Finite Element Analysis
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Module No. # 01
Lecture No. # 12

In the last class, we have seen there, how to calculate the displacements, transverse displacements, bending moment shear force in beam subjected to thermal stresses or temperature stresses. In today's class, we are going to solve an example using whatever we learnt. The displacement equation to calculate displacement at any point along the beam length and also bending moment at any point along the beam length and shear force at any point along the beam length when beam is subjected to thermal stresses and let us go back and see what we have done in the last class.

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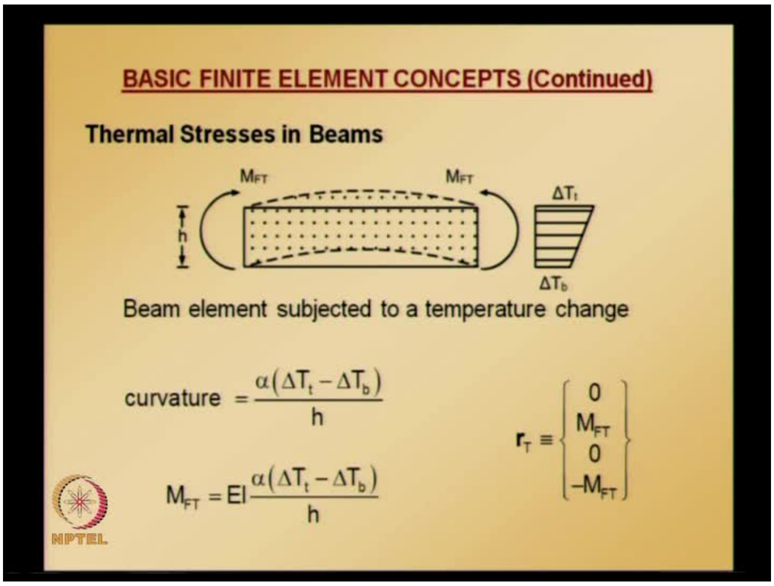

Considering a beam element which is subjected to a linearly varying temperature over its depth, we developed the equations and this is the beam element subjected to a temperature change at delta capital T subscript t, is the change in the top surface temperature delta capital T subscript b is the change in the bottom surface temperature; h is the depth of the beam.

When a beam element like this is subjected to temperature change, its going to be subjected to, it is going to experience curvature. Because of that, it is going to also be subjected to uniform moment which is going to be constant along its span and when we apply end constraints for this kind of elements, fixed end moments are going to be developed which are indicated there - M FT and whether is a change in the temperature of the top surface and bottom surface curvature, that this beam element is going to experience is given by this and corresponding ones, we know the curvature; we know the relation between the curvature and bending moment; so we can find what is M FT.

Once we get M FT and please note that the solution procedure for beam subjected to the temperature changes is similar to that we adopted for beams subjected to uniformly distributed loads and because, here moment is a constant uniform moment along length of the beam. As beam span it is you are going to have uniform moment. So, instead of uniformly distributed load we have uniform moment over the length of the beam.

So the solution, procedure is similar to that we adopted for a uniformly distributed load and if you recall, what we did for uniformly distributed load is try to find for a beam span for the given loading conditions tries to find what are the fixed end solutions and using those fixed end solutions, put them in a vector form following the sign conventions for applied loads and moments and degrees of freedom. Put all these moments and forces in a vector form and the equivalent load vector for solving the displacements transverse displacements. Rotation is going to be a sine; we need to apply a sine change to the vector that we get from fixed end solutions.

So, using that approach the equivalent load vector turns out to be this, because, there is no shear along the span of the beam only bending moment will be there and the equivalent load vector using the procedure that putting the fixed end solution in a vector form and reversing this sign we are going to get equivalent load vector.

Once we get the equivalent load vector rest of the solution procedure is similar to what we did for beam subjected to uniformly distributed load or concentrated loads. So, we need to assemble the element equations and we need to apply or using the element connectivity or a node numbers and elementary numbers. We can get the global equation system and applying the boundary condition. We can solve for the unknowns; that is rotations and transverse displacements. Once we get these values we can go back to each element and now, since in thermal stresses the beam subjected to thermal stresses; it is subjected to uniform moment. We need to apply fixed end correction for this final solution that we get from finite element.

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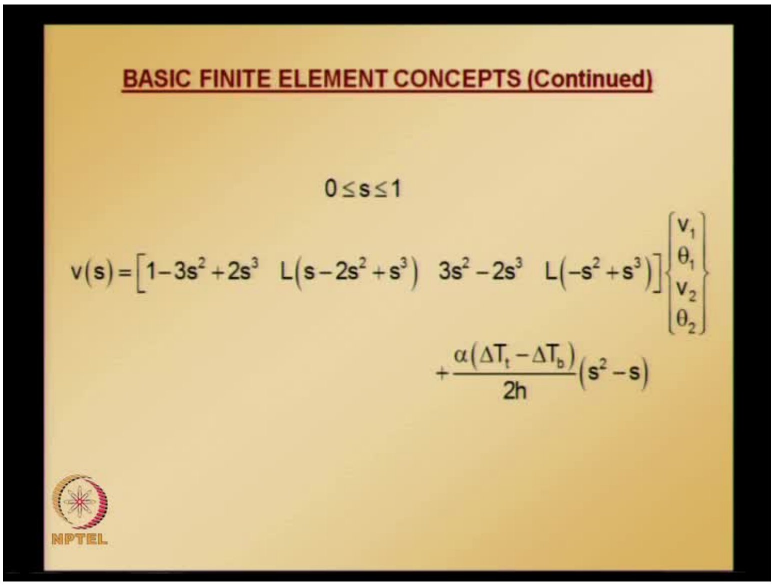

Let us see in detail all the steps by taking an example. Once, we get the nodal values we get, we can interpolate by using this equation displacements and this is what I mentioned. The first term is finite element interpolation; interpolation using finite element shear functions and second term is the correction term which is obtained by integrating the curvature; that is expressed by the beam element twice.

If you integrate curvature twice and when this beam element is restrained at the ends this is fixed end solutions. So, a displacement at the ends at s is equal to 0 and s is equal to 1 its value is 0. So, applying the condition that we transverse displacement is equal to 0 at s is equal to 1 and s is equal to 0 and integrating twice the curvature equation we are going to get the fixed end solution which acts like a correction term and that is what is written there as a second term and that acts like a correction term to the transverse displacement obtained from finite element interpolations.

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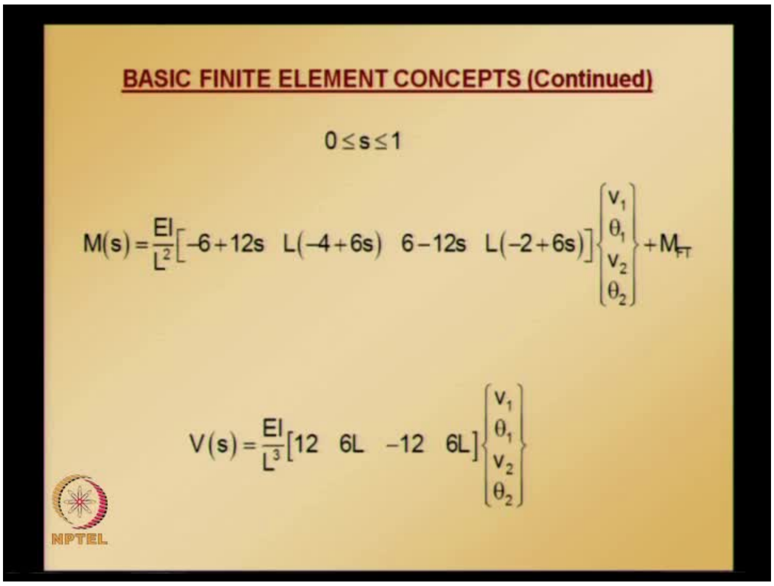

Now, moments similarly, correction term is there, in addition to the finite element interpolated values and please note, that there is no shear because of these thermal stresses and fixed end condition. So, there is no correction term for shear force to get clear idea about how to approach a problem using all these concepts; let us take an example.

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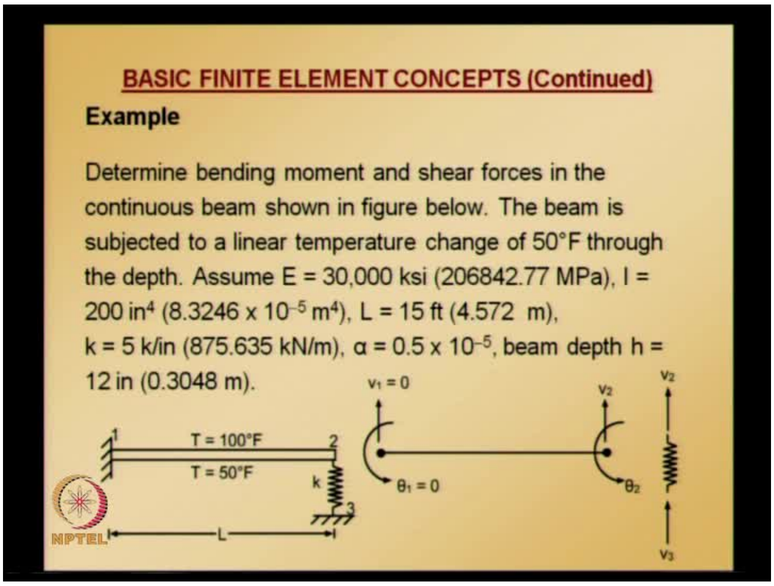

Determine bending moment and shear force in the continuous beam shown in the figure. And, the beam is actually, it is a beam and a spring combination is there. The beam is subjected to linear temperature change and all units are given in both FPS and SI units, since the numbers given in FPS units are round numbers.

Let us approach this problem using the FPS values. So, the linear temperature change of 50 degrees fahrenheit through the depth that is delta capital T subscript t minus delta capital T subscript b, that value is 50 fahrenheit and rest of the material properties, Young’s modulus moment of inertia length and spring constant coefficient of thermal expansion, height of the beam all quantities are given there.

This problem as you can notice, there are two kinds of elements. One is beam element; another is a spring element. So, we will take two elements and three nodes the node numbers are shown in the figure 1, 2, 3 and beam element comprises of nodes 1 and 2 and spring element comprises of nodes 3 and 2.

In this problem, you can notice that node 1 and node 3 are constraints. So, all degrees of freedom at node 1, since this is a beam element at node 1, we have 2 degrees of freedom v 1 theta 1 and that node 3 we have only 1 degree of freedom. The displacement along the axial direction of the spring and at node 2 since, it is a part of both beam and spring element, it is going to have 2 degrees of freedom rotation and also transverse displacement. So, v 2 theta 2 are the degrees of freedom at node 2 and now node 1 1 node 3 are fixed. The corresponding degrees of freedom are 0 that is v 1 is equal to 0 theta 1 is equal to 0 and also the displacement at node 3 is going to be 0; v 3 is going to be 0. So, now let us take this discretization and then we need to assemble the element equations.

Before doing that, we need to find imagining when there is a temperature change. It is a beam element going to be subjected at an uniform bending. So, imagining this beam element to be fixed at both ends, we need to calculate what is the fixed end moment solution or fixed end solution imagining. This beam element to be fixed at both ends and that is used to calculate equivalent nodal vector for beam element. So, we need to calculate what is M FT. M FT is the fixed end moment because of the temperature change and that can be concluded using the EI values and also temperature change 50 fahrenheit alpha coefficient of thermal expansion and the depth of the beam is also given there.

