear, we can draw the shear force diagram and bending moment for element 1 at any point along the length of element 1.

Again, using finite element shape functions we can calculate once we know the nodal values; but, here we need to apply fixed and correction which we already calculated - M FT. So, applying that correction we get this and this is a function of s s is equal to 0 corresponds to node 1 s is equal to 1 corresponds to node 2.

So, bending moment value at these two ends can be obtained by substituting s is equal to 0 and s is equal to 1; again, using the sign convention for internal moments we can draw the bending moment diagram for element 1.

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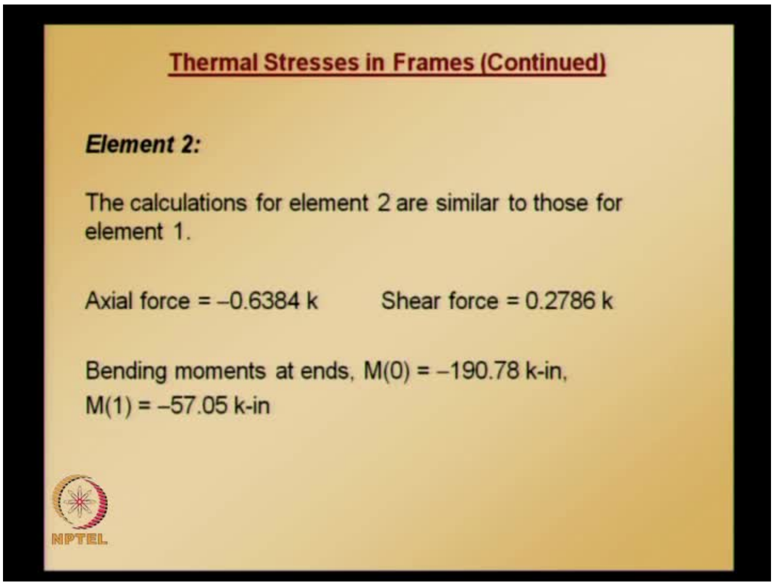

Similar calculations we can also repeat for element 2 - calculations for element 2 are similar to those for element 1 - the details are not given here explicitly, but the final axial force value shear force value and bending moment value are given.

Here also, when you are calculating the forces - element forces - for element 2 for axial force and bending moment we need to apply the fixed end corrections; whereas, for shear force no such correction is require.

Using these values we can draw free body diagram showing bending moment, shear force, and axial forces for element 1 and element 2 and we can easily verify these two elements independently or in equilibrium and also when they are put together whether they are in equilibrium - we can do that kind of verification.

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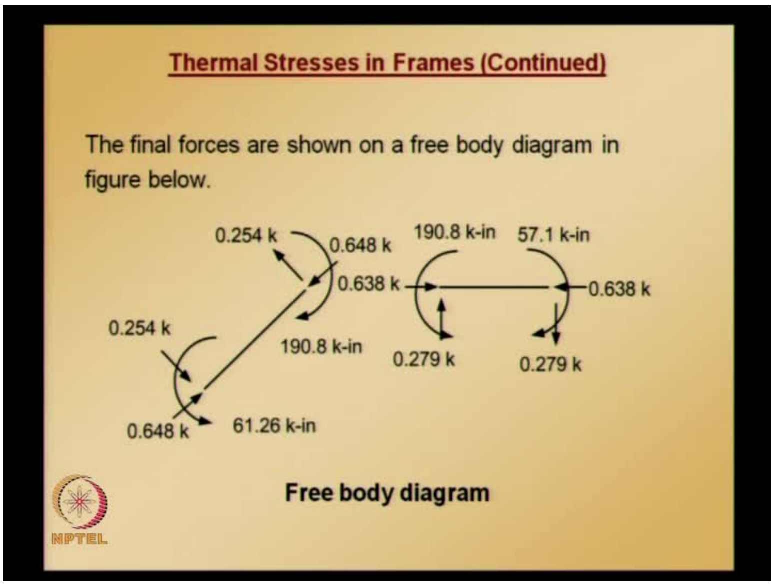

So, final forces are shown on the free body diagram and figure below. It can be easily verified that indeed each element itself is in equilibrium and also both elements put together - entire structure as in equilibrium, which is necessary for solution to be correct.

Now, will proceed; we have seen here 2D plane frames both subjected to uniformly distributed load and subjected to temperature changes and now let us move forward and let us look at 3D space frame element.