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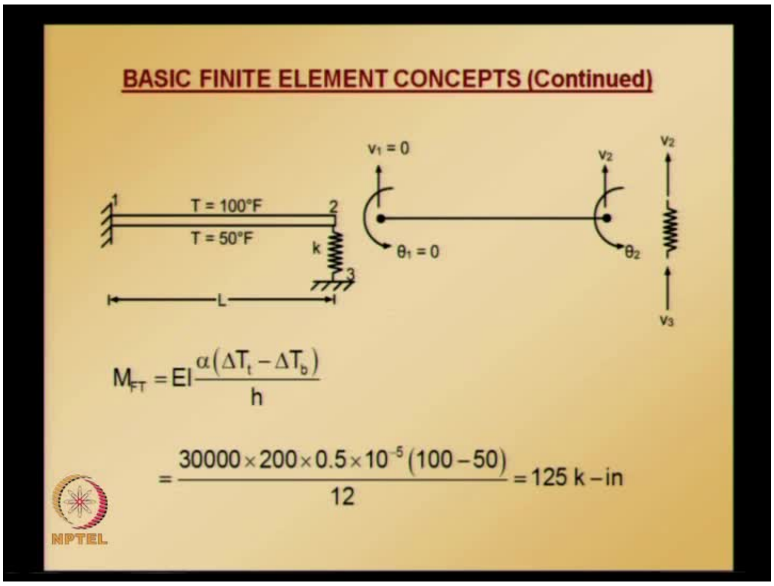

So, using these values we can calculate what is M FT and this is what is shown here. Please note, that we are working out in FPS units and we got M FT. Following the sign conventions for applied moments and shears and putting the fixed end solution in a vector form and reversing this sign, we are going to get equivalent nodal vector for beam element.

We got equivalent nodal vector for beam element and now, we can write the element equations for beam element. The stiffness matrix is similar to what we derived earlier. So, substituting all the quantities that is Young's modulus, moment of inertia and length of the beam, all these numerical values into the stiffness matrix that you already have for beam element, we are going to get element equations for beam.

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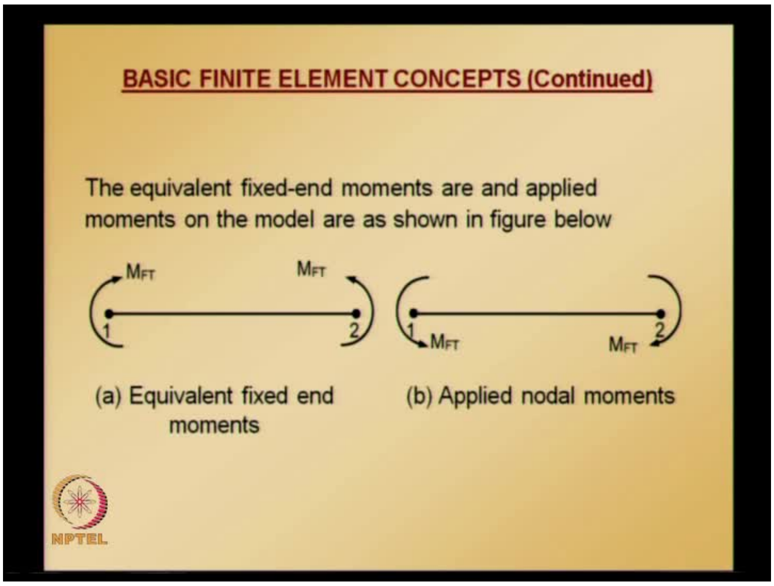

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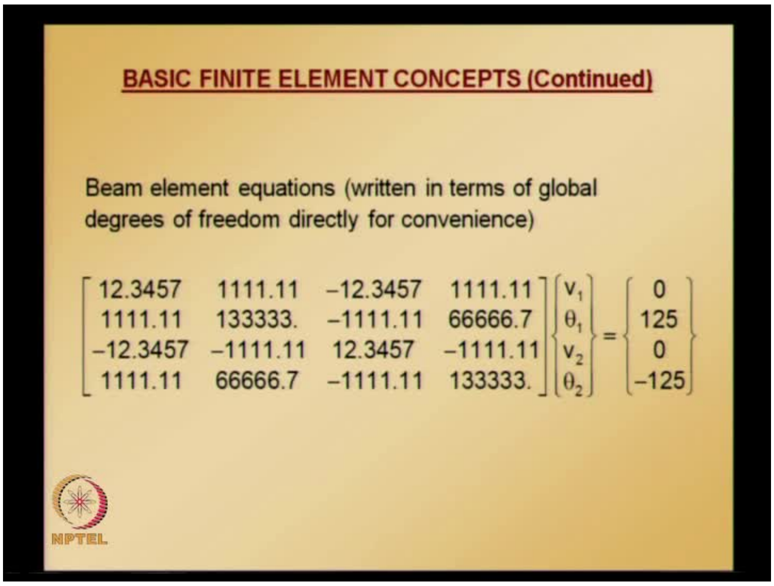

Here the equivalents between the fixed end moments and the equivalent nodal vector are shown here. So, once we get the fixed end moment M FT, we need to reverse the sign change; we need to reverse the sign and we get the equivalent nodal forces and then we can write the element equations for beam element. Please note that beam element the degrees of freedom are v 1 theta 1 v 2 theta 2.

Now, coming back to the spring element spring element has 2 degrees of freedom v 3 and v 2. And, we need to decide whether, how we number these nodes? Whether node 3 is local node 1 or whether node 3 is local node 2? Here the notation that I am following is global node 3 is local node 1; global node 2 is local node 2.

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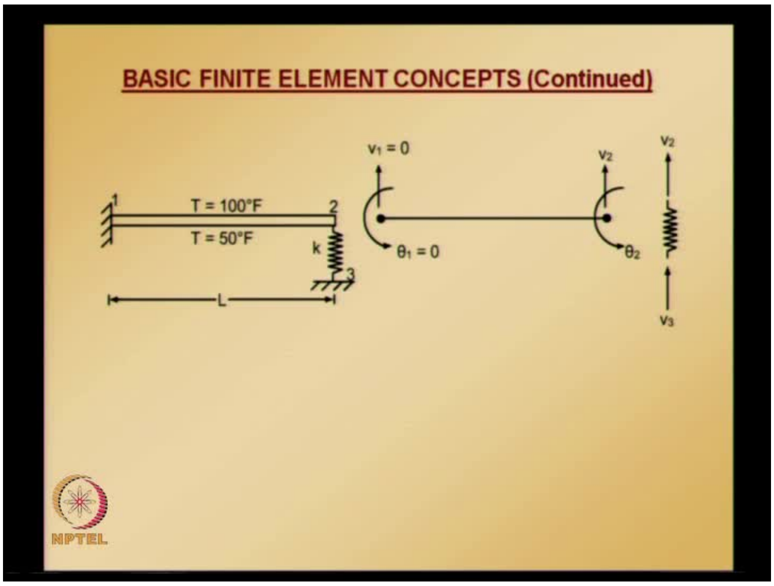

So, using with that understanding we get element equations for spring element like this. So, local node 1 for spring element is 3 local node 2 for spring element is global node 2 and now, we got element equations for beam element and element equations for spring element.

Now, we need to assemble the global equation system and before doing that we need to have a clear idea where the contribution from each of these elements will be going. The element contribution from beam element goes into, actually there will be total 5 degrees of freedom for this particular problem because, at node 1 we have 2 degrees of freedom; at node 2 we have 2 degrees of freedom; at node 3 we have only 1 degree of freedom. so total global equation system will be a 5 by 5 and let us see how the contribution goes in into this global equation system.

A beam element contribution goes into 1, 2, 3, 4 rows and columns of the global equation system and the tricky part is, spring element where the contribution goes in to decide that actually spring is connecting node 3 and a node 2. At node 2 we need to note that the common degree of freedom between spring element and beam element is transverse displacement v 2.

So, the spring element goes into the location or locations 5 and 3 rows and columns of the global equation system. So now, since we numbered node 3 are as a local node 1 and node 2 as local node 2 for spring element here in the spring equations that is 5 minus 5 minus 5 5 the contribution at 1 1 location of this equation system goes into a 5 5 location of the global equation system. The quantity that is minus 5 which is at 1 2 location of this equation system it goes into 5 3 location.

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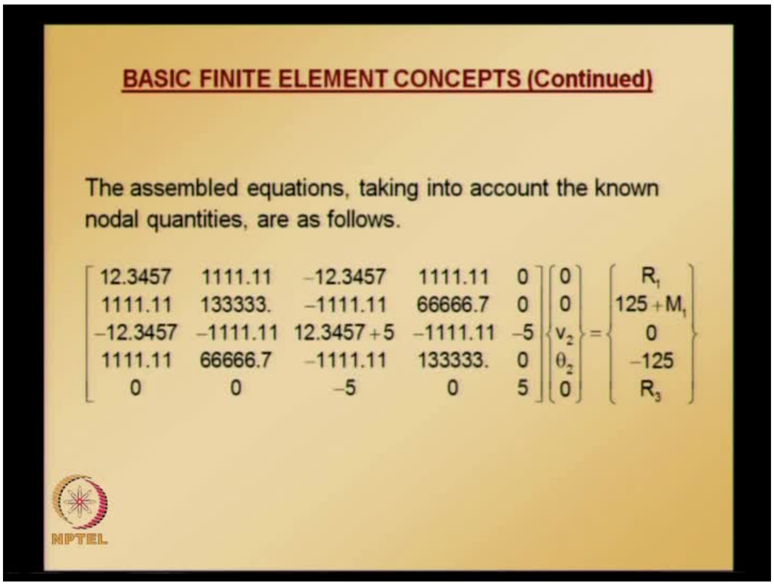

Similarly, the minus 5 which is at the location 2 1 it goes into 3 5 location of global equation system and the component which is at 2 2 location; it goes into 3 3 location of the global equation system with this understanding. We can get the final assembled global equation system and please note that l node L is fixed so v 1 theta 1 are 0.

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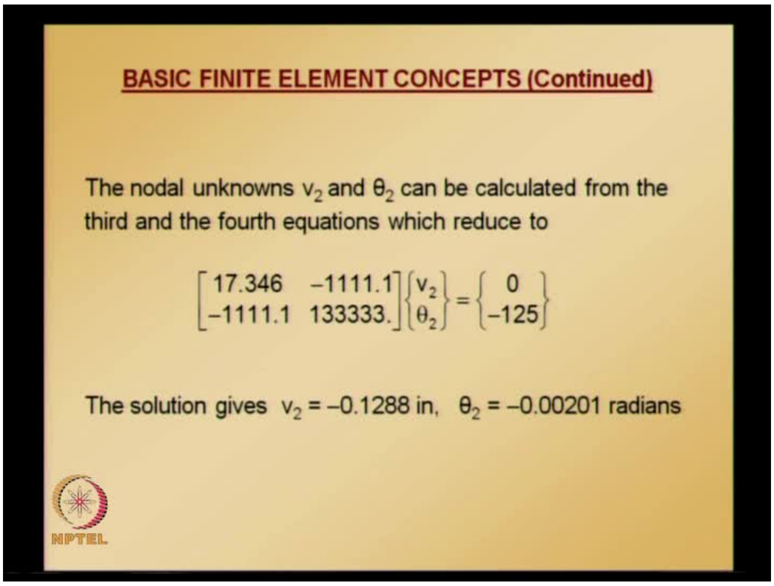

Wherever the displacements and rotations are constraint, reactions will be there the reactions will be developed. So, the reaction at because of the constraint of v 1 is R 1 reaction because of constraining theta 1 is M 1 and also v 3 is constraint so, corresponding reaction is R 3. Eliminating or eliminate cancelling the rows and columns corresponding to the degrees of freedom which are 0, we are going to get reduced equation system which is going to be 2 by 2 equation system and which we can solve and get the values of v 2 and theta 2. That is, v 2 is nothing but transverse displacement at node 2; theta 2 is nothing but rotation at node 2.

Once we get the v 2 theta 2 values which are given here, once where we get these values we can go back to each element that is a beam element and spring element and find the element quantities like the bending moment and shear force in the beam element ends. A spring element will have only axial force. We can find what is the axial force in spring element using these values.