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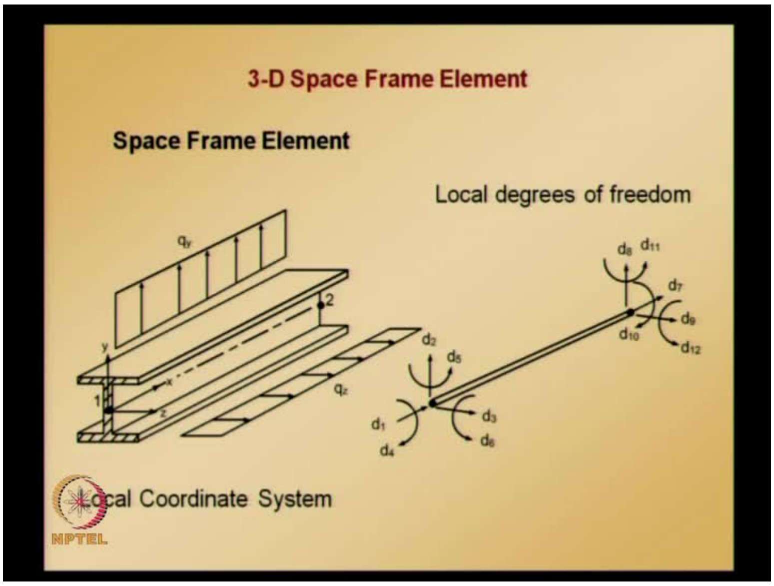

The plane frame element can be generalized to analyze space frame structures. The local coordinate system for space frame element is chosen such that member is located along local x axis - you can see in the figure the member is located along local x axis.

Principle axis of its cross section are oriented along y and z axis; local z axis is along the axis of maximum moment of inertia and local y axis is along minimum moment of inertia; element must include axial force effects and bending effects due to loads applied in x-z plane and x-y plane.

In addition, element must also be able to resist torsional forces and similar to plane frame - 2D plane frame - element case, here also we assume with in small displacement theory all these effects - that is, axial effects, bending effects and torsional effects are assume to be uncoupled; final element equations are then just combination of equations treating these effects individually.

Note that each node has 6 degrees of freedom now - three translations and three rotations and displacements and forces are positive along positive coordinate directions; positive directions for applied moments and rotations are based on right hand rule.

Here, what I mean by right hand rule is - suppose if you put your thumb pointing in the positive direction of axis and the direction in which your fingers curl indicates the positive direction for moment or the rotation; so, that is what is right hand rule.

When the thumb of your right hand is pointing towards positive coordinate direction the curl of your fingers defines positive direction for applied moments and rotations in right hand rule.

Now, the 3D space frame element we shown here in both local coordinate system, and also all the local degrees of freedom are shown, and if you see the figure d 1 d 2 d 3 or x y z displacement at node 1.

Similarly, d 7 d 8 d 9 or the x y z displacements at node 2 and d 4 d 5 d 6 are rotations about x y z axis at node 1; and d 10 d 11 d 12 are rotations about x y z axis at node 2; and q x are the q z and q y are the loads applied in x-z plane and x-y plane.

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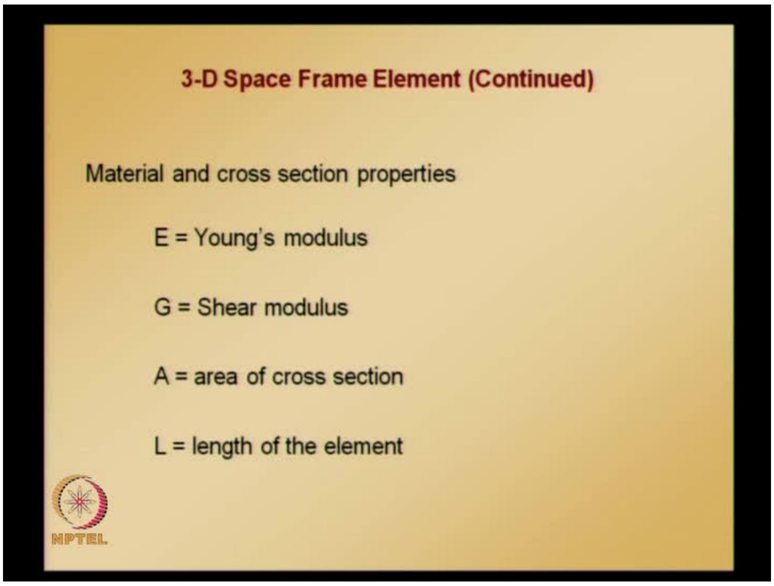

Let us see what are the notations that will be using in these 3D space frame element; material and cross sectional properties notation for that - you are familiar with this Young’s modulus denoted with E.

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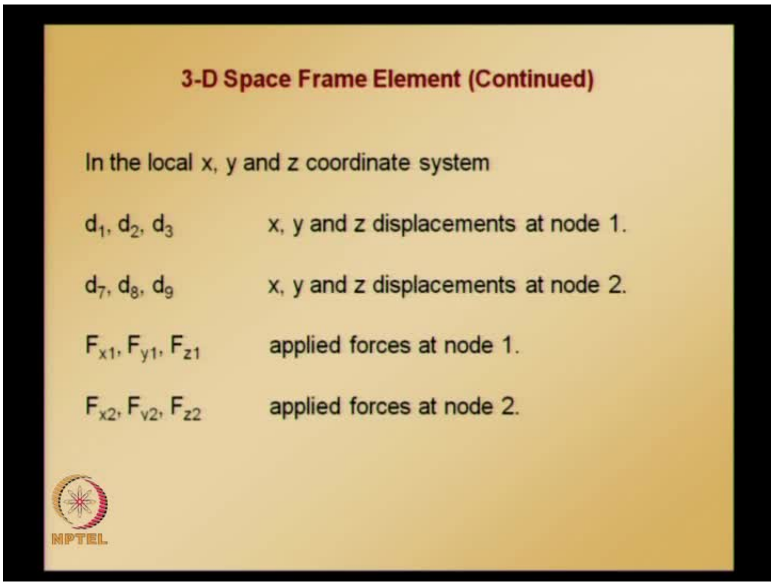