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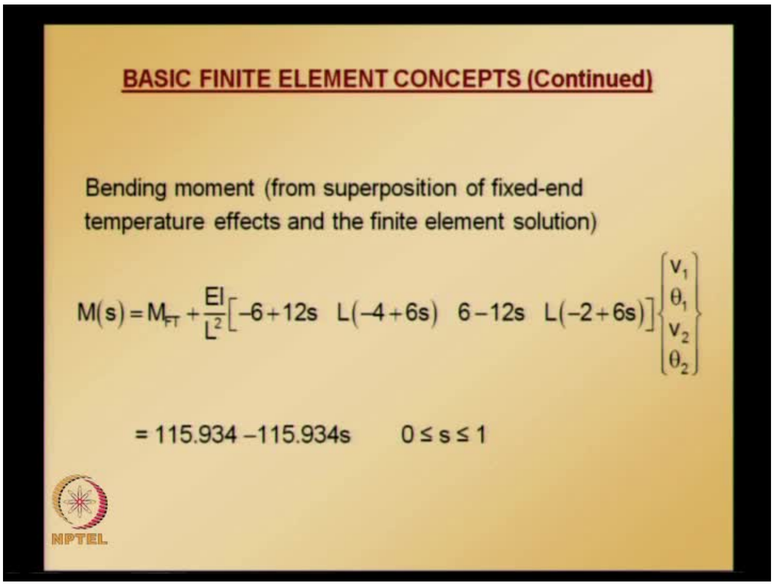

So bending moment here, we need to apply the fixed end solution correction. That is what is done; bending moment from superposition of fixed end temperature affects in finite element solution.

The first part is the correction term and the second part is finite element interpolation using second derivative of shear functions. Now, we know what is v 1 theta 1, v 2 theta 2 values; and v 1 theta 1 are 0 by virtue of the boundary condition and v 2 theta 2, we just calculated. So, substituting all these quantities we are going to get s and by varying s from 0 to 1, we can sweep over the entire span of the beam and we can get an idea how bending moment is varying over the span of the beam.

Similarly, we can calculate shear force but noting that because of the temperature affects more shear force or fixed end shear is not going to be there.

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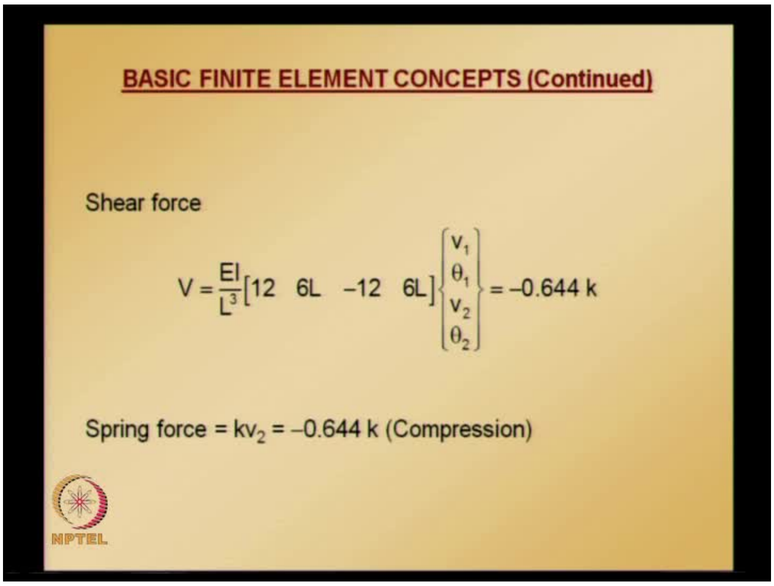

So no shear for shear force node correction, because of the temperature affects is required and the shear force is given by this formula using the third derivative of shear functions, finite element shear functions for beam element and using the nodal values we get this shear force value.

Please note that the bending moment and shear force that we just calculated using these values or expressions, we can actually draw the free body diagram for beam element using the sign convention for moments and internal shears. And, now let us calculate, what is the spring force? Spring force is, please note, that local node 1 for spring element is 3 and local node 2 for spring element is 2; so, the formula is spring constant times v 2 minus v 3 and since v 3 is 0 because of the virtue of the constraint that is applied at that location. So, spring force is given by k times spring constant times transverse displacement at node 2. That is what is used to compute the spring force there and you can see that the spring force turns out to be negative value. Negative value indicates it is in compression; spring is in compression and it is expected because, when there is when the top temperature of the beam element is higher than bottom temperature of the beam.

It is going to bend in such a manner that spring is going to be in compression. If the temperature changes in the reverse direction that is, bottom temperature is higher than top temperature then, spring is going to be in tension.

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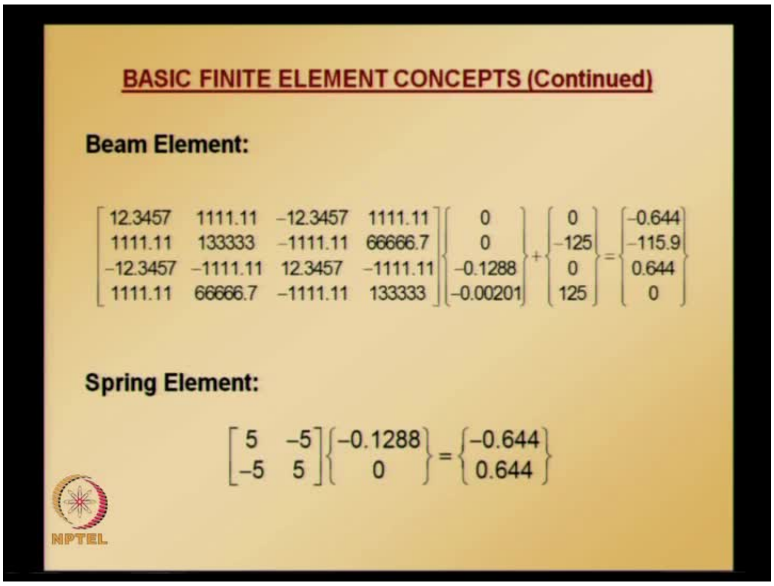

Now, we are ready to draw the free body diagrams using the sign conventions and here the end that is, the moment and shear at the ends or the applied curves. Suppose, if you want these the value of the rotations and the transverse displacements we calculated v 2 theta 2; if you want that, what is the equivalent and nodal values of moments and shear that we need to apply, that can be calculated using these. The element equations for beam element multiplied with v 1 theta 1, v 2 theta 2 and applying the fixed end correction, we are going to get the corresponding shear and moments.

Similarly, for spring element and using either these values or the previous values that we calculated for moment and shear, using the corresponding sign conventions for if you are using the expressions that we used, that we computed or the values that we computed for internal moments and shears using the nodal values a while back, using those you can actually draw the free body diagram. Or, you can use these values and you can draw the free body diagram and using the appropriate sign conventions.

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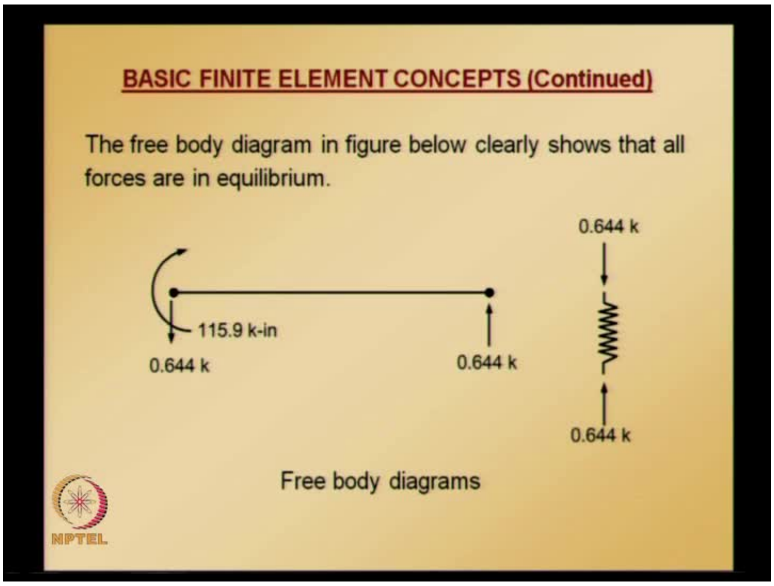

So, the free body diagram indicating all the forces are shown here. You can easily check that each of these elements that is beam element and spring element is itself in equilibrium. Now, this completes the topic on beams and now we will start looking at a slightly different topic which is analysis of structural frames and for the structural frame axial for analysis of structural frame, we will be using whatever we learnt till now. That is, the beam element equations and also the equations that we developed for axial deformations.

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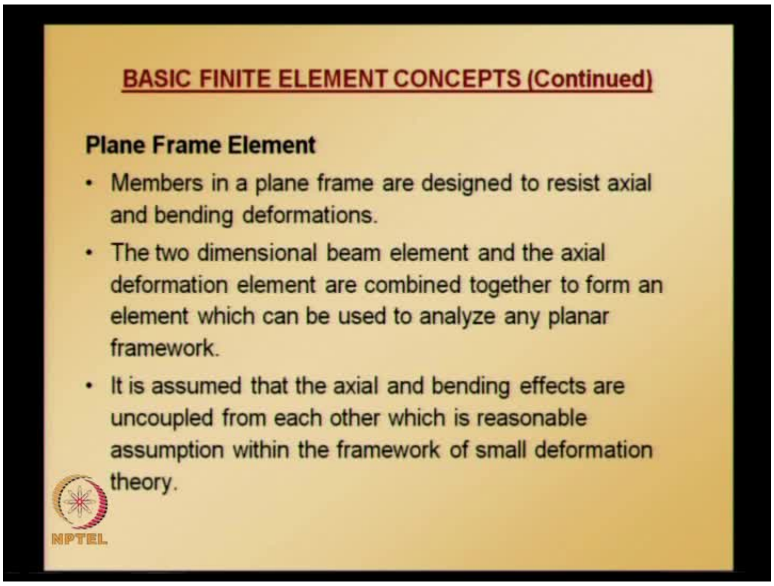

So, the beam element and axial deformation element presented in the earlier lectures are combined together to form a general element for analysis of 2 and 3 dimensional frames. Members in a plane frame are designed to resist axial and bending deformations. So, it is a combination of a beam element and an axial deformation element. The 2 dimensional beam element and axial deformation element are combined together to form an element which can be used to analyze any planar frame work. And, while doing this it is assumed that the axial and bending effects are uncoupled from each other, which is reasonable assumption within the framework of small deformation theory.

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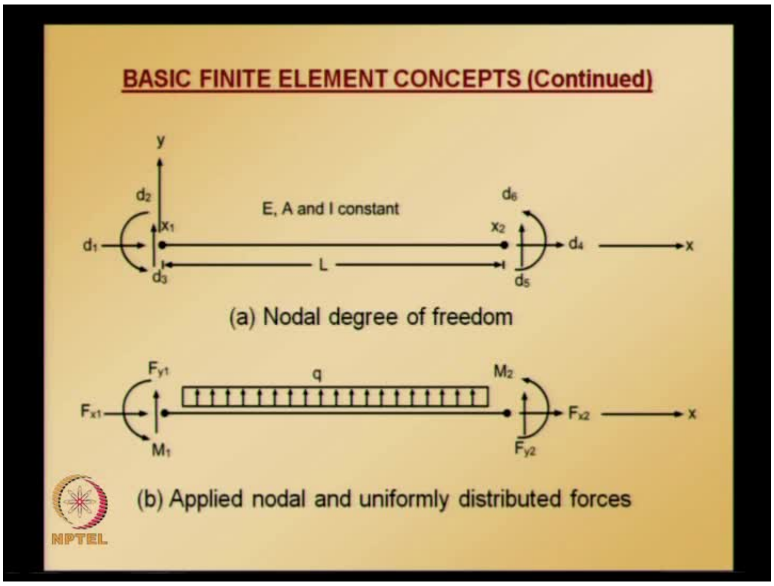

So, the equations for a plane frame element can be written directly from axial deformation and beam bending equations that we just looked at in the earlier lectures. If, the element is like this as shown in the figure (Refer Slide Time: 25:50) and please note that this coordinate system which is indicated in the figure is local coordinate system, each node has a 3 degrees of freedom; two translations and 1 rotation at each node, you have two translations and 1 rotation. d 1 and d 4 are the degrees of freedom corresponding to the axial deformation and d 2 and d 5 are corresponding to the transverse displacement and d 3 d 6 are corresponding to the rotations.

Similarly, these are the nodal degrees of freedom similarly apply nodal and uniformly distributed load. Because of that, what are the equivalent nodal forces and moments that are going to be developed? Those are also shown in figure b.

So, d 1 and d 4 are the degrees of freedom corresponding axial deformation. We can write the element equations for these degrees of freedom d 1 and d 4 using the axial deformation element equations that we learnt. And, d 2, d 3, d 5, d 6, these are the degrees of freedom corresponding to the bending. So, we can use element equation system corresponding to the beam bending and write the element equations for these degrees of freedom.

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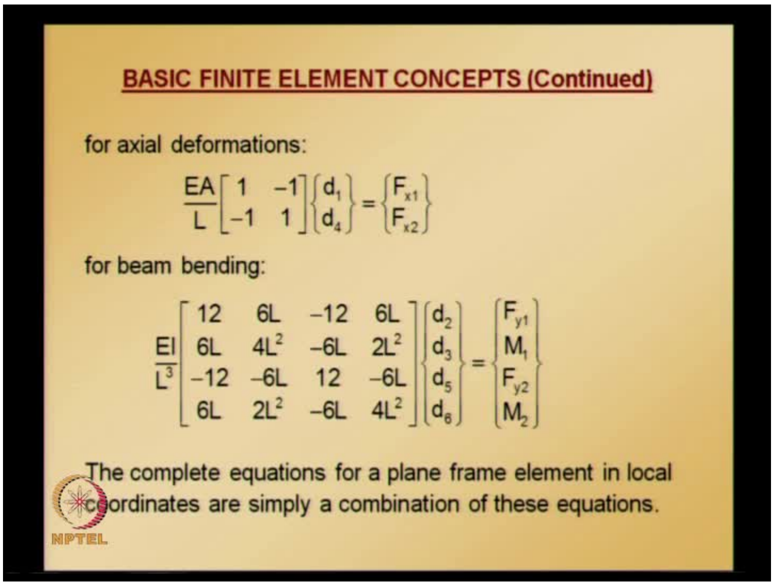

So, for axial deformation these are the element equations for beam bending. These are the element equations and we already started out with an assumption that axial deformation and beam bending effects are uncoupled. Carefully noting down the degrees of freedom in the order of they appear, we can get the global equation system for frame element. The complete equation for a planar frame element in the local coordinate system are simply a combination of these two sets of equation; that is, element equations corresponding to axial deformation and element equations corresponding to beam bending.

We need to appropriately place the contributions from each of these two things; that is, axial deformation of beam bending to get the global equation or complete element equation for frame element in the local coordinate system. And, that is what is shown here.

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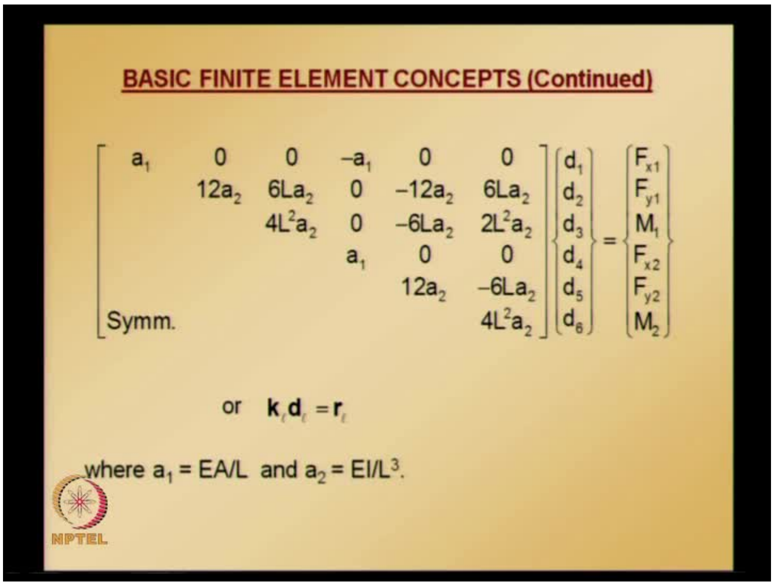

Where a 1 only upper triangular stiffness matrix is shown; lower triangular, because of this symmetry of the stiffness matrix, lower triangular components can be easily be found and please note that in this equation system a 1 is nothing but EA over L which is nothing but it is coming from axial deformation effects. And, a 2 is EI over L cube that is coming from bending effects.

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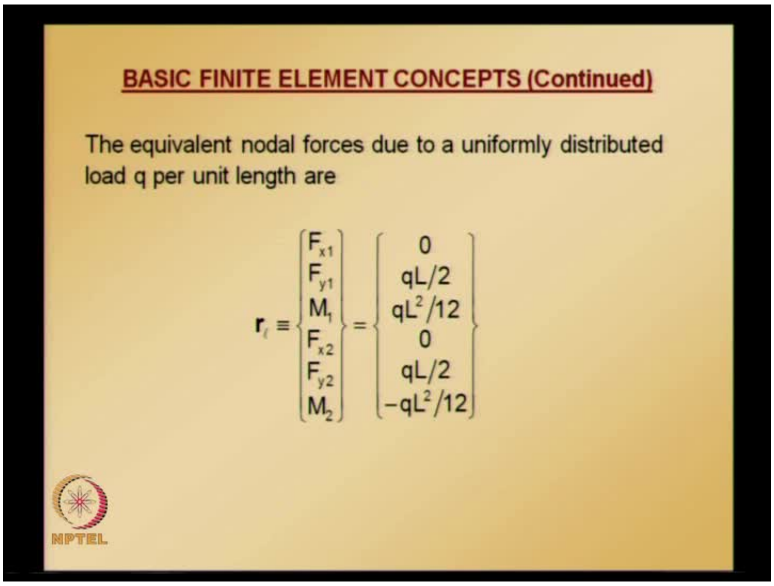

Now, what are the equivalent nodal force vector forces due to the uniformly distributed load applied on the frame element? That is shown in the figure when we started out this frame elements and frame element is subjected to uniformly distributed load acting the transverse direction. And, with no axial forces the equivalent nodal vector is similar to what we have for beam element except that, the locations corresponding to the axial forces is equal to 0. Rest of the components, are similar to what we have for a beam element. So, the equivalent nodal forces due to uniformly distributed load of q per unit length is given here.

We have the element equations for a frame element in the local coordinate system but, usually this frame element will be arbitrarily oriented in space. If you take any structure we need to even look at that transformations and other things. So, for proper assembly, a global x y coordinate system is chosen for the entire frame. All element nodal coordinates are defined with respect to this coordinate system; an arbitrarily oriented plane frame is shown here.

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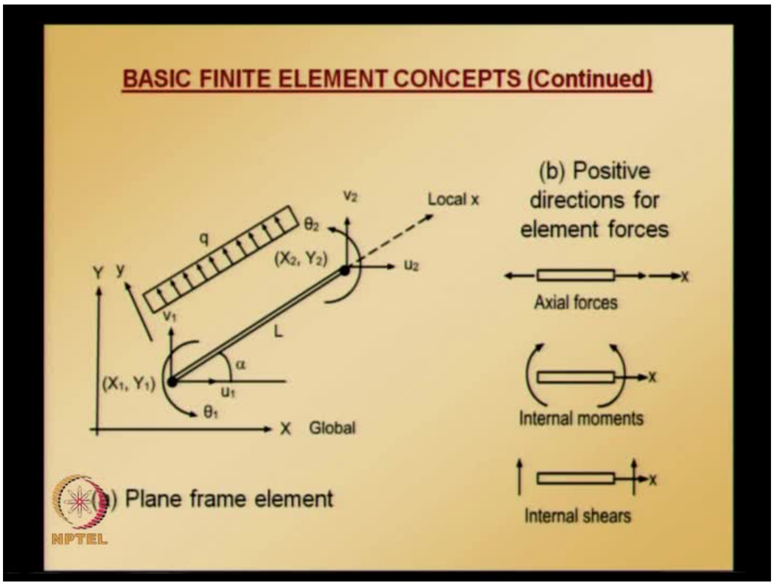

Now, whatever equations that we have in the local coordinate system, that we need to see how they get transformed into this global coordinate system x y. local x axis is defined as a vector or defined by a vector going from node 1 to node 2 and local y axis is perpendicular to local x axis or local y axis is at 90 degrees counter, clockwise from the local x axis.

The applied forces and moments are assumed to be positive along global coordinates, coordinate directions, the distributed loads applied over an element of positive if, they are acting in the positive local y direction positive directions for internal forces moments and shears are shown here.

The sign convention: so the sign convention is similar to what we adopted for beam element and also what we adopted for axial deformation elements. So, the same sign convention is carried forward because basically, what we are doing is a frame element. We are assembling the element equations for a frame element based on element equation for axial deformation element and bending element.

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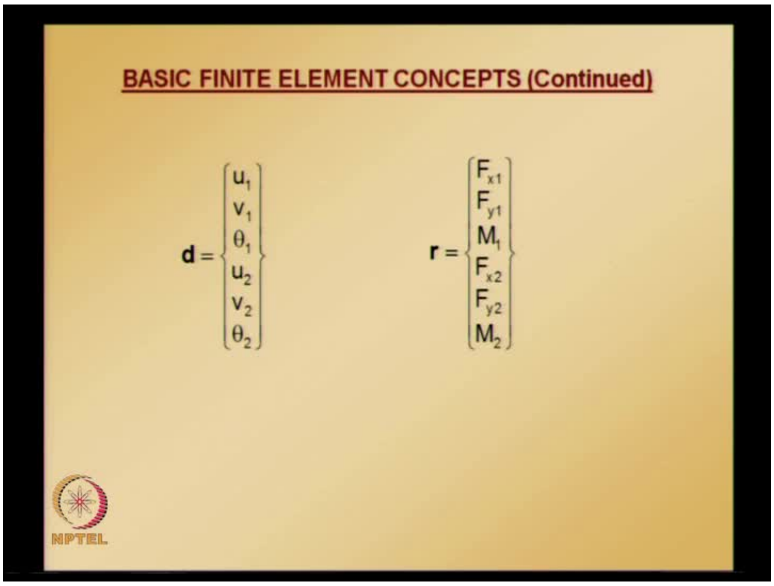

So, the degrees of freedom, global degrees of freedom are u 1 v 1 theta 1, u 2 v 2 theta 2; u 1 is going to be the axial displacement at node 1; v 1 is transverse displacement at node 1 and theta 1 is rotation at 1. Similarly, u 2 v 2 theta 2, are the corresponding degrees of freedom at node 2 and this is a global load vector. Global load vector comprises of axial force at node 1 shear force at node 1, moment at node 1. Similarly, F x2 F y2 M 2 are the corresponding forces and moments at node 2. A coordinate transformation between local x y coordinate system and global x y coordinate system, gives equation that can be assemble directly for any plane frame problem.