Now, one more thing is coming here - shear modulus, because torsional effects are also included for 3D space frame case and capital A is area of cross section - this also you are familiar with; length L is the length of element; and this is what I mentioned - the corresponding forces at node 1 or F x1 F y1 F z1 similarly at node 2 are F x2 F y2 F z2 in the directions x y z respectively.

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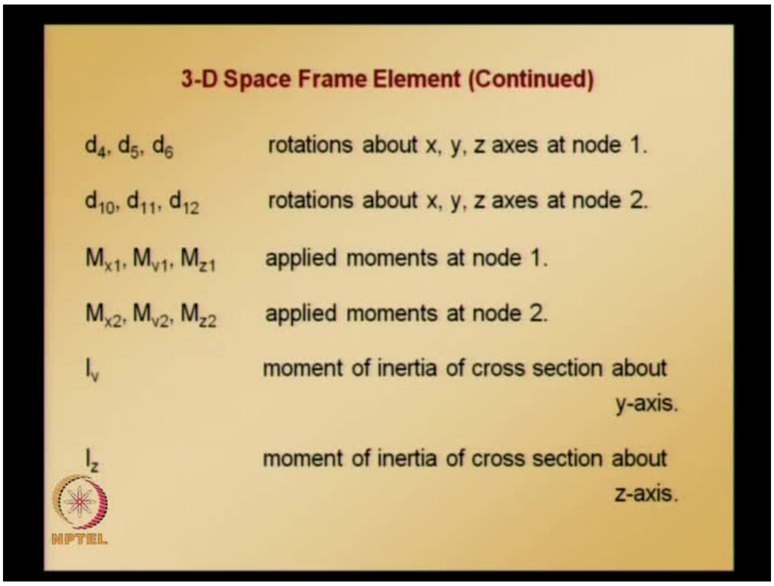

These are the rotations at node 1 node 2 and corresponding moments at node 1 node 2 are M x1 M y1 M z1 M x2 M y2 M z2; since we have bending above both about the x axis - sorry z axis - and y axis we need corresponding moments of inertia.

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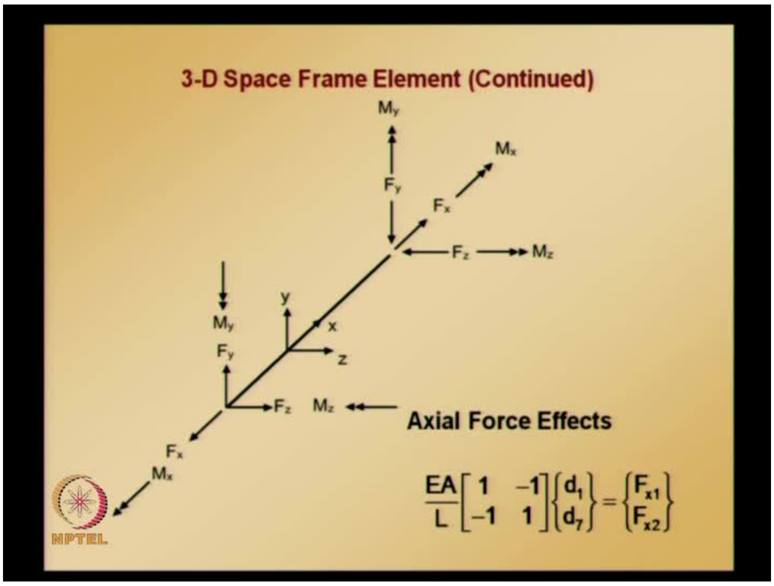

Moments of inertia of cross section about y axis and moment of inertia of cross section about z axis is that denoted with I y and I z; and here, the positive directions for internal forces and moments are shown in this figure. If you observe this figure, a double added arrow convention is used to show moments; with this convention, if thumb of right hand is pointed in the direction of the arrow, then curl of fingers shows the direction of bending moment; so, these are the positive directions - the convention for positive directions - for internal forces and moments.

Because, finally we will be using this sign convention after we calculate all the internal forces - that is, all the three component of forces at node 1 and node 2 and also all the three components of moment set node 1 and node 2.

We can use the sign conventions to draw the free body diagram and also to draw bending moment, shear force, axial force and torsional moment, such kind of things we can draw using the sign convention.

So, the positive directions for internal moments and internal shears are assumed to be in the direction, which is indicated in this figure; now, let us put together all the effects for assembling the final element equation for 3D space frame element which we will be using for solving problems of 3D space frame element.