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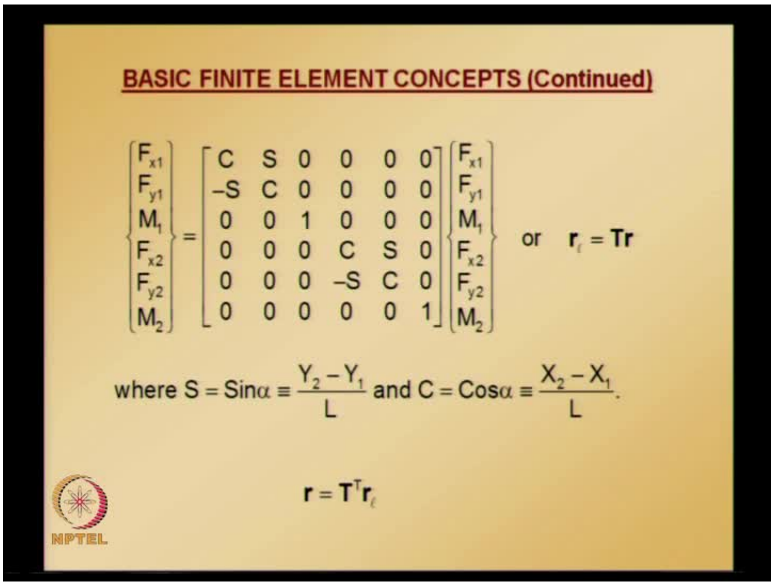

The transformation matrix, that is local force vector; how it is related to global force vector is given by this transformation and the force vector on the right hand side is the force vector in the local coordinate system and the force vector on the left hand side is the force vector in the local coordinate system. What you have on the right side is transformation matrix and the force vector in the global coordinate system. This can be obtained; this transformation matrix can be obtained using or noting that only forces get transformed moments, are not going to get transformed within the coordinate system.

The transformation of forces is similar to what we adopted for axial deformation elements and this C and S C is nothing but cosine alpha. Alpha is nothing but the angle between the local x coordinate or local x axis and global x axis measured in the counter clockwise direction. Alpha, once we know alpha, we can find what is cosine alpha. Cosine alpha is, once we know the coordinates a global x y coordinates of the nodes, we can find what is cosine alpha and sine alpha is in the formulas there. And, this equation can be written compactly as r l is equal to T times r; where r l is the local force vector or local load vector and r is global load vector; T is the transformation matrix and also as in the axial deformation problem, or truss problems, the displacements also get transformed in the same manner as the forces. The inverse relation between the global load vector and local load vector is given by r is equal to T transpose r l because this transformation matrix is orthogonal.

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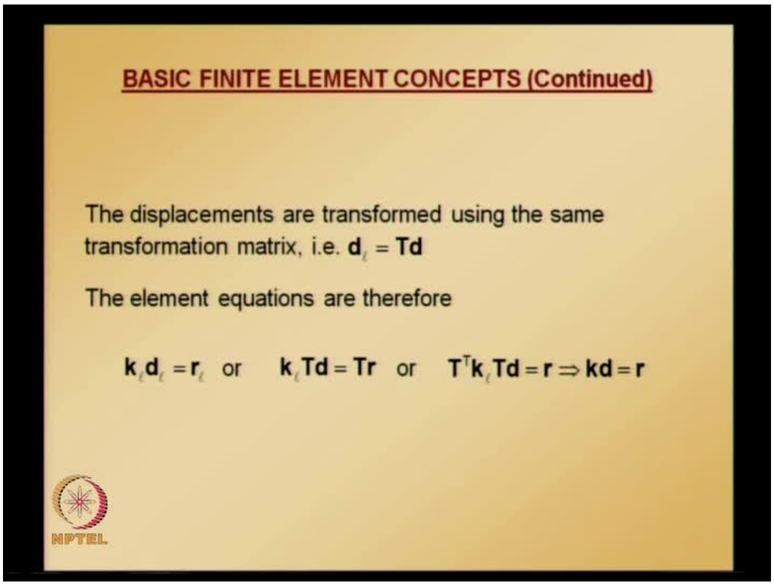

Orthogonal matrix is a matrix for which T transpose or inverse is same as transpose. As I mentioned, displacements are transformed using the same transformation matrix and the inverse relation that is d is equal to T transpose d l element equations. The local coordinate system can be written as k l d l is equal to r l and d l is equal to T d is substituted and both left hand side and right hand side are multiplied with T transpose and k is defined as t transpose k l T. Then finally, it can be written as kd equal to r k is equal to T transpose k l T.

Earlier we have looked at this kind of thing when we are solving 2 dimensional or 2D plane truss and 3D space truss problems. We have similar kind of transformations except that the transformation matrix here is different. So, after carrying out the indicated matrix multiplication, the plane frame element equations in the global coordinate system are obtained for 1 element. We obtained and similar process we can repeat for other elements.

Assembly and solution procedure for nodal degrees of freedom remain same as what we are doing till now. Once nodal displacements are known, element quantities displacements, axial forces, moments, shears at any point along an element, can be computed using shear functions and their derivatives for each element. The global displacements must be transformed into local displacement before performing the element level calculations for an element with uniform load. The equation in terms of local coordinate s that is s going from 0 to 1 s value is equal to 0; at node 1 s value is equal to 1 at node 2.

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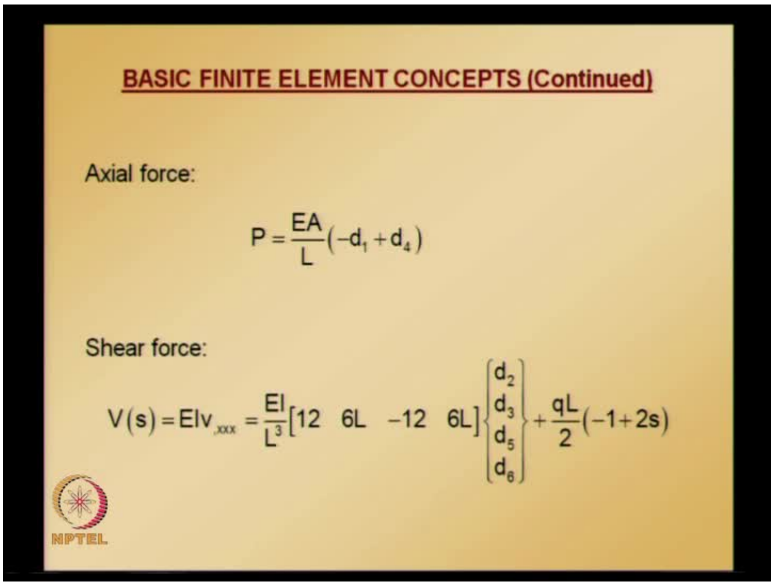

For that kind of element axial degrees of using the computed degrees of freedom, we can find axial force in frame element and also shear force. Since this frame element is subjected to distributed load, we need to apply fixed end correction and s goes from 0 to 1 s, 0 s is equal to 0, corresponds to node 1 s is equal to node 1 corresponds to node 2.

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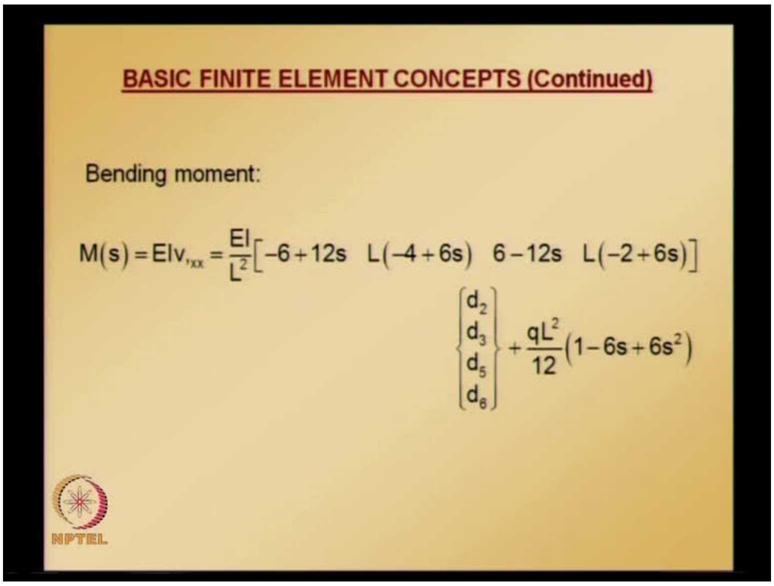

Similarly, we can also calculate bending moment where also, we need to apply fixed end solution correction. So, this is the procedure for solving frame problems to illustrate this procedure; let us take an example.

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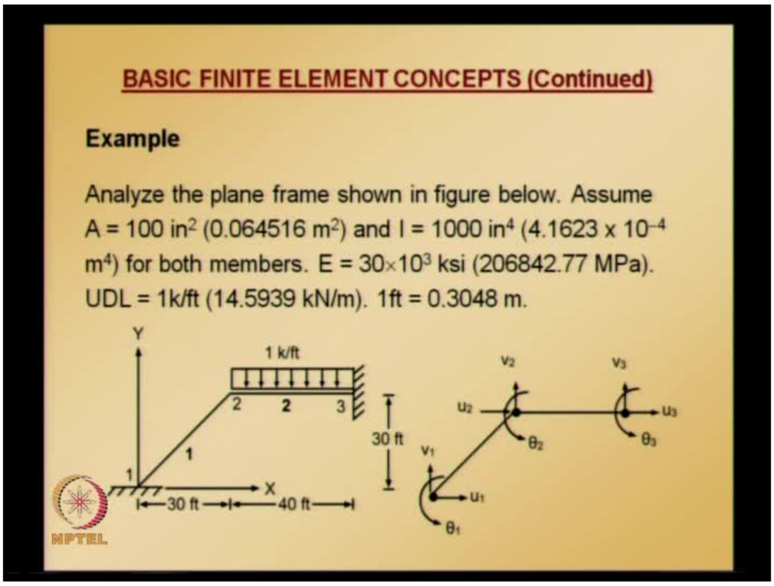

Analyze the plane frame shown in figure below: cross sectional areas, moment of inertia of cross section; there are two members here. For both the members, cross sectional area and moment of inertia of cross section same; the values are given both in FPS units and SI units and material properties and distributed load, the value magnitude and also the direction, is also given. So, its acting, the distributed load in the acting in the downward direction and since the values given in FPS units are in round numbers, let us proceed and solve this problem in FPS units. Please note that for the element numbers and the node numbers that are given in the figure, node 1 is fixed. All degrees of freedom are constrained, that is, u 1 v 1 theta 1.

Similarly, all degrees of freedom at node 3 are constrained, that is, u 3 v 3 and theta 3 are constrained and only node which as non-zero degrees which is going to have non-degree 0 degrees of freedom is node 2 - which is going to be u 2 v 2 and theta 2. We need to solve for these values and element 1 is assumed to go, or assumed to be along the vector which is going from node 1 to node 2. Element 2 is assumed to be along a vector which is going from node 2 to node 3 and all the degrees of freedom are indicated there.

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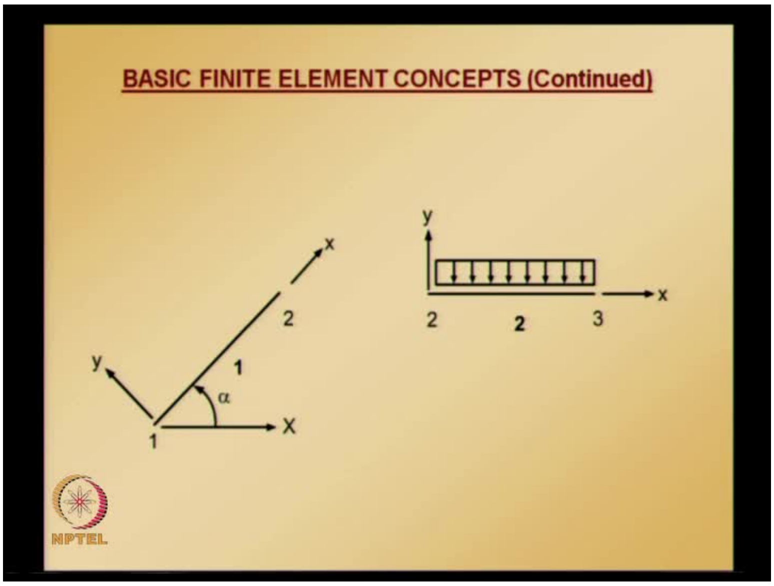

So, now by choosing arbitrarily, element 1 goes from node 1 to node 2 and element 2 goes from node 2 to node 3. The local coordinate system for each element shown here: x axis is oriented along the axis of the member and y axis is at 90 degrees with respect to x axis in the counter clockwise direction.