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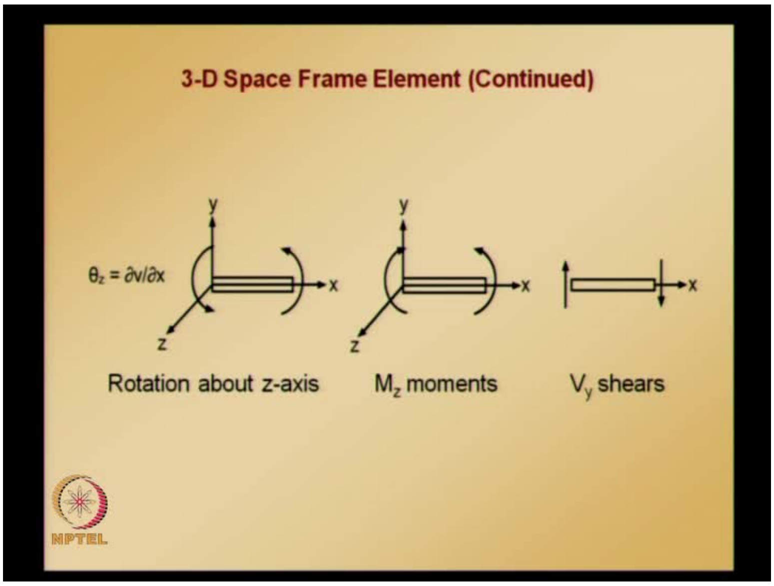

Now, let us see what are the axial force effects. Axial force effects, you are already familiar with - it is given by this equation, except that we need to include the degrees of freedom and forces corresponding to the axial effects; so, using those we get this equation; and, bending forces in x-y plane and figure illustrates the forces applied in x-y plane produce bending about z axis and using right hand rule rotation about z axis is equal to partial derivative of v with respect to x.

The situation is exactly same as that for two-dimensional beam element; so, the equations that you already have for beam bending can be directly used here or the equations that you… If you adopt the same approach as we did for developing beam element equations - if you adopt that procedure for this case - you get exactly same equations.

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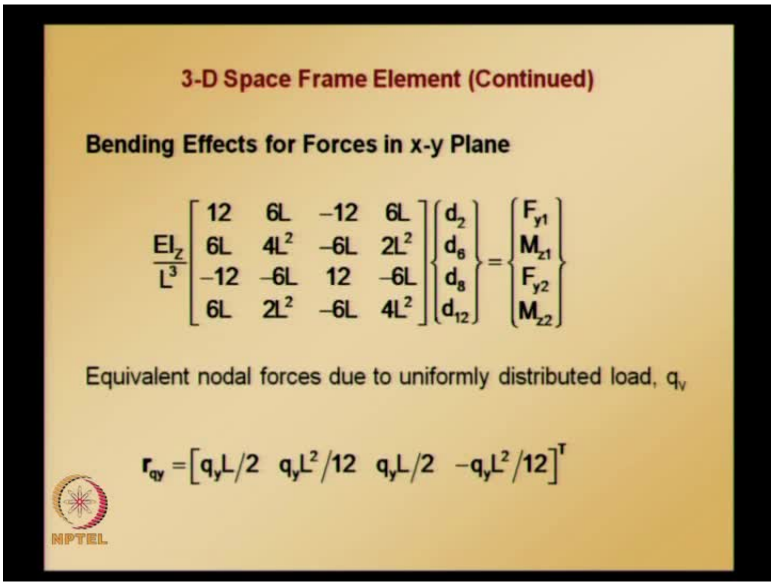

So, for bending effects that are in the x-y plane it is given by this one - this equation. Since the rotation is about z-axis we need to use corresponding moment of inertia and we are calculating flexural rigidity EI.

That is what is difference - there I z is used and in the load vector and also in the displacement vector corresponding values to the corresponding degrees of freedom values are shown in the equation; and, equivalent load vector for uniformly distributed load is given by this - this is also coming from the element equations that we developed for beam bending problem; because, the direction in which the rotation is taking place and also the direction in which the load is applied is the same as that from beam bending problem.

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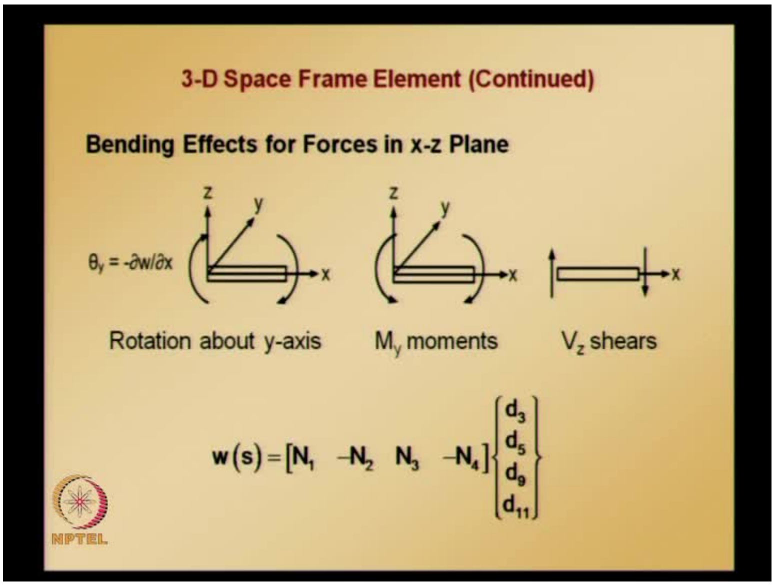