Similarly, for element 2 also the x axis and y axis are shown. So, now we know the global x axis y axis with respect to global x axis and y axis. We can find what are the coordinates of node 1, node 2 and node 3 using the global coordinates - global x, y coordinates of node 1, node 2, node 3.

We can calculate what are the direction cosines and once you know the direction cosine, since material properties and the length of the members even for element two length is already given straight forward for element 1, we need to use the nodal coordinates of nodes; the global coordinate of node, we can find what is the length of the element and material properties are given cross sectional areas are given.

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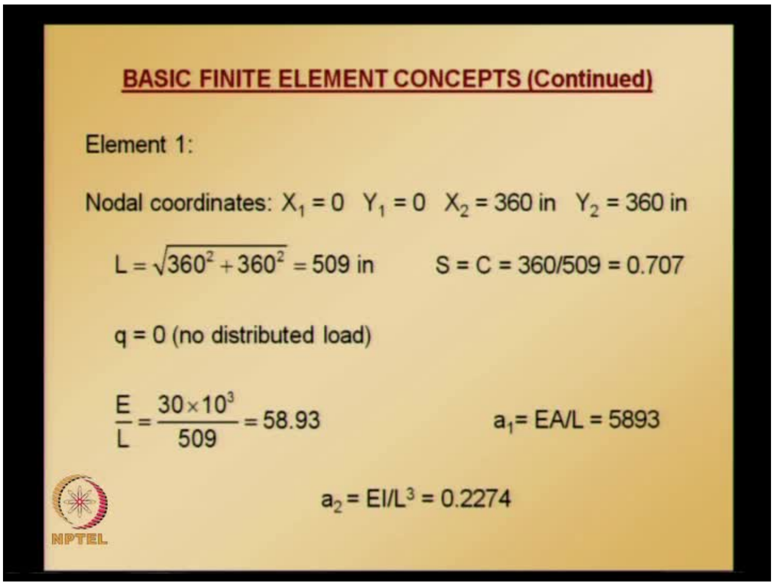

So, we are ready to assemble the element equations. Substituting numerical values into the element equation that we derived and noting the nodal coordinates, the global nodal coordinates, we can calculate length and also direction cosines. Hence, since there is no distributed load on element 1, we do not need to assemble the load vector for element 1. Also, at the end when we are calculating the element quantities, we do not need to apply any correction fixed end solution correction for element 1 and using the material property and length values, we can calculate what is E by L and similarly, a 1 and a 2 values can be calculated.

Once these numerical values are known, we can find what is the local stiffness matrix and also using direction cosines we can find what is the transformation matrix and also what is the using the load value that is given which is 0. Local load vector is going to be 0 anyway and that value can also be found. Once we have all these, we can find what is the global equation system by knowing the global stiffness matrix.

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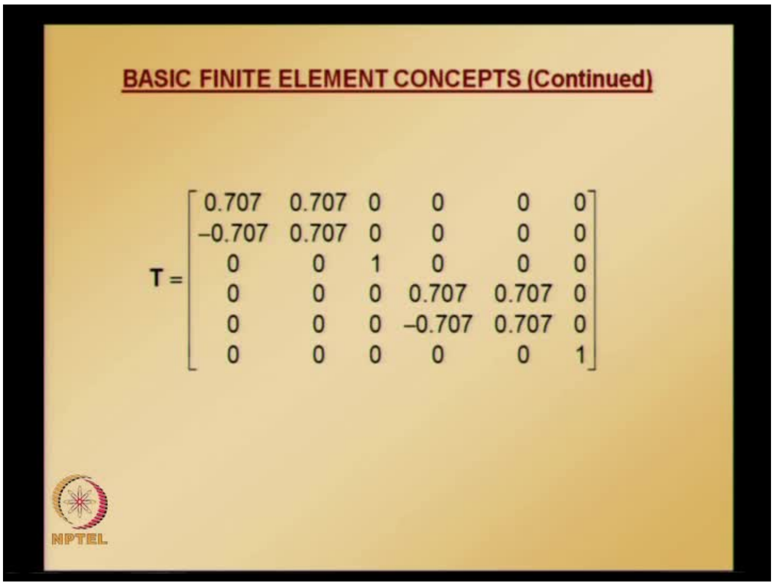

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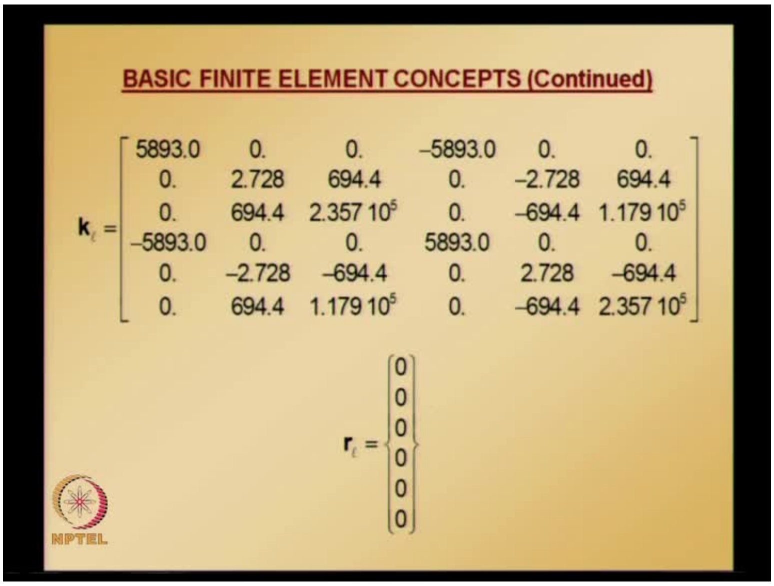

Global stiffness matrix for this element 1 is going to be, or stiffness matrix for element 1 in global coordinate system is going to be T transpose k l T. All the computations are shown here; this is transformation matrix and local stiffness matrix and local load vector.

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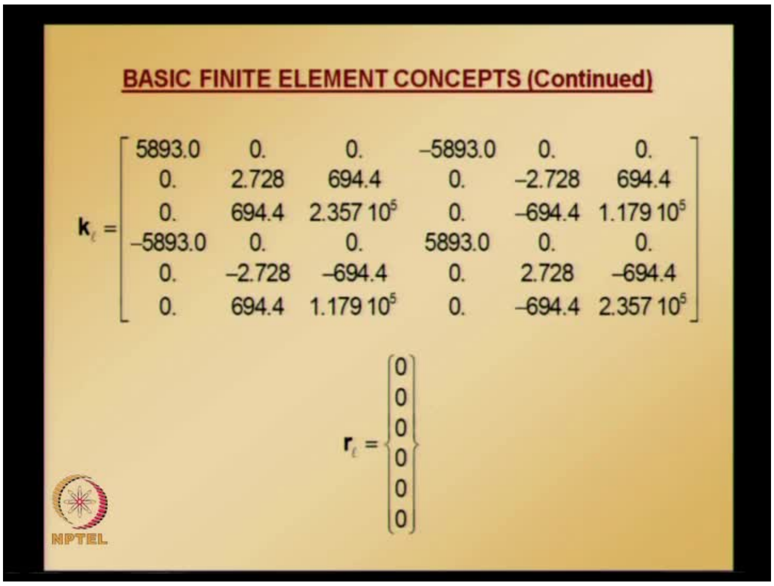

Stiffness matrix for element 1 in the global coordinate system is given by T transpose k l T. Please note that this element 1 contribution goes into the locations and at node 1, you have, or at each node you have 3 degrees of freedom. So, element frame element 1 is connecting nodes 1 and 2; so the contribution goes into 1 2 3 4 5 6 rows and columns of the global equation system.

Element 2 is connecting nodes 2 and 3; so the contribution goes into 4 5 6 7 8 9 rows and columns. So, total global equation system will be 9 by 9 equation system but instead of assembling 9 by 9 equation system, we can adopt a smart way and we can at the time of assembly of element global equation system itself, we can directly assemble the reduced equation system noting the degrees of freedom which are fixed.

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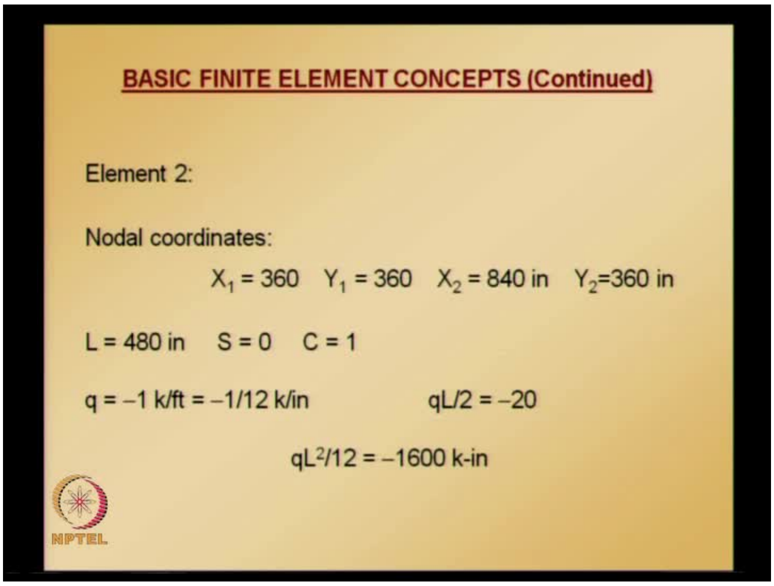

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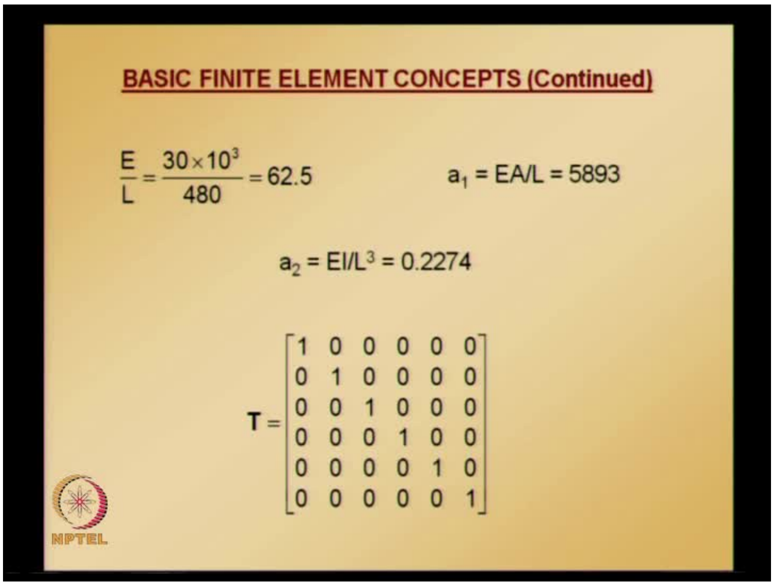

Anyway, before doing that let us see element 1 nodal coordinates. Using these values we can find what are the directions. Cosines length is straight forward; it is given and element 2 is subjected to UDL (uniformly distributed load). So, we need to calculate the fixed end solutions because, those values are required for assembling the equivalent load vector. Once we have all these values, we can write what is transformation matrix and what is the local or stiffness matrix for element 2 in the local coordinate system.