Now, this is how bending effects or bending effects in the x-y plane - because of that we can calculate the contributions. Now, bending effects for forces in x-z plane: forces applied in x-z plane produce bending about y axis using right hand rule, the clockwise rotations about y axis are positive; so, the rotation about y axis is equal to now minus partial derivative of w with respect x.

The shape functions for w are same as those for beam bending about z axis, which we have already seen; except that change in sign for rotation terms is required here; because, if you see here, our sign convention says that counter clockwise rotations are positive whereas here we have clockwise rotation - so theta z theta y sorry theta y is minus partial derivative of w with respect x.

Because of this clockwise rotation, even when we are interpolating, the shape functions for w are same as those for beam bending; but, for rotation, that is, the shape functions corresponding to partial derivative of w with respect x we need to have this negative sign.

So, this is how w can be interpolating using finite element shape functions. If you see, only difference here is N 1 and N 4 are appended with negative sign and where N 1 N 2 N 3 N 4 are same as what you have already… - when we are solving a beam bending shape functions.

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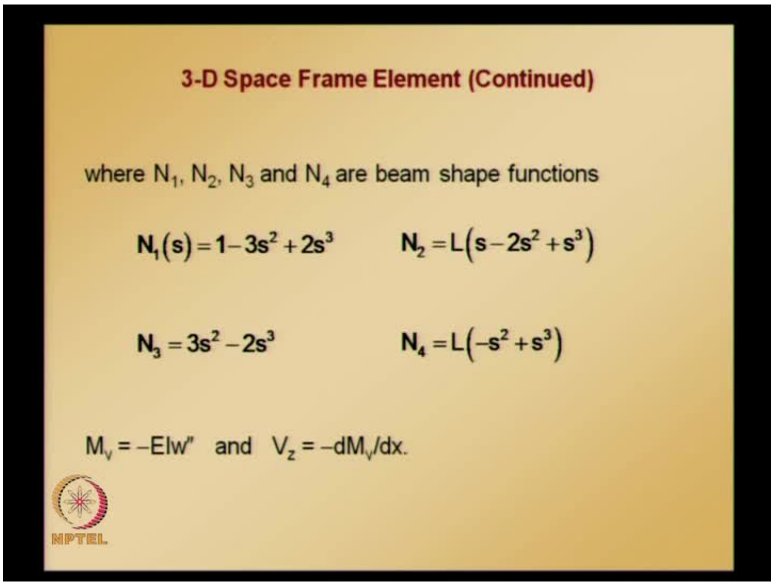

Using the approach similar to what we did when we were deriving equations for beam bending problem - it can be easily shown that bending moment is given by this, and shear force is given by this; only difference is a negative sign is getting appended and substituting shape functions into beam bending potential - beam bending potential energy functional - the following equations can be easily derived by applying the stationarity condition.

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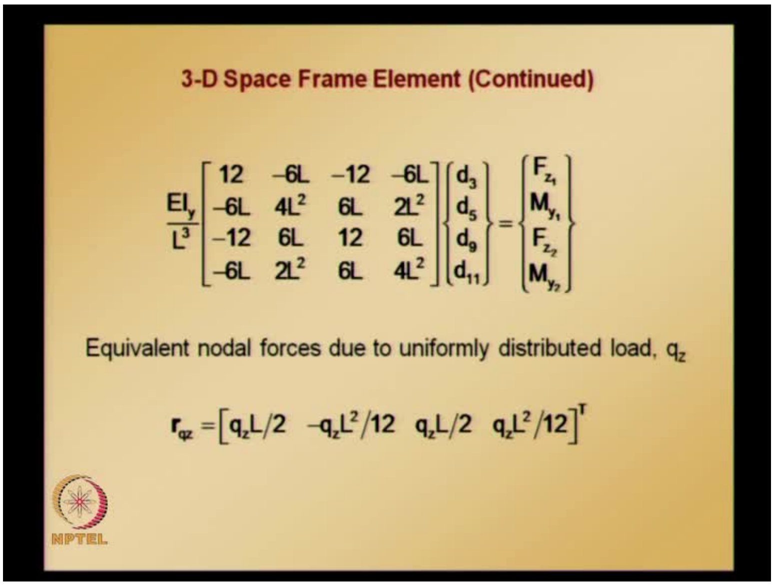

So, this equation gives us bending effects in x-z plane - because of forces in x-z plane; and, equivalent load vector due to uniformly distributed load is given by this; if it is some other load that we need to find the fixed end moments and shears and putting them together with a sign change we get the equivalent load vector similar to what we did for when we are solving beam bending case.

So, we have already noted down the equations with corresponding degrees of freedom and forces for bending about for axial effects and also bending about x-y plane and x-z plane.

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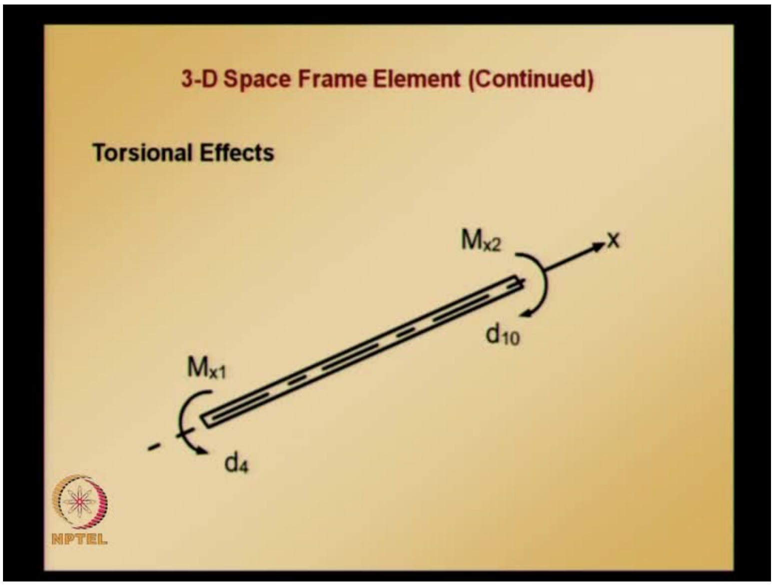

Now we are left with one more thing, which is torsional; so, torsional effects - a bar of cross section subjected to torsional moments or twisting moment at its end is shown in this figure, and the twisting we know that twisting moment per unit length is related to angle of twist through this equation.

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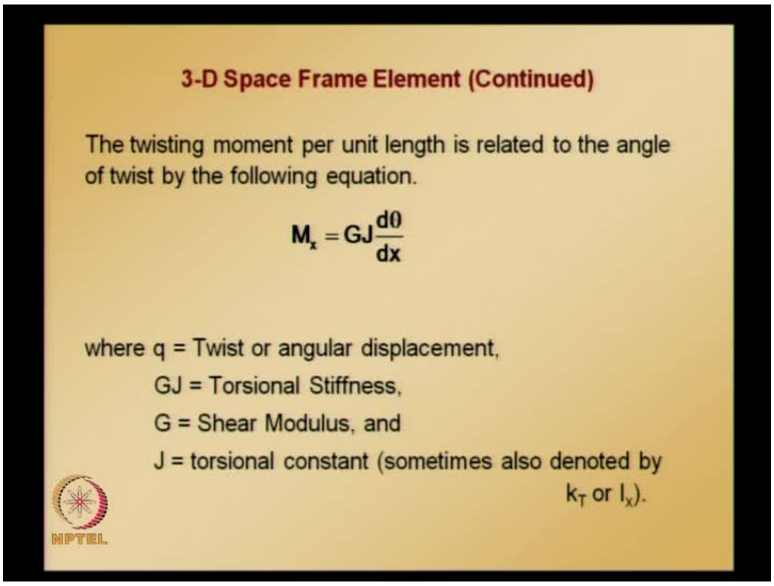

Where q is the twist or angular displacement, GJ is torsional stiffness similar to EI, EI is flexural rigidity, whereas, GJ is torsional rigidity or torisonal stiffness, and G is the shear modulus and J is torsional constant - sometimes denoted by k T or I x.

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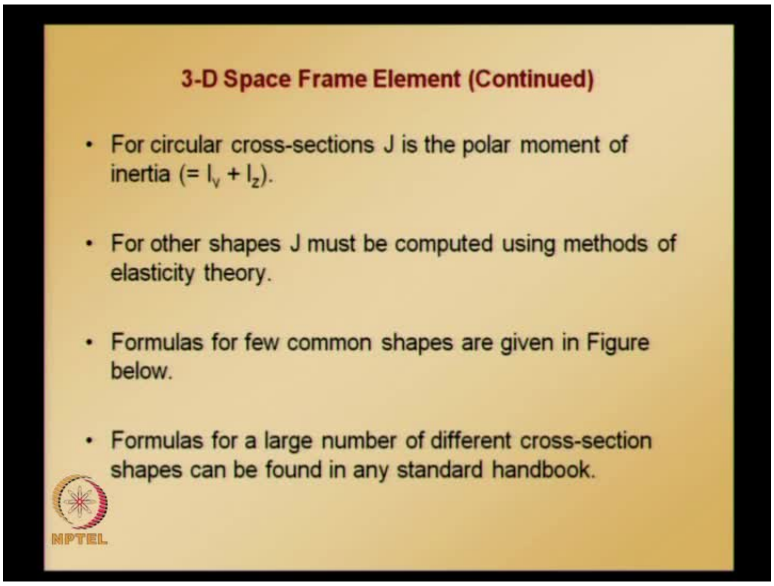

This J value for various cross sections - you can refer any standard handbook; but, for simple cross sections you must have already learnt these things in your mechanics of material class circular cross section; J is polar moment of inertia which is some of I y and I z for other shapes J must be computed using methods of elasticity theory.