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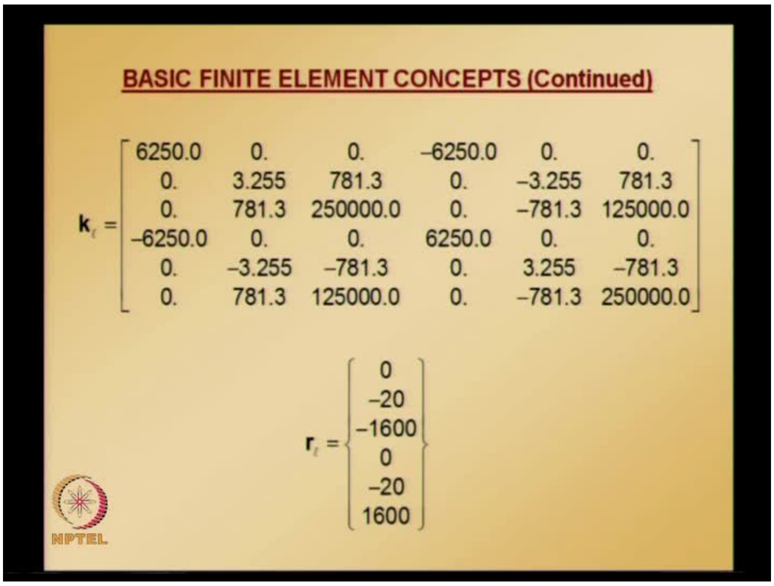

Please note that this element is oriented along the direction of a global x axis. So, local x axis and global x axis are in the same direction. Transformation matrix is going to be identity matrix; so local the stiffness matrix for element 2 in the local coordinate system will be same as stiffness matrix for element 2 in the global coordinate system. So, now this is a load vector for element 2 in the local coordinate system.

Since local coordinate system is same as global coordinate system, even the load vector in the global coordinate system is going to be same as this one. We do not need to apply any transformation because the transformation matrix is identity matrix. Because of the boundary conditions, all the degrees of freedom at node 1 and 3 are 0. There is no need to assemble corresponding rows and columns further since, the specified values are 0. The corresponding columns will not contribute anything either; so, the following…, thus in the following global equations, only the terms associated with node 2 are written.

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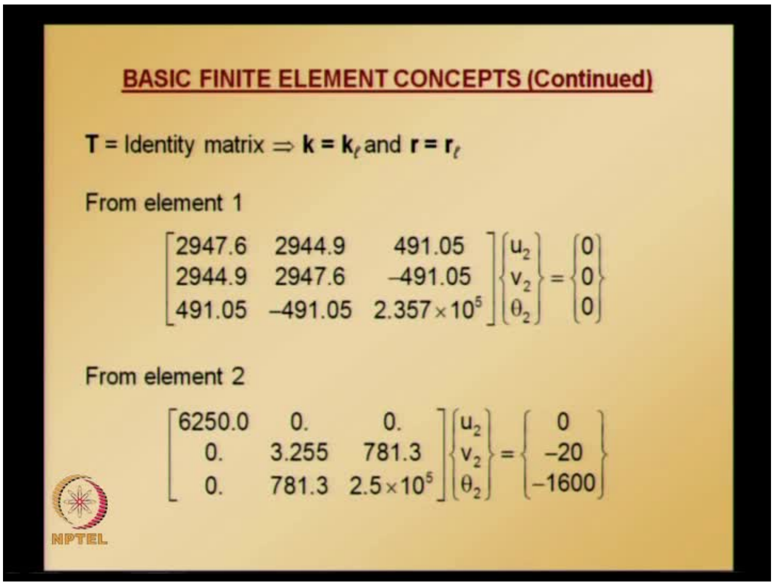

So, for element 1, the terms associated with node 2 are and they if you look at the stiffness matrix, is corresponding to element 1. We have written whatever is there in the lower quadrant; those components or those values contribute to the global or reduced equation system; so those values are written here.

From element 2 also, an element 2 is connecting nodes 2 and 3 and it is in a direction. Element 2 is in a direction of vector which goes from node 2 to node 3. So, whatever is there, the values in the upper quadrant of element 2 stiffness matrix, that contribution comes into the reduced equation system and that is what is written here for element 2 and adding these two equations, global equations are obtained. Global equations are simply sum of these two equations , two element equations (Refer Slide Time: 53:00).

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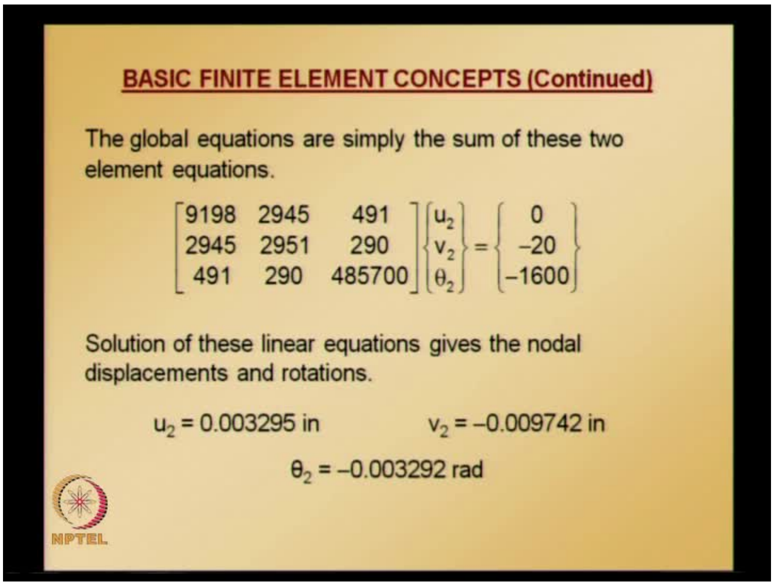

So, adding the corresponding component value we are going to get reduced global equations which we can solve for u 2 v 2 and theta 2. The solution gives us the value of u 2 v 2 and theta 2 and once we get u 2 v 2 theta 2, we know the rest of the degrees of freedom are 0. We can go to each element and find element forces. Element forces can easily be obtained by first determining the local displacements because, whatever we calculated, these are all global degrees of freedom.

We need to back calculate what are the local degrees of freedom or local displacements and then using shear functions and their derivatives we can obtain what are the moments and what are the shear forces and axial forces. So, let us go and do it for element 1 and element 2 separately.

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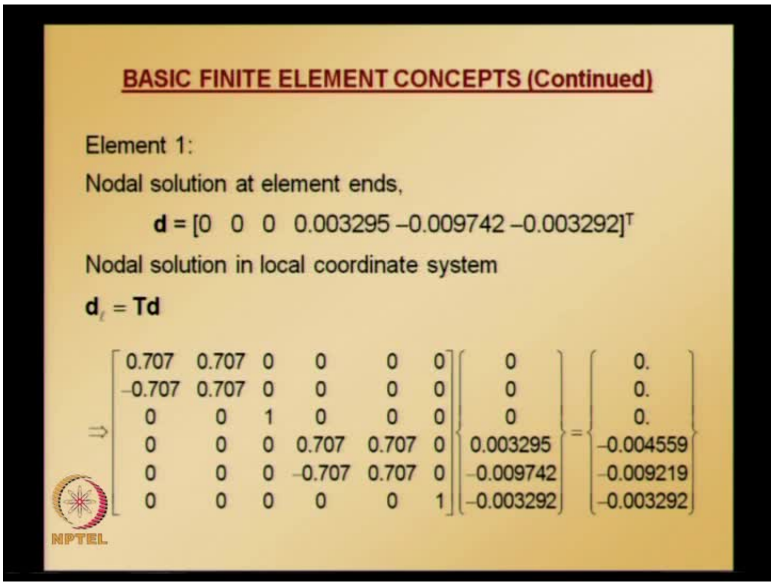

Element 1, the nodal solution that we get, that we obtained just now is written here. That is, u 1 v 1 theta 1, u 2 v 2 theta 2 values. This is the global displacement vector and we need to find local displacement vector using the relation coming from transformation matrix; that is d l is equal to T times d.

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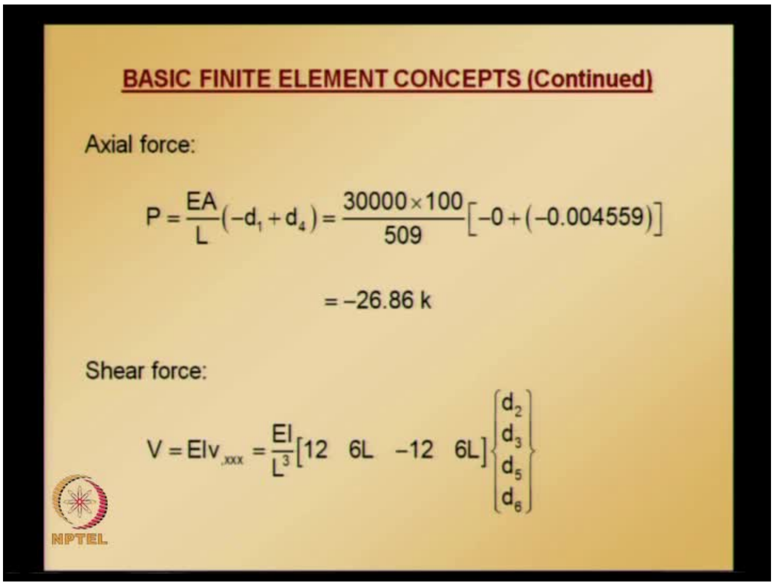

So, nodal solution in the local coordinate system because, we require when we are calculating axial force shear force and bending moment, we require local nodal values nodal values in the local coordinate system and once we get the local displacement vector or the displacement vector in the local coordinate system, we can apply the formula for axial force.

Please note that they give the local degree of freedom in the axial direction for element 1 is 0. Similarly, we can obtain for shear force substituting the values of d 2 d 2 d 5 d 6. This d 2 is going to be the local displacement in the transverse direction; d 3 is moment at node 1 and d 2 is transverse displacement at node 1 and d 5 is transverse displacement at node 2 and d 6 is rotation at node 2.

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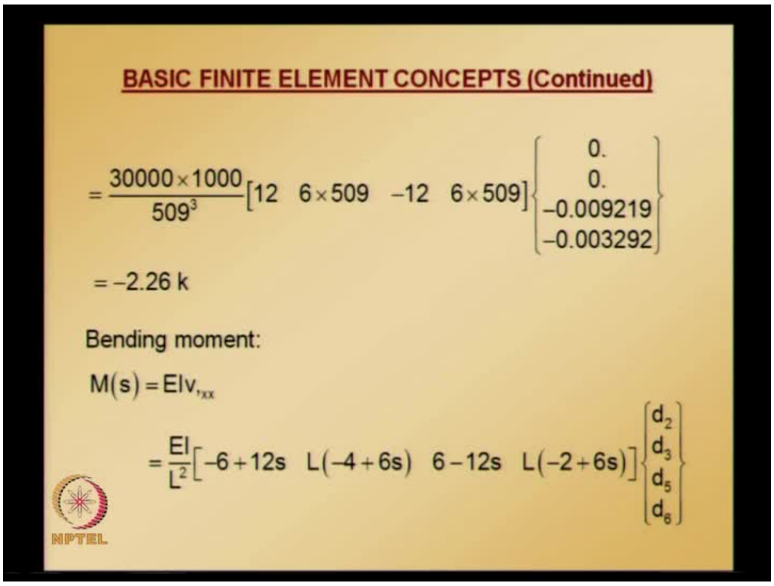

Substituting these values d 2 d 3 d 5 d 6, we get shear and bending moment is given by this using the transverse displacement values and rotations. Please note that when we are calculating either shear force or bending moment, we are not applying any fixed end solution correction because, element 1 is not subjected to any distributed load.