Formulas for few common shapes are given below; formulas for large number of different cross section shapes can be found in any standard handbook.

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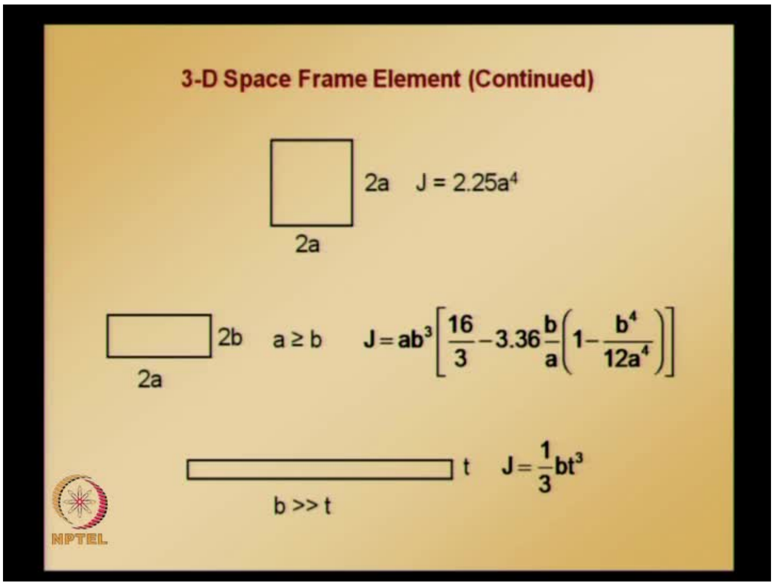

So, for a square having dimensions 2a 2a J is given by this polar moment of inertia; similarly, for a rectangular section with the dimension shown that 2a 2b where a is greater than or equal to b polar moment of inertia is given by that equation, and if you have a section in which width is much larger than thickness - J is given by this.

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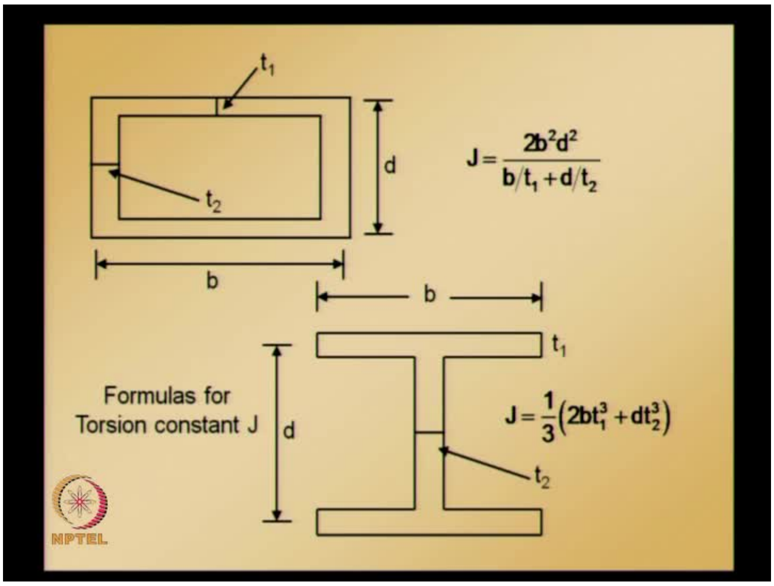

For angular rectangular section with dimensions b and d, thicknesses t 1 t 2, polar moment of inertia is given by this; and, for an I section with dimension shown there - width b depth d and thickness of flange and web t 1 t 2 J is given by this.

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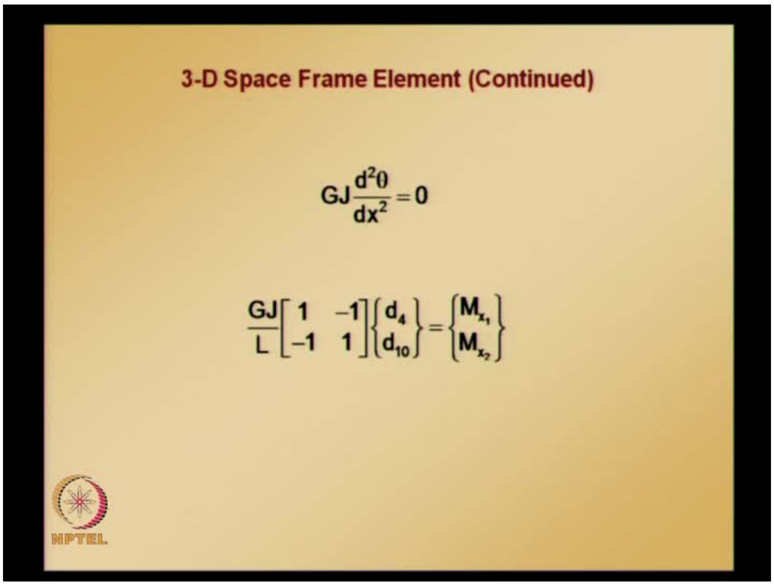

So, this is how you can get the torsional constant J; now, we need to look at the governing differential equations before we get element equations, because of the torsional effects. So, this equation, this is the governing differential equation for a bar subjected to twist; and, you can see this is a second order differential equations similar to 1 for axial deformation problem, and 0 is there on the right hand side and that is based on the assumption that there is no applied distributed twisting moment along the span of the member.