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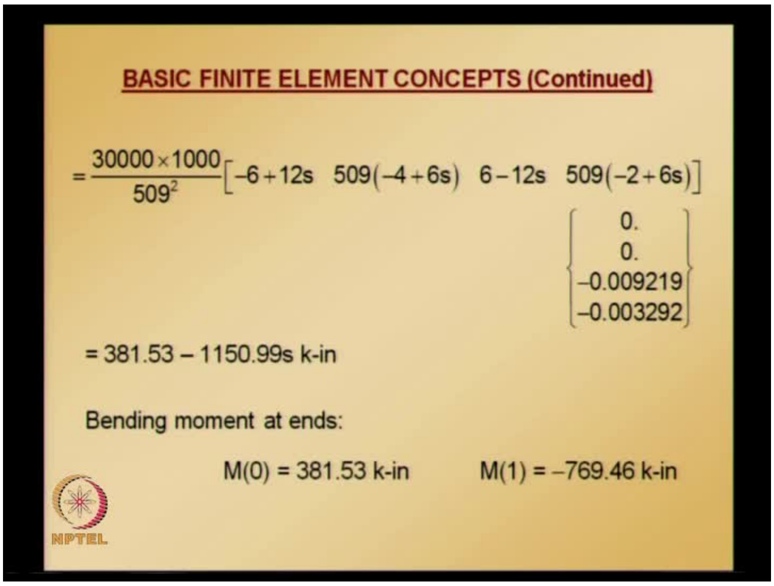

Substituting the numerical values, we get moment and we can calculate from this what is the bending moment value. Shear is constant whereas, bending moment is having linear variation. So, bending moment value at node 1 is obtained by substituting s is equal to 0; bending moment at node 2 is obtained by substituting s is equal to 1.

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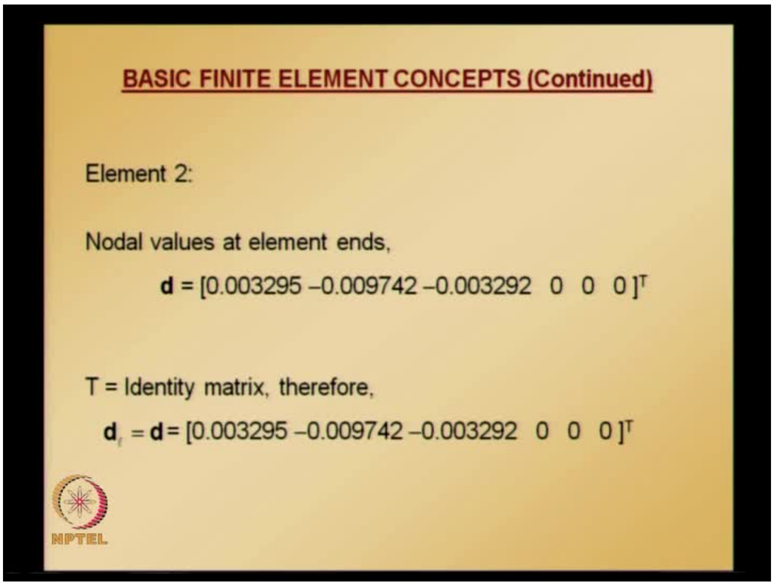

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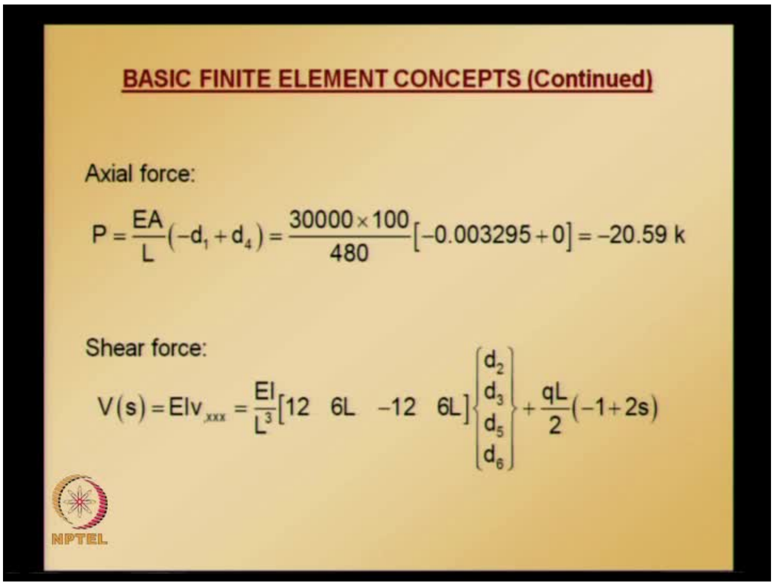

Similar calculations we can repeat for element 2. Only thing is we need to take corresponding degrees of freedom for element 2. The nodal values at element ends and transformation matrix here is identity matrix. So local nodal values or nodal values in the local coordinate system will be same as nodal values in the global coordinate system. Using these nodal values, we can calculate axial force and please note that element 2 is subjected to uniformly distributed loads. So, we need to apply fixed end correction for element 2 for shear and bending moment.

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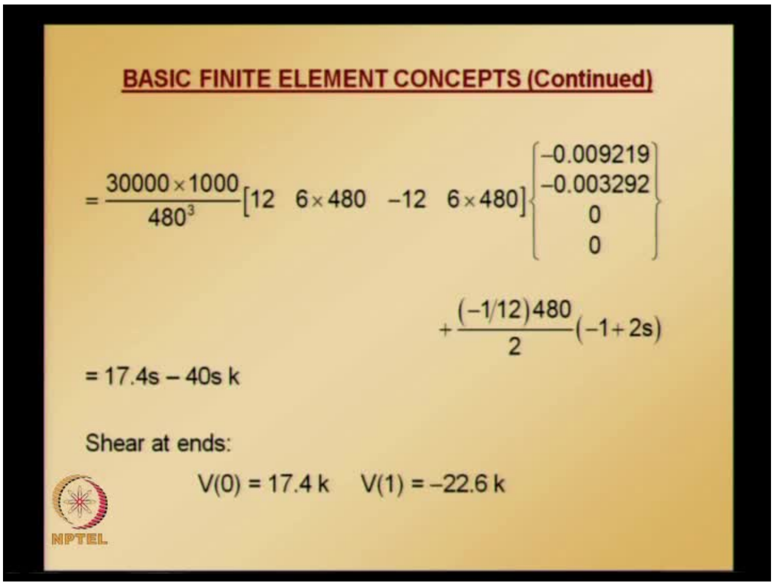

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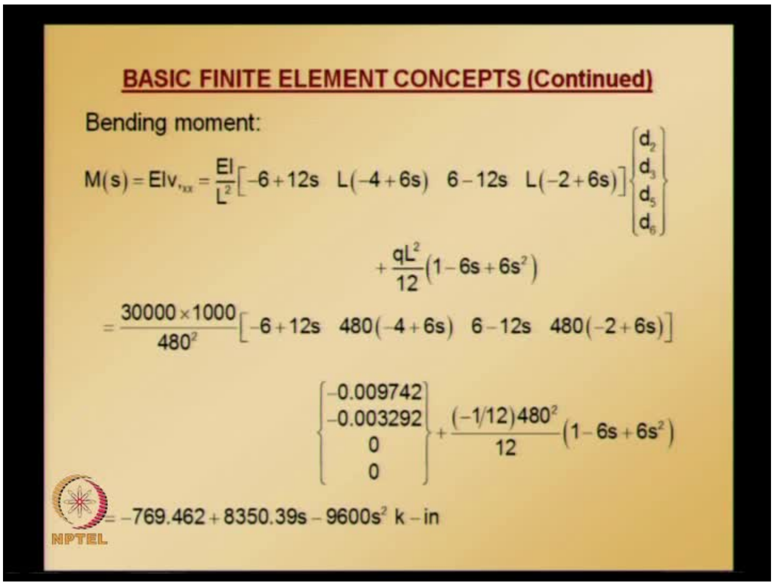

So, the first part of this calculation is coming directly from the nodal values and finite element interpolation and the second term is fixed end solution correction and substituting the numerical values we are going to get shear which is linearly varying. Shear force at node 2 is given by substituting s is equal to 0. Shear force at node 3 is obtained by substituting s is equal to 1 in this, so shear force at the ends.

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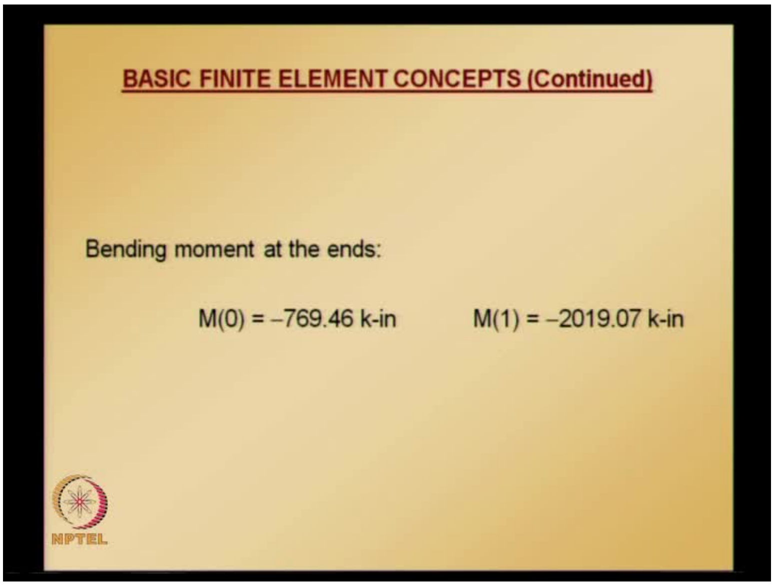

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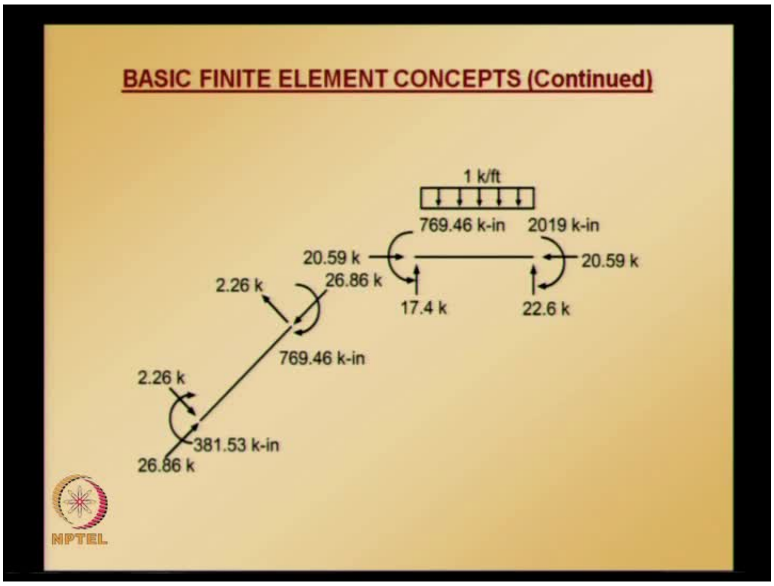

Now, bending moment again, the first term is coming from finite element interpolation and second term is coming from fixed end correction. Substituting the numerical values, we get bending moment expression in which we can substitute s is equal to 0 and s is equal to 1, to get the bending moment at node 2 and node 3, respectively. We can draw using these values; we can draw free body diagram of each of these elements and see whether each element is in equilibrium or not and these are the bending moments at the ends and this is the free body diagram of both elements.

The forces at element ends are shown in the free body diagram and it can be easily noted that these forces are in equilibrium except that, small round of errors may be there because of missing some significant digits and you can see each element in itself is in equilibrium; hence, this structure is equilibrium.