Once we have this differential equation you can draw a similarity between this and the axial deformation problem; and, from its similarity to axial deformation problem it is easy to see that linear trial solution would give the following finite element equations; so, this is the element equations to capture the torsional effects.

Now, we put together the element equations to capture axial force effects and bending effects - both in x-y plane and x-z plane. Now, we are able to derive for element equations for torsional effects, and when we are deriving all these equations we use the corresponding degrees of freedom and also corresponding forces; and, when we are starting out with this 3D space frame element the details - we made an assumption that all these effects that is axial effects bending effects and torsional effects are all uncoupled; so, combining all force 4 effects - now combining all the 4 effects - the following equations are obtained for 3 dimensional frame element in its local coordinate system.

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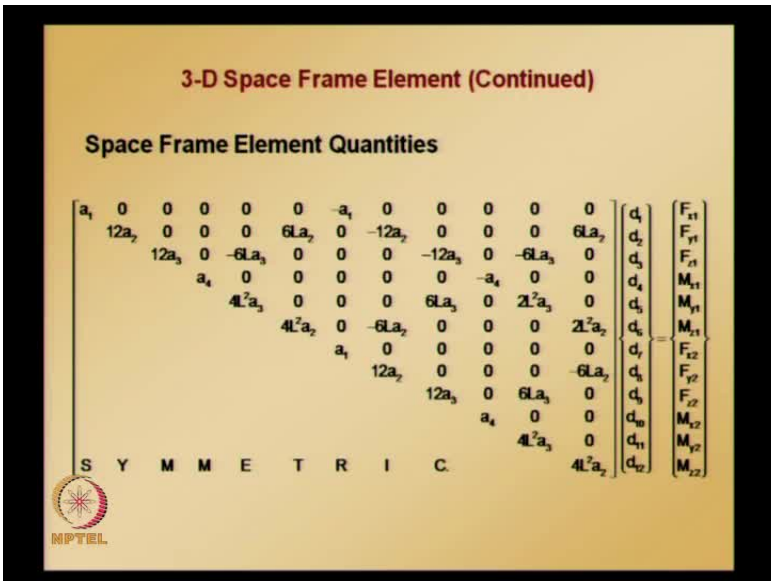

Here, if you see all the displacement vector having all the degrees of freedom from all kinds of effects - all the 4 effects - and also the force vector has the corresponding forces due to all effects; and, this is similar to 2D plane frame element equation, except that bending about 2 planes is considered here and torsional effects are also considered here.

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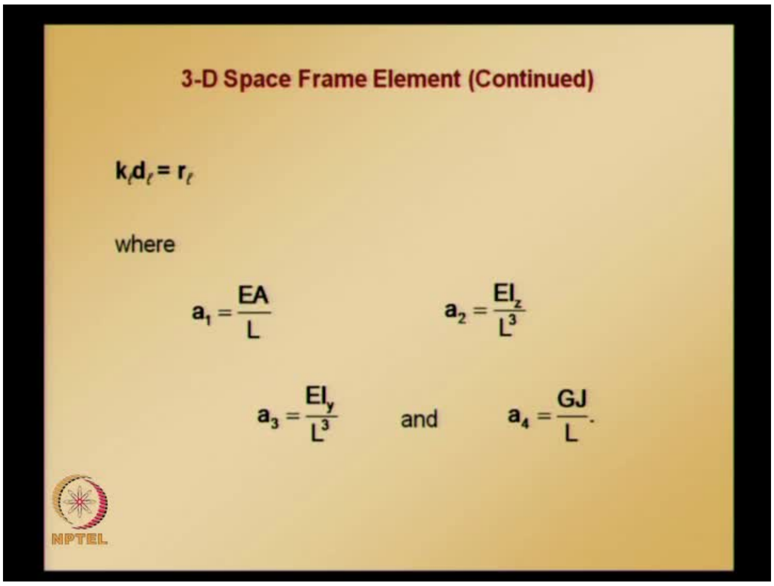

If you see here, we have some coefficients - a 1 a 2 a 3 and a 4 and this equation can also be compactly written as k l d l is equal to r l; so, that equation can be written compactly in this manner, and also various coefficients are defined here; coming from axial effects and bending effects in both frames, and also torsional effects.

In a next class we will see some numerical problems and before that we need to also look at - because we got the element equations in the local coordinate system, we need to see the transformation matrix details and how to transform this element equations into the global coordinate system - this local element equations into the global coordinate system - and then will solve a problem to understand various details, in the next class